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SOLUTION OF SHIFF SYSTEMS BY USING DIFFERENTIAL TRANSFORM METHOD

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ÖZET

Bu çalışmada, stiff adi diferansiyel denklemleri çözmek için diferansiyel dönüşüm metodu kullanıldı ve teorisi tartışıtıldı. Metodun, lineer ve lineer olmayan diferansiyel denklem sistemlerine etkinliğini göstermek için, bazı örnekler verildi. Sayısal hesaplamalarda MAPLE bilgisayar cebiri sistemleri kullanıldı.

Anahtar Kelimeler: *stiff sistem, diferansiyel dönüşüm metodu, adi diferansiyel denklemlerin sayısal çözümü, MAPLE*

ABSTRACT

In this paper, we use the differential transform method to solve stiff ordinary differential equations of the first order and an ordinary differential equation of any order by converting it into a system of differential of the order one. Theoretical considerations have been discussed and some examples were presented to show the ability of the method for linear and non-linear systems of differential equations. We use MAPLE computer algebra systems for numerical calculations [13].

Key Words: *Stiff system, the differential transform method, Numerical solution of the ordinary differential equations, MAPLE.*

1. INTRODUCTION

A system of first order differential equations can be considered as:

$$\begin{cases} y_1' = f_1(x, y_1, \dots, y_n) \\ y_2' = f_2(x, y_1, \dots, y_n) \\ \quad \vdots \\ y_n' = f_n(x, y_1, \dots, y_n) \end{cases} \tag{1.1}$$

with initial condition

$$y_i(x_0) = y_{0,i}, \quad i = 1, 2, \dots, n$$

where each equation represents the first derivative of one of the unknown functions as a mapping depending on the independent variable x, and n unknown functions $f_1, \dots, f_n$. Since every ordinary differential equation of order n can be written as a system consisting of n ordinary differential equation of order one, we restrict our study to a system of differential equations of the first order. That is, f and y are vector functions for which we assumed sufficient differentiability [19,12,5,16,25,2,1,21].

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A special class of initial-value problems is those for which the solutions contain rapidly decaying transient terms and the numerical solution of slow smooth movements is considerably perturbed by nearby rapid solutions. Such problems are called stiff systems of differential equations. A potentially good numerical method for the solution of stiff systems differential equations must have good accuracy and some reasonably wide region of absolute stability [20]. In generally, the methods designed for non-stiff problems when applied to stiff systems of differential equations tend to be very slow and can be anomalies results in solution because stiff problems need small step lengths to avoid numerical instability. For example, when exact solution contains terms of the form e^{kx}, where k is negative real part of a complex number problem give meaningless results. [7,11,26,27,17,24,4,29,286,20]. Because the exact solution consists of a steady-state term that does not grow significantly with x, together with a transient term that decay rapidly to zero[7,27,17] and the error will increase since the error associated with the decreasing transient part[7].

Many algorithms are available for solving non-stiff systems [19,12,5,16,25,2,1,21] but most of these algorithms are numerically unstable for stiff systems unless the chosen step size is taken to be extremely small [7,17]. Since around 1969, numerous works have been carrying on the development of accuracy of numerical solution and efficient of methods for stiff systems because of the wide variety of applications of stiff problems occur in many areas such as chemical engineering, non-linear mechanics, biochemistry, the analysis of control system, the study of spring and damping and life sciences. Some of these are; for example, [11] discussed briefly methods of investigating the stability of particular systems and recommended implicit step methods. [26] compared two methods, GEAR and STİFF3, which were developed specifically for stiff differential equations according to accuracy of numerical solution and efficiency of method and that GEAR is the preferred algorithm for stiff differential equations. In [17], stiff differential equations are solved by Radau Methods.

Numerical methods related to stiff systems are also given in different types [11,26,27,17,24,4,29,28,6,20,22,14,15].

In the work, stiff system of differential equations is considered by differential transform method. Differential transform method can easily be applied to stiff systems of differential equations. Series coefficients can be formulated very simply.

The concept of differential transform with one-dimensional was first introduced by Zhou[30], who solved linear and non-linear initial value problems in electrical circuit analysis. Chen and Ho [9,10] proposed the method to solve eigenvalue problems and develop the basic theory of two- dimensional transform method. In the literature, different types of partial differential equations problems and differential algebraic equations problems with low index are solved by the differential transform method [30,9,10,23,8,18,3].

The method gives an analytical solution in the form of a polynomial. But, It is different from Taylor series method that requires computation the high order derivatives. The differential transform method is an iterative procedure that is described by the transformed equations of original functions for solution of differential equations.

2. THE DIFFERENTIAL TRANSFORM METHOD

The differential transform of the k th derivate of function $y(x)$ in one dimensional is defined as follows:

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0} \tag{2.1}$$

where $y(x)$ is original function and $Y(x)$ is transformed function and the differential inverse transform of $Y(k)$ is defined as

$$y(x) = \sum_{k=0}^{\infty} (x - x_0)^k Y(k) \tag{2.2}$$

From (2.1) and (2.2) is defined

$$y(x) = \sum_{k=0}^{\infty} (x - x_0)^k \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=x_0} \tag{2.3}$$

Equation (2.3) implies that the concept of the differential transformation is derived from Taylor series expansion at $x = x_0$. From definitions of equations (2.1) and (2.2) it is easy to obtain the basic definitions and operations of the one-dimensional differential transformation shown in Table 1. There are introduced in [9,10,23,8,18,3]. In real applications, the function $y(x)$ given in (2.3) is expressed by a finite series and equation (2.3) can be written as

$$y(x) = \sum_{k=0}^{n} (x - x_0)^k Y(k) \tag{2.4}$$

where $y(x) = \sum_{k=n+1}^{\infty} (x - x_0)^k Y(k)$ is negligibly small.

Table 1
The fundamental operation of one–dimensional differential transform method

Original function	Transformed function
$y(x) = u(x) \pm v(x)$	$Y(k) = U(k) \pm V(k)$
$y(x) = cw(x)$	$Y(k) = cW(k)$
$y(x) = dw/dx$	$Y(k) = (k+1)W(k+1)$
$y(x) = d^j w/dx^j$	$Y(k) = (k+1)(k+2)...(k+j)W(k+j)$
$y(x) = u(x)v(x)$	$Y(k) = \sum_{r=0}^{k} U(r)V(k-r)$
$y(x) = x^j$	$Y(k) = \delta(k-j) = \begin{cases} 1, & k = j \\ 0, & k \neq j \end{cases}$

3. Numerical Examples

The Differential transform Method applied in this study is useful in obtaining approximate solutions of stiff ordinary differential equations of first order and an ordinary differential equation of any order. We illustrate it by the following examples using MAPLE computer algebra systems [13].

Example 1. We consider the following differential equation system[15]

$$\begin{aligned} y_1' &= -1002 y_1 + 1000 y_2^2 \\ y_2' &= y_1 - y_2(1 + y_2) \end{aligned} \tag{3.1}$$

with initial condition

$y_1(0) = 1$ and $y_2(0) = 1$.

The exact solution is

$y_1(x) = e^{-2x}$ and $y_2(x) = e^{-x}$.

By using the basic properties of differential transform method from table 1 and taking the transform of Eqs. (3.1) we can obtain that

$$Y_1(k+1)=\frac{1}{(k+1)}\left(-1002.Y_1(k)+1000.\sum_{r=0}^{k}Y_2(r)Y_2(k-r)\right)$$
$$Y_2(k+1)=\frac{1}{(k+1)}\left(Y_1(k)-Y_2(k)+\sum_{r=0}^{k}Y_2(r)Y_2(k-r)\right) \tag{3.2}$$

Substitute $k=0,1,...,m$ into (3.2), series coefficients can be obtained that

$Y_1(1)=-2,\quad Y_1(2)=2,\quad Y_1(3)=-\frac{4}{3},\quad Y_1(4)=\frac{2}{3},\quad Y_1(5)=-\frac{4}{15},$

$Y_1(6)=\frac{4}{45},\quad Y_1(7)=-\frac{8}{315},\quad Y_1(8)=\frac{2}{315},\quad Y_1(9)=-\frac{4}{2835}$

$Y_2(1)=-1,\quad Y_2(2)=\frac{1}{2},\quad Y_2(3)=-\frac{1}{6},\quad Y_2(4)=\frac{1}{24},\quad Y_2(5)=-\frac{1}{720},$

$Y_2(6)=\frac{1}{720},\quad Y_2(7)=-\frac{1}{5040},\quad Y_2(8)=\frac{1}{40320},\quad Y_2(9)=-\frac{1}{362880}$

By substituting the values of $Y_1(k)$ *and* $Y_2(k)$ into (2.2), we obtain $y_1(x)$ and $y_2(x)$ as

$$y_1(x)=1-2x+2x^2-\frac{4}{3}x^3+\frac{2}{3}x^4-\frac{4}{15}x^5+\frac{4}{45}x^6-\frac{8}{315}x^7+\frac{2}{315}x^8-\frac{4}{2835}x^9$$
$$y_2(x)=1-x+\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{24}x^4-\frac{1}{720}x^5+\frac{1}{720}x^6-\frac{1}{5040}x^7+\frac{1}{40320}x^8-\frac{1}{362880}x^9$$

The numerical results are illustrated in table 2 and 3.

Table 2.
Comparison of theoretical and numerical values of y_1 in example 1.

t	Theoretical (y_1)	Numerical(y_1)	Error
0.1	0.8187307531	0.8187307532	0.0000000001
0.2	0.6703200460	0.6703200461	0.0000000001
0.3	0.5488116361	0.5488116345	0.0000000016
0.4	0.4493289641	0.4493289365	0.0000000276
0.5	0.3678794412	0.3678791888	0.0000002524
0.6	0.3011942119	0.3011926747	0.0000015372
0.7	0.2465969639	0.2465899006	0.0000070633
0.8	0.2018965180	0.2018701030	0.0000264150
0.9	0.1652988882	0.1652144772	0.0000844110
1.0	0.1353352832	0.1350970018	0.0002382814

Table 3.
Comparison of theoretical and numerical values of the y_2 in example 1.

t	Theoretical (y_2)	Numerical(y_2)	Error
0.1	0.9048374180	0.9048374181	0.0000000001
0.2	0.8187307531	0.8187307532	0.0000000001
0.3	0.7408182207	0.7408182206	0.0000000001
0.4	0.6703200460	0.6703200461	0.0000000001
0.5	0.6065306597	0.6065306595	0.0000000002
0.6	0.5488116361	0.5488116345	0.0000000016
0.7	0.4965853038	0.4965852966	0.0000000072
0.8	0.4493289641	0.4493289365	0.0000000276
0.9	0.4065696597	0.4065695710	0.0000000887
1.0	0.3678794412	0.3678791887	0.0000002525

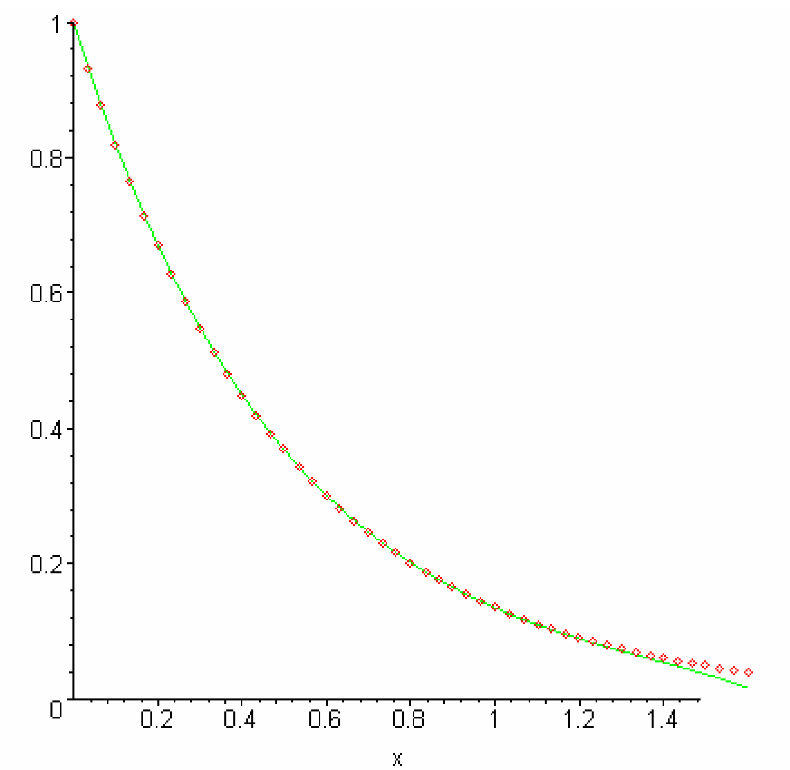

Fig 1. Values of y_1 and y_1^*, y_1^* is numerical solution of y_1.

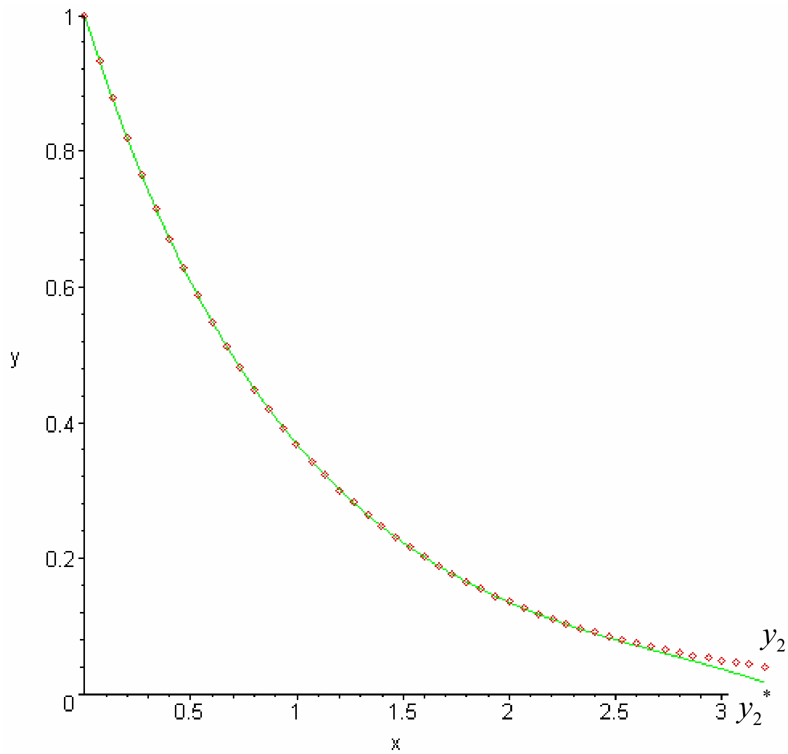

Fig 2. Values of y_2 and y_2^*, y_2^* is numerical solution of y_2.

Example 2. Let us consider following system of differential equation[15]

$$\begin{aligned} y_1' &= -20y_1 - 0.25y_2 - 19.75y_3, \quad y_1(0) = 1, \\ y_2' &= 20y_1 - 20.25y_2 + 0.25y_3, \quad y_2(0) = 0, \\ y_3' &= 20y_1 - 19.75y_2 - 0.25y_3, \quad y_3(0) = -1. \end{aligned} \tag{3.3}$$

with initial values

$$y_1(0) = 1 \quad y_2(0) = 0 \quad y_3(0) = -1$$

The analytic solution of the problem is

$$\begin{aligned} y_1 &= [e^{-1/2t} + e^{-20t}(\cos(20t) + \sin(20t))]/2, \\ y_2 &= [e^{-1/2t} - e^{-20t}(\cos(20t) - \sin(20t))]/2, \\ y_3 &= -[e^{-1/2t} + e^{-20t}(\cos(20t) - \sin(20t))]/2. \end{aligned}$$

If we apply differential transform method to the given equation system, it is obtained that

$$\begin{aligned} Y_1(k+1) &= \frac{1}{(k+1)}\left(-20Y_1(k) - 0.25Y_2(k) - 19.75Y_3(k)\right) \\ Y_2(k+1) &= \frac{1}{(k+1)}\left(20Y_1(k) - 20.25Y_2(k) + 0.25Y_3(k)\right) \\ Y_3(k+1) &= \frac{1}{(k+1)}\left(20Y_1(k) - 19.75Y_2(k) - 0.25Y_3(k)\right) \end{aligned} \tag{3.4}$$

For $k=0,1,...,m$ $Y_1(k), Y_2(k), Y_3(k)$ coefficients can be calculated from (3.4) as

$$Y_1(1)=-\frac{1}{4},\quad Y_1(2)=-\frac{3199}{16},\quad Y_1(3)=\frac{85333}{32},\quad Y_1(4)=-\frac{3413333}{256},\quad Y_1(5)=-\frac{1}{7680},$$

$$Y_1(6)=\frac{3640888889}{10240},\ Y_1(7)=-\frac{2621440000001}{1290240},\ Y_1(8)=\frac{104857600000001}{315},\ Y_1(9)=-\frac{1}{371589120}$$

$$Y_2(1)=\frac{79}{4},\quad Y_2(2)=-\frac{3199}{16},\quad Y_2(3)=-\frac{1}{96},\quad Y_2(4)=\frac{10240001}{768},\quad Y_2(5)=-\frac{273066667}{2560},$$

$$Y_2(6)=\frac{3640888889}{10240},\quad Y_2(7)=-\frac{1}{1290240},\quad Y_2(8)=-\frac{4993219047619}{983040},\quad Y_2(9)=\frac{838860799999}{371589120}$$

$$Y_3(1)=\frac{81}{4},\quad Y_3(2)=-\frac{3201}{16},\quad Y_3(3)=\frac{1}{96},\quad Y_3(4)=\frac{3413333}{256},\quad Y_3(5)=-\frac{819199999}{7680},$$

$$Y_3(6)=\frac{32767999999}{92160},\ Y_3(7)=\frac{1}{1290240},\ Y_3(8)=-\frac{104857600000001}{20643840},\ Y_3(9)=-\frac{27962026666667}{123863040}$$

By substituting the values of $Y_1(k)$, $Y_2(k), Y_3(k)$ into (2.2), we obtain $y_1(x)$, $y_2(x)$ and $y_3(x)$ as

$$y_1(x)=1-\frac{1}{4}x-\frac{3199}{16}x^2+\frac{85333}{32}x^3-\frac{3413333}{256}x^4-\frac{1}{7680}x^5+\frac{3640888889}{10240}x^6...$$

$$y_2(x)=\frac{79}{4}x-\frac{3199}{16}x^2-\frac{1}{96}x^3+\frac{10240001}{768}x^4-\frac{27306667}{2560}x^5+\frac{3640888889}{10240}x^6...$$

$$y_3(x)=-1+\frac{81}{4}x-\frac{3201}{16}x^2+\frac{1}{96}x^3+\frac{3413333}{256}x^4-\frac{819199999}{7680}x^5+\frac{32767999999}{92160}x^6....$$

The numerical results are illustrated in table 3, 4 and 5.

Table 4.
Comparison of theoretical and numerical values of the y_1 in example 2.

t	Theoretical (y_1)	Numerical(y_1)	Error
0.1	0.9476815826	0.9476815825	0.0000000001.
0.2	0.9968025985	0.9968025984	0.0000000001.
0.3	0.9935823806	0.9935823807	0.0000000001.
0.4	0.9893856850	0.9893856852	0.0000000002.
0.5	0.9842872353	0.9842872356	0.0000000003
0.6	0.9783588395	0.9783588396	0.0000000001
0.7	0.9716694173	0.9716694174	0.0000000001
0.8	0.9642850327	0.9642850329	0.0000000002
0.9	0.9562689318	0.9562689318	0.0000000000
1.0	0.9476815826	0.9476815825	0.0000000001

Table 5.
Comparison of theoretical and numerical values of the y_2 in example 2.

t	Theoretical (y_2)	Numerical(y_2)	Error
0.1	0.0338613302	0.0338613302	0.0000000000
0.2	0.0665062521	0.0665062520	0.0000000001
0.3	0.0979389656	0.0979389656	0.0000000000
0.4	0.1281662413	0.1281662413	0.0000000000
0.5	0.1571972304	0.1571972304	0.0000000000
0.6	0.1850432835	0.1850432834	0.0000000001
0.7	0.2117177754	0.2117177754	0.0000000000
0.8	0.2372359366	0.2372359366	0.0000000000
0.9	0.2616146929	0.2616146930	0.0000000001
1.0	0.2848725110	0.2848725109	0.0000000001

Table 6.
Comparison of theoretical and numerical values of the y_3 in example 2.

t	Theoretical (y_3)	Numerical(y_3)	Error
0.1	0.9652663858	0.9652663857	0.0000000001
0.2	0.9317499408	0.9317499409	0.0000000001
0.3	0.8994464644	0.8994464644	0.0000000000
0.4	0.8683491855	0.8683491855	0.0000000000
0.5	0.8384489520	0.8384489520	0.0000000000
0.6	0.8097344126	0.8097344126	0.0000000000
0.7	0.7821921922	0.7821921921	0.0000000001
0.8	0.7558070592	0.7558070590	0.0000000002
0.9	0.7305620874	0.7305620874	0.0000000000
1.0	0.7064388095	0.7064388093	0.0000000002

Example 3. Consider following system [15]

$$\begin{aligned} y_1' &= -21y_1 + 19y_2 - 20y_3 \\ y_2' &= 19y_1 - 21y_2 + 20y_3 \\ y_3' &= 40y_1 - 40y_2 + 40y_3 \end{aligned} \tag{3.5}$$

with initial values

$$y_1(0) = 1 \quad y_2(0) = 0 \quad y_3(0) = -1.$$

The analytic solution of the problem is

$$y_1 = \left[e^{-2t} + e^{-40t}\left(\cos(40t) + \sin(40t)\right)\right]/2$$
$$y_2 = \left[e^{-2t} - e^{-40t}\left(\cos(40t) - \sin(40t)\right)\right]/2$$
$$y_3 = -\left[e^{-2t} + e^{-40t}\left(\cos(40t) - \sin(40t)\right)\right]/2.$$

By using the basic properties of differential transform method and taking the transform of Eqs. (3.5) we have

$$\begin{aligned} Y_1(k+1) &= \frac{1}{(k+1)}\left(-21Y_1(k) + 19Y_2(k) - 20Y_3(k)\right) \\ Y_2(k+1) &= \frac{1}{(k+1)}\left(19Y_1(k) - 21Y_2(k) + 20Y_3(k)\right) \\ Y_3(k+1) &= \frac{1}{(k+1)}\left(40Y_1(k) - 40Y_2(k) + 40Y_3(k)\right). \end{aligned} \tag{3.6}$$

For $k = 0,1,...,m$ $Y_1(k), Y_2(k), Y_3(k)$ coefficients can be calculated from (3.6) as

$$Y_1(1) = -1, \quad Y_1(2) = -799, \quad Y_1(3) = \frac{63998}{3}, \quad Y_1(4) = -213333, \quad Y_1(5) = -\frac{2}{15},$$
$$Y_1(6) = \frac{113777778}{5}, \; Y_1(7) = -\frac{27306666668}{105}, \; Y_1(8) = \frac{409600000001}{315}, \; Y_1(9) = -\frac{2}{2835}$$
$$Y_2(1) = -1, \quad Y_2(2) = 801, \quad Y_2(3) = -21334, \quad Y_2(4) = \frac{640001}{3}, \quad Y_2(5) = -\frac{2}{15},$$
$$Y_2(6) = -\frac{1023999998}{45}, \; Y_2(7) = \frac{819999996}{315}, \; Y_2(8) = -\frac{45511111111}{35}, \; Y_2(9) = -\frac{2}{2835}$$
$$Y_3(1) = 80, \quad Y_3(2) = -1600, \quad Y_3(3) = 0, \quad Y_3(4) = -\frac{1280000}{3}, \quad Y_3(5) = -\frac{20480000}{3},$$
$$Y_3(6) = \frac{409600000}{9}, \; Y_3(7) = 0, \; Y_3(8) = -\frac{163840000000}{63}, \; Y_3(9) = \frac{13107200000000}{567}$$

By substituting the values of $Y_1(k)$, $Y_2(k)$, $Y_3(k)$ into (2.2), we obtain $y_1(x)$, $y_2(x)$ and $y_3(x)$ as

$$y_1(x) = 1 - x - 799x^2 + \frac{63998}{3}x^3 - -213333x^4 - \frac{2}{15}x^5 + \frac{113777778}{5}x^6...$$
$$y_2(x) = -x + 801x^2 - 21334x^3 + \frac{640001}{3}x^4 - \frac{2}{15}x^5 - \frac{1023999998}{45}x^6...$$
$$y_3(x) = -1 + 80x - 1600x^2 - \frac{1280000}{3}x^4 - \frac{20480000}{3}x^5 + \frac{409600000}{9}x^6...$$

The numerical and theoretical results of y_1 and its error are illustrated in table 7.

Table 7.
Comparison of theoretical and numerical values of the y_1 in example 3.

t	Theoretical(y_1)	Numerical(y_1)	Error
0.1	0.9959322154	0.9959322153	0.0000000001
0.2	0.9876494697	0.9876494696	0.0000000001
0.3	0.9757613240	0.9757613237	0.0000000003
0.4	0.9608307308	0.9608307306	0.0000000002
0.5	0.9433749893	0.9433749894	0.0000000001
0.6	0.9238669934	0.9238669939	0.0000000005
0.7	0.9027367174	0.9027367204	0.0000000030
0.8	0.8803729025	0.8803729145	0.0000000120
0.9	0.8571248954	0.8571249329	0.0000000375
1.0	0.8333046046	0.8333047113	0.0000001067

Example 4. We consider a system representing a nonlinear reaction was taken from Hull [21].

$$\frac{dy_1}{dt} = -y_1, \quad \frac{dy_2}{dt} = y_1 - y_2^2, \quad \frac{dy_3}{dt} = y_2^2. \tag{3.7}$$

The initial conditions are given by $y_1(0)=1,\ y_2(0)=0$ and $y_3(0)=0$. By taking the transform of Eqs (3.7) we have

$$Y_1(k+1) = -\frac{1}{(k+1)} Y_1(k)$$
$$Y_2(k+1) = \frac{1}{(k+1)}\left(Y_1(k) - \sum_{r=0}^{k} Y_2(r) Y_2(k-r) \right) \tag{3.8}$$
$$Y_3(k+1) = \frac{1}{(k+1)} \sum_{r=0}^{k} Y_2(r) Y_2(k-r)$$

For $k = 0,1,\ldots,m$ $Y_1(k), Y_2(k), Y_3(k)$ coefficients can be calculated from (3.8) and substituting these values into (2.2), we have $y_1(x)$, $y_2(x)$ and $y_3(x)$ that

$$y_1(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7 + \frac{1}{40320}x^8 - \frac{1}{362880}x^9$$
$$y_2(x) = x - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{5}{24}x^4 + \frac{1}{40}x^5 - \frac{71}{720}x^6 + \frac{19}{1008}x^7 + \frac{1469}{40320}x^8 - \frac{329}{17280}x^9$$
$$y_3(x) = \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{60}x^5 + \frac{7}{72}x^6 - \frac{47}{2520}x^7 - \frac{7}{192}x^8 + \frac{691}{36288}x^9$$

As it seen from above examples, the differences between the exact and numerical solutions are quite small.

5. CONCLUSION

The differential transform method has been applied to the solution of stiff systems of differential equations. The numerical examples have been presented to show that the approach is promising and the research is worth to continue in this direction. Using the differential transform method, the solution of the stiff systems of differential equations can be obtained in Taylor's series form. All the calculations in the method are very easy. The calculated results are quite reliable and are compatible with many other methods such as Pade Approximation that we studied in [15]. The method is very effective to solve most of differential equations system.

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