# DISCUSSION OF SCHRÖDINGER WAVE EQUATION IN THE MAXWELL EQUATION SYSTEM

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ABSTRACT. — Light quantum is known to exist in the general structure of light which is concidered in the electromagnetic spectrum.

It appears paradoxial when lihgt is considered to propagate as waves and at the same time carry discreat quanta.

In this article Maxvell equations are treated in the context of Helmholtz theorem and it is shown that the solution of this system is the Schrödinger wave equation.

**Thus a new demention to the paradoxial situation has been added.**

## INTRODUCTION

It is well known that the light propagates in the form of Electromagnetic wave. In this context, maxwell equations are often used in the treatment of the propagation of optical wave (Bateman, 1955).

On the other hand, the light has been the subject of quantum mechanics due to the presence of light quantum in its structure. The Importance of Schrodinger wave equation comes from the fact that it explains one aspect of the nature of light. Sommerfeld, starting with the wave - Optic differential equation derived Schrodinger wave equation (Sommerfeld, 1928).

In this article, discussion of Schrodinger wave equation in the context of the treatment of maxwell equation system in the light of Helmholtz theorem is attempted. Thus a new demension has been added to the dualty of wave - descreat mass paradox.

## THEORY

### Maxwell equations in the context of Helmholtz theorem

If an F vector field complies with all the general mathematical conditions, this vector field may be considered as the sum of two vectors equation (1).

$$\vec{F} = -\Delta\,\varphi + \Delta \times \vec{A} \qquad \text{(1)}$$

In this equation φ is obtained by the differentiation of the scalar potential function and it is an irrotational vector. *(A)* is a potential vector an it is solenoidal.

**In the context of this theorem the displacement - curreint density vector $\frac{\partial \vec{D}}{\partial t}$ may be theoretically wretten in a general form as in equation (2).**

$$\frac{\partial \vec{D}}{\partial t} = -\Delta\,\varphi + \Delta \times \vec{H} \qquad \text{(2)}$$

It is also possible to assume a medium where φ is hot zero.

If we now arrange Maxwell's equations in accordance with the above conditions and relations we get equation system (3).

$$\frac{\partial \vec{D}}{\partial t} = -\Delta \varphi + \Delta \times \vec{H} \quad \Delta \cdot \vec{D} = \rho$$
$$\frac{\partial \vec{B}}{\partial t} = -\Delta \times \vec{E} \qquad \Delta \cdot \vec{B} = 0 \qquad \ldots\ldots (3)$$

**(The units used here are M.K.S. system).**

**Here:**

**φ : Scalar potential function**

**$\vec{H}$ : Magnetic field intensity vector**

**$\vec{B}$ : Magnetic field induction vector**

**$\vec{E}$ : Electrical field intensity vector**

**$\vec{D}$ : Electrical displacement vector**

**ρ : Is defined by the relation $\Delta \cdot \vec{D} = \rho$**

The differential equations of the scalar function (φ):

The solutions of equation system (3) are the differential equations of the scalar function (φ).

If we take the divergence of both sides of the first equation. In equations system (3) we get equation (4).

$$-\Delta \cdot \Delta \rho = \frac{\partial \rho}{\partial t} \quad \ldots\ldots (4)$$

**in an explicit form this is**

$$-\Delta \cdot \left( \frac{\partial \rho}{\partial x} \vec{i} + \frac{\partial \varphi}{\partial y} \vec{j} + \frac{\partial \varphi}{\partial z} \vec{k} \right) = \frac{\partial \rho}{\partial t}$$

**on the other hand $\frac{\partial \rho}{\partial t}$ is equal to:**

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial \rho}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial \rho}{\partial z} \frac{\partial z}{\partial t}$$

**Therefore equation (5) can be written**

$$-\Delta \cdot \left( \frac{\partial \varphi}{\partial x} \vec{i} + \frac{\partial \varphi}{\partial y} \vec{j} + \frac{\partial \varphi}{\partial z} \vec{k} \right) = \frac{\partial \rho}{\partial x} V_x + \frac{\partial \rho}{\partial y} V_y + \frac{\partial \rho}{\partial z} V_z \ \ldots (5)$$

**as**

$$r^2 = V^2 = V_x^2 + V_y^2 + V_z^2 \quad \ldots\ldots (6)$$

and $V$ velocity may be assumed canstant the variation of $V_x$ with respect to x, $V_y$ with respect to $y$, and $V_z$ with respect to $z$ is zero.

Under these conditions equation (5) may be written as

$$-\Delta \cdot \left(\frac{\partial \varphi}{\partial x} \vec{i} + \frac{\partial \varphi}{\partial y} \vec{j} + \frac{\partial \varphi}{\partial z} \vec{k}\right) = \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z)$$

we may rearrange this as follows:

$$-\Delta \left(\frac{\partial \varphi}{\partial x} \vec{i} + \frac{\partial \varphi}{\partial y} \vec{j} + \frac{\partial \varphi}{\partial z} \vec{k}\right) = \Delta \cdot (\rho V_x \vec{i} + \rho V_y \vec{j} + \rho V_z \vec{k})$$

One solution of this equation is equation (7).

$$-\left(\frac{\partial \varphi}{\partial x} \vec{i} + \frac{\partial \varphi}{\partial y} \vec{j} + \frac{\partial \varphi}{\partial z} \vec{k}\right) = \rho V_x \vec{i} + \rho V_y \vec{j} + \rho V_z \vec{k} \qquad (7)$$

multiplying both sides of equation (7) by velocity vector in scalar form we get

$$-\left(\frac{\partial \varphi}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial \varphi}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial \varphi}{\partial z} \frac{\partial z}{\partial t}\right) = \rho (V_x^2 + V_y^2 + V_z^2)$$

from this relation equation (8) may be derived.

$$-\frac{\partial \varphi}{\partial t} = \rho V^2 \qquad (8)$$

from equation (4) and (8) we can derive the wave equation (9).

$$\Delta^2 \varphi = \frac{1}{V^2} \frac{\partial^2 \varphi}{\partial t^2} \qquad (9)$$

On the other hand, from equation (7) and (8) Hamiltonian equation (10) may be derived

$$\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 = \frac{1}{V^2}\left(\frac{\partial \varphi}{\partial t}\right)^2 \qquad (10)$$

(Bateman, 1955).

Thus, from the discussion of maxwell's equations in the context of Helmholtz theorem, the wave equation and the important equation of geometrical optics and wave mechanics namely the Hamiltonian equation were derived for the function

$$\varphi = \varphi (x, y, z, t)$$

**Derivation of the Schrödinger wave equation from φ function**

Up to this point we derived the solutions of maxwell's equations purely an theoretical basis and in the context of Helmholtz theorem.

Thus we have shown that φ potential function has a solution giving the following wave function

$$\Delta^2 \varphi = \frac{1}{V^2} \frac{\partial^2 \varphi}{\partial t^2}$$

and that this wave propagates with a phase velocity ($V$) it is also shown that Hamiltonian equation.

$$\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 = \frac{1}{V^2}\left(\frac{\partial \varphi}{\partial t}\right)^2$$

is also a solution of the φ function which is the principle function of physical optics and wave mechanics.

Now we will attempt to ellucedate the physical implications in the wave mechanics of our findings.

For a monochromatic light lets assume that (φ) varies in accordance with equation (11)

$$\varphi = \psi\ (x, y, z)\ e^{i\omega t} \quad \ldots\ldots \quad (11)$$

From equation (10) and (11) we can derive equation (12)

$$-\left[\frac{1}{k_o{}^2\psi^2}\left(\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2\right)\right]=n^2 \quad \ldots\ldots \quad (12)$$

**Substituting relation (13)**

$$\frac{1}{i\,k_o\,\psi}\quad\frac{\partial\psi}{\partial q}=\frac{\partial S}{\partial q} \quad \ldots\ldots \quad (13)$$

**in the equations we get**

$$\psi = A\,e^{i\,k_o\,S} \quad \ldots\ldots \quad (14)$$

$$\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2=n^2 \quad \ldots\ldots \quad (15)$$

Here (S) is action function or Hamiltonian characteristic function. Difining the relation

**where** $k = n\,k_o$

$\frac{\omega}{V} = k$ **the wave number, ($k_o$) is the volue of k in vacuum (n) refrective index with respect** to vacuum.

We also know that relation (16) exists.

$$\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2=2\,m\,(E\text{-}V) \quad \ldots\ldots \quad (16)$$

(A. Sommerfeld, Wave - Mechanics, New - York 1928 p. 3)

Here *m* is the point mass, E energy constant, V potential energy. All is the function of x, y, z coordinats.

From equations (9), (11), (15) and (16) relation (17) can be derived.

$$\Delta^2\,\psi + 2\,m\,(E\text{-}V)\,k_o^2\,\psi = 0 \quad \ldots\ldots \quad (17)$$

**here $k_o$ is a universal value and can take the value of**

$$k_o = \frac{2\,\pi}{h} \quad \ldots\ldots \quad (18)$$

**where *h* is plank's canstant (Sommerfeld, 1928, p. 5).**

If we substitude (18) in (17) we get Schrödineer's wave equation for micro mechanics for single point mass. Equation (19)

$$\Delta^2 \psi + 2\, m\, (E\text{-}V) \left(\frac{2\,\pi}{h}\right)^2 \psi = 0 \quad \ldots\ldots\ldots\ldots (19)$$

This is the fundemental equation of wave mechanics. Here ( y) is the wave function.

If we assume that external forces are nanexistant than (V) may be assumed zero. Under these conditions Schrödinger wave equation may be expressed by equation (20).

$$\Delta^2 \psi + m\, E\, \frac{8\,\pi^2}{h^2}\, \psi = 0 \quad \ldots\ldots\ldots\ldots (20)$$

**As**

$$m\, E\, \frac{8\,\pi^2}{h^2} = k^2$$

equation (20) may be written in the form of equation (21)

$$\Delta^2 \psi + k^2\, \psi = 0 \quad \ldots\ldots\ldots\ldots (21)$$

If this function is integrated for plane wave conditions, for positive x direction we obtain relation (22)

$$\psi = A\, e^{ik\,x} \quad \ldots\ldots\ldots\ldots (22)$$

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