A COMPUTER METHOD FOR PREPARING BETA DIAGRAMS (PROGRAM IN FORTRAN IV-H, USING AN IBM 360/67 COMPUTER)

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ABSTRACT.— A method for preparing beta diagrams has been programmed for the IBM OS/360 (FORTRAN H) computer. The 570 CALCOMP system has been used to output the resultant plots. The program finds spherical coordinates of intersections of traces of individual segments, then projects them on an equatorial plane by using stereographic projection principles. It makes necessary corrections to fit the projected points to an equal area polar net (Billing net). The program, then, to calculate intensity of distribution of projected points performs the following tasks: (a) distributes the additive value of each point to a one percent domain by using the Mellis method; (b) takes care of the values which might fall outside the primitive circle; (c) chooses a suitable contour interval and integrates the values between contours. The final distribution is displayed both by an output from line printer and an output from CALCOMP plotter on a 20-cm Billing net. The computer program described in this report may be used, by slight alterations, to prepare various diagrams used in structural geology and in fabric data analysis in structural petrology.

INTRODUCTION

This study was carried out by the author at Stanford University during the spring of 1968.

The program is prepared to draw beta diagrams, but by slight alterations can be used to prepare other useful diagrams.

Programs in different languages and using somewhat different techniques have been provided by Robinson (1963) and by Noble (1964). The method used in this program is believed by the present author to give more accurate results than the above-mentioned works; however no attempts will be made to compare between them.

In the following chapters the meaning of beta diagrams, the technique in their preparation and the computer program itself will be discussed rather briefly. A listing of the program and outputs from a test run are presented . at the end of the text. Operating instructions for the program are given in the program as comment statements in logical steps.

EQUAL AREA NET

The following paragraph which summarizes the use of equal area net is quoted from Turner and Weiss (1963) :

«... Structural analysis is concerned especially with orientation data relating to planes and lines (fabric elements) and their intersections. In study of crystal morphology the relative orientations of planes and lines are conventionally represented and their geometric relations determined by means of the familiar stereographic projection —a tool which is commonly employed, too, in graphic solution of many problems in structural geology. In structural analysis, the necessity to evaluate preferred orientations of fabric elements imposes a peculiar limitation on graphic procedure. All equal area on the surface of the reference sphere must remain equal on the projection itself. This is not true of the stereographic projection, in which centrally situated areas are diminished relatively to peripheral areas of equivalent size on the reference sphere. To obviate this difficulty it is customary in structural analysis to use a type of equal-area projection — also known as the Lambert projection (after its inventor) or the Schmidt projection (after W. Schmidt who first used it in structural geology).

Both types of projection employ a reference sphere in which planes and lines passing through the center intersect the surface as great circles and points respectively. In equal-area projection as in stereographic projection, these are projected — but from the lower hemisphere only onto the equatorial plane; but the graphic procedure employed maintains the desired equal-area specification which is absent from stereographic projection. Because of this property, density distribution of points on the projection faithfully reflects the preferred orientation of the corresponding lines passing through the center of the reference sphere. Stereographic projection is from a point source, and circles on the reference sphere appear as arcs of circles on the projection. This is not true of the equal-area projection; circles are projected as elliptical arcs, save where they lie in or normal to the plane of projection (the boundary circle and diameters of the projection respectively)...

Figure 1 illustrates the principles of the stereographic projection. For the stereographic projection can the transfer of configuration on sphere of a plane be carried out by direct geometrical construfction. In equal area type of projection the configuration is obtained by computational methods for which such concepts as the plane of projection, the center of projection and line of projection do not have a simple graphical meaning. Both, stereographic net (Wulff net)

and equal-area net (Schmidt net or Lambert net) can be prepared as equatorial or polar projection. Equatorial projection is more often used because it enables the user to evaluate the coordinates of the intersections. In the present program the polar variety of equal-drea net (this net is also called Billing equal-area net) is used, because it can be drawn much easier by plotter, and coordinates of intersection vectors are calculated by mathematical methods. Polar and equatorial nets of stereographic and equal-area projection are illustrated in Figure 2. Scale and area distortion on these nets are also presented on the same figure.

STATISTICAL USE OF EQUAL - AREA NET

The nature and degree of preferred orientation of a given type of planar or linear fabric element are expressed graphically by the distribution on an equalarea projection of points (poles, in the case of planar elements), representing the individual orientations of a large number of representative measured elements of

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the kind in question. The points so plotted constitute a pattern of preferred orientation or an orientation diagram of the given element. A statistically random orientation is expressed by a pattern in which there is no obvious tendency for reproducible local concentration of plotted points. Most orientation diagrams representing tectonite subfabric show marked reproducible local concentration and complementary voids or sparcely populated areas. These are graphic evidence of preferred orientation.

To sharpen the pattern of preferred orientation expressed by points on a net, it is customary in structural analysis to draw density contours on the point diagrams. Each contour delineates an area within which the density of distribution of plotted points exceeds some minimum value, e.g. 5 % of the total points per 1 *%* of the projection area.

To count the points lying within any one percent circular area of the projection and to draw the contour, several contouring procedures are currently used: *Schmidt method* is by far widely used. In this method a contouring counter (Fig. 3a) is the principle device. It is a card with a circular hole 1/100 of the area of the complete equal-area net. This counter is placed on the net and the number of points which can be seen within the circle are counted and the percentage concentration is calculated (Fig. 3b). For example if there were 500 poles on the net, and

at one position of the counter 10 occurred within the hole, then the percentage at the center of the counting circle would be $(10 \text{ X } 100/500)$ 2 percent. These percentages are recorded from place to place (usually controlled with a grid) on the net and the density distribution contoured. At the periphery of the net the percentage are calculated with a special counter (Fig. 36) so that a complete one percent area is used. *Mellis method* or circle method is another tool to contour the population density. Circles of 1 percent are drawn around each point taking the point as the center (Fig. 4a and 46). Overlapping area between two circles have a 2 percent concentration, those between

Fig. 3 (a-b).

three circles have 3 percent concentration, etc. This method has sounder statistical meaning; also, it is the least subjective method of contouring: the only one by which strictly identical diagrams will be produced from the same point diagram by independent workers. But its application is limited because it becomes almost impossible to count the number of overlapping circles. However this inconvenience is overcome by using computer (in our program we have used this method).

The object of counting is to draw density contours connecting centers of circular areas within which the measured points are equally concentrated. Contouring is an arbitrary procedure the statistical significance of which has not been rigorously evaluated. The following rules are recommended by Turner and Weiss (1963): (1) On any diagram the number of contours should not be more than six. (2) The highest contour is chosen to emphasize and differentiate maxima large enough to stand out clearly on a projection 2 to 3 inches in diameter. For instance, if the areas of local concentration of 10 to 12 percent are small, it may be advisable to draw the highest contour at 8 percent. (3) In any diagram contour intervals should preferably be uniform.

BETA DIAGRAMS

The following paragraph is quoted from Ramsay (1967) :

«...Surfaces within a cylindrical fold contain a line parallel to the rectilinear generator and therefore the intersections of any two observations of measured tangent planes to the folded surface intersect in a line parallel to this generator. This line is known as a beta axis and is parallel to the fold axis. The stereogram or equal-area projection offers a very convenient way for graphic determination of the orientation of such beta axes. All the beta axes computed in this manner should have a parallel orientation (i.e., all the great circles representing individual surfaces in the fold should pass through a single point). In practice, however, folds rarely have a perfectly cylindrical form and the measurements of the fold surfaces are always subject to a certain error. This means that the computed beta axes do not generally coincide, but they are unimodally grouped around a point which gives the best fit orientation for the mean beta axis or fold axis. The number of beta intersection produced by the intersection of n observed planes (tangent planes) in the fold is given by the arithmetic progression:

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$$
S=0+1+2+3+\ldots+\ldots+\ (n-1)=\frac{n(n-1)}{2}.
$$

If $n> 3$, the number of beta intersections outnumbers the observation and in practice it may be almost impossible to determine their position in a single diagram (for example, if $n = 500$, then $S = 124,750...$

However usage of high speed digital computers overcomes this impracticability.

Figures *5a* and *5b* illustrate the use of beta diagram.

Beta diagrams can also be used to solve complex folding by dividing it to cylindrical portions. A fan of cleavage can be analysed by using beta diagrams. Other orientation studies can be performed by making slight changes in the present program or/and using only parts of it. (The function of every part in the program is explained by comment statements in the program itself and also in the next chapter so that one could modify the program.)

MAJOR STEPS IN THE PROGRAM

A real $(100 \text{ X } 100)$ array is used to fill in the values to be used 0.100 in plotting, and an integer array KAR $(100 \tX 100)$ is used to transfer values in real array to the corresponding percentage.

In PART ONE¹ the cells of the array outside the projection circle (net) are filled with 10,000 (Fig. 6).

In PART TWO angles representing measured form surfaces are read in (see chapter explaining the

¹ The beginning and end of parts are indicated in the program by means of comment statements.

inputs to the program). Pole of these surfaces are computed and direction cosines of the computed poles are stored in X, Y, Z arrays. (Positive X direction is to the north, Y to the east and Z downward.) These arrays are declared with 200 spaces each. This means one has to adjust this size if he wants to deal with more than 200 measurements. No other change in the program is required for such a modification. But it should be remembered that number (IS) of intersection is related to the number of the measurements by the formula mentioned above. IS might be a little smaller than the value predicted by the formula, because for surfaces having the same attitude, intersection is not possible. This fact is taken into account in the program (see Part Three).

In PART THREE direction cosines of vectors generated by the intersection of every two surfaces are evaluated using the following relations :

X' A + Y'B + Z'C = 0 (cross product)
\nX'' A + Y''B + Z''C = 0 (cross product)
\n
$$
A^2 + B^2 + C^2 = 1
$$
 (theorem for direction cos.)

where X', Y', Z' are direction cosines of one and X'' , Y'', Z'' are direction cosines of another vector. A, B, C are the corresponding direction cosines of the vector generated by intersection. Then these coordinates on the sphere are projected to an equatorial plane. Billing equal-area net is used in this case. For the equal-area projection the basic equation connecting the radius vector o with the polar distance (-) has the form

$$
\varrho' = 2 \text{ R } \sin \frac{\theta}{2} \quad \text{(Vistelius 1966)}
$$

In the program A and B are transformed using the above formula and geometrical relations (Fig. 7).

$$
0 = \overline{OR}^{2} - C^{2} = 1 - C^{2} \quad \text{(OR is radius and equals 1)}
$$
\n
$$
0' = 2 \sin \frac{\theta}{2} \qquad \text{(OR = 1)}
$$
\n
$$
2 \sin \frac{\theta}{2} = 2 \sqrt{\frac{1 - \cos \theta}{2}} \qquad \text{trigonometric relation}
$$
\n
$$
\frac{\theta'}{\theta'} = \frac{2 \sqrt{\frac{1 - \cos \theta}{2}}}{\sqrt{1 - C^{2}}}
$$

But Cos $\Theta = C$ and the formula takes the following form:

$$
\frac{\varrho'}{\varrho} = \sqrt{\frac{2}{1+C}}
$$

From similar triangles shown on Fig. 8 AA/A and BB/B ratios can be calculated

$$
AA = A \sqrt{\frac{2}{1+C}} \qquad BB = B \sqrt{\frac{2}{1+C}}
$$

The next step is to fill the AR array by using AA and BB coordinates

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Diameter of the projection net is 90 cells as shown on Figure 6. One percent circle will have a diameter of 9 cells. The filling operation is illustrated in Figure 9.

x,x' are the cells defined by AA and BB and (AA)' (BB)'. The values are added to previous values of the neighboring cells (occurring in 1 percent influence circle). It could be figured out that several (even thousands) of circles may comprise the same cell.

In PART FOUR the value in cells outside the net are added to their corresponding portion inside the net (Fig. 10). Outside cells are left filled with 10,000 to differentiate them from all of the inner cells.

In PART FIVE values of AR array are transported to integer KAR array and in the mean time values in each cell are divided by (total number of points in the net/100) to find representative percentage value.

In PART SIX the final value of each cell is printed out. Coordinate axes are also printed. The circle is deformed to an ellipse, due to rectangular form of the characters. Area outside of the primitive circle is filled with *=* sign. The values smaller than 1 are not printed. Values larger than

9 are represented by alphabetical symbols. A table of these symbols is given at the bottom of the map. The maximum value on the map is also printed below them. The main objective of printing out such a map, despite the great distortion, is to furnish user with a reference table showing actual values in each cell.

In PART SEVEN 'CALCOMP' plotter subroutines are called to draw a 20-cm Billing equal-area net. 10° interval is used.

In PART EIGHT 'CALCOMP' plotter subroutines are called to plot the cell values by classifying them as it is shown in the table below. Symbols used in plotting are :

N, S, E, W for north, south, east, west.

for values $> 4/5$ of the max. value. \bigcap for values $>$ 3/5 but \leq 4/5 of max. Δ for values $> 2/5$ but \leq 3/5 of max. \star for values > 1/5 but \leq 2/5 of max.

 \Box for values \leq 1/5

values less than one are ignored and left blank.

INPUTS TO THE PROGRAM

The number N of measurements to be used should be punched in the very first data card. 15 format is used for this purpose. The upper limit for this N number is limited with the size of X, Y, Z arrays and it is set to 200 in this program. It could be augmented by simply changing the size of these arrays,.

N data cards must follow the very first card bearing the value of N. 215 format is used in these cards. First number being the azimuth and the second latitude of vectors representing dips. Azimuth value is the angle (from 0 to 360°) which the direction of the dip of measured plane makes with the north (clockwise direction being positive). It is expressed as integer degrees. Latitude is also expressed as integer degrees. It is the angle that the dip vector makes with the horizontal (from.0 to 90°).

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JCL card must contain a special sign to cause page ejects to be supressed, thus permitting to obtain a map printed without gap across page margins.

Another job control statement should designate that plotter is going to be used. One should examine the listing of the program for detail, but should also keep in mind that these formats are subject to change any time.

The program consumed maximum 2 minutes computer time for a data set of 50 measurements.

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COMPUTER PROGRAM LISTING

PARE 004

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BETA DIAGRAM DRAWN BY LINE PRINTER DURING A TEST RUN OF THE PROGRAM

Beta diagram entirely drawn by CALCOMP plotter during a test run of the program. (Original diameter was 20 cm. The symbols are darkened by hand for printing purposes)