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Generalized Dynamic Codebook Design for Non-Coherent Short-Packet Communications: Radius Control and Placement of Complex-Conjugate Reciprocal Zero Pairs

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Abstract – Modulation on conjugate-reciprocal zeros (MOCZ) is a non-coherent communication scheme designed for the reliable transmission of short data packets over unknown wireless multipath channels. Unlike traditional methods that require channel estimation or training sequences, MOCZ encodes information onto the zeros of the transmitted signal's z-transform, enabling the receiver to decode data without prior channel knowledge. To support multiple users, the multi-user MOCZ (MU-MOCZ) scheme was introduced, employing time-division multiplexing (TDM). However, the use of a fixed codebook for all users in MU-MOCZ can degrade performance due to variations in channel zero distributions. This paper extends previous work by introducing a dynamic codebook design for MU-MOCZ. The design incorporates a rotation parameter into the angular positions of zero pairs, allowing the transmitter to adjust the codebook to each user's unique channel conditions and optimize performance. Simulation results demonstrate significant improvements in bit error rate (BER), throughput, and packet delivery ratio (PDR) compared to the fixed codebook MU-MOCZ scheme.

Keywords *– MOCZ, dynamic codebook, non-coherent, short packet communication, z-transform*

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I. Introduction

Modern wireless communication systems face significant challenges in ensuring efficient and reliable data transmission, particularly in non-coherent communication scenarios where the receiver operates without prior knowledge of channel state information (CSI). Non-coherent communication methods are gaining attention for their ability to reduce transceiver complexity by eliminating the requirement for channel estimation [1], [2], [3]. This makes them highly suitable for applications demanding low latency and high reliability, such as Internet of Things (IoT) networks, machine-type communications (MTCs), and ultra-reliable low-latency communication (URLLC) systems in 5G and beyond [4]. Furthermore, non-coherent schemes are inherently robust against channel dynamics, making them ideal for rapidly changing communication environments. Their simplified design also leads to lower power consumption, which is critical for energy-constrained devices in IoT applications. Furthermore, removing the dependency on CSI, these schemes enable faster data processing, ensuring efficient and consistent performance in mission-critical systems.

One key aspect of non-coherent communication is the design of the codebook [5], which plays a significant role in determining system performance metrics such as bit error rate (BER), spectral efficiency (SE), and power consumption. The effectiveness of the codebook directly affects how well the system can detect and decode information in the absence of CSI at the receiver. Unlike coherent systems that use CSI to optimize transmission methods, non-coherent systems depend on well-designed codebooks to ensure reliable data detection under varying channel conditions. These systems rely on the codebook to facilitate detection even when the channel is unknown or changes over time [6]. However, static codebook designs may not perform optimally in dynamic environments [7], as they lack the ability to adjust to changes in channel behavior and interference. As a result, system performance may suffer, causing higher error rates and inefficient use of resources.

To address these limitations, dynamic codebook design enables adaptation to real-time channel variations, improving the overall performance of non-coherent communication systems. Modifying the codebook structure based on current channel conditions allows these systems to maintain optimal performance even under changing environments. This adaptability is particularly crucial for ensuring reliable communication in dynamic conditions, where static codebooks may struggle to provide consistent performance. The study in [8] highlights the importance of designing individual constellations and their corresponding bit-mapping schemes for non-coherent communication systems, which helps optimize data detection and transmission. Furthermore, [9] introduces suboptimal constellations specifically designed for multiuser environments, which rely on energy detection techniques to address the challenges of detecting signals in the presence of noise and interference. The study in [6] introduces a structured Grassmannian constellation called Grass-Lattice for noncoherent communications over SIMO Rayleigh block-fading channels, utilizing a measure-preserving mapping from the unit hypercube to the Grassmannian of lines.

In recent work, the authors in [10], [11] introduced a non-coherent communication scheme known as modulation on conjugate-reciprocal zeros (MOCZ), where data is mapped onto the zeros of the z-transform of the transmitted sequences. This new communication scheme eliminates the need for CSI, simplifying the receiver structure. The study in [12] proposed a time-domain user multiplexing method using MOCZ by incorporating a guard period between downlink users to avoid interference. On the other hand, the work in [13] introduced a new frame design technique called z-domain user multiplexing (ZDM-MOCZ), which allows for multiplexing in the z-domain instead of sequentially in the time domain, providing a more efficient use of resources. While these techniques represent significant advancements in non-coherent communication, a common challenge in these studies is the reliance on fixed codes or static codebooks for user transmission. This lack of flexibility in adapting to changing channel conditions limits the system's ability to achieve optimal performance in varying scenarios.

This paper builds upon our previous work presented in [20], where we introduced a novel approach for constellation design in the z-domain by dynamically controlling the radius of circles on the z-plane. In that initial study, we focused on modifying the radius to optimize key performance metrics, such as BER and packet delivery ratio (PDR).

Although the proposed formulation for the zero-pair radius demonstrated significant performance improvements, full adaptation of the positioning of modulation zeros to changing channel conditions remained unfulfilled.

In this paper, we extend the previous work by repositioning the zero-pairs with the addition of an extra rotation parameter.

The main contributions of this paper are as follows:

- We build upon our previous work, where the radius parameter α is dynamically scaled, by introducing a rotation parameter β to modify the angular position of the zero-pair, in addition to the radius scaling.
- We demonstrate how this two-parameter control enables flexible positioning of modulation zeros on the z-plane to mitigate the effect of channel zeros.
- We analyze the performance of the proposed scheme within a non-coherent communication framework, highlighting its adaptability to varying channel conditions and robustness without the need for CSI.
- We present a theoretical analysis and simulations to evaluate the impact of the proposed design on key performance metrics, including BER, PDR, and throughput.

The remainder of this paper is structured as follows: Section II presents the system model and defines the problem statement. In Section III, we introduce the generalized dynamic codebook design framework and discuss its implementation. Section IV provides simulation results to illustrate the performance improvements achieved with the dynamic codebook. Finally, Section V concludes the paper and outlines potential directions for future research.

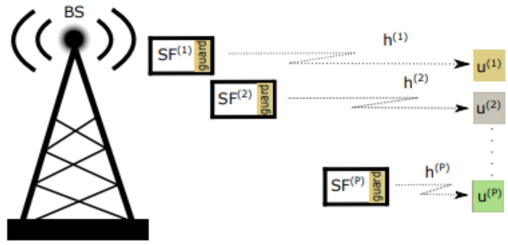

Fig. 1. Time-domain multiplexing [13]

II. SYSTEM MODEL

In this study, we consider a multiuser downlink communication system where a base station (BS) serves the number of P user equipments (UEs), as depicted in Fig. 1. The p-th UE is denoted by $u^{(p)}$, where $p = 1, 2, \ldots, P$. The BS encodes each user's message of length $N-1$ into a sequence of length N, represented as $s = [s_0, s_1, \ldots, s_{N-1}]^T$, where $s_n \in C$ for each user. Each user is served independently in separate time slots, following a time-division multiplexing (TDM) scheme [12]. To prevent interference between the transmissions of different UEs, each transmission is preceded by a guard interval, which is designed to be greater than or equal to the maximum delay spread of the channel.

In the discrete-time domain, the received signal at the p-th UE, denoted as $y^{(p)}[n]$, is modeled as the linear convolution of the transmitted signal s[n] with the multipath channel impulse response $h^{(p)}[n]$, along with additive noise $w^{(p)}\}[n]$. This can be expressed as

$$y^{(p)}[n] = \sum_{k=0}^{L-1} h^{(p)}[k]s[n-k] + w^{(p)}[n], \quad (1)$$

where $n = 0, 1, \ldots, N+L-1$, and $h^{(p)}[n]$ is the discrete-time impulse response of the channel for the p-th user, with a maximum delay spread of L. s[n] is the transmitted signal of length N. $w^{(p)}[n]$ is additive white Gaussian noise (AWGN) with zero mean and variance σ^2 .

The received signal consists of $N+L-1$ samples due to the convolution operation. To ensure no inter-symbol interference (ISI) occurs between consecutive transmissions, a guard interval is introduced at the transmitter, which is at least L samples long. This guard interval accommodates the channel's delay spread and prevents overlap between signals of adjacent users in the time-division multiplexing (TDM) scheme. The received signal can also be represented using matrix notation as

$$\boldsymbol{y}^{(p)} = \boldsymbol{H}^{(p)}\boldsymbol{s} + \boldsymbol{w}^{(p)}, \quad (2)$$

where $\mathbf{y}^{(p)} \in C^{(N+L-1)\times 1}$ is the $p-$th user received signal vector. $\mathbf{s} \in C^{N\times 1}$ is the transmitted signal vector. $\mathbf{w}^{(p)} \in C^{(N+L-1)\times 1}$ is the noise vector. $\mathbf{H}^{(p)} \in C^{(N+L-1)\times N}$ is the Toeplitz channel matrix formed by $h^{(p)}[n]$.

$$\boldsymbol{H}_{(N+L-1)\times N} = \begin{bmatrix} h_0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 \\ \vdots & h_1 & \ddots & 0 \\ h_{L-1} & \vdots & \ddots & h_0 \\ 0 & h_{L-1} & \ddots & h_1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & h_{L-1} \end{bmatrix}. \quad (3)$$

The receiver processes $\mathrm{y}^{(\mathrm{p})}$ to recover s, employing the specific demodulation scheme and taking advantage of the structure of the transmitted signal encoded in the z-domain.

The wireless channel is modelled as a Rayleigh frequency-selective channel with L propagation paths. Its impulse response is given by

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \quad (4)$$

where h_l and τ_l represent the complex-valued path gain and time delay of the l-th path, respectively, and $\delta(\cdot)$ is the Dirac delta function. This model captures the effects of multipath propagation, where each path contributes a delayed and scaled version of the transmitted signal.

Assume that in the discrete-time domain, the channel impulse response $\mathrm{h(n)}$ has a length of $\mathrm{N} + 1$ samples. Its z-transform, or transfer function, is expressed as

$$H(z) = \sum_{n=0}^{N} h(n) z^{-n}, \quad (5)$$

This can be further factored as

$$H(z) = h(0) \prod_{k=1}^{N} (1 - c_k z^{-1}). \quad (6)$$

where $c_k$denotes the k-th zero of $\mathrm{H(z)}$.

The zeros c_k represent the roots of the channel transfer function and are critical in determining the channel's spectral characteristics. This formulation connects the time-domain representation of the channel to its z-domain behavior, which is particularly useful in non-coherent communication schemes that encode information directly onto the zeros of the transmitted signal's z-transform.

Convolution in the time domain corresponds to multiplication in the z-domain. Therefore, the z-transform of the received signal $\mathrm{y}^{(\mathrm{p})}[\mathrm{n}]$ is given by

$$\boldsymbol{Y}^{(p)}(z) = \boldsymbol{H}^{(p)}(z)\boldsymbol{S}(z) + \boldsymbol{W}^{(p)}(z). \quad (7)$$

where $\mathbf{Y}^{(\mathrm{p})}(\mathrm{z})$ is the z-transform of the received signal $\mathrm{y}^{(\mathrm{p})}[\mathrm{n}]$. $\mathbf{H}^{(\mathrm{p})}(\mathrm{z})$ is the z-transform of the channel impulse response $\mathrm{h}^{(\mathrm{p})}[\mathrm{n}]$. $\mathbf{S}(\mathrm{z})$ is the z-transform of the transmitted signal $\mathrm{s}[\mathrm{n}]$. $\mathbf{W}^{(\mathrm{p})}(\mathrm{z})$ is the z-transform of the additive noise $\mathrm{w}^{(\mathrm{p})}[\mathrm{n}]$.

This relationship highlights how channel response and noise affect the transmitted signal in the z-domain. From $\mathbf{Y}^{(\mathrm{p})}(\mathrm{z})$, the receiver can decode the transmitted message by detecting these zeros, even without explicit knowledge of the channel.

Unlike traditional coherent communication systems, which rely on channel estimation and equalization at the receiver to address distortions introduced by the channel, non-coherent schemes avoid the need for these processes. Channel estimation and equalization are computationally demanding and contribute to increased latency. To address these challenges, modulation on conjugate-reciprocal zeros (MOCZ) has been introduced as an efficient non-coherent communication method [10].

A. Modulation on Conjugate-Reciprocal Zeros (MOCZ)

Consider a polynomial $\mathrm{Q(z)}$ defined as

$$Q(z) = z^N + a_1 z^{N-1} + \cdots + a_N. \quad (8)$$

where $\mathrm{a_i} \in \mathrm{C}$ for $\mathrm{i} = 1, 2, \ldots, \mathrm{N}$, and N is the degree of the polynomial.

A root of the polynomial $\mathrm{Q(z)}$ is a value $\gamma \in \mathrm{C}$ that makes the polynomial expression equal to 0, such that $\mathrm{Q}(\gamma) = 0$. According to the fundamental theorem of algebra, a polynomial of degree N has exactly N roots [14]. Therefore, any polynomial can be factored as $\prod_{i \le N}(x - a_i)$, where each a_i denotes a root of the polynomial. In non-coherent communication schemes, these roots correspond to the zeros of the transmitted signal and play a crucial role in encoding the information directly onto the z-transform of the signal.

In MOCZ scheme, the p −th user's message at the BS is divided into K subgroups, denoted by $\boldsymbol{m}^{(\boldsymbol{p})} = \left[m_1^{(p)}, m_2^{(p)}, \cdots, m_K^{(p)}\right]$ [10]. Each $m_k^{(p)}$ message is derived from M-ary alphabet $[M]$. Subsequently, the vector $\boldsymbol{s}^{(\boldsymbol{p})} = \left[s_0^{(p)}, s_1^{(p)}, \cdots, s_K^{(p)}\right]^T \in C^{K+1}$ is generated by the inverse z-transform of $S^{(p)}(z)$ which is given by

$$\boldsymbol{S}^{(p)}(z) = \prod_{k=1}^{K} \left(z - \alpha_k^{(p)}\right), \quad p = 1,2, \ldots, P. \quad (9)$$

where $\alpha_k^{(p)} \in C$ is the k −th root of the polynomial $S^{(p)}(z)$, and $\alpha_\mathrm{k}^{(\mathrm{p})}$ is chosen by mapping m_k to an element in the zero-alphabet given by

$$\Xi = \left(\left(\alpha_1, \frac{1}{\overline{\alpha_1}}\right), \left(\alpha_2, \frac{1}{\overline{\alpha_2}}\right), \ldots, \left(\alpha_K, \frac{1}{\overline{\alpha_K}}\right)\right). \quad (10)$$

where the zero-pairs $\left(\alpha_k, \frac{1}{\overline{\alpha_k}}\right)$ for $\mathrm{k} = 1,2, \ldots, \mathrm{K}$ in the zero codebook are conjugate-reciprocal pairs.

In [15], MOCZ is applied to enhance communication security by introducing artificial noise injection, while [16] integrates MOCZ with faster-than-Nyquist (FTN) signalling to achieve higher spectral efficiency. A novel method is presented in [17] for computing the majority vote (MV) function by employing modulation on conjugate-reciprocal zeros (MOCZ). In [18], a noncoherent SIMO framework with MOCZ is proposed to minimize pilot overhead and fading effects, along with a low-complexity Viterbi-like detector.

B. Existing Work and Problem Definition

Multi-user modulation on conjugate-reciprocal zeros (MUMOCZ), introduced in [12], expands the original MOCZ scheme [10] to enable support for multiple users. In the standard MOCZ method, the data for each user is encoded onto the complex conjugate-reciprocal zeros of the z-transform of the transmitted signal, providing reliable communication without requiring channel estimation or equalization. The MUMOCZ framework is designed by facilitating the sharing of the communication channel among multiple users in a time division multiplexing (TDM) manner.

The time-division multiplexing (TDM) technique enables multi-user communication while preserving the non-coherent nature of the system, as it does not require channel estimation. However, using a fixed codebook for all users poses a challenge. Each user experiences distinct multipath environments and channel conditions, leading to variations in the distribution of zeros associated with their respective channels. A fixed codebook lacks the flexibility to accommodate these differences, potentially leading to performance degradation and affecting the system's overall reliability and efficiency.

This study extends our previous work [20], which introduced a dynamic codebook design that reformulates the radius equation through scaling. In this paper, we build upon that work by adding an additional parameter, β, along with the existing parameter α, to reposition the zero-pairs on the z-plane within the codebook. This extension allows the transmitter to better adapt to the unique channel conditions of each downlink user by using a specific codebook for every user.

III. PROPOSED DYNAMIC CODEBOOK DESIGN

In this section, we propose a dynamic codebook design that builds upon and significantly extends the capabilities of our previously introduced z-domain modulation scheme. The key enhancement lies in the modification of the radius equation, now incorporating an additional parameter for rotation. While the earlier approach solely focused on controlling the radius of the modulation zeros through the scaling parameter α, the current design introduces the rotation parameter β, enabling dynamic angular positioning of the modulation zeros on the z-plane. The parameter α controls the magnitude (or radius), while the parameter β determines the angular positioning of the modulation zero-pairs on the z-plane. The two-parameter design (α and β) allows for flexible and effective modifications to meet varying channel conditions:

- In some scenarios, the effect of the channel zeros on detection performance can be minimized by modifying only the scaling parameter α.
- In other cases, modifying only the rotation parameter β is sufficient.
- Under more complex conditions, both parameters (α and β) need to be jointly optimized to achieve the desired performance.

The multipath channel for each p-th user can be represented as $h^{(p)}(n)$, where p denotes the user. The corresponding z-transform of the channel for the p-th user is given as [19].

$$H^{(p)}(z) = \sum_{n=0}^{L^{(p)}-1} h^{(p)}(n) z^{-n}. \qquad (11)$$

where $L^{(p)}$ denotes the maximum discrete delay spread of the channel between the BS and the p-th user. This can be expressed as $L^{(p)} = \left\lfloor \frac{\tau_{max}}{T_s} \right\rfloor$, where $\lfloor \cdot \rfloor$ represents the floor function, τ_{max} is the maximum delay spread, and T_s denotes the sampling period, respectively. The $h^{(p)}(n)$, for $n = 0,1,\ldots,L^{(p)}-1$, contain the channel taps.

The zeros of $H^{(p)}(z)$, denoted by $z_i^{(p)}$, are the points in the z-plane where the channel transfer function equals zero given by

$$H^{(p)}(z_i) = 0 \quad for\ i = 1,2,\cdots,L^{(p)}-1. \qquad (12)$$

The receiver detection involves identifying the zeros transmitted from the received signal, which is impacted by the channel conditions of each user. The dynamic codebook improves performance of the detection by modifying the zero-pairs to align with the channel's characteristics. This adaptation enables the receiver to better distinguish the transmitted zeros from noisy channel zeros, improving detection efficiency, reducing errors, and improving overall communication performance and reliability in varying channel conditions.

In [10], the radius R is designed to optimize the Euclidean distances between zeros and their conjugate-reciprocal counterparts, along with the distances between neighboring zero pairs. The radius R is given by

$$R(K) = \sqrt{1 + sin\left(\frac{\pi}{K}\right)}. \qquad (13)$$

and the k-th root of the polynomial $S^{(p)}(z)$ corresponding to the p-th user is given by

$$\begin{aligned} \alpha_k^{(p)} &= R(K) \cdot e^{j\left(\frac{2\pi(k-1)}{K}\right)} \\ &= \sqrt{1 + sin\left(\frac{\pi}{K}\right)} \cdot e^{j\left(\frac{2\pi(k-1)}{K}\right)}. \end{aligned} \qquad (14)$$

However, using a fixed radius and hence a static codebook for all users with varying channel conditions can lead to performance degradation, as different channel characteristics cause the zeros to be randomly distributed across the z-plane. To address this challenge, the equation in (13) is modified based on the specific channel conditions of each user, aiming to position the transmitted zeros farther away from the randomly distributed channel zeros, thus enhancing detection accuracy. We introduce the α and β parameters into the equation to scale the radius and reposition the zero pair's angle, respectively.

$$R(K) = \sqrt{1 + sin\left(\frac{\pi}{\alpha K}\right)}. \qquad (15)$$

where α ∈ R. This modified radius R effectively adjusts the number of zero pairs on the z-plane, allowing a more flexible arrangement of the transmitted zeros among the channel zeros based on the parameter α. For instance, as shown in Fig. 1, setting α to 2 can be interpreted as doubling the number of zero pairs on the z-plane, which would be evenly distributed around the circles at radii R and $\frac{1}{R}$, according to the modified radius equation.

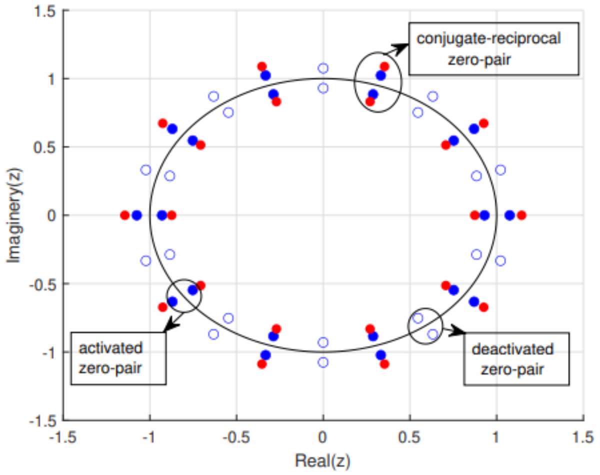

Fig. 2. Red circles indicate the complex zero pairs in the conventional codebook with $\alpha = 1$, whereas the white circles, which are deactivated, and blue circles represent the zero pairs in the proposed codebook with $\alpha = 2$.

We also introduce the rotation parameter β into the root $\alpha_k^{(p)}$ of the polynomial associated with the p-th user. The inclusion of β allows for a controlled rotation in the placement of the roots. The k-th root of the polynomial $S^{(p)}(z)$ corresponding to the $p-th$ user, denoted as $\alpha_k^{(p)}$, is expressed as

$$\alpha_k^{(p)} = R(\alpha, K) \cdot e^{j\left(\beta+\frac{2\pi(k-1)}{K}\right)} = \sqrt{1+\sin\left(\frac{\pi}{\alpha K}\right)} \cdot e^{j\left(\beta+\frac{2\pi(k-1)}{K}\right)}. \quad (16)$$

In this expression, $R(\alpha, K)$ denotes the magnitude or radius of the root, K is the total number of roots, k indexes the individual root, and β introduces the angular rotation applied to all roots. This parameter provides an additional degree of freedom, allowing the system to optimize the placement of roots in the complex plane to meet specific communication requirements. The proposed dynamic codebook converges to the fixed codebook configuration when the parameters $\alpha = 1$ and $\beta = 0$.

A. Assumptions and Limitations

The objective of the proposed dynamic codebook design is to enable adaptation to channel zero disturbances for each user. This approach modifies the placement of modulation zeros based on the specific characteristics of the channel experienced by each user. The method involves adjusting both the radius and angular position of the existing zero-pairs in the codebook.

It is assumed that the transmitter has prior knowledge about the distribution of channel zeros, which can be obtained through feedback from the receiver. The feedback mechanism involves exchanging information about the distribution of channel zeros rather than full CSI. This information is sent from the receiver to the transmitter periodically, with the frequency of feedback depending on the channel dynamics. The receiver provides statistical information on the characteristics of the channel, including the distribution of the channel zeros. This feedback is typically in the form of Channel Quality Indicator (CQI) data, which allows the transmitter to evaluate the channel conditions before transmission. The CQI data play a key role in enabling dynamic adjustments to the modulation zeros, ensuring the system can adapt to varying channel conditions.

It is crucial to maintain enough distance between the modulation zeros and the channel zeros to improve detection reliability. As shown in Fig. 2, increasing the phase separation between zero pairs improves performance by keeping the transmitted zeros at a safe distance from the channel zeros. This strategy ensures better communication performance across different channel conditions.

The packet delivery ratio assumes no frame retransmission, focusing only on data frames transmitted over the wireless channel. Once a frame is transmitted and an acknowledgment is received, there is no need for retransmission. This assumption simplifies the analysis by considering only the successful delivery of frames within a single transmission attempt, and the impact of potential retransmissions or error recovery mechanisms is not included in the scope of the simulations.

The proposed method is most effective in scenarios with moderate channel variability, where feedback about channel zero distributions can be timely and accurate. However, in extremely fast-varying channels, the timeliness of this feedback may become a bottleneck, limiting the effectiveness of codebook updates. In such cases, the delay in reporting channel zero distributions could result in outdated or suboptimal codebook configurations, reducing the potential performance gains in terms of BER, throughput, and PDR.

B. Performance Analysis

The Direct Zero-Testing (DiZeT) technique [10] is employed by each user to detect transmitted zeros by analysing the roots of the polynomial $Y^{(p)}(z)$.

User throughput represents the amount of data successfully received by a user over a specific time interval, and in [13], it is defined as

$$T_{MU\text{-}MOCZ} = \frac{N_{bits}^{(p)}}{N_{sym}\left[P\left(K_p + 1 + L\right)\right]}(1 - BER), \quad (17)$$

where $N_{bits}^{(p)}$ represents the total number of bits transmitted, while N_{sym} denotes the number of symbols transmitted. P refers to the number of downlink users, and K_p, where $p \in \{1, 2, \dots, P\}$, denotes the number of zeros assigned to each p-th user. The term L corresponds to the length of the channel or the channel delay spread. BER stands for the Bit Error Rate, and $T_{MU\text{-}MOCZ}$ represents the throughput.

PDR is defined as the ratio of the number of packets successfully received by the destination node to the number of packets sent by the source node. It serves as a key metric for evaluating network reliability, providing insight into the effectiveness of communication across the network. A high PDR indicates reliable data transmission, while a low PDR may suggest issues such as packet loss, network congestion, or interference. PDR is expressed as [13]

$$PDR = \frac{P_{rec}^{(p)}}{P_{snd}}. \quad (18)$$

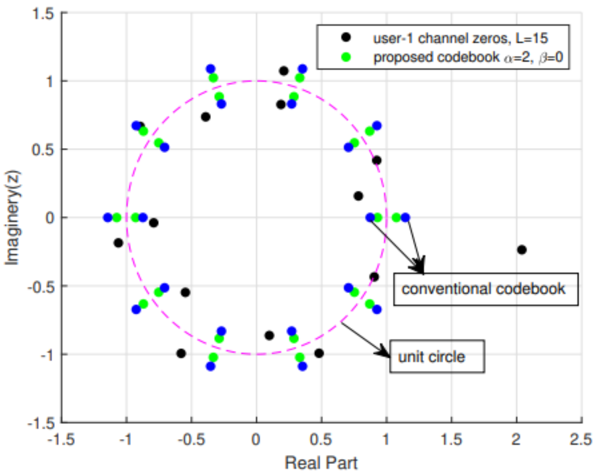

Fig. 3. Black circles represent the complex zeros of H(z) for the first user, blue circles indicate complex zero-pairs from the conventional codebook, and green circles denote the complex zero-pairs when $(\alpha, \beta) = (2,0)$ based on the distribution of the complex zeros of H(z).

where $P_{rec}^{(p)}$ indicates the count of packets that have been successfully received by the p-th user, and P_{snd} refers to the overall number of packets sent from the source.

Adjusting the parameters α and β does not introduce additional computational overhead, as these adjustments do not impact the complexity of modulation or demodulation.

IV. SIMULATIONS AND RESULTS

In this section, we present the simulation setup and results to evaluate the performance of the proposed dynamic codebook design in a multiuser time-division multiplexing (TDM) system. The simulations are designed to reflect practical scenarios where each user experiences unique multipath channel conditions. Specifically, the transmitter serves multiple users in a TDM manner as illustrated in Fig. 1, and the signal propagation for each user experiences unique multipath channels, leading to varying numbers and distributions of channel zeros on the z-plane. A feedback mechanism is assumed to be in place where each user sends channel quality information (CQI) to the transmitter. The CQI includes information about the statistical distribution of the channel zeros on the z-plane. Hence, the transmitter has the knowledge about the distribution of the channel zeros for the related downlink users. Based on the feedback received from each user, the transmitter dynamically adjusts the codebook dedicated to that user. The adjustment involves the parameters α and β.

To illustrate the effectiveness of the proposed dynamic codebook design, we consider two users in the TDM system. Each user is assigned 10 transmitted zeros, and we assume that the channel's z-transform exhibits 14 randomly distributed zeros on the z-plane. The performance of the proposed design is compared to the conventional MU-MOCZ scheme. In the conventional MU-MOCZ scheme, the scaling parameter α is set to 1, and the rotation parameter β is set to 0 (i.e., no additional adjustments are made).

Fig. 3 illustrates the distribution of channel zeros on the z-plane as experienced by the first user. The channel for this user is characterized by L = 15 multipath components, which result in a set of channel zeros being randomly distributed

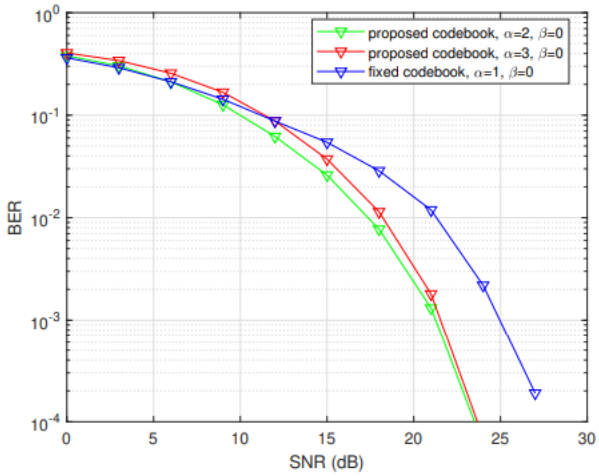

Fig. 4. BER performance comparison between conventional and proposed schemes for the first user.

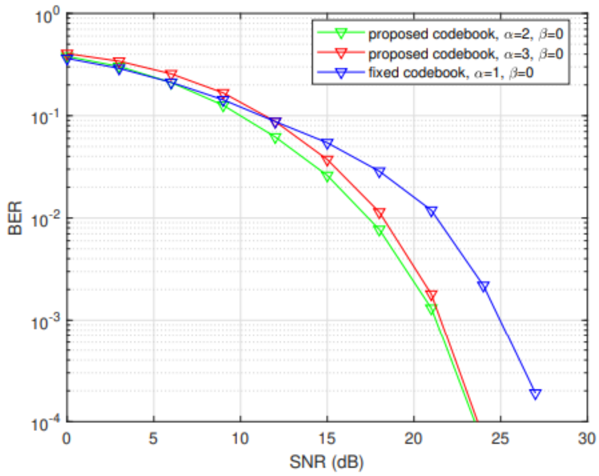

Fig. 5. Throughput performance comparisons between conventional and proposed schemes for the first user.

across the z-plane. These randomly placed zeros can cause distortions in the transmitted signal, especially in noisy environments, significantly impacting the receiver's ability to correctly detect the transmitted data. To mitigate this challenge, the transmitter relies on feedback from the user regarding the distribution of channel zeros. This feedback, assumed to be provided in the form of channel quality indicators (CQI) or similar metrics, enables the transmitter to dynamically place the zero-pairs in the codebook. Specifically, in this simulation, the modification involves setting the scaling parameter to $\alpha = 2$.

This increases the radius of the modulation zeros, ensuring that they are well-placed relative to the channel zeros on the z-plane. The increased separation between modulation zeros and channel zeros reduces the likelihood of detection errors at the receiver, even in noisy conditions. The resulting improvement in BER performance is significant, as shown in Fig. 4. These results demonstrate the effectiveness of the proposed dynamic codebook design in adapting to varying channel conditions and enhancing the reliability of non-coherent communication systems. Fig. 5 illustrates the throughput performance of the user for different values of the α scaling parameter.

Similarly, Fig. 6 shows the distribution of channel zeros experienced by the second user, whose channel is also

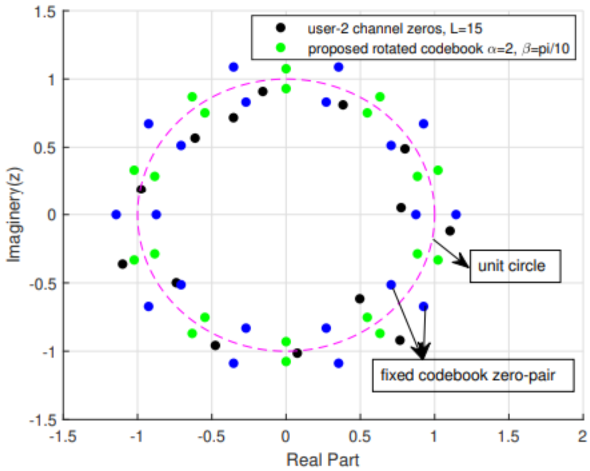

Fig. 6. Black circles represent the complex zeros of H(z) for the second user, blue circles indicate complex zero-pairs from the conventional codebook, and green circles denote the complex zero-pairs when $(\alpha, \beta) = (2, \text{pi}/10)$ based on the distribution of the complex zeros of H(z).

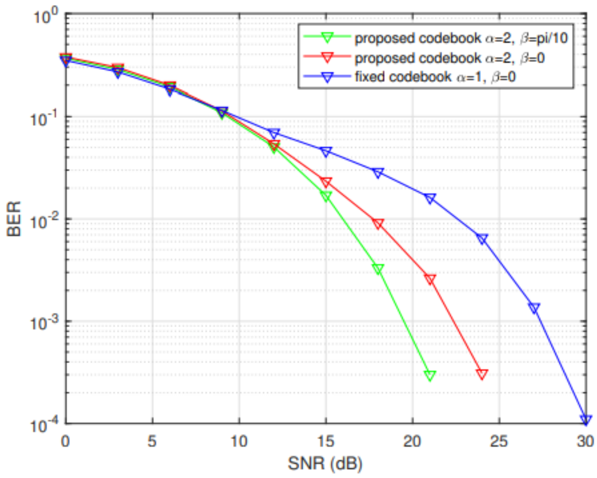

Fig. 7. BER performance comparison between conventional and proposed schemes for the second user.

characterized by L = 15 multipath components, resulting in 14 channel zeros randomly distributed across the z-plane.

Based on the assumption that the second user provides feedback to the transmitter about the distribution of its channel zeros, the transmitter adapts the positioning of the zero-pairs in the codebook. In this case, the transmitter utilizes both the scaling parameter α and the rotation parameter β. The codebook's complex conjugate reciprocal zero-pairs are adjusted around the unit circle, enabling better placement relative to the channel zeros. This combined adjustment of α and β results in improved BER performance as depicted in Fig. 7, compared to the conventional MU-MOCZ scheme which uses a fixed codebook. The improvement highlights the flexibility and effectiveness of the proposed dynamic codebook design in handling diverse channel conditions and enhancing the performance of non-coherent communication systems.

Fig. 8 and Fig. 9 show the packet delivery ratios (PDRs) for user-1 and user-2, respectively. Our simulations clearly demonstrate that the proposed dynamic codebook design offers significant improvements in PDR compared to the conventional MU-MOCZ scheme. Setting the scaling

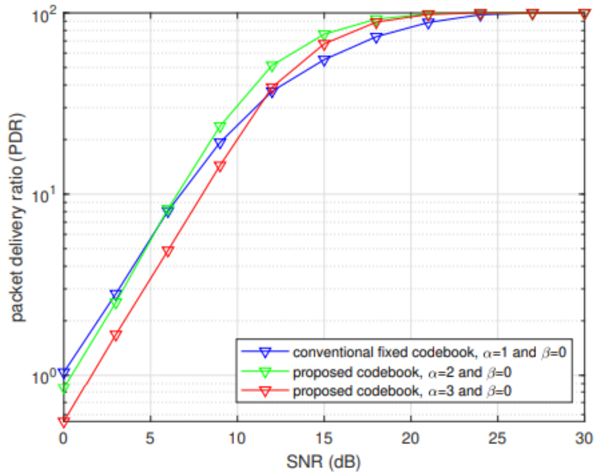

Fig. 8. PDR performance for the first user under different configurations of the scaling parameter α and the rotation parameter β.

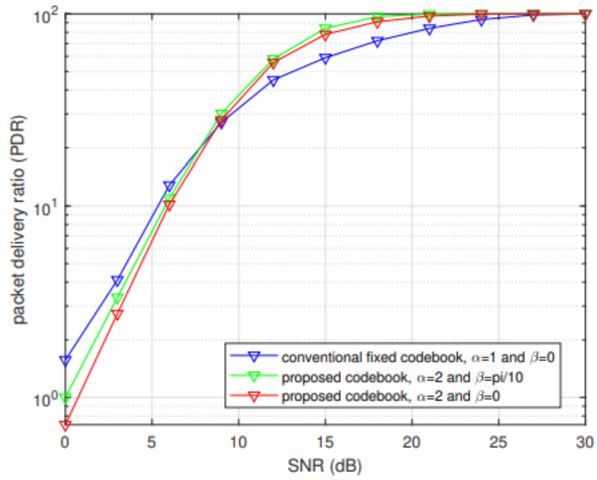

Fig. 9. PDR performance for the second user under different configurations of the scaling parameter α and the rotation parameter β.

parameter α = 2 and the rotation parameter β =0 for user-1 results in an improvement in PDR, while for user-2, setting α = 2 and β = π/10 enhances the performance.

These configurations highlight the flexibility and effectiveness of the dynamic codebook design in adapting to varying channel conditions for different users.

The simulation results confirm that the proposed dynamic codebook design for non-coherent short-packet communication systems improves overall system performance, including BER, throughput, and PDR. Modifying the radius and angular positioning of the conjugate-reciprocal zeros based on channel characteristics, the system effectively reduces the impact of channel zeros, leading to more reliable communication.

V. CONCLUSION

The present work is an extension of our paper presented at the ISAS 2024 Conference [20]. In this extended version, we introduce an additional parameter to control the positioning of zero pairs on the z-domain. This new parameter provides a more flexible and precise method for placing modulation zeros, significantly improving the system's performance under various channel conditions. This development helps mitigate the impact of channel distortions, ensuring more robust communication in changing environments. In the conventional scheme, MU-MOCZ uses a fixed codebook for all users. This means that the modulation zeros are predefined and do not adapt to the specific channel conditions or characteristics of individual users. As a result, the system may not fully optimize for the varying channel conditions experienced by different users, leading to potential performance inefficiencies.

The dynamic codebook design proposed in this paper is developed for the MU-MOCZ scheme, where multiplexing of users occurs in the time domain. Since each codebook is assigned to one user within a specific time frame, the zeros in the z-plane are assigned to that user, allowing flexibility to adjust the radius R and rotate the circles at R and 1/R, which affects the placement of all zeros simultaneously. In contrast, the ZDM-MOCZ scheme allocates zeros across multiple users for z-domain multiplexing, which is not addressed in this paper. In future work, we plan to extend the dynamic codebook design to support ZDUM-MOCZ users.

The proposed dynamic codebook design reduces the impact of channel effects, leading to significant improvements in BER, throughput, and PDR performance. In future work, we plan to develop efficient techniques for reporting channel zero distributions to the transmitter.

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Authors' Contributions

The authors' contributions to the paper are equal.

Statement of Conflicts of Interest

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The authors declare that this study complies with Research and Publication Ethics

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