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THE DISCUSSION OF SCHRODINGER WAVE EQUATION ON THE CONCEPT OF POINT MASS PAIRS

Sırrı KAVLAKOĞLU *

ABSTRACT— As it is known, the light emission has both of wave and quantum nature. Because of the latter, it can be handled as a point mass in the case of necessity. The wave properties of the light are investigated by Maxwell wave equations and the point mass properties by Schrödinger wave equation. The purpose of the paper is to show that these two properties of the light can be treated by a single equation system. To realize this aim, "Extended Maxwell Equation System" is proposed and it is shown that, this system, under acceptable limitations, comprises both electromagnetic wave equations and the scalar wave equation from which Schrödinger wave equation can be derived. Thus it is revealed that the symetry which is present between electric field vector and magnetic field vector in the "Extended Maxwell Equation System", also exists between wave front surface normals of y and φ of two scalar Schrödinger wave equation. Since, the coefficients of Schrödinger wave equation pair can be different, it is possible to assume the existence of two different mass points, associated to these two wave front surface which arc perpendicular to each other.

INTRODUCTION

Maxwell equation system provides most effective means to investigate the electromagnetic fields (Straton, 1941). The investigation of the electromagnetic wave properties of the light could be done with these equations (Bateman, 1955).

But, the direct investigation of the "light-quantum" properties of the light cannot be done with Maxwell equation systems. Only, after introducing the concepts of singular points, singular curves, the investigation of the electron model of the light is done with Maxwell equation systems (Bateman, 1955).

As it is known, "pointmass-wave" properties of the light are investigated by using Schrödinger "wave-equation" (Sommerfeld. 1928).

Events of the light comprise of "pointmass-wave" events as well as electromagnetic wave events. Therefore, it is thought to be interesting to investigate all the events of light within the framework of one equation system. Maxwell equation system is considered a good approach to realise this aim.

As the first step, it has been shown that Maxwell equation system's can be extented by using Helmholtz theorem (Phillips, 1933). Secondly, it has been shown that scalar wave function of the extension term, satisfies Schrödinger wave equation. Since the coefficients of Schrödinger equation pair can have different values, there must be two point mass, differing from each other, connected to two orthogonal surfaces. Therefore, in any light emission event, point mass pair concept has to be a concept which should be considered seriously.

Thus, it has become possible to investigate an electromagnetic wave event together with the associated pointmass-wave event by using proposed extented Maxwell equation system. In this way , it has been proved trial Maxwell symetry between electric field intensity vector and magnetic field intensity vector exist, also, exists between the wave surface normals of y and φ of the scalar wave equations of the system.

Therefore a concept of seperate pointmass related to the two orthogonal surface, y and φ may be accepted.

THEORY

EXTENDED MAXWELL EQUATION SYSTEM

Maxwell equation system (Bateman, 1955) is:

$$\nabla X E = -\frac{1}{C}\frac{\partial H}{\partial t} \qquad \nabla . H = 0 \tag{1}$$

$$\nabla X H = \frac{1}{C}\frac{\partial E}{\partial t} \qquad \nabla . E = 0$$

E : vector of electric field intensity
H : vector of magnetic field intensity
C : velocity of light
t : time

E and H are assumed as

$$E = E_0 (x, y, z) e^{i \omega t}$$
$$H = H_0 (x, y, z) e^{i \omega t} \tag{2}$$

Here, (ω) is angular velocity, (x,y,z) are cartesion coordinates. According to Hclmoltz theorem, a vector F can be ex pressed as the sum of one selonoidal and one irrotalional vectors as:

$$F = -\nabla\Psi + \nabla X A \tag{3}$$

Scalar (ψ) and vectorial (A) functions arc the functions of time as.

$$\Psi = \Psi_0 (x, y, z) e^{i \omega t} \qquad A = A_0 (x, y, z) e^{i\omega t}$$

Mathematically, according to the Helmoltz theorem, the following equation of

$$\frac{1}{C}\frac{\partial E}{\partial t} = \nabla X H$$

can be written as

$$\frac{1}{C}\frac{\partial E}{\partial t} = -\nabla\psi + \nabla X H \tag{4}$$

or as

$$\frac{1}{C}\frac{\partial}{\partial t}(E - i\frac{C}{\omega}\nabla\psi) = \nabla X H \tag{5}$$

Let's assume

$$S = E - i\frac{C}{\omega}\nabla\Psi$$

Under this situation, a modified, mathematically extented for of Maxwell equation system (1) is defined as

$$\nabla X S = -\frac{1}{C}\frac{\partial H}{\partial t} \qquad \nabla . H = 0$$

$$\nabla X H = \frac{1}{C}\frac{\partial S}{\partial t} \qquad \nabla . S = 0 \qquad (6)$$

In the paper, the Equation system (6) is called "Extented Maxwell Equation System".

PHYSICAL MEANING MAXWELL EQUATION SYSTEM

The following wave equations, can be written easily, from equations (6), as

$$\nabla^2 S - \frac{1}{C^2}\frac{\partial^2 S}{\partial t^2} = 0$$

$$\nabla^2 H - \frac{1}{C^2}\frac{\partial^2 H}{\partial t^2} = 0 \qquad (7)$$

From them

$$\nabla^2 S_0 + \frac{\omega^2}{C^2} S_0 = 0 \qquad (8)$$

$$\nabla^2 H_0 + \frac{\omega^2}{C^2} H_0 = 0 \qquad (9)$$

differential equations can be written.

Since

$$S_0 = E_0 - i\frac{C}{\omega}\nabla\Psi_0$$

(E_0, Ψ_0 are reel), and substituting it in (8), it is got

$$\nabla^2 (E_0 - i\frac{C}{\omega}\nabla\psi_0) + \frac{\omega^2}{C^2} \left[\nabla^2 (\nabla\psi_0) + \frac{\omega^2}{C^2}\nabla\psi_0 \right] = 0$$

By separating reel and imaginary terms in this equation the following differential equation is got.

$$\nabla^2 E_0 + \frac{\omega^2}{C^2} E_0 - i\frac{C}{\omega}\left[\nabla^2 (\nabla\psi_0) + \frac{\omega^2}{C^2}\nabla\psi_0\right] = 0 \qquad (10)$$

To find the solution, these are written as

$$\nabla^2 E_0 + \frac{\omega^2}{C^2} E_0 = 0$$

$$\nabla^2 (\nabla\psi_0) + \frac{\omega^2}{C^2}\nabla\Psi_0 = 0$$

are written.

Therefore the solution of the equation system (6), is reduced to the solution of the following three differential equations.

$$\nabla^2 E + \frac{\omega^2}{C^2} E = 0 \tag{11}$$

$$\nabla^2 H + \frac{\omega^2}{C^2} = 0 \tag{12}$$

$$\nabla^2 (\nabla\psi) + \frac{\omega^2}{C^2} \nabla\psi = 0 \tag{13}$$

Equation (13) is the part of the "Extented Maxwell Equation Eystem".

Because of (11) and (12) equations, (1) and (6) equations systems have all the properties of the electromagnetic wave propagation. Additionally, (6) equation system contains a scalar (ψ) function.

To reveal the properties of (ψ), (13) differential equation is written as:

$$\nabla\left(\nabla^2 \psi + \frac{\omega^2}{C^2} \psi \right) = 0$$

From this,

$$\nabla^2 \psi + \frac{\omega^2}{C^2} \psi = l$$

(l) is an integration constant.

The latter equation can also be written as:

$$\nabla^2 \psi + \left(\frac{\omega^2}{C^2} - \frac{l}{\psi} \right) \psi = 0 \tag{14}$$

Let's assume $l \neq 0$ and,

$$\frac{\omega^2}{C^2} - \frac{l}{\psi} = \frac{\omega^2}{a^2} \tag{15}$$

Under this situation the equation (14) will be transformed into the form of,

$$\nabla^2 \psi + \frac{\omega^2}{a^2} \psi = 0 \tag{16}$$

"a" is the phase velocity and because of (15), it is the function of position,

Let's put $\frac{\omega}{a} = k, \frac{C}{a} = n$ and $k = k_0 n$

Here, (k) is wave number, (n) is the refractive index of the space.

Then, the equation (16) becomes as,

$$\nabla^2 \psi + n^2 k_0^2 \psi = 0 \qquad (17)$$

To investigate the properties (n^2), let's assume

$$\psi = B\, e^{ik_0 T}$$

where (B) and (T) are the functions of (x,y,z) cartesian coordinates. In this case

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \Delta T_1$$

This is Hamilton differential equation (Sommerfeld, 1928).

Here $\Delta T_1 = n^2$

On the other hand, ΔT_1 is

$$\Delta T_1 = 2m \,[\, e - U(x,y,z)]$$

Here, m, point mass; e, energy constant and U is potential energy.

Then differential equation becomes,

$$\nabla^2 \psi + 2m (e-U)\, k_0^2 \psi = 0$$

or

$$\nabla^2 \psi + 2m (e-U) \left(\frac{2\pi}{h}\right)^2 \psi = 0$$

where (h) is Planck's constant (Sommerfeld, 1928). This is the well-known Schrödinger's wave equation.

As, it has been shown, the (ψ) function in the equation (6) justifies Schrödinger wave equation.

Therefore, it becomes possible to investigate all the light events comprising of wave, pointmass-wave characteristics of the light, by using "Extended Maxwell Equation System".

On the other hand, if the similar operations are done in the $\frac{1}{C}\frac{\partial H}{\partial t}$ field, one wave front (ψ) function orthogonal to wave front function (ψ) will be specified. Consequently, it will be possible to infer the presence of two different mass points associated with these functions.

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