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Research Article

The New Analysis for the Caputo-Fabrizio Fractional Gas Dynamics Model with the Robust Technique

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Abstract"

Clarity and consistency are increased if the Caputo-Fabrizio fractional gas dynamics equation is analyzed using the Caputo-Fabrizio q-Elzaki homotopy analysis transform technique (CFq-EHATT). Numerical solutions of this equation are represented in 2D and 3D graphs using Maple software. Table 1 demonstrates the effectiveness and consistency of the proposed method. The $\hbar$-curves of the obtained solutions are plotted. The derived solutions facilitated a physical understanding of the gas dynamics equation.

Keywords: Caputo-Fabrizio fractional derivative, gas dynamics equation, Elzaki transform.

Caputo-Fabrizio Kesirli Gaz Dinamiği Modeli için Güçlü Yöntemle Yeni Sayısal Analiz

Öz

Caputo-Fabrizio kesirli gaz dinamiği denklemi, Caputo-Fabrizio q-Elzaki homotopi analiz dönüşüm tekniği (CFq-EHATT) kullanılarak analiz edilirse, netlik ve tutarlılık artmaktadır. Bu denklemin sayısal çözümü Maple yazılımı kullanılarak 2D ve 3D grafiklerle gösterilmiştir. Tablo 1 önerilen yöntemin etkinliğini ve tutarlılığını göstermektedir. Elde edilen çözümlerin $\hbar$ eğrileri çizilmiştir. Türetilen çözümler gaz dinamiği denkleminin fiziksel olarak anlaşılmasını kolaylaştırmıştır.

Anahtar Kelimeler: Caputo-Fabrizio kesirli türevi, gaz dinamik denklemi, Elzaki dönüşümü.

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1. Introduction

Fractional calculus (FC) is a branch of applied mathematics that focuses on derivatives and integrals of arbitrary orders. The primary advantage of employing fractional partial differential equations (FPDEs) is their nonlocal characteristic. FPDEs play a significant role in modeling many problems. Numerous FPDEs have been identified in the literature [1, 2, 3, 4, 5, 6].

FPDEs offer generalized forms of differential equations of integer order. FPDEs have been extensively studied in recent years due to their significant applications in several fields. FC is an unparalleled discipline for investigating several complicated processes that display nonlinearity. A multitude of established approaches have been proposed to ascertain precise solutions to FPDEs [7, 8, 9, 10, 11, 12, 13].

The principles of energy, momentum, and mass conservation are expressed through the mathematical formulation of gas dynamics equations (GDEs). Gas dynamics is a branch of fluid dynamics that investigates the movement of gases and its effects on physical structures. The examination of gas dynamics has numerous practical applications in various scientific and technical issues. Recently, many academics have investigated the GDE in different works because of its significance in multiple physical processes [2, 14].

Recently, numerous researchers have examined the GDE due to its importance in diverse physical processes. The various techniques, such as semi-analytical and numerical methods, have been employed to analyze the gas-dynamics equation [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 38].

In recent years, various definitions of fractional derivatives have been proposed to model memory and hereditary properties of complex systems. Among these, the Caputo and Riemann–Liouville derivatives are widely used due to their effectiveness in capturing power-law memory effects through singular kernels. However, these classical operators often lead to mathematical and computational challenges due to their nonlocal and weakly singular nature. In contrast, the Caputo-Fabrizio (CF) derivative introduces a non-singular exponential kernel, offering improved analytical and numerical properties. The CF operator avoids the singularity at the origin, facilitating the modeling of systems with fading memory and allowing for more stable numerical implementations. This distinction makes the Caputo-Fabrizio derivative particularly attractive for applications where regularity and well-posedness of the solutions are essential [4, 31].

The Elzaki transform (ET) is a mathematical instrument employed in the resolution of differential equations, especially within engineering and applied sciences. It is an integral transform akin to the Laplace transform, possessing distinctive qualities that render it advantageous for specific issue types. The ET is employed to resolve linear and nonlinear ordinary differential equations. ET has been augmented to proficiently manage fractional-order derivatives. It is advantageous for assessing systems when conventional transformations may not facilitate efficient calculations. It diminishes the intricacy involved in resolving differential equations. It offers alternative alternatives when other transformations may be unwieldy. It is congruent with fractional calculus and other sophisticated mathematical systems [24, 25, 26, 27, 28, 29, 30].

The Caputo-Fabrizio fractional derivative has emerged as an alternative to conventional fractional derivative formulations and has gained extensive application. This derivative definition was established to enhance the modeling of memory effects. The Caputo-Fabrizio fractional derivative facilitates a more precise examination of features like viscosity, energy dissipation, and diffusion processes in modeling intricate physical phenomena, such as gas dynamics equations [31].

In [2, 14], the subsequent GDE has been examined utilizing diverse methodologies.

$$\frac{\partial^{\mu} u(x,t)}{dt^{\mu}} + \frac{1}{2}(u^2(x,t))_x = u(x,t) - u^2(x,t), 0 < \mu \leq 1, t > 0, \tag{1}$$

via the initial condition (IC) $u(x,0) = \Theta(x)$.

In this article, this equation is examined in the Caputo-Fabrizio (CF) fractional sense as

$${}^{CF}D_t^{\mu} u(x,t) + \frac{1}{2}\left(u^2(x,t)\right)_x = u(x,t) - u^2(x,t), 0 < \mu \leq 1, t > 0. \tag{2}$$

The aim of the paper is to obtain the novel numerical solution of the Caputo-Fabrizio fractional GDE (CFFGDE) by a novel semi-analytical method. The first innovation in the paper is to present the CF q-Elzaki homotopy analysis transform technique (CFq-EHATT), which is a new semi-analytical method that combines the CF Elzaki transform (CFET) with the q-homotopy analysis transform technique (q-HATT). This method provides faster, easier and more effective numerical solutions of Caputo-Fabrizio FPDEs (CFFPDEs).

The rest of this work is as follows. The Part 2 presents the fundamentals of CF fractional calculus and the ET. CFq-EHATT is introduced in Section 3. The CFFGDE is analyzed in Part 4. The conclusion is given in Part 5.

2. Preliminaries

In this section, main definitions, theorems and methods are introduced.

Definition 2.1. [24] The ET of the function $\psi(t)$ is described as

$$E[\psi(\tau)] = T(s) = s \int_0^{\infty} \psi(\tau)\, exp\left[-\frac{\tau}{s}\right] d\tau, \tau > 0. \tag{3}$$

Definition 2.2. [31] The CF fractional derivative of order $\mu > 0$ for the function $\psi(t)$ is defined by

$$ {}^{CF}_{t_0}D^{\mu}_{0,t}[\psi(t)] = \frac{\varpi(\mu)}{1-\mu}\int_0^t \psi'(\tau)\, exp\left[-\frac{\mu(t-\tau)}{1-\mu}\right] d\tau, \tau > 0, \tag{4} $$

where $\psi(\mu)$ is the normalization function such that $\varpi(0) = \varpi(1) = 1$, and $\psi \in H^1(a_1, a_2), a_2 > a_1$.

Definition 2.3. [32] The Caputo-Fabrizio (ET) (CFET) of the CF derivative (CFD) for the function $\psi(t)$ is defined by

$$ {}^{CF}_{t_0}E_{\mu}\{{}^{CF}_{t_0}D^{\mu}_{t}[\psi(t)]\} = \frac{1}{1-\mu+\mu w}\left({}^{CF}_{t_0}E_{\mu}[\psi(t)] - w^2\psi(0)\right), \mu \in (0,1]. \tag{5} $$

2.1. The Main Idea of CFq-EHATT

In this part, it is presented the CFq-EHATT **[33]**, which combines via CFET and q-HATT.

Consider the Caputo-Fabrizio time-fractional nonlinear partial differential equation (CFTFNPDE)

$$ {}^{CF}_{t_0}D^{\mu}_{\tau}\psi(\xi,\tau) + A\psi(\xi,\tau) + H\psi(\xi,\tau) = \zeta(\xi,\tau), n-1 < \mu \leq n, \tag{6} $$

where A and H are linear and nonlinear operators, $\zeta(\xi,\tau)$ is a nonhomogeneous function, and ${}^{CF}_{t_0}D^{\mu}_{t}$ is CFD of order μ.

Step 1. By applying the CFET to CFTFNPDE with the IC, Eq (7) is identified as

$$ \frac{\left({}^{CF}_{t_0}E_{\mu}[\psi(\xi,\tau)] - w^2\psi(\xi,0)\right)}{1-\mu+\mu w} + {}^{CF}_{t_0}E_{\mu}[A\psi(\xi,\tau) + H\psi(\xi,\tau)] = {}^{CF}_{t_0}E_{\mu}[\zeta(\xi,\tau)]. \tag{7} $$

Rewriting the Eq (7), Eq (8) is acquired as

$$ {}^{CF}_{t_0}E_{\mu}[\psi(\xi,\tau)] - w^2\psi(\xi,0) + (1-\mu+\mu w){}^{CF}_{t_0}E_{\mu}[A\psi(\xi,\tau) + H\psi(\xi,\tau) \tag{8} $$

$$ -\zeta(\xi,\tau)] = 0. $$

Step 2. Via the HAM, the nonlinear operator (NO) of $\omega(\xi,\tau;q)$ is described by

$$ N^1[\omega(\xi,\tau;q)] = {}^{CF}_{t_0}E_{\mu}[\omega(\xi,\tau;q)] - w^2\omega(\xi,\tau;q)(0^+) $$

$$ +(1-\mu+\mu w)\left[{}^{CF}_{t_0}E_{\mu}[A\omega(\xi,\tau;q) + H\omega(\xi,\tau;q) - \zeta(\xi,\tau)]\right], \tag{9} $$

where $q\epsilon\left[0,\frac{1}{n}\right]$.

A homotopy is constructed as

$$(1-nq)\,{}_{t_0}^{CF}E_\mu[\omega(\xi,\tau;q)-\psi_0(\xi,\tau)]=\hbar q\Lambda(\xi,\tau)\,{}_{t_0}^{CF}E_\mu[\omega(\xi,\tau;q)], \tag{10}$$

where ${}_{t_0}^{CF}E_\mu$ signifies CFET and $\hbar\neq 0$. The outcomes of Eq (10) are determined for $q=0$ and $q=\frac{1}{n}$ by

$$\omega(\xi,\tau;0)=\psi_0(\xi,\tau),\omega\left(\xi,\tau;\frac{1}{n}\right)=\psi(\xi,\tau). \tag{11}$$

With q increasing from 0 to $1/n$, $\omega(\xi,\tau;q)$ converges to the solution $\psi(\xi,\tau)$ from $\psi_0(\xi,\tau)$.

When the Taylor theorem is applied to q, the result is found by

$$\omega(\xi,\tau;q)=\psi_0(\xi,\tau)+\sum_{\mathrm{c}=1}^{\infty}\psi_c(\xi,\tau)q^c, \tag{12}$$

where

$$\psi_c(\xi,\tau)=\frac{1}{c!}\frac{\partial^c\omega(\xi,\tau;q)}{\partial q^c}|_{q=0}. \tag{13}$$

For $\psi_0(\xi,\tau)$, n and $\hbar$, which are convenient, Eq (12) converges at $\mathrm{q}=\frac{1}{n}$.

By dividing by $c!$ and differentiating the 0-th order deformation Eq (10) c times with regard to q, for $q=0$ it is acquired as

$${}_{t_0}^{CF}E_\mu[\psi_c(\xi,\tau)-\chi_c\psi_{c-1}(\xi,\tau)]=\hbar\Lambda(\xi,\tau)\mathcal{R}_{1,c}(\vec{\psi}_{c-1}). \tag{14}$$

Step 3. When inverse CFET (ICFET) is used on Eq (14), the outcome result is obtained by

$$\psi_c(\xi,\tau)=\chi_c\psi_{c-1}(\xi,\tau)+\hbar\left({}_{t_0}^{CF}E_\mu\right)^{-1}\left[\Lambda(\xi,\tau)\mathcal{R}_{1,c}(\vec{\psi}_{c-1})\right], \tag{15}$$

where

$$\mathcal{R}_{1,c}(\vec{\psi}_{c-1})={}_{t_0}^{CF}E_\mu[\psi_{c-1}(\xi,\tau)]-w^2\left(1-\frac{\chi_c}{n}\right)\psi_0(\xi,\tau)$$
$$+(1-\mu+\mu w)\left[{}_{t_0}^{CF}E_\mu[A\psi_{c-1}(\xi,\tau)+\Lambda_{c-1}(\xi,\tau)-\zeta(\xi,\tau)]\right] \tag{16}$$

and

$$\chi_c=\begin{cases}0, & \mathrm{c}\le 1,\\ n, & c>1,\end{cases} \tag{17}$$

where Λ_c is homotopy polynomial and it is presented as

$$\Lambda_c = \frac{1}{c!}\frac{\partial^c \omega(x,t;q)}{\partial q^c}|_{q=0}, \qquad \omega(\xi,\tau;q) = \omega_0 + q\omega_1 + q^2\omega_2 + \cdots \tag{18}$$

Via Eqs (16)-(17), it is acquired as

$$\psi_c(\xi,\tau) = (\chi_c + \hbar)\psi_{c-1}(\xi,\tau) - w^2\left(1 - \frac{\chi_c}{n}\right)\psi_0(\xi,\tau)$$
$$+\hbar\left({}_{t_0}^{CF}E_\mu\right)^{-1}\left[\left((1-\mu+\mu w)\,{}_{t_0}^{CF}E_\mu[A\psi_{c-1}(\xi,\tau) + \Lambda_{c-1}(\xi,\tau) - \zeta(\xi,\tau)]\right)\right]. \tag{19}$$

Step 4. Thus, the solution of Eq. (6) via CFq-EHATT is as follows.

$$\psi(\xi,\tau) = \psi_0(\xi,\tau) + \sum_{c=1}^{\infty}\psi_c(\xi,\tau)\left(\frac{1}{n}\right)^c. \tag{20}$$

2.2. Convergence Analysis

In this section, theorems related to the existence and convergence of the method are given. **Theorem 4. 1. (Uniqueness Theorem) [34, 35, 36]** The solution for the CFTFNPDE (6) acquired by CFq-EHATT is unique for $\forall \boldsymbol{\mu} \boldsymbol{\epsilon} (\mathbf{0}, \mathbf{1})$, where $\boldsymbol{\mu} = (\mathbf{n} + \hbar) + \hbar(\boldsymbol{\rho} + \boldsymbol{\nu})\boldsymbol{\Phi}$.

Theorem 4. 2. (Convergence theorem) [35, 36] Let **Ω** is a Banach space (BS) and **Z**: **Ω** → **Ω** is a nonlinear mapping. Assume that the inequality

$$\|Z(a) - Z(h)\| \le \eta\|a - h\|, \forall a, b \in X \tag{21}$$

holds, then **Z** has a fixed point (FP) in BS. Therefore, for the arbitrary choice of $\mathbf{a_0}, \mathbf{b_0} \in \boldsymbol{\Omega}$, the sequence created by the CFq-EHATT converges to a FP of **Z** and the following inequality holds

$$\|\varrho_m - \varrho_n\| \le \frac{\eta^n}{1-\eta}\|\varrho_1 - \varrho_0\|, \forall a, b \in \Omega. \tag{22}$$

3. Application

In the part, the numerical solution of the CFFGDE are acquired with CFq-EHATT.

Example 3.1. Consider the CFFGDE **[2, 14]**

$$^{CF}D_t^{\mu}u(x,t) + \frac{1}{2}\left(u^2(x,t)\right)_x = u(x,t) - u^2(x,t), \mu \in (0,1], t > 0, \tag{23}$$

with the IC

$$u(x,0) = e^{-x}. \tag{24}$$

Implementing the CFET to Eq. (23) via IC, then it is acquired as

$${}^{CF}_{t_0}E_\mu[u(x,t)] - w^2 u(x,0) + (1-\mu+\mu w)\,{}^{CF}_{t_0}E_\mu\left[\frac{1}{2}\left(u^2(x,t)\right)_x - u(x,t)\right. \tag{25}$$

$$\left. + u^2(x,t)\right] = 0.$$

The NO is constructed with Eq. (25) as

$$N^1[\varpi(x,t;q)] = {}^{CF}_{t_0}E_\mu[\varpi(x,t;q)] - w^2 e^{-x} + (1-\mu+\mu w) \tag{26}$$

$$\times\, {}^{CF}_{t_0}E_\mu\left[\frac{1}{2}\frac{\partial \varpi^2(x,t;q)}{\partial \mathrm{x}} - \varpi(x,t;q) + \varpi^2(x,t;q)\right].$$

The c-th order deformation equation is identified as

$${}^{CF}_{t_0}E_\mu[u_c(x,t) - \chi_c u_{c-1}(x,t)] = h\mathcal{R}_{1,c}[\vec{u}_{c-1}], \tag{27}$$

where

$$\mathcal{R}_{1,c}[\vec{u}_{c-1}(x,t)] = {}^{CF}_{t_0}E_\mu[\vec{u}_{c-1}\,(x,t)] - e^{-x}w^2\left(1 - \frac{\chi_c}{n}\right) \tag{28}$$

$$+(1-\mu+\mu w)\,{}^{CF}_{t_0}E_\mu\left[\sum_{s=0}^{c-1} u_s \frac{\partial u_{c-1-s}}{\partial x} - u_{c-1}(x,t) + \sum_{s=0}^{c-1} u_s u_{c-1-s}\right].$$

Implementing the ICFET to Eq. (27), we have

$$u_c(x,t) = \chi_c u_{c-1}(x,t) + h\left({}^{CF}_{t_0}E_\mu\right)^{-1}\left\{\mathcal{R}_{1,c}[\vec{u}_{c-1}\,(x,t)]\right\}. \tag{29}$$

By employing IC, we obtain

$$u_0(x,t) = e^{-x}. \tag{30}$$

Substituting $c = 1$ in Eq (29), then $u_1(x,t)$ is acquired as

$$u_1(x,t) = -he^{-x}(1-\mu+\mu t). \tag{31}$$

In the similar way, substituting $c = 2$ in Eq (29), then $u_2(x,t)$ is obtained as

$$u_2(x,t) = (n+h)\left(-he^{-x}(1-\mu+\mu t)\right)$$

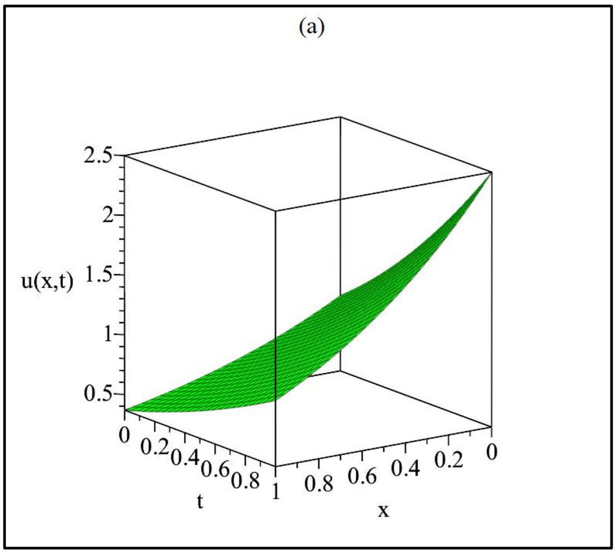
$$+h^2 e^{-x}\left[(1-\mu)^2 + 2\mu(1-\mu)t + \frac{\mu^2 t^2}{2}\right]. \tag{32}$$

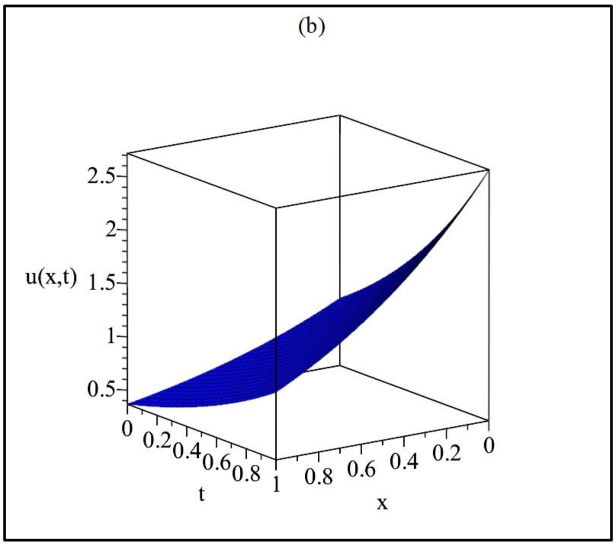
This way the remaining terms are found. The solution of the CFFGDE is identified via the CFq-EHATT

$$u(x,t) = u_0(x,t) + \sum_{c=1}^{\infty} u_c(x,t)\left(\frac{1}{n}\right)^c. \tag{33}$$

Putting $\mu = 1, n = 1,\ h = -1$ in Eq. (33), $\sum_{c=1}^{\Theta} u_c(x,t)\left(\frac{1}{n}\right)^c$ converge to the exact solution (ES) $u(x,t) = e^{-x+t}$ of the CFFGDE when $\Theta \to \infty$.

Fig. 1 indicates the 3D graphs of CFq-EHATT, the ES, and the absolute error (AE) for $u(x,t)$.





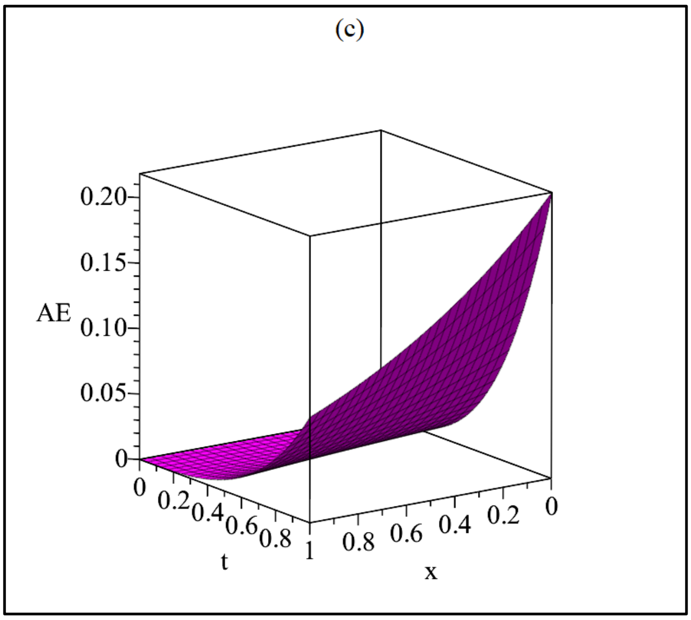


Figure 1: (a) CFq-EHATT solution (b) ES (c) AE for CFFDE at $n = 1, h = -1, \mu = 1$.

Fig. 2 illustrates the 2D plots of CFq-EHATT for the $u(x,t)$ solution with the ES for various μ values.

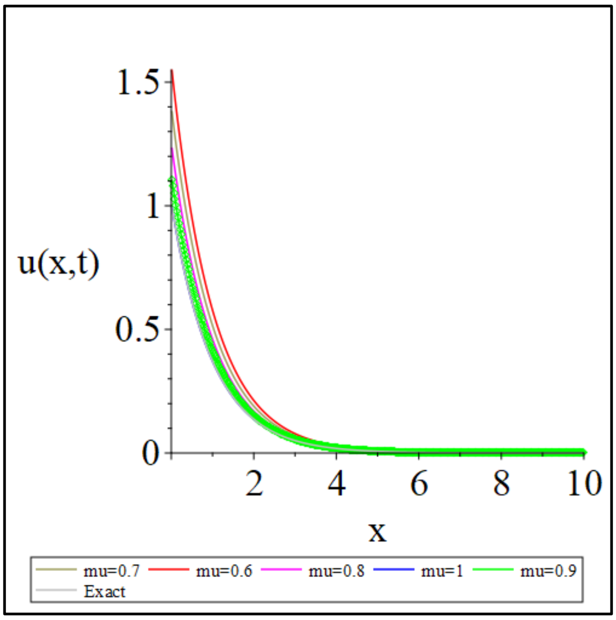
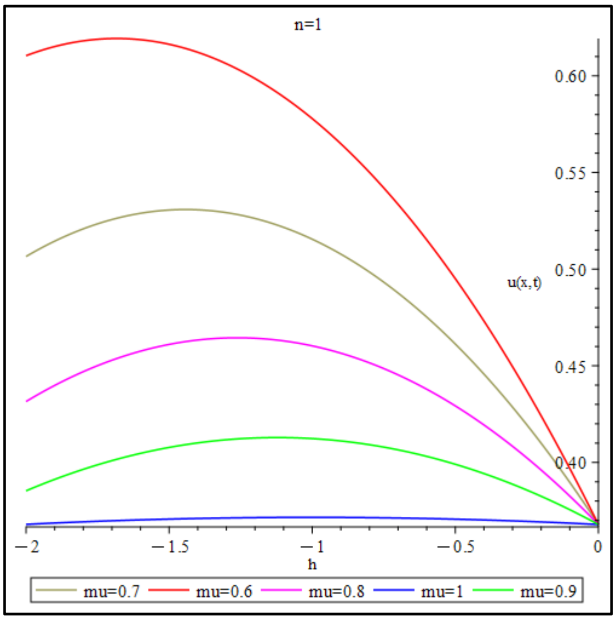
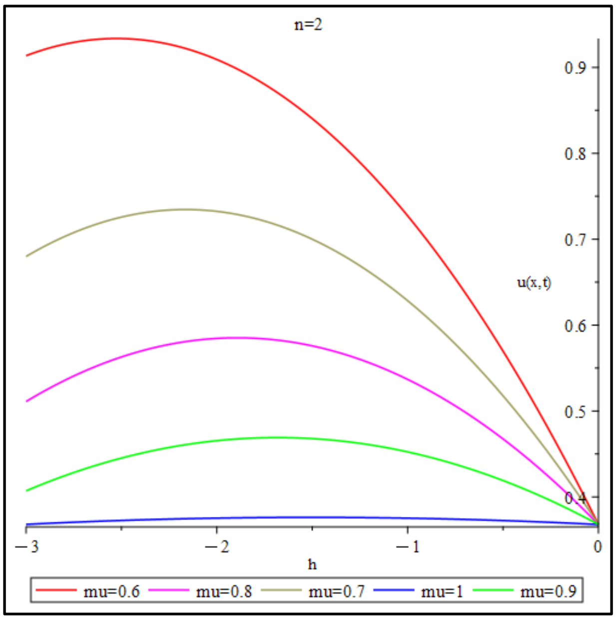


Figure 2: The behaviour of the CFq-EHATT solutions for $u(x,t)$ and ES for $h = -1, n = 1, t = 0.01$ and various μ values.

Fig. 3 indicates the $\hbar$-curves for $u(x,t)$ of CFFGDE with varying μ and n.


n=1
0.60
0.55
0.50
u(x,t)
0.45
0.40
−2
−1.5
−1
−0.5
0
h
mu=0.7
mu=0.6
mu=0.8
mu=1
mu=0.9


n=2
0.9
0.8
0.7
u(x,t)
0.6
0.5
0.4
−3
−2
−1
0
h
mu=0.6
mu=0.8
mu=0.7
mu=1
mu=0.9

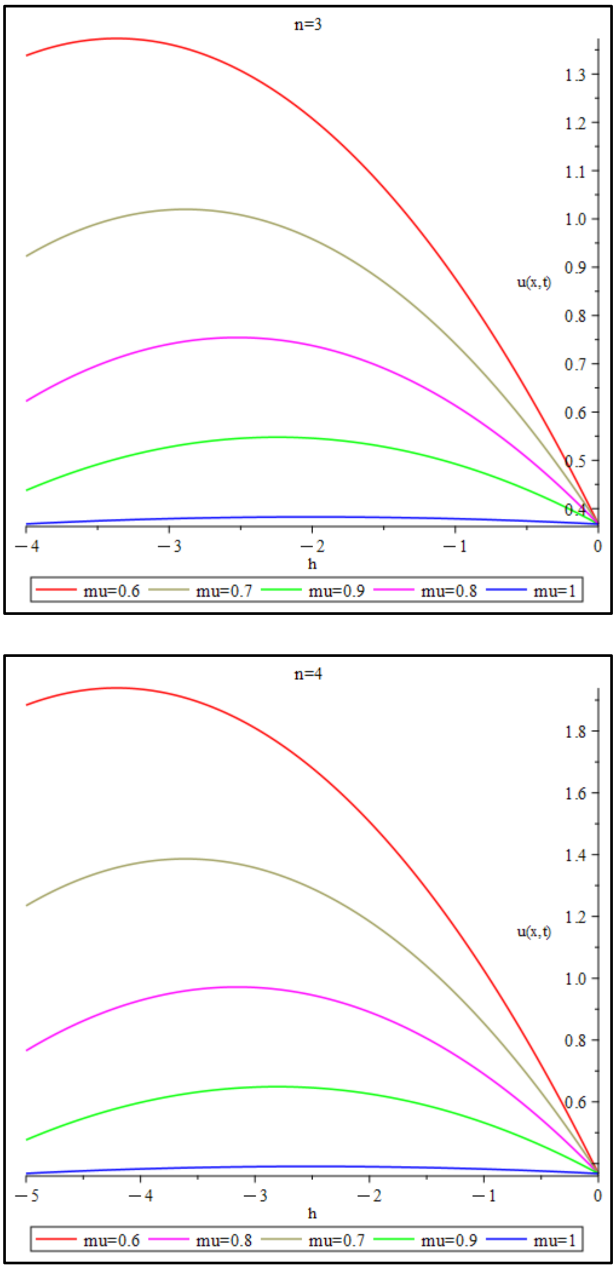


Figure 3: $\hbar$-curves for $u(x,t)$ of CFFGDE with varying μ at $x = 1$ and $t = 0.01$ for varied n.

4. Results and Discussion

The numerical simulations performed for the CFFGDE are presented. The simulations were conducted for different values of $x, t,$ and μ. Fig. 1 illustrates the comparison of results generated by Cq-HATM, ES, and the AE. Fig. 2 illustrates the two-dimensional graphical depiction of the results obtained for the CFFGDE, with varied values of μ. Fig. 3 illustrates the $\hbar$-curves for the CFFGDE, with each curve representing a distinct set of μ and n values for CFFGDE. These curves are essential in altering the convergence region.

In the context of the Caputo-Fabrizio q-Elzaki homotopy analysis transform technique, the $\hbar$-curve serves as a critical tool for analyzing and controlling the convergence of the series solution. It provides a graphical representation of how the auxiliary parameter $\hbar$ influences the convergence behavior of the solution series. A flat segment in the $\hbar$-curve indicates a valid range of $\hbar$ values where the method converges to a reliable solution. Conversely, steep or oscillatory behavior signifies regions of instability or divergence. Therefore, plotting $\hbar$ -curves helps identify optimal $\hbar$ values that ensure faster and more stable convergence. In this study, Fig. 3 presents the $\hbar$ -curves for various values of the fractional order α and time t. These curves reveal the sensitivity of the method's convergence to the choice of $\hbar$ and demonstrate the robustness of the proposed CFq-EHATT.

Table 1 presents the numerical solutions of $u(x,t)$ derived from the CFFGDE solution with CFq-EHATT for distinct values of t at $\mu = 1,\ h = -1, n = 1, x = 0.001$.

Table 1: Comparision EDM **[14]** and CFq-EHATT solutions $u(x,t)$ for CFGGDE.

t	$\lvert u_{exact} - u_{EDM} \rvert$	$\lvert u_{exact} - u_{CFq-EHATM} \rvert$
0.001	4.99×10^{-7}	2.00×10^{-10}
0.002	1.99×10^{-6}	1.00×10^{-9}
0.003	4.49×10^{-6}	4.00×10^{-9}
0.004	7.98×10^{-6}	1.10×10^{-8}
0.005	1.24×10^{-5}	2.10×10^{-8}
0.006	1.79×10^{-5}	3.60×10^{-8}
0.007	2.44×10^{-5}	5.70×10^{-8}
0.008	3.1×10^{-5}	8.50×10^{-8}
0.009	4.03×10^{-5}	1.22×10^{-7}
0.010	4.97×10^{-5}	1.67×10^{-7}

Although Table 1 presents numerical comparisons between the CFq-EHATT and the EDM, a more in-depth analysis can provide greater insight into the strengths of the proposed method.

From a theoretical standpoint, the CFq-EHATT combines the advantages of the Caputo-Fabrizio exponential kernel with the flexibility of the q-homotopy approach, resulting in improved convergence behavior. In contrast, EDM relies on recursive decomposition, which may be computationally less stable for complex nonlinearities. Regarding computational complexity, CFq-EHATT demonstrates reduced iteration count and improved convergence rates due to its analytical construction and tunable auxiliary parameter $\hbar$. Furthermore, while EDM often requires symbolic integration or repeated convolution operations, CFq-EHATT benefits from the Elzaki transform's compatibility with the Caputo-Fabrizio derivative, which simplifies the inversion process and reduces computational burden. Thus, the proposed method not only yields more accurate results but also offers greater computational efficiency, making it preferable for solving nonlinear fractional models.

5. Conclusion

This study analyzes CFFGDE using CFq-EHATT. This approach is a recently introduced semi-analytical technique. Furthermore, the two and three dimensional plots depicting the solutions to these equation for different values μ were produced with the Maple. The overall configuration of the surface plots for CFFGDE is observed to differ. Furthermore, Table 1 indicates that CFq-EHATT outperforms EDM with AE. It can be concluded that the newly proposed approaches for addressing CFTFNPDE are both beneficial and efficient.

Ethics in Publishing

There are no ethical issues regarding the publication of this study.

Author Contribution

Aslı Alkan did all the work on the article.

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There is no any conflict of interest.

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