OKU Fen Bilimleri Enstitüsü Dergisi
8(4): 1711-1726, 2025

OKU Journal of The Institute of Science and Technology, 8(4): 1711-1726, 2025

Osmaniye Korkut Ata Üniversitesi
Fen Bilimleri Enstitüsü
Dergisi

Osmaniye Korkut Ata University
Journal of The Institute of Science
and Technology

Potansiyel İlave Cisimlere Sahip Çift Yıldız Sistemi UZ Lyr ve Z Dra'nın Dinamik Kararlılık Analizi

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Araştırma Makalesi

Makale Tarihçesi:
Geliş tarihi: 09.12.2024
Kabul tarihi:17.04.2025
Online Yayınlanma:16.09.2025

Anahtar Kelimeler:
Örten çift yıldız sistemi
Yörünge kararlılık
Yörünge dönem değişimi
UZ Lyr
Z Dra

ÖZ

Bu çalışmada, REBOUND paketi ile N-cisim simülasyonları kullanarak UZ Lyr ve Z Dra sistemlerinin yörünge kararlılığı araştırılmıştır. Özellikle, Mean Exponential Growth Factor of Nearby Orbits (MEGNO) ve Wisdom-Holman Symplectic Integrator (WHFast) kullanılarak, ilave gezegensel cisimlere ev sahipliği yapabilecek bu çift yıldız sistemlerinin dinamik kararlılığı analiz edilmiştir. UZ Lyr için elde edilen sonuçlar, sistemin yaklaşık 10^4 yıl sonra dinamik olarak kararsız hale geldiğini ve MEGNO kararlılık haritasının gösterdiği gibi kaotik bir davranışa girdiğini göstermektedir. Ancak daha kısa zaman ölçeklerinde sistem yarı-kararlı görünmektedir. Sonuçlar, kısa vadeli simülasyonlara dayanarak uzun vadeli yörünge kararlılığını tahmin etmenin doğasında var olan zorlukları ortaya koymaktadır. Benzer şekilde, Z Dra sistemi için, önceki çalışmalarda önerilen dört model incelenmiştir. Simülasyonlarımız, tek ışık zaman etkisi (LTT) içeren modellerin kararlı kaldığı, iki LTT'li modellerin ise hem WHFast hem de MEGNO analizlerinde kısa zaman ölçeklerinde belirgin bir kararsızlık sergilediğini göstermektedir. Sonuçlar, ilave cisimlerin gözlemlenen zamanlama değişimlerine daha iyi bir ampirik uyum sağlayabileceğini, ancak bu tür konfigürasyonların genellikle dinamik olarak kararsız olduğunu göstermektedir. Bu çalışma, ikili sistemlerde ek cisimlerin potansiyel varlığını değerlendirirken uzun vadeli dinamik analizin önemini vurgulamaktadır.

Dynamical Stability Analysis of UZ Lyr And Z Dra Binary Systems with Potential Additional Bodies

Research Article

Article History:
Received: 09.12.2024
Accepted: 17.04.2025
Published online:16.09.2025

Keywords:
Eclipsing binary
Orbital stability
Orbital period variation
UZ Lyr
Z Dra

ABSTRACT

In this study, the orbital stability of the UZ Lyr and Z Dra systems was investigated using the REBOUND package with N-body simulations. In particular, the dynamical stability of these binary star systems, each potentially hosting additional planetary bodies by using the Mean Exponential Growth Factor of Nearby Orbits (MEGNO) and the Wisdom–Holman Symplectic Integrator (WHFast) was analyzed. The results for UZ Lyr show that the system becomes dynamically unstable after about 10^4 years, entering a chaotic regime as indicated by the MEGNO stability map. On shorter timescales, however, the system appears to be quasi-stable. The results reveal the inherent difficulties in forecasting long-term orbital stability based on short-term simulations. Similarly, four models proposed in previous studies were examined for the Z Dra system. Our simulations show that models containing a single light-time effect (LTT) remain stable, whereas models with two LTT terms exhibit pronounced instability on short timescales in both WHFast and MEGNO analyses. The results suggest that while additional bodies may provide a better empirical fit to observed timing variations, such configurations are often dynamically unstable. This study highlights the importance of long-term dynamical analysis when evaluating the potential presence of additional objects in binary systems.

To Cite: Er H., Kenger ME., Özdönmez A., Tekkeşinoğlu M. Dynamical Stability Analysis of UZ Lyr And Z Dra Binary Systems with Potential Additional Bodies. Osmaniye Korkut Ata Üniversitesi Fen Bilimleri Enstitüsü Dergisi 2025; 8(4): 1711-1726.

1. Introduction

The exoplanet field has made remarkable progress (Mayor and Queloz, 1995; Beuermann et al., 2010; Doyle et al., 2011; Horner et al., 2012; Lohr et al., 2014; Nasiroglu et al., 2017; Sonbas et al., 2022; Er et al., 2024a) since the groundbreaking detection of the planet around the millisecond pulsar PSR B1257+12 nearly three decades ago (Wolszczan and Frail, 1992). Over 5000 exoplanets have been confirmed various methods, leading to significant advances in both ground- and space-based instruments. In addition, more than 7000 exoplanet candidates are currently awaiting confirmation, demonstrating the rapid and continuing progress in our understanding of planetary systems other than our own. Among exoplanet detection methods, the most notable method for detecting the existence of additional bodies in orbits around binary stars is the timing method (Goździewski et al., 2012; Marsh et al., 2014; Goździewski et al., 2015; Deeg and Belmonte 2018; Özdönmez et al., 2023). A circumbinary planet orbiting a binary system can induce sinusoidal variations in the O–C diagram, the differences between observed (O) and calculated (C) eclipse timings, due to the light travel time (LTT) effect (Irwin, 1952; Beuermann et al., 2012; Er et al., 2021).

There are several categories of eclipsing binary (EB) systems, primarily classified according to their Roche lobe configurations into three main groups: detached, semi-detached, and contact binaries (Kopal 1959). In classical Algol-type systems, the components are generally in close and exhibit a semi-detached, where one star fills its Roche lobe while the other does not. Typically, the primary star is a more massive main-sequence star of spectral type B–A that remains within its Roche lobe, whereas the less massive secondary is often an F–K type subgiant that fills its Roche lobe (Wang and Zhu 2019; Ma et al., 2022). The O–C diagrams of Algol-type binaries reveal variations caused by different mechanisms, demonstrating the diverse applications of LTT effect analyses in these systems (Zasche et al., 2008; Yuan and Qian 2019; Shi et al., 2021; Bakış et al., 2022; Yıldırım et al., 2023). The LTT effect can reveal the presence of substellar mass orbital companions such as planets and brown dwarfs in such systems (Lee et al., 2009; Qian et al., 2012; Wolf et al., 2021). However, definitive claims about CBPs require independent evidence, as other mechanisms such as the Applegate mechanism, apsidal motion, mass transfer, and angular momentum loss due to magnetic braking can also cause variations in orbital periods (Applegate, 1992; Claret and Giménez, 2010; Parsons et al., 2010; Schreiber et al., 2010; Zorotovic and Schreiber, 2013; Almeida et al., 2019; Burdge et al., 2019; Almeida et al., 2020).

UZ Lyr was considered an Algol-type eclipsing binary system (Nijland, 1931). However, it is labelled as "forgotten" by Koch et al., (1979) because a detailed light curve analysis of the system was not available. The system has been included in the catalogues of variable stars (Malkov et al., 2006; Pigulski et al., 2009) and has since been studied as photometric and spectroscopic by various researchers (Prša et al., 2011; Slawson et al., 2011; Armstrong et al., 2014; Frasca et al., 2016; Matson et al., 2017; Roobiat and Pazhouhesh, 2022). The orbital period variation of UZ Lyr was first proposed by Rafert (1982). Hoffman et al., (2006) and later Gies et al., (2015) analyzed the O-C diagram and suggested that the observed period variations could be due to a third body or stellar spots. Borkovits et al., (2016) suggested that the third body could have a period of ~15.16 yr and a minimum mass of $0.17M_{\odot}$. Recently, Roobiat and Pazhouhesh (2022) constructed the most recent O-C diagram of the system to study its orbital period variation. The O-C diagram of the system was modelled using several approaches, including linear, parabolic, and cubic functions, in addition to one or two LTTs. It was concluded that the best fit was obtained with a linear function combined with two LTTs.

The eclipsing binary Z Dra (V=10.8^{m}, P=1.357456^{d}) was first discovered by Ceraski (1903) using photographic observations of Blajko (1903) and identified as an Algol-type variable. The system was studied photometrically and spectroscopically by many researchers, and physical parameters such as mass and radius were determined (Dugan, 1912; Dugan, 1915; Struve, 1947; Ishchenko, 1947; Hill et al., 1975; Terrell, 2006). The orbital period variation of the system was first discovered by Dugan, 1915 and was later confirmed by many studies (Kopal, 1936; Ishchenko, 1947; Kreiner, 1971; Frieboes-Conde and Herczeg, 1973; Herczeg and Frieboes-Conde, 1974; Rafert, 1982). Khaliullina (2016) investigated the orbital period variation of the system by adding a sinusoidal term to the quadratic model and detected a cyclic variation with a period of 60.2 years, which was attributed to the presence of a third body with mass of $M_3 > 0.70M_{\odot}$. Finally, Yuan et al., (2016) published new minimum times for Z Dra between April 2014 and February 2015 using different telescopes. To investigate the cyclic behavior in the system, Yuan et al., (2016) modelled the updated O-C diagram with different models by adding additional object(s) to the quadratic and cubic terms. To explain the observed timing variations, Yuan et al., (2016) investigated three different models and analysed the Z Dra O-C diagram. According to their results, the model that best fit the data included two additional companions with orbital periods of ~59.4 and 29.8 years. In other words, the combination of these two periodicities best fitted the cyclic variations in the O-C residuals among the three solutions they examined. This implies that the most plausible explanation for the observed light travel time effects in the system is the presence of two companions with these particular periods.

Although it is possible to derive a statistical model that describes the O-C diagram, the orbits of the systems must be stable for at least several thousand years (Mai and Mutel, 2022). Essentially, for an orbital solution to be physically plausible, it must both statistically replicate the O-C variations and maintain dynamic stability over thousands of years. Understanding the nature of these systems requires studying their stability and dynamics. In recent years, significant progress has been made in detecting additional bodies in binary systems through the analysis of orbital period variations (Brown-Sevilla et al., 2021; Gajdoš and Vaňko, 2023; Er et al., 2024). However, many studies have primarily relied on statistical models to fit these variations, often without performing long-term dynamical stability assessments. The results of our simulations reveal that while additional bodies may statistically account for the observed timing variations, their corresponding orbital configurations may be dynamically unstable. Thus, our study provides a critical methodological improvement and deeper insight into the complex dynamical behavior of binary systems, underscoring the necessity of coupling statistical modelling with long-term dynamical analyses (Horner et al., 2012; Mai and Mutel, 2022).

This study aims to investigate the orbital stability for Z Dra and UZ Lyr, respectively, for which dynamical stability simulations have not been performed in the literature, using new system parameters derived from the LTT models reported by Yuan et al., (2016) and Roobiat and Pazhouhesh (2022).

2. Materials and Methods

We used the N-body orbital integration package of REBOUND (Rein and Liu, 2012) to examine the orbital stability of the proposed planetary system. This package consists of a Mean Exponential Growth Factor of Nearby Orbits (MEGNO, Cincotta and Simó, 2000) indicator and a Wisdom–Holman symplectic integrator (WHFast, Rein and Tamayo, 2015). MEGNO is an important tool for studying the dynamics of planetary systems and other celestial bodies. By generating MEGNO stability maps, it is possible to examine how two initially close orbits behave over time based on a set of parameters such as semi-major axis and eccentricity (Cincotta and Simó, 2000; Rein and Tamayo, 2015; Livesey et al., 2024).

The following formula defines the MEGNO time-averaged value:

$$\langle Y(t)\rangle = \frac{1}{t-t_0}\int_{t_0}^{t} Y(t')dt'$$

Here, t_0 and t denote the start and end times of the integration, respectively. Y is a time-weighting factor derived from the variational equations applied to the orbit, which is used to determine whether the system behaves chaotically or regularly (for further details, see Morbidelli 2002; Hinse et al., 2010; Livesey et al., 2024). If the MEGNO indicator $\langle Y\rangle$ is ≤ 2, the system is stable, while values above 2 indicate chaos, with a value of 10 being assigned when a particle is ejected or collides (Cincotta and Simó, 2000; Goździewski et al., 2001; Brown-Sevilla et al., 2021; Gajdoš and Vaňko, 2023; Özdönmez et al., 2023; Livesey et al., 2024). WHFast is an advanced version of the symplectic Wisdom-Holman integrator (Wisdom and Holman, 1991), optimized for systems with a dominant central body and minor perturbations to Keplerian orbits. This integrator supports the kernel method as well as first- and second-order symplectic correctors. WHFast efficiently integrates orbits over a specified duration, demonstrating the evolution of orbital parameters such as the semi-major axis and eccentricity as functions of time. This is crucial for analyzing planetary interactions, predicting when a planet may escape the system or collide, and assessing the long-term stability of orbits (Wisdom and Holman, 1991).

3. Results

We used the N-body orbital integration package of REBOUND (Rein and Liu, 2012) to investigate the orbital stability of the UZ Lyr and Z Dra systems, where possible additional objects may exist. Dynamical stability simulations were performed using the best-fit planetary masses and orbital elements determined by Roobiat and Pazhouhesh (2022) for UZ Lyr and Yuan et al., (2016) for Z DRA in Tables 1 and 2. In Tables 1 and 2, the parameters are defined as follows: P_i represents the orbital period of the *i*th body, while $e_3$denotes its orbital eccentricity and ω_3 indicates the longitude of pericentre (periastron). The parameter $K_3$is defined as the semi-amplitude of the LTT signal observed in the O–C diagram, which arises due to the gravitational perturbation by the *i*th body. Moreover, $a_{12}sin_i$represents the projected semi-major axis of the binary system around the barycentre, and $M_{3,min}$is the minimum mass of the *i*th body as derived from the LTT model. The central binary star is regarded as a single mass in both simulations of the two systems, and all orbits were restricted to co-planar. The choice of parameters for our N-body simulations was guided by the two previous observational studies. The orbital parameters used in our analysis were obtained from LTT models fitted to the O-C diagrams, with the semi-major axis (a) and eccentricity (e) being the most critical parameters; these values were taken directly from published LTT solutions. The integration timestep was set to 0.01% of the shortest orbital period (in years) to preserve the symplectic nature of the Wisdom-Holman integrator and to minimise the accumulation of numerical errors (Wisdom and Holman, 1991; Rein and Tamayo, 2015). We systematically evaluated alternative initial conditions and configurations. Specifically, the semi-major axis and eccentricity were varied over 50 equally spaced intervals within a defined range (Ngrid), and the effects of these variations on the dynamical behavior of the system were analysed using MEGNO stability maps. This comprehensive exploration of the parameter space allowed us to account for potential uncertainties and to determine the sensitivity of the long-term stability results to the chosen initial conditions and observational constraints. Under these conditions, dynamic stability simulations were first performed with a time interval of 10^7 yr to obtain both the MEGNO value and the orbital stability timeline. In the event of structural instability in the system's orbit over shorter time periods, the application of MEGNO was performed for analysis at these shorter timescales.

Figure 1 shows that the system consisting of the parameters of the best-fit model (linear function with two LTTs) proposed by Roobiat and Pazhouhesh (2022) for UZ Lyr, exhibits chaotic orbital behavior, with its configurations becoming destabilized and significantly perturbed after about 10^4 years. In Figure 2, the system is in the unstable region ($\langle Y \rangle \geq 2$) in the MEGNO stability map analyzed in the 10^6 year time interval, while it is in the stable region ($\langle Y \rangle \leq 2$) in the 10^4 year MEGNO stability map in Figure 3.

Table 1. Orbital parameters of UZ Lyr obtained from two LTT models by Roobiat and Pazhouhesh (2022).

Parameters	
$P_3(yr)$	23.140(20)
e_3	0.06(2)
$\omega_3(°)$	63(2)
$K_3(day)$	0.00523(39)
$a_{12}sin_i(au)$	13.645(17)
$M_{3,min}(M_\odot)$	0.31494(13)
$P_4(yr)$	360(85)
e_4	0.60(11)
$\omega_4(°)$	56(6)
$K_4(day)$	0.05011(69)
$a_{123}sin_i(au)$	88.344(21)
$M_{4,min}(M_\odot)$	0.55187(29)
Rebound Parameters	
Time-Step (yr)	0.02
Ngrid	50

Table 2. Orbital parameters of Z Dra obtained from the two LTT models with the quadratic term (solution 3 in them) and the two LTT models with the cubic term (solution 4 in them) by Yuan et al., (2016)

Parameters	Solution 3	Solution 4
$P_3(yr)$	29.81 ± 0.08	29.05 ± 0.08
e_3	0.43 ± 0.01	0.11 ± 0.03
$\omega_3(°)$	285.6 ± 3.9	83.1 ± 26
$M_{3,min}(M_\odot)$	0.39 ± 0.03	0.33 ± 0.04
$A_3(au, i_3 = 90^\circ)$	12.74 ± 0.3	12.3 ± 0.2
$P_4(yr)$	59.41 ± 0.12	58.07 ± 0.12
e_4	0.62 ± 0.02	0.56 ± 0.01
$\omega_4(°)$	76.8 ± 4	240.5 ± 1.6
$M_{4,min}(M_\odot)$	0.77 ± 0.02	0.77 ± 0.03
$A_4(au, i_4 = 90^\circ)$	22.3 ± 0.3	21.9 ± 0.2
Rebound Parameters		
Time-Step (yr)	0.03	0.03
Ngrid	50	50

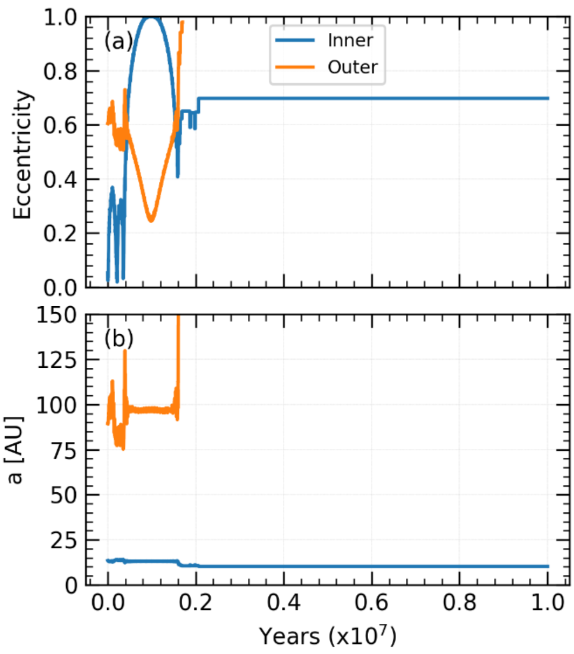

Figure 1. Orbital stability analysis of the UZ Lyr system based on the two-companion LTT model proposed by Roobiat & Pazhouhesh (2022). This simulation, performed with the WHFast integrator over 10^7 years, illustrates the long-term dynamical evolution of the system. (a) shows the time evolution of the eccentricities for the inner (blue) and outer (orange) companions over 10 Myr, while (b) presents the corresponding variations in their semi-major axes. Note that the outer companion is eventually ejected from the system, whereas the inner companion maintains a stable orbit. These results suggest that the UZ Lyr system is unlikely to sustain a dynamically stable configuration with two additional bodies.

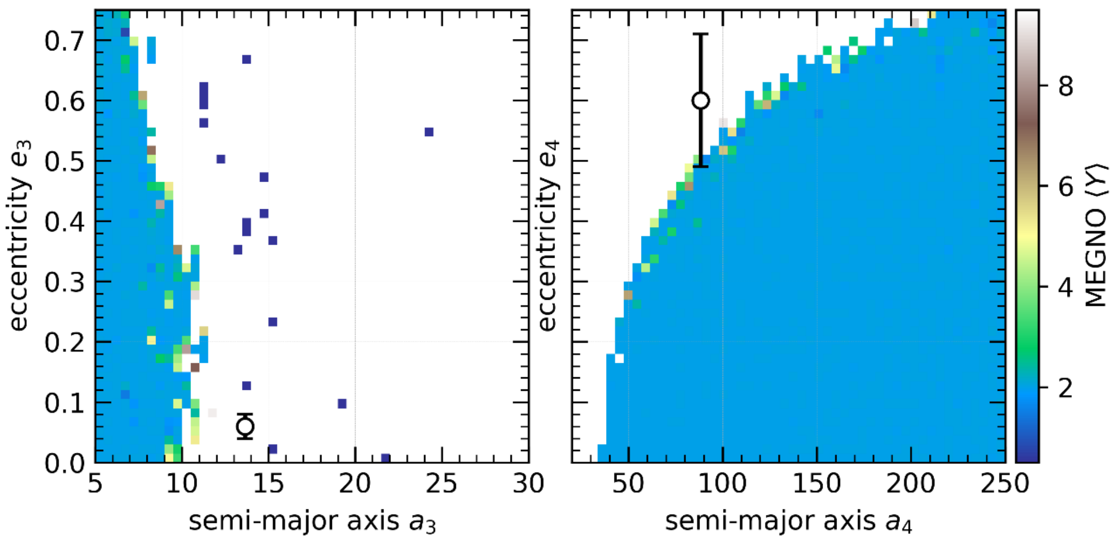

Figure 2. MEGNO chaos parameter surface map for a range of eccentricity and semi-major axis values for the inner companion (left panel) and the outer companion (right panel) of the two-companion LTT solution by Roobiat & Pazhouhesh (2022) for a duration of 1 Myr. The white circles denote the best-fit model parameters along with their uncertainties (refer to Table 1). In this context, the white circles indicate that, for this solution, both companions are located in highly chaotic regions of the parameter space.

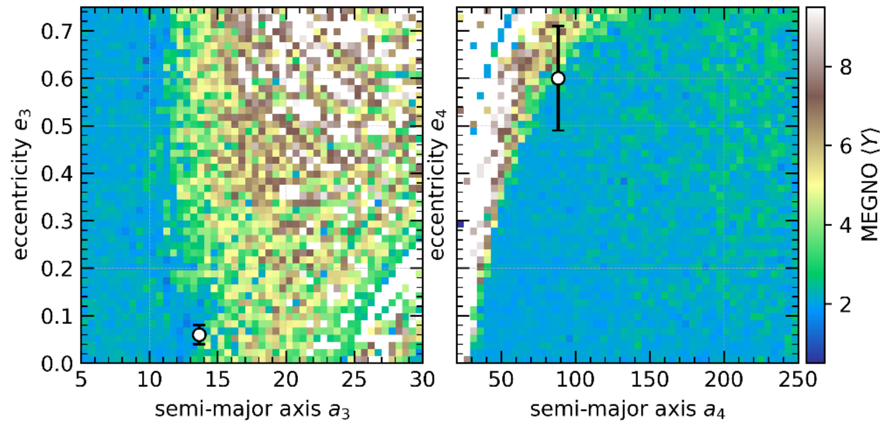

Figure 3. MEGNO chaos parameter surface map for a duration of 10^4 year for the inner companion (left panel) and the outer companion (right panel) corresponding to the two-companion LTT solution by Roobiat & Pazhouhesh (2022). Here, the white circles again mark the best-fit parameters (and their uncertainties) as listed in Table 1. In contrast to Figure 2, the white circles in this figure fall within regions that are comparatively stable, suggesting a more regular orbital behavior for the alternative solution.

Since Yuan et al., (2016) modelled the updated O-C diagram of Z Dra with 4 different functions by adding additional object(s) to the quadratic and cubic terms, we also performed dynamic stability simulations on 4 different models. Solutions 1 and 2 by Yuan et al., (2016) involve single LTT models, and we conclude that they are stable, as expected for a system with a single LTT. However, dynamic stability simulation graphs are not presented in this study because Yuan et al., (2016) stated that these two solutions are not very compatible with the O-C diagram and that the best solution is solution 3 (quadratic term + two LTT). When other models are examined, it becomes clear that the system exhibits instability on short timescales for both solution 3 and solution 4, as shown in Figure 4. Moreover, an analysis of the MEGNO stability map, as illustrated in Figure 5, indicates that the system parameters of both solutions are situated within unstable regions.

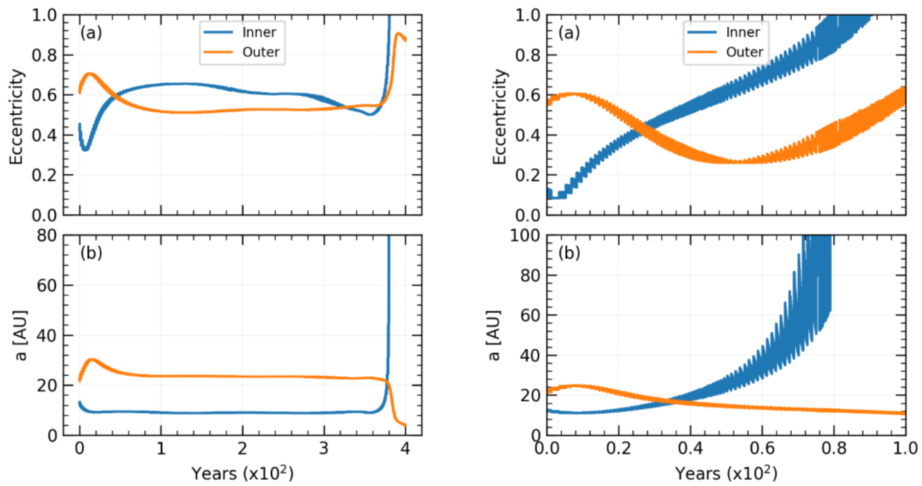

Figure 4. Orbital stability timeline for the Z Dra system using two LTT models (Solutions 3 and 4) from Yuan et al. (2016). The left panel shows the stability timeline for Solution 3, while the right panel presents that for Solution 4. In both cases, short-timescale interactions lead to rapid mutual perturbations, resulting in either collisions or ejections from the system. This outcome demonstrates that the Z Dra configuration, as modelled by these solutions, is dynamically unstable.

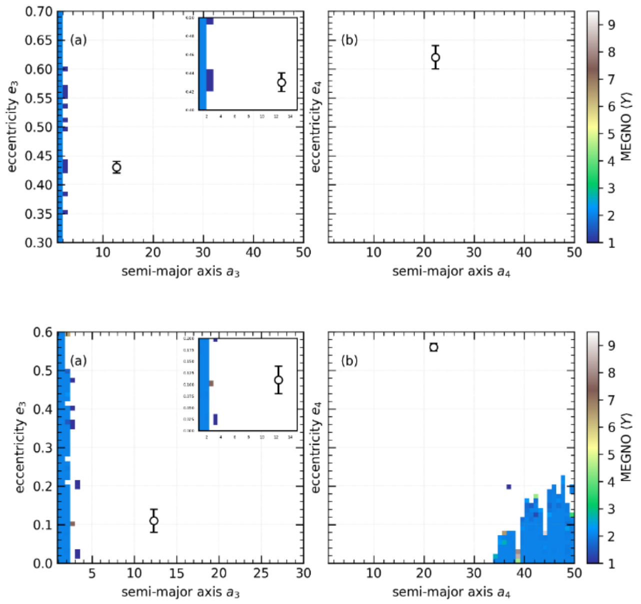

Figure 5. Orbital stability analysis of the Z Dra system using the two LTT models provided by Yuan et al. (2016). The upper panel illustrates the stability timeline for Solution 3, and the lower panel shows the results for Solution 4. In both models, the short-term dynamics reveal that the inner and outer companions are quickly ejected from the system. The accompanying MEGNO stability maps, constructed according to the two-companion solution, further underscore instability of the system. On short timescales, both inner and outer bodies escape the system in both solutions.

4. Discussion and Conclusions

The dynamic stability of UZ Lyr and Z Dra was analyzed using N-body simulations of REBOUND (Rein and Liu, 2012), focusing on the possible presence of additional objects in these systems. The simulation timescales were chosen using a stepwise approach with increasing time intervals to thoroughly capture the long-term dynamic behavior of the system. We first examined the stability of the system over a relatively short timescale (about 10^3 years). If the system remained stable over this interval, we extended the integration period to longer durations (10^4, 10^5, 10^6 years) and continued this process until the system either exhibited chaotic behavior. If instability occurred before the maximum planned duration was reached, the simulation was terminated early. When the dynamic stability simulation results for the UZ Lyr system based on the model parameters proposed by Roobiat and Pazhouhesh (2022) are examined, the eccentricity of the inner planet approaches 1.0. This suggests that the inner planet is moving towards orbital instability, which could disrupt the overall balance of the system. The eccentricity of the outer planet initially remained constant at about 0.6 but decreased over time to the level of 0.2. The semi-major axis of the inner planet shows only minor variations and remains relatively stable at a low level. On the other hand, the semi-major axis of the outer planet exhibited oscillatory behavior in the early stages of the simulation and later detached from the system. The dynamic stability simulation results for the UZ Lyrae system indicate significant instability. The orbit of the system becomes chaotic after about 10^4 years. This behavior is consistent with expectations for systems experiencing nonlinear perturbations from multiple bodies. Further analysis using the MEGNO strengthens this instability. The MEGNO value exceeds 2 in a 10^6 year stability map, indicating that the system is entering a chaotic regime. Interestingly, in a shorter 10^4 year simulation, the MEGNO remains below 2, indicating that the system initially appears stable. This suggests that while UZ Lyr is dynamically stable on short timescales, its long-term evolution is dominated by chaotic interactions that lead to instability. This chaotic transition is likely driven by several intertwined mechanisms. First, the cumulative effect of gravitational perturbations from the additional companion gradually alters the orbital elements of UZ Lyr, such as the eccentricity, the semi-major axes and the argument of pericentre. These small perturbations, while negligible in the short term, can lead to significant changes over time due to the system's sensitivity to initial conditions. Second, overlapping mean-motion resonances and secular resonances are expected to play a significant role in destabilizing the system. The sensitivity of chaotic systems to initial conditions means that even minor variations within the observational uncertainties can lead to markedly different evolutionary outcomes. Furthermore, unstable orbits are not always implied by a chaotic system. However, chaotic time evolution is always implied by unstable orbits. The integration length is the crucial factor to take into account. There is a chance that the quasi-period will be incorrectly concluded if the dynamical system's chaotic onset moment necessitates a significantly longer time span than the integration time. Thus, it is important to integrate the system long enough for it to potentially display chaotic behavior (Horner et al., 2012, 2013; Hinse et al., 2014). Our results show that short-term stability can be misleading, because a configuration that appears stable over a limited number of orbits may be dynamically unstable on astrophysically important timescales. Long-term integrations are therefore important to reliably assess the feasibility of multibody configurations in binary systems. More precise orbital parameters are needed to improve our orbital stability models. This finding highlights the challenges of definitively characterizing systems with multiple potential additional objects. Short-term stability does not guarantee long-term stability, and systems can transition from quasi-stable to unstable on different timescales, depending on the specific configurations of additional bodies. These results are also seen in other systems (Goździewski et al., 2001; Horner et al., 2013; Borkovits et al., 2016; Brown-Sevilla et al., 2021; Er et al., 2021; Mai and Mutel, 2022; Özdönmez et al., 2023).

The Z Dra system presents a more complex scenario due to the variety of models proposed by Yuan et al., (2016), who tested four different solutions for the O-C diagram. Of these, solutions 1 and 2 represent single LTT models. As expected for single LTT systems, both solutions remain dynamically stable in our simulations. This is consistent with the general expectation that simple perturbations from a single additional body are less likely to induce chaotic behavior in such systems. It is important to note, however, that Yuan et al., (2016) concluded that these solutions do not fit the O-C diagram satisfactorily, suggesting that they are less likely to represent the true configuration of Z Dra. In contrast, solutions 3 and 4, which involve two LTTs, exhibit pronounced instability on short timescales. The presence of two additional bodies significantly perturbs the system, pushing it into a chaotic regime. This instability is confirmed by the MEGNO simulation, where both solutions exhibit values indicative of unstable orbits. According to our numerical simulations, the system disintegrates rapidly, with the hypothetical companions either colliding with the central binary or being ejected in a matter of thousands of years. These findings suggest that either the proposed companions do not exist, or they must occupy a markedly different orbital configuration than previously proposed. The results suggest that while adding more bodies may provide a better empirical fit to the O-C diagram, it also leads to dynamic instability. Therefore, while models with two LTTs may provide better agreement with observations, they are unlikely to be physically realistic on long timescales due to the rapid onset of chaotic behavior. Future studies, particularly for UZ Lyr, should include additional observational data to revise the system parameters and orbital stability.

Conflict of Interest

The authors stated that there are no conflicts of interest regarding the publication of this article.

Researchers' Contribution Rate Statement

The contribution rates of the authors in the study are equal.

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