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DURGUN POROZ ORTAMDAN LİNEER VE LİNEER OLMAYAN VİSKOELASTİK AKIŞKANLARIN AKIŞI VE ISI TRANSFERİ İÇİN MATEMATİKSEL MODEL DENKLEMLERİ

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ÖZET

Geçirgen heterojen ortamdaki akışkanın matematiksel modellemesi, Resin Transfer Molding, Injection Molding, vs. gibi birçok imalat proseslerinde önemli bir yer tutar. Bu araştırmada, Local Volume Averaging Technique metodu ile elde edilen 3 boyutlu, zamana bağlı izotermal olmayan genel bir matematiksel model elde edildi. Model, ortalama hız, basınç, polimerik gerilmeleri ve sıcaklık değerlerini kütle, momentum ve enerji korunum kanunlar ile polimerik gerilme modellerini kullanarak elde etmektedir. Polimerik akışkan gerilmeleri için lineer (UCM ve Oldroyd-B modelleri) ve lineer olmayan (Giesekus ve PTT modelleri) modeller kullanılarak polimerik akışkanın viskoelastik karakteristikleri ortaya konulabilir.

GOVERNING EQUATIONS FOR QUASI-LINEAR AND NONLINEAR VISCOELASTIC FLUID FLOWS AND HEAT TRANSFER THROUGH STATIONARY POROUS MEDIA

ABSTRACT

Mathematical modeling involving porous heterogeneous media is important in a number of composite manufacturing processes, such as resin transfer molding (RTM), injection molding and the like. In this research, a mathematical model by utilizing the local volume averaging technique to establish 3-D, time dependent and non-isothermal governing equations is presented. The developments should be able to predict the averaged velocity, pressure, polymeric stress and temperature fields by modeling the conservation laws (e.g. mass, momentum and energy) of the flow field coupled with constitutive equations for polymeric stress field. The governing equations of the flow are averaged for the fluid phase. Furthermore, the model target a variety of viscoelastic models (e.g. Newtonian model, Upper-Convected-Maxwell Model, Oldroyd-B model, Giesekus model and PTT modl) to provide a fundamental understanding of the elastic effects on the flow field. The present research is focused on non-isothermal considerations and a variety of constitutive models accounting for the viscoelastic flow behaviors.

Key Words: *Mathematical modeling, porous media,non-isothrmal viscoelastic fluid flows*

1.INTRODUCTION AND MOTIVATION

Flow through heterogeneous porous media is, in general, of importance in many practical industrial applications. Some of the examples include composites process modeling, petroleum industrial applications, groundwater flow, chemical reactors, heat exchangers and the like. As related to composites process modeling applications, which is the main motivation of the current research, flow reinforced polymeric composite materials are increasingly being used in many applications. Liquid Composite Molding (LCM) processes such as Resin Transfer Molding (RTM) and Structural Reaction Injection Molding (SRIM) are two manufacturing methods to fabricate composite structures.

In recent years, RTM and its derivative processes have been gaining popularity in the aerospace, infrastructure, automotive, and military industries. The selection of a manufacturing technique for producing polymeric composite parts must be based on consideration of a number of factors, including the size and shape of the part, microstructural control, reinforcement and matrix type, required performance, market economics, and so on. RTM appears uniquely capable of satisfying the low-cost/high-volume parts of the automotive industry as well as the higher performance/lower volume parts of the aerospace industry. Moreover, variations of the RTM process make it well suited for the production of large, complex, thick-sectioned structures for infrastructure and military applications. For example, the glass-fiber/vinyl-ester bridge deck and the lower hull of the Army Composite Armored Vehicle (CAV). The automotive industry has used resin transfer molding (RTM) for decades.

Perhaps the greatest benefit of RTM relative to other polymer composite manufacturing techniques such as autoclave curing, filament winding, hand-layup and pultrusion is the separation of the molding process from the design of the fiber architecture. Having the fiber preform stage separate from the injection and cure stages enables the designer to create uniquely tailored material to fit precisely a specific demand profile. This is accomplished by combining a variety of fiber types and forms. In fact, liquid molding enables attainment of high levels of microstructural control and part complexity compared with processes such as injection molding and compression molding. Other benefits afforded by RTM include; low capital investment, good surface quality, tooling flexibility, large and complex shapes, parts integration, range of available resin systems, range of reinforcements, controllable fiber volume fraction and the like.

In RTM processes, dry reinforcement (i.e., unimpregnated) is pre-shaped and oriented into a skeleton of the actual part known as the preform, which is inserted into a matched die mold. The mold is then closed, and a low-viscosity reactive fluid is injected into the tool. The air is displaced and escapes from vent ports placed at the high points. During this time, known as the injection or infiltration stage, the resin "wets out" the fibers. Heat applied to the mold activates polymerization mechanisms that solidify the resin in the step known as cure. The resin cure begins during filling and continues after the filling process. Once the part develops sufficient strength, it is moved or demolded. Flow progression and distribution, and fiber impregnation during the process are influenced by elastic effects of the polymeric fluid and the drag force exerted by the fluid as it flows through the fiber network.

So far, computational models for the flow through porous media have been strictly restricted to Newtonian flows and even to this day, Darcy’s law has been adapted as the primary governing mathematical model because of its simplicity. As has been stated in the literature, this type of formulation is very much inadequate in terms of representing the physics of the problem in several situations causing inaccurate simulations. Extensions to Darcy’s model in the form of Brikman’s equation [37] and the like also exist, but have been primarily centered on Newtonian flows. Quite often, the resin is non-Newtonian. In particular, the resin is characterized as viscoelastic and not much work appears in the literature, which accounts for such flows through complex porous preforms. This is the focus of the present research and attention is focused on isothermal considerations and a variety of constitutive models accounting for the viscoelastic flow behaviors.

2.PREVIOUS WORK

The liquid fluid flow through porous media has been previously studied by researchers interested in geological problems [1, 2]. The governing equation to model the flow through porous media is the empirical Darcy's law [3, 4] which states the flow rate of Newtonian fluids through saturated porous media to be proportional to the pressure drop across the medium.

Recently, this law is being used in modeling the flow of polymeric composite materials. It is assumed that the fiber network is constrained in such a way that it acts as an incompressible porous medium [5, 6]. Research has consistently used Darcy's law as the governing equation for low Deborah numbers [7]. Relatively very little experimental research has been conducted to support the use of Darcy's law in fiber composite processes involving non-Newtonian resins and in developing an objective and effective method for measuring the fiber permeabilities and the dependence of permeability on the fluid, local pressure and nature of porous media [8, 9, 10].

Kozeny [11] proposed an expression for the permeability based on an idealized porous medium geometry and assumed that a porous medium can be represented by an assemblage of channels of various cross-sections. The permeability, then, can be obtained in terms of a hydraulic radius. Carman [12] developed a modification to Kozeny's theory using the specific surface area of the porous medium particles. The resulting expression, the Carman-Kozeny equation, gives the permeability as a function of the porosity of the medium [13].

Almost a century after Darcy proposed his equation, Brinkman [27] described a more accurate equation and model flow through porous media. Other relevant early works appear by Lundgren [28], Beavers and Joseph [29], Beavers et al [30], and Sparrow and Loeffler [31].

The capillary model was originally used for flow through isotropic porous medium, and its extension to flow across cylinders is developed by only redefining the hydraulic radius (Lam and Kardos [14]; Dave et al. [15]). The anisotropy of the medium was not included explicitly. Therefore, no consistent experimental results have been obtained using this approach [Lam and Kardos [14]; Skartis and Kardos [16]; Chimleewski et al. [17]; Gutowski et al. [18]. Bruschke [19] has shown that the results of the capillary model deviate significantly from the numerical solution for flow across an aligned bed of cylinders.

A more realistic approach to predict the permeability used for Darcy's law is to consider the geometry as an array of aligned cylinders and calculate the drag forces across them. Flow perpendicular to regular cylinder arrays has been analyzed extensively for Newtonian fluids, using a number of different mathematical treatments [20-23]. The effect of random fiber spacing has been addressed by a limited number of authors [24-26].

There are analytical and computational methods to approximate the permeabilities used in Darcy’s law. Sangani and Acrivos [32] and Gebart [33] used analytical expressions for the permeability in the transverse direction of a unidirectional mat. Self-consistent method to predict permeability analytically in aligned fiber bundles has been developed by Berdichevsky and Cai [34] and Cai and Berdichevsky [35]. Some researchers [36, 37] have also performed experimental techniques to determine permeability for anisotropic mats. As an alternative to the analytical representations of the permeability, computational methods have been used using the unit cell. After obtaining an approximation for the permeability, a second level analysis for which the solid circular fibril used for the unit cell is replaced with a porous tow and Brinkman’s theory is used is performed [38]. Several researchers [38, 39] incorporeted Gebart’s geometrical assumptions as related to packing of the fabric preform structure and Darcy’s law in their calculations.

The method of multiple scales has been considered as an alternative to approximate the details of the flow [40, 41]. The method has been used primarily by mathematicians and is referred to by “homogenization” [42]. Some references have referred to this method as the “asymptotic expansion” technique [43] and the “two-space method” [41]. The implementation of the method of multiple scales to porous media flow has been carried out by Keller [41]. Ene [42] assumed the flow through porous media is governed by an elliptic differential equation. Recently, Chang and Kikuchi [43, 40] applied the homogenization technique to the porous media flow They first calculated the permeability from the first homogenization level. Using this value of the permeability, they solved the homogenized forms of the flow equations.

Vafai and his coworkers [44, 45, 46, 47, 48, 49] employed a general model for the momentum equation based on the local volume averaging technique. In their work, they investigated the effects of a solid boundary and inertial forces on flow and heat and mass transfer in porous media. They also studied the effects of variable porosity

3.GOVERNING CONSERVATION EQUATIONS

In order to develop a mathematical development for flow through porous media, it is useful to give a formal description of the microscopic geometry of the porous medium. Following Tucker and Dessenberg [50] who wrote a review chapter on governing equations for flow and heat transfer, this is done by defining phase functions, as follows:

$$X_{\mathrm{f}}(\boldsymbol{x}) = \begin{cases} 1 & \text{if } \boldsymbol{x} \text{ lies in the fluid} \\ 0 & \text{if } \boldsymbol{x} \text{ lies in any other phase} \end{cases} \tag{1}$$

The solid phase function can also be defined in a similar fashion. If the solid media is not stationary, the phase functions will be time-dependent as well. When solving real flow problems, the character of the porous medium is never considered in detail, but the phase functions are necessary functions to obtain the governing conservation and constitutive equations of the flow field. The two phases under consideration will be indicated by subscripts “f” for the fluid phase and “s” for the solid phase. Terms associated with either phase will be denoted with the corresponding subscript, while the terms associated with both phases will have no subscript. For example, ρ is the density of both phases, while ρ_{f} is the density of the fluid and $\boldsymbol{\sigma}_{\mathrm{s}}$ is the total stress tensor in the solid.

3.1. The Local volume averaging technique

The basic concepts in porous media theory are the local volume averaging technique and the use of averaged variables instead of instantaneous ones in field equations. Rather than modeling the microscopic resin flow around each fiber, porous media theory predicts averaged velocity, pressure and stress fields for the flow phase. Any average of any variable can be related to a point in the medium. The point $\boldsymbol{x}$ that has the average value is determined by the location of the averaging volume V (Figure 1).

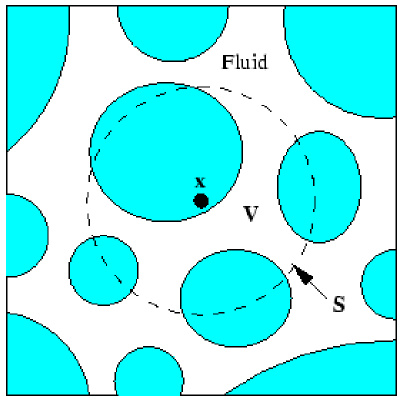

Figure 1 Microscopic View of a Porous Medium, showing the averaging volume V and its surface S associated with a point $\boldsymbol{x}$

This approach is similar to the averaging procedure used in continuum mechanics for homogenous materials. Instead of computing the position and velocity of every atom in a gas (i.e. the Lagrangian viewpoint), one develops the continuum theories to calculate the velocity, pressure and temperature for small regions that contain many atoms (i.e. the Eularian viewpoint). Within the mathematical aspects of continuum mechanics, it is possible to consider the velocity at a point. However, the results of continuum mechanics loose their meaning when the size of the point approaches the mean free path-line (streak-lines) of an atom. Similarly, porous media theory gives the averaged velocity, pressure and stress at each point, but these results lose their meanings when we examine the material on the scale of the individual pores and fibers. When the porous solid mat has a regular structure, like a woven fabric, then a unit cell like the one shown in Figure 2 (a) can be used as the representative volume. If the medium has a random structure then, the representative volume will contain many particles in an averaged sense (Figure 2 (b)). Details and derivations can be found in Referance [51].

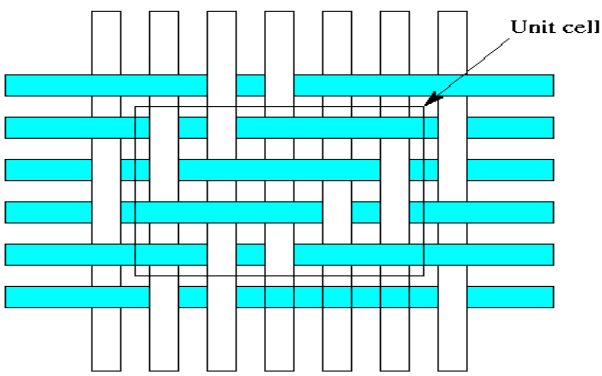

Figure 2 (a) A unit cell (heavy line)
in a 2 : 2 twill weave fabric

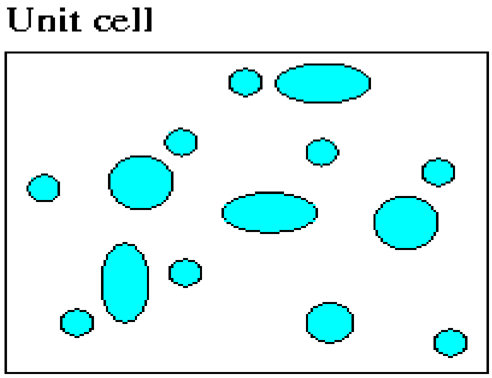

Figure 2 (b) A unit cell (heavy line)

for many particles in an averaged sense

To obtain a relationship between the phase average and the intrinsic phase average, first we define the volume fractions of the phases as

$$\varepsilon_{\mathrm{f}} = \frac{V_{\mathrm{f}}}{V} = \frac{1}{V}\int_V X_{\mathrm{f}}\,\mathrm{d}V \qquad \text{and} \qquad \varepsilon_{\mathrm{s}} = \frac{V_{\mathrm{s}}}{V} = \frac{1}{V}\int_V V_{\mathrm{s}}\,dV \tag{2}$$

where the fluid portion ε_{f} is often called the porosity and the solid fraction is ε_{s}. From these definitions, we see that the phase and intrinsic averages are now related by

$$\varepsilon_{\mathrm{f}}\langle B_{\mathrm{f}}\rangle^{\mathrm{f}} = \langle B_{\mathrm{f}}\rangle \tag{3}$$

The derivations that follow assumes that the solid phase is not moving (i.e. stationary solid phase), there is no mass transfer between the solid and the fluid and the densities of the solid and fluid are constant. The averaging technique is outlined in Figure 3.

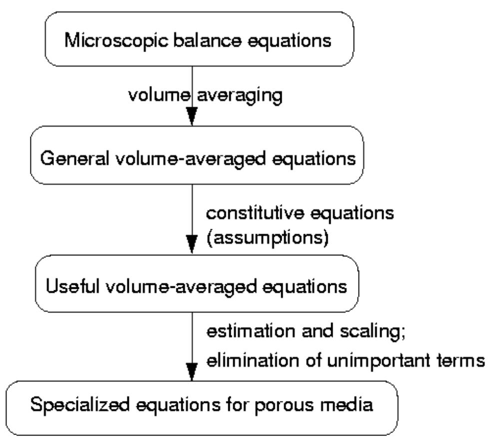

Figure 3 Outline of procedure for deriving balance equations for resin transfer molding

3.2 The conservation of mass: Continuity equation

The continuity equation or conservation of mass equation states that matter is conserved. Since the solid phase does not move, it automatically satisfies the continuity, and we need to develop an equation for the fluid only. Following Alakus [51], the continuity equation is obtained as

$$\nabla \cdot \langle \boldsymbol{v}_{\mathrm{f}} \rangle = 0 \tag{4}$$

where $\boldsymbol{v}_{\mathrm{f}}$ is the velocity vector of the flow field. This is the continuity equation used in calculations of fluid flow through porous medium.

3.3 The Conservation of momentum equations

The equation of motion (conservation of momentum) balances the forces applied to each material particle against the particle's acceleration. For each point in a homogenous material the equation of motion (microscopic conservation of momentum equation) is

$$\frac{\partial(\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v}) = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g} \tag{5}$$

where $\boldsymbol{\sigma}$ is the total fluid stress tensor which is comprised of the pressure and viscous and polymeric stresses, and $\mathbf{g}$ is the gravitational body force per unit mass. Following Alakus [51], the averaged momentum equations are given by

$$\rho_{\mathrm{f}} \frac{\partial \langle \boldsymbol{v}_{\mathrm{f}} \rangle}{\partial t} + \rho_{\mathrm{f}} \nabla \cdot \left[\frac{1}{\varepsilon_{\mathrm{f}}} \langle \boldsymbol{v}_{\mathrm{f}} \rangle \langle \boldsymbol{v}_{\mathrm{f}} \rangle \right] + \rho_{\mathrm{f}} \nabla \cdot \left[\boldsymbol{M}_{\mathrm{D}} : \nabla \langle \boldsymbol{v}_{\mathrm{f}} \rangle \right]$$

$$= -\varepsilon_{\mathrm{f}} \nabla \left\langle P_{\mathrm{f}} \right\rangle^{\mathrm{f}} + \nabla \cdot \langle \boldsymbol{\tau}_{\mathrm{f}} \rangle - \varepsilon_{\mathrm{f}} \left[\frac{\eta}{S} + \frac{\rho_{\mathrm{f}} b \left| \langle \boldsymbol{v}_{\mathrm{f}} \rangle \right|}{\sqrt{S}} \right] \langle \boldsymbol{v}_{\mathrm{f}} \rangle \tag{6}$$

where P_f is the intrinsic phase pressure of the fluid, η is the viscosity, b is a dimensionless material parameter, $\boldsymbol{M}_{\mathrm{D}}$ is a fourth-order "dispersive viscosity" tensor and its value would depend on the average velocity $\langle \boldsymbol{v}_{\mathrm{f}} \rangle$ and other flow parameters such as viscosity, porosity, particle shape and size, etc and the quantity S ($\equiv \varepsilon_{\mathrm{f}}\, l_{\mathrm{o}}^2\, k^*$) is called the permeability. This equation assumes a general non-Newtonian fluid with constant viscosity and density flowing in an isotropic porous medium. It reduces to the isotropic version of the Brinkman equation for slow Newtonian flows. It should be noted that one must either have an expression for the inertial dispersion tensor $\boldsymbol{M}_{\mathrm{D}}$ or else ignore dispersion entirely.

3.4 The Conservation of energy equation

The averaged energy equation and relevant boundary conditions with detailed explanation and notation for flow through porous media have been presented by Tucker and Dessenberger [50] as;

$$\begin{gathered}\left\{ \varepsilon_{\mathrm{f}} \left(\rho C_p \right)_f + \varepsilon_{\mathrm{s}} \left(\rho C_p \right)_s \right\} \frac{\partial \langle T \rangle}{\partial t} + \left(\rho C_p \right)_f \langle \boldsymbol{v}_{\mathrm{f}} \rangle . \nabla \langle T \rangle = \\ \nabla . \left\{ \left(k_e + K_D \right) . \nabla \langle T \rangle \right\} + \varepsilon_{\mathrm{f}} \rho_f H_R \left\langle f_c \left(c_f , T_f \right) \right\rangle^f + \eta \langle \boldsymbol{v}_{\mathrm{f}} \rangle . S^{-1} . \langle \boldsymbol{v}_{\mathrm{f}} \rangle\end{gathered} \tag{7}$$

4. CONSTITUTIVE EQUATIONS FOR TOTAL EXTRA STRESS FIELD: FLOW MODELS

The present model deals with both Newtonian and non-Newtonian (e.i. viscoelastic) fluid flows through porous media. The former include fluids that are characteristics of water and most gases. The latter include fluids with a complicated molecular structure, e.g. polymers, emulsions and rubber, and have a great deal of industrial significance.

4.1. Volume-Averaged Constitutive Equations

The averaged Newtonian stres field is given by [51]

$$\langle \boldsymbol{\tau}_{\mathrm{f}} \rangle = \eta \left(\nabla \langle \boldsymbol{v}_{\mathrm{f}} \rangle + \left(\nabla \langle \boldsymbol{v}_{\mathrm{f}} \rangle \right)^{\mathrm{T}} \right) \tag{8}$$

where τ_f is the total extra stress, $\dot{\boldsymbol{\gamma}} = \nabla \boldsymbol{v}_f + (\nabla \boldsymbol{v}_f)^T$ is the rate of strain tensor, η is the viscosity. The phase average of UCM and Oldroyd-B models [52] for the fluid phase to obtain the averaged UCM and Oldroyd-B constitutive model [51] is obtained as follows: The Elastic-Viscous-Split-Stress (EVSS) formulation of the UCM and Oldroyd-B models are derived based on the change of variables as follows. Define the elastic stress as Let $\boldsymbol{\sigma}$ denote the Cauchy (total) stress tensor, then the extra stress $\boldsymbol{T}_f$ is defined by $\boldsymbol{\sigma}_f = -p\boldsymbol{I} + \boldsymbol{T}_f$ where p is the indeterminate pressure and the extra-stress $\boldsymbol{T}_f$ is given by a differential constitutive model. For EVSS formulation, $\boldsymbol{T}_f = \boldsymbol{T}_1 + \eta\dot{\boldsymbol{\gamma}}$ and

$$\begin{aligned}&\langle \boldsymbol{T}_1\rangle + \lambda_1\left(\left\langle \frac{\partial \boldsymbol{T}_1}{\partial t}\right\rangle + 2\eta(1-\beta)\left\langle \frac{\partial \boldsymbol{D}}{\partial t}\right\rangle\right) + \\ &\lambda_1(2-\varepsilon_f)\Big[(\langle \boldsymbol{v}_f\rangle\cdot\nabla)(\langle \boldsymbol{T}_1\rangle + 2\eta(1-\beta)\langle \boldsymbol{D}\rangle) - (\langle \boldsymbol{T}_1\rangle + 2\eta(1-\beta)\langle \boldsymbol{D}\rangle)\cdot\nabla\langle \boldsymbol{v}_f\rangle - \\ &(\nabla\langle \boldsymbol{v}_f\rangle)^T\cdot(\langle \boldsymbol{T}_1\rangle + 2\eta(1-\beta)\langle \boldsymbol{D}\rangle)\Big] = 0 \qquad (9)\end{aligned}$$

where $\boldsymbol{D} = \frac{1}{2}\dot{\boldsymbol{\gamma}}$ is the deformation rate tensor, β is the ratio of the retardation time to the relaxation time; $\beta=\lambda_2/\lambda_1$. When $\beta=0$ the EVSS form of the UCM model is recovered. By definition $\boldsymbol{T}_{(1)} = \frac{\partial \boldsymbol{T}_f}{\partial t} - \nabla\boldsymbol{v}_f^T\cdot\boldsymbol{T}_f - \boldsymbol{T}_f\cdot\nabla\boldsymbol{v}_f$ is the Upper-Convected derivative of the polymeric stress, $\boldsymbol{v}_f$ is the velocity vector, λ_1 is relaxation time and η is the total viscosity.

Following [51], the averaged Giesekus constitutive model[52] is given by

$$\begin{aligned}&\langle \boldsymbol{T}_1\rangle + \lambda_1\left(\left\langle \frac{\partial \boldsymbol{T}_1}{\partial t}\right\rangle + 2\eta(1-\beta)\left\langle \frac{\partial \boldsymbol{D}}{\partial t}\right\rangle\right) + \\ &\lambda_1(2-\varepsilon_f)\Big[(\langle \boldsymbol{v}_f\rangle\cdot\nabla)(\langle \boldsymbol{T}_1\rangle + 2\eta(1-\beta)\langle \boldsymbol{D}\rangle) - (\langle \boldsymbol{T}_1\rangle + 2\eta(1-\beta)\langle \boldsymbol{D}\rangle)\cdot\nabla\langle \boldsymbol{v}_f\rangle - \\ &(\nabla\langle \boldsymbol{v}_f\rangle)^T\cdot(\langle \boldsymbol{T}_1\rangle + 2\eta(1-\beta)\langle \boldsymbol{D}\rangle) + \\ &\alpha\frac{1}{(1-\kappa)\eta_1}\{\langle \boldsymbol{T}_1\rangle\cdot\langle \boldsymbol{T}_1\rangle\} - 2\alpha(\langle \boldsymbol{T}_1\rangle\cdot\langle \boldsymbol{D}\rangle + \langle \boldsymbol{D}\rangle\cdot\langle \boldsymbol{T}_1\rangle) + 4\alpha(1-\kappa)\eta\langle \boldsymbol{D}\rangle\cdot\langle \boldsymbol{D}\rangle\Big] = 0\end{aligned}$$

(10)

where α is a dimensionless Giesekus model mobility factor , η_2 is the solvent viscosity, η_1 is the polymeric viscosity and $\eta = \eta_1 + \eta_2$ is the total viscosity and $\kappa = \eta_2/\eta$.

The volume averaged PTT model is obtained in a similar fashion [51] as follows:

$$\begin{aligned}&\langle \boldsymbol{T}_1\rangle + \lambda_1\left(\left\langle \frac{\partial \boldsymbol{T}_1}{\partial t}\right\rangle + 2\eta\left\langle \frac{\partial \boldsymbol{D}}{\partial t}\right\rangle\right) + \\ &\lambda_1(2-\varepsilon_f)\Big[(\langle \boldsymbol{v}_f\rangle\cdot\nabla)(\langle \boldsymbol{T}_1\rangle + 2\eta\langle \boldsymbol{D}\rangle) - (\langle \boldsymbol{T}_1\rangle + 2\eta\langle \boldsymbol{D}\rangle)\cdot\nabla\langle \boldsymbol{v}_f\rangle - \\ &(\nabla\langle \boldsymbol{v}_f\rangle)^T\cdot(\langle \boldsymbol{T}_1\rangle + 2\eta\langle \boldsymbol{D}\rangle) + \frac{\varepsilon_m}{\eta_1}\left[\mathrm{trace}(\langle \boldsymbol{T}_1\rangle)\right]\cdot\langle \boldsymbol{T}_1\rangle\Big] = 0 \qquad (11)\end{aligned}$$

where, ε_m is a model parameter.

4.2 Boundary Conditions

Let Ω be the flow domain and $\partial\Omega$ be its boundary. On $\partial\Omega$, we identify several partial boundaries as follows:

(1) $\partial\Omega_T$, on which one imposes essential boundary conditions for the stress tensor at the inlet

$$\boldsymbol{T}_f = \boldsymbol{T}_{fINLET} \quad \text{on} \quad \partial\Omega_T \tag{12}$$

(2) $\partial\Omega_V$, with essential boundary conditions for the velocity fields

$$\langle \boldsymbol{v}_f \rangle = \boldsymbol{U}_{in} \quad \text{on} \quad \partial\Omega_{Vin} \tag{13}$$

$$\langle \boldsymbol{v}_f \rangle = \boldsymbol{0} \quad \text{on} \quad \partial\Omega_{Vwall} \tag{14}$$

(3) $\partial\Omega_N$, with homogeneous natural boundary conditions

$$\langle \boldsymbol{\sigma} \rangle \cdot \boldsymbol{n} = 0 \quad \text{on} \quad \partial\Omega_N \tag{15}$$

where $\boldsymbol{U}_{in}$ is the prescribed velocity field and its boundary is $\partial\Omega_{Vin}$. $\partial\Omega_T$ is the boundary on which the extra stress tensor is imposed as a boundary condition for the stress field. Boundary conditions used for the calculations are shown in Figure 4.

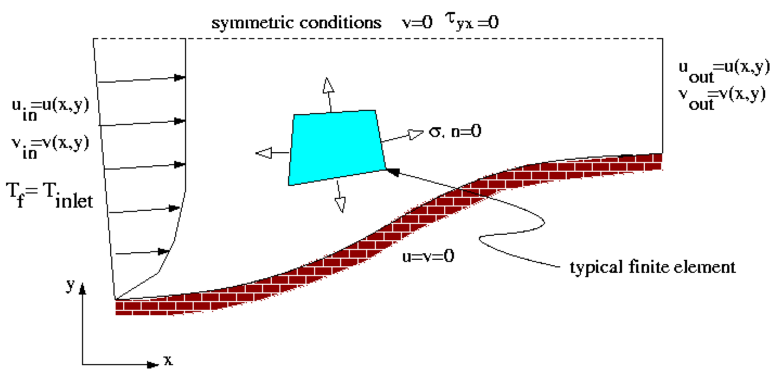

Figure 4. Typical boundary conditions used for calculations of viscoelastic fluid flows

5. CONCLUSIONS

In this research, a set of balance equations specific to non-ısothermal viscoelastic flow through porous media has been extracted from the general equations, derived using the local averaging technique. The advantages of this approach are that notions such as average pressure, average velocity and average temperature can be precisely defined and the governing equations are derived from the accepted principles of conservation laws. Mass and momentum equations have been obtained for both quasi-linear and nonlinear viscoelastic fluids of constant parameters and density in a stationary porous medium. A general energy balance has also been given and the resulting energy equation is based on the assumption of local thermal equilibrium. Using this model, the average temperature of both the fluid and solid is determined from the solution of a single energy equation.

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