Submission Date:
15.01.2025
Acceptance Date:
14.04.2025
Publication Date:
29.06.2026

To cite this article: Telci, İ. & Aykaç Alp, E. (2026). Portfolio optimization: Application of the Black-Litterman Model. *İstanbul Ticaret University Journal of Social Sciences, 25*(55), 1-33, doi: 10.46928/iticusbe.1620460

# PORTFOLIO OPTIMIZATION: APPLICATION OF THE BLACK-LITTERMAN MODEL*

*Research*

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* This article was derived from the PhD thesis titled "Portfolio Optimization: Application of the Black-Litterman Model," prepared by İsmail Telci under the supervision of Prof. Dr. Elçin Aykaç Alp within the Financial Economics PhD Program at Istanbul Commerce University, Institute of Trade and Finance (June 2025).

# Portfolio Optimization: Application of the Black-Litterman Model

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**Abstract**

This research article proposes an approach for using the Black-Litterman model to optimize the asset allocation of a financial investment portfolio consisting entirely of investment fund shares, essentially a fund portfolio. In this study, the investment fund market was modeled financially using indices generated by grouping investment funds traded in the Turkish investment fund market, which was considered as an application. In this market, while potential performance improvements were identified for determining efficient portfolios based on nonlinear programming, i.e., portfolio optimization processes, it was also demonstrated that results comparable to traditional asset indices could be produced. Furthermore, this research highlights the advantages of incorporating investor (capital owner) and fund/portfolio manager views (market direction forecasts) into the optimization process and suggests that investment fund indices can be used as effective substitutes for optimization models that use traditional/official market benchmark composite indices. In conclusion, this study expands the application of the Black-Litterman model to investment fund investments, providing a structured approach to portfolio management.

**Keywords:** Portfolio selection, investment funds, optimization techniques.

**JEL Code:** G11, G23, C61

# Portföy Optimizasyonu: Black Litterman Modeli Uygulaması


**Özet**

İlgili bu araştırma makalesi çalışması, yatırım aracı olarak tamamı yatırım fonu katılım paylarından oluşan bir finansal yatırım portföyünün yani aslında bir fon sepeti portföyünün, varlık dağılımının optimizasyonu uygulaması için Black-Litterman modelinin kullanım yöntemine dair bir yaklaşım önerisi sunmaktadır. Bu çalışmada, uygulama olarak ele alınan Türkiye yatırım fonu piyasasında işlem gören yatırım fonlarının gruplanması ile üretilen endekslerin kullanımıyla yatırım fonu piyasası finansal olarak modellenmiştir. Bu piyasada doğrusal olmayan programlamaya dayalı etkin portföylerin belirlenmesi yani portföy optimizasyonu süreçleri için, potansiyel performans iyileştirmeleri ortaya konulurken, aynı zamanda da geleneksel varlık endeksleriyle karşılaştırılabilir sonuçlar üretilebildiği gösterilmiştir. Yine ilgili bu araştırma çalışmasında, yatırımcı (sermayedar) ve fon/portföy yöneticisi görüşlerinin (piyasa yönü tahminlerinin) optimizasyon sürecine dahil edilebilmesinin avantajlarını öne çıkarırken ve yatırım fonu endekslerinin geleneksel/resmi piyasa gösterge bileşik endekslerinin kullanıldığı optimizasyon modellerine göre etkili ikameler olarak kullanılabileceğini öne sürmektedir. Sonuç olarak, bu çalışma, Black-Litterman modelinin yatırım fonu yatırımlarına uygulanması alanını genişleterek portföy yönetimine yapılandırılmış bir yaklaşım sağlar.



**Anahtar Kelimeler:** Portföy seçimi, yatırım fonu, optimizasyon teknikleri.

**JEL Kodu:** G11, G23, C61

## Introduction

The management of a financial investment portfolio consisting of investment fund participation shares is the subject of this study. The optimization process of a fund basket portfolio managed with this approach based on a financial model is the research method and focus of this study.

For modern portfolio theory and the optimization approach developed by Black and Litterman during their time at Goldman Sachs Investment Bank in the 1990s, the literature has largely focused on the analysis of portfolios consisting of financial investment instruments belonging to main asset classes such as stocks and bonds. Since the 2000s, studies have mostly focused on developments in the mathematical infrastructure of the model, and no significant change has been observed in the data sets used.

In this relevant study, an approach is presented on how portfolios that only include investment funds should be analyzed within the Black Litterman financial/mathematical optimization model. Since investment funds also contain many asset classes within themselves, a method is proposed on how each type and type of investment fund should be addressed within the Black Litterman model.

At this point, it is important for the investor to make a more efficient selection of the type and kind of investment funds to be invested in. Thus, the type and type selection of investment funds also replaces the main asset preference.

Our hypothesis is that in an ecosystem where there are many financial markets and investment instruments, savers who want to make their investments by purchasing fund participation shares will be able to create a fund basket and thus put diversification into practice. It may also be necessary to determine the content distribution of such a fund basket portfolio by using optimization methods. At this stage, price time series will be needed to be included in the mathematical optimization process. At this point, since traditional main asset price time series will not be sufficient, the use of price time series representing these investment fund types and types in the operations process will increase the expected efficiency from the relevant optimization. In addition, as an optimization approach, the use of the Black Litterman model was preferred in this research study because it allows the investor/manager views and estimates to be added to the process. To include investment funds classes that contain more than one main asset class in the portfolio, the use of price time series representing these funds is recommended in the studies in literature.

With the use of new subtype and type indices created for this purpose, the deficiencies encountered in the process of adapting the optimization result to the real portfolio as a result of the use of traditional main asset indices are eliminated, and the calculated asset weights - primarily with preferred investment funds - are provided, thus the expected return can be revealed more accurately. Another benefit here is that the optimization based on a more accurate expected average return calculation with the market provides a higher performance adjusted for risk.

**The Literature Review**

The Black Litterman Model, which emerged as a result of the academic studies of Fisher Black and Robert Litterman in 1991 and 1992, has gained a considerable place in literature on mathematical optimization for investment portfolio optimization. The model, which emerged during the research period of Black and Litterman on portfolio management during their time at Goldman Sachs Investment Bank, generally advanced the developments on Markowitz's Mean Variance model and Sharpe's Mean Variance model to an advanced level. Bevan and Winkelmann (1998), who work at Goldman Sachs Investment Bank, prepared various fixed income security portfolio optimization examples using the Black Litterman Model and data from the period 1995-1998. With these studies, they showed that portfolios created with the Black Litterman model produced higher risk-adjusted return performance than other portfolios.

He and Litterman (1999) explained the Black Litterman Model in more depth in their articles based on their studies conducted at the Goldman Sachs Investment Bank and presented various comparisons with the Markowitz Mean Variance Model. Drobetz (2001) showed an optimization problem example solution using the Black Litterman model and the sector-index data in the European stock market between the years 1993-2000. As a result of the research, they showed that the model eliminated many weaknesses as an alternative to the Markowitz Mean Variance model. Herold (2003) conducted studies on the variance matrix of investor opinions in the Black Litterman Model. The study emphasized the need to calculate the probability density function for each investor estimate in order to calculate the matrix. Idzorek (2002) proposed an alternative approach for the use of investor/manager market expectation estimates in his published article and thus conducted research based on the step-by-step development of the Black Litterman model. Manela and Gofman (2012) took the Black Litterman Model to a further stage and integrated the linear multi-factor asset pricing method into it. Wang (2012) added a solution layer to the Black Litterman Model where the inflation calculation is included.

There are studies in literature where indices are used as data, as in this study. However, the aim of this study is to expand the current application area of the Black Litterman method over the investment fund market. An optimization approach is presented for portfolios investing in investment funds. The indices used for this are investment funds.

The literature review below reveals that there are no studies in this direction in the literature in the context of the originality of this relevant study.

Kooli and Selam (2010) used S&P/TSX, S&P500, MSCI EM, MSCI EAFE main asset indices and used the Fund Weighted Composite Index from Hedge Fund Research (HFR) to model the entire market universe. A new optimization approach presented by Black and Litterman is being reexamined. Mahrivandi et al. (2017) used historical weekly closing price data of Bank Central Asia (BCA), Bank Rakyat Indonesia (BRI), Bank Mandiri and Bank Negara Indonesia (BNI) between 2014-2016. In this paper, the BL model presents an alternative solution where stock returns and investor opinions are modeled from non-normal distribution. Karan (2017) used Apple Inc. Coca-Cola Bottling Co. Walt Disney Co. Ford Motor Co. Goldman Sachs Inc. Johnson & Johnson McDonald’s Corp. Wal-Mart Stores Inc. Exxon Mobil Corp. Yahoo! Inc. Stocks use closing data. Arisena et al. (2018), BBCA, GGRAM, WSKT stock closing prices were used as data. Bertsimas et al. (2020), stocks in the energy, financial services, and telecommunications sectors were also used. Malm (2021), showed the optimization process by creating a global stock portfolio consisting of ETFs. It examines Idzorek's extension of the Black-Litterman model in terms of confidence levels and makes a general comparison with the Canonical Reference Model.

## Portfolio Management, Optimization and Black Litterman Model

### *Portfolio Management*

Portfolio management is the activity of investing in various investment instruments by directing financial wealth (asset) at a certain level of risk perception and appetite that both parties agree on and that the party receiving the portfolio management service will tolerate and managing this diversified basket.

The area where the interaction resulting from the coexistence of risky investments is taken into consideration and examined is evaluated as the portfolio approach.

Portfolio management is defined in its broadest sense as the process of managing money. William F. Sharpe has determined three basic functions and steps related to portfolio management in this process: portfolio analysis, portfolio revision and performance evaluation (Özçam, 1997).

### Optimization and Markowitz's Mean Variance Model

Optimizing, i.e. optimization or mathematical optimization, defines the process of finding -reaching- the best possible solution for a problem. In Markowitz's Mean Variance Model, which opens the doors to Modern Portfolio Theory, a rational investor avoids risk. This investor must make a choice on the efficient frontier. When we consider that there is more than one efficient portfolio on the efficient frontier, the determination of which of these efficient portfolios will be preferred will be determined largely by the investor's risk preference or appetite, i.e. the relatively inversely proportional relationship between return and risk. Accordingly, the selection of the optimal portfolio will emerge according to the expected return, risk and asset allocation parameters of each efficient portfolio (Sarıoğlu, 2021).

Quadratic programming is a solution option in calculating the efficient portfolio (İbrahimov, 2007). The general expression formula for this is as follows.

Objective: max/min

$$f(x_1, x_2, x_3, \dots, x_n) \tag{1}$$

Constraint:

$$g_1(x_1, x_2, x_3, \dots, x_n)\ (\leq, = \text{or} \geq)\ b_1 \tag{2}$$

$$g_2(x_1, x_2, x_3, \dots, x_n)\ (\leq, = \text{or} \geq)\ b_1 \tag{3}$$

$$g_m(x_1, x_2, x_3, \dots, x_n)\ (\leq, = \text{or} \geq)\ b_1 \tag{4}$$

Here, in accordance with all existing restrictions $(x_1, x_2, x_3, \dots, x_n)$ the relevant points have the status of feasible solution set, all these feasible solutions are called feasible solution region. For this maximization process of all feasible solutions.

$$f(x_1^*, x_2^*, x_3^*, \dots, x_n^*) \geq f(x_1, x_2, x_3, \dots, x_n) \tag{5}$$

The suitable points $(x_1^*, x_2^*, x_3^*, \dots, x_n^*)$ that satisfy the condition are called optimal solutions.

The same logic is written for the minimization process as follows.

$$(x_1^*, x_2^*, x_3^*, \dots, x_n^*) \leq f(x_1, x_2, x_3, \dots, x_n) \tag{6}$$

When all the constraint functions $f, g_1, g_2, g_3, \dots, g_m$ in the model are linear, the current optimization problem is called a linear programming model, but if any of these functions are nonlinear, the problem is called a nonlinear programming problem (Bazaraa et al., 1993).

Since the objective function in Harry Markowitz's Means Variance Model is not linear, the equation itself becomes a non-linear equation problem.

**Black Litterman Model**

One of the weak links of the Markowitz Mean Variance Optimization method, which initiated Modern Portfolio Theory, is that its mathematical optimization structure has

1. Sensitive structure to data (matrix multiplication operations).
2. Estimation errors are also added to this structure.

Due to its characteristics, it produces over-concentrated portfolio asset allocation results. This also causes a lack of under-diversification. The Black Litterman model was produced as a solution alternative to this problem of over-concentration in asset class allocation weights.

Theoretical structure of the Black Litterman Model (BLM): The model is created by combining 4 theories (Idzorek, 2002):

1. Markowitz; Mean Variance Model, 1952.
2. Sharpe; Capital Assets Pricing Model - CAPM, 1964.
3. Sharpe; Reverse optimization.
4. Teil; Mixed estimation method, 1978.

Although the Black Litterman model stands out as an asset weight distribution model throughout literature, it is primarily an equation that recalculates the expected excess returns in the original function. The feature of this equation is that it allows investors and financial investment portfolio managers to add their estimates of the returns of the main asset classes and financial assets to the optimization process.

The Black-Litterman model combines two sources in calculating expected excess returns.

Source 1: Average information data on expected returns from financial markets, namely implied excess equilibrium returns vector.

Source 2: Estimated excess equilibrium returns vector within the subjective expectations of the treasury, fund manager, and investor regarding the return expectations of main asset classes, countries, sectors, and individual financial assets.

Black Litterman Model Master Equation Formula.

$$[R] = \left(r - r_f\right) \tag{7}$$

$$E[R] = [(\tau\Sigma)^{-1} + P^I\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P^I\Omega^{-1}Q] \tag{8}$$

Formula’s 1st part:

$$E[R] = [(\tau\Sigma)^{-1} + P^I\Omega^{-1}P]^{-1} \dots \tag{9}$$

$\tau$ ; Tau coefficient. $(\tau\Sigma)$; Variance-Covariance matrix. $P$ ; P-link matrix. $\Omega$ ; Investor forecast uncertainty coefficient.

The first part of the formula is the multiplication part where the weights assigned to the market residual equilibrium return vector and the function of assigning market opinions to financial assets are realized.

Formula's 2nd part:

$$E[R] = \cdots [(\tau\Sigma)^{-1}\Pi + P^{I}\Omega^{-1}Q] \tag{10}$$

$\Pi$ ; Indirect residual equilibrium return. $Q$ ; Investor market expectation/opinion.

In the second part of the formula, this is the part where the market residual equilibrium return vector comes together with investor estimates and is combined. Since the final value is a weighted average information, the weights of the components in this combination are important. These weights indicate how confident the investor is about his/her own views compared to the expected equilibrium return estimate confidence level.

**The Method and the Material**

This study focuses on the optimization of a portfolio that can be considered a kind of diversification by bringing together already diversified portfolios, that is, by creating a whole fund basket. Since the portfolio subject to optimization is an investment fund and since the investment funds are separated in terms of type and type, the performances of investment funds that vary periodically in the markets can sometimes exceed the market average and sometimes remain below the average.

The reason for this, of course, depends on the performance of the manager who manages the portfolio, that is, the portfolio management company. For this reason, it is thought in the main idea of this study that it would be healthier to follow the optimization process and processes of investment funds through indices calculated based on the price performances of investment funds, which constitute a significant part of the markets based on total portfolio values.

The Black Litterman model makes significant progress in eliminating the deficiencies of the Mean Variance model in asset allocation selection. The fact that it allows the inclusion of investor/manager expectations and views regarding the expected returns, which are important parameters in the optimization of an investment portfolio, into the variance-covariance based calculation matrix also played an important role in the selection of this model for the main framework of the study.

The application of the Black Litterman model to the fund basket portfolios, consisting of investment-free-private and retirement investment fund participation shares traded in the investment fund markets, was carried out by following the steps below.

1. The funds that constitute the entire investment fund market are determined. This determines the investment fund, special funds, free funds and retirement funds, for which the local market specific analysis will be made.
2. The daily price changes of individual funds are calculated logarithmically.
3. The funds are classified and grouped according to structural similarity, risk and investment characteristics. Thus, the investment fund market is modeled as a market.
4. The relevant fund type-type composite indices are created from the investment funds brought together according to their similarities and common aspects.
5. BIST KYD Reverse Repo Net Index is used as risk-free interest rate.
6. Variance and covariance matrices ($\Sigma$) of calculated investment fund indices are created.
7. Weight share (w) of investment fund index within the entire investment fund market is calculated.
8. Risk aversion coefficient ($\delta$) is calculated.
9. Expected -average- returns are calculated by performing reverse optimization on Capital Asset Pricing Model (CAPM).
10. Matrix (P) containing investor-manager expectations regarding investment fund indices is created.
11. New -equilibrium expected return values (Q) based on subjective estimates of investor-manager regarding fund indices are specified.

12. Variance matrix (Q) of investment-manager estimated expectations is calculated.
13. Portfolio optimization is performed on the estimated expected equilibrium return and covariance results created with Black Litterman model. Thus, the equilibrium expected returns (Π) and estimated expectations (Q) are combined at the matrix level via the Black Litterman model.
14. The combined -new- equilibrium expected return vector (E(R)) and the weights of the fund index assets' share distribution in the portfolio are renewed.

**The Relevant Data Set of The Study**

The data set of the research consists of the following data belonging to a total of 565 mutual funds founded by 68 portfolio management companies operating in the Turkish money and capital markets and located in the TEFAS-Turkey Electronic Fund Purchase and Sale Platform as of 31.08.2024.

1. Unit/share price – Turkish lira-₺ (the division of the total portfolio value of the participation shares of the investment funds by the number of shares in circulation of the fund).
2. Total portfolio value – Turkish lira (net asset value-₺).

Time series were used. The relevant time series covers the 252-business days trading period between 29.08.2023 and 29.08.2024.

For information, only the traded investment funds were evaluated in the analysis, the number of which was 565 as of the date of the study.

The public data distribution system of the Turkish Electronic Fund Trading Platform TEFAS website was used to obtain the data. (Takasbank, 2024)

Data processing, calculations, drawing and writing of graphics were carried out in Microsoft Office 365 Excel and Word software.

**Limitations and Assumptions of the Study**

In this study, the restriction on main asset distribution was not used in the optimization processes carried out by using the investment fund type and type sub-indices created for the Black Litterman model and the indices of official market operators for comparison purposes. Thus, the problem of determining the Efficient portfolio becomes a mathematical unconstrained nonlinear multivariate programming process.

In the application provided by the Black Litterman model and the possibility of adding investor manager views to the matrix, only 2 views were added to the system. These views were limited in value. BIST KYD Reverse Repo Net Index was used as risk-free return.

The process begins with the calculation of the indexes of the funds according to their subtype, type and theme to create time series and model the investment fund market.

Knowing the entire investment fund market, while expressing the main mass of data at this stage, is necessary for the calculation of the Market risk premium (unit risk cost) Lambda parameter, market variance and market average excess return parameters for this market whole with a Beta of '1'.

In this context, the concept of entire market means entire investment fund market for a fund basket portfolio to be optimized over investment funds. For example, in the optimization of a stock portfolio, the corresponding indexes and parameters to the stock market composite index and its total market value parameters in the optimization of an investment fund are the entire investment fund market index and the total portfolio value of all investment funds.

After this stage, the fund market is classified and divided into groups according to fund subtypes, types or themes. Accordingly, the components that constitute the whole of the fund market should first be separated under the sub-detail of umbrella fund type and then fund title type.

While conducting this study, the question on what basis can this separation and re-classification be created? should be answered by selecting from the following options.

1. According to the portfolio content within the framework of legal regulations.
2. According to the risk degrees of fund portfolios (in TEFAS, there are stages from 1 to 7 starting from low to high risk).
3. According to portfolio management strategies.

4. According to portfolio investment themes.

5. According to portfolio comparison criteria, investment strategy band ranges.

***TEFAS Umbrella and Sub-Detailed Fund Classes:***

The basic and standard Umbrella Fund Classes traded on TEFAS – Turkey Electronic Fund Purchase and Sale Platform, according to the Capital Markets Board (CMB) and Capital Markets Law (CML) regulations, are as follows.

**Table 1.** TEFAS Umbrella Fund Classes

| No. | TEFAS Umbrella Fund Class | Fund Size TRY | Market Share (%) | Number of funds |
|---|---|---|---|---|
| 1 | Exchange Traded Funds | 83,607,042,364.90 | 5.78 | 22 |
| 2 | Debt Instruments (TEFAS) | 84,597,895,292.38 | 5.85 | 73 |
| 3 | Fund Basket (TEFAS) | 71,176,873,445.08 | 4.92 | 78 |
| 4 | Money Market (TEFAS) | 780,408,821,698.94 | 53.94 | 54 |
| 5 | Mixed (TEFAS) | 4,819,042,789.08 | 0.33 | 9 |
| 6 | Precious Metals (TEFAS) | 69,608,593,877.25 | 4.81 | 18 |
| 7 | Participation (TEFAS) | 119,761,876,450.50 | 8.28 | 63 |
| 8 | Stock (TEFAS) | 174,380,872,651.65 | 12.05 | 117 |
| 9 | Variable (TEFAS) | 58,574,839,531.10 | 4.05 | 131 |
| | TOPLAM | 1,446,935,858,100.88 | 100.00 | 565 |

**Source:** Takasbank (*İstanbul Takas ve Saklama Bankası A.Ş.*) (n.d.). Turkey Electronic Fund Trading Platform – TEFAS (*Türkiye elektronik fon alım satım sistemi*). https://www.takasbank.com.tr/

In the fund prospectuses which are the most important official documents published on the Capital Markets Board (CMB) regulations and Public Disclosure Platform (PDP) regarding investment funds, the type and type of the fund is determined primarily according to the umbrella fund class under which the relevant fund falls. After this main classification, more detailed sub-groupings are made.

Accordingly, the total portfolio value of a total of 565 investment funds traded in 9 umbrella classes in the investment fund market in Turkey as of 31.08.2023 is TRY 1,446,935,858,100.88.

Although the above 9 main group classifications seem sufficient in the context of legislation, when it comes to a portfolio optimization process, in an ecosystem where there are many investment funds that exhibit very different structural features from each other and have detailed portfolio diversification within themselves and in the market, this level of grouping does not provide the desired sensitivity in optimization results. Because, when we assume that such general indices containing many funds are created, there will be a decrease in the efficiency of the optimization process since there will be a high variance in the calculation of the expected balance -average- returns. Therefore, there will be a need for fund groupings that are more consistent and have similar characteristics within themselves.

As can be seen in Table 1 and Table 2, a total of 565 investment funds in 9 Umbrella classes, again a total of 565 in Table 4, and 27 sub-classes under the same total portfolio value market size were separated; by bringing together funds that are more similar to each other according to their features such as legislation, asset distribution, risk levels and investment strategies, index groups to be optimized were created. Thus, since 27 sub-groups were formed instead of 9 groups, the number of investment funds in each group also decreased.

**Table 2.** Detailed Sub-Fund Type Classes

| No. | Detailed Sub-Fund Class | Fund Size TRY | Market Share (%) | Number of funds |
|---|---|---|---|---|
| 1 | Money Market | 780,408,821,698.94 | 53.94 | 54 |
| 2 | Exchange Traded Funds | 83,607,042,364.90 | 5.78 | 22 |
| 3 | Debt Securities-Foreign Debt Securities (Eurobond) | 17,960,409,388.72 | 1.24 | 10 |
| 4 | Debt Securities-Foreign Debt Securities | 1,189,181,032.02 | 0.08 | 3 |
| 5 | Debt Securities-Private Sector Debt Securities | 4,826,570,091.40 | 0.33 | 13 |
| 6 | Debt Securities-Long Term | 575,409,342.24 | 0.04 | 2 |
| 7 | Debt Securities-Medium Term | 550,296,221.56 | 0.04 | 4 |
| 8 | Debt Securities-Short Term | 45,134,407,430.16 | 3.12 | 23 |
| 9 | Fund Basket-Commodity Foreign | 2,319,235,490.52 | 0.16 | 4 |
| 10 | Fund Basket-Foreign | 3,509,438,250.22 | 0.24 | 4 |
| 11 | Fund Basket-Other | 65,348,199,704.34 | 4.52 | 70 |
| 12 | Mixed & Variable-Mixed | 4,819,042,789.08 | 0.33 | 9 |
| 13 | Balanced Variable Funds | 13,126,904,510.77 | 0.91 | 22 |
| 14 | Conservative/Cautious Variable Funds | 3,597,305,392.85 | 0.25 | 6 |
| 15 | Attack/Dynamic/Growth Variable Funds | 8,027,237,770.39 | 0.55 | 9 |
| 16 | Aggressive Variable Funds | 4,066,453,334.03 | 0.28 | 13 |
| 17 | Other Variable Funds | 29,756,938,523.06 | 2.06 | 81 |
| 18 | Debt Securities-Other Debt Securities | 14,361,621,786.28 | 0.99 | 18 |
| 19 | Precious Metals-Gold | 69,608,593,877.25 | 4.81 | 18 |
| 20 | Participation-Gold | 35,380,306,591.95 | 2.45 | 11 |

| | | | | |
|---|---|---|---|---|
| 21 | Participation-Lease Certificate (TRY) | 29,173,250,741.16 | 2.02 | 19 |
| 22 | Participation-Lease Certificate (Foreign Exchange) | 297,235,460.56 | 0.02 | 2 |
| 23 | Participation-Share | 7,132,081,674.90 | 0.49 | 7 |
| 24 | Participation-Other | 54,911,083,656.83 | 3.79 | 25 |
| 25 | Share-Foreign Share | 29,020,646,864.56 | 2.01 | 10 |
| 26 | Share-Index Share | 18,652,848,871.89 | 1.29 | 15 |
| 27 | Share-Share | 119,575,295,240.30 | 8.26 | 91 |
| | TOTAL | 1,446,935,858,100.88 | 100.00 | 565 |

**Source:** Takasbank (*İstanbul Takas ve Saklama Bankası A.Ş.*) (n.d.). Turkey Electronic Fund Trading Platform – TEFAS (*Türkiye elektronik fon alım satım sistemi*). https://www.takasbank.com.tr/

***Detailed Investment Fund Market Classification Requirement***

As of 31.08.2024, the number of types of financial investment instruments traded in Turkish money and capital markets and subject to legal securities accounting is 36.

**Table 3.** List of Investment Instruments in Turkey

| No. | Investment Tool |
|---|---|
| 1 | Shares |
| 2 | Government Bonds |
| 3 | Treasury Bonds |
| 4 | Foreign Currency Public Domestic Debt Instr. |
| 5 | Financing Bonds |
| 6 | Private Sector Bonds |
| 7 | Asset-Backed Securities |
| 8 | Public External Debt Instruments |
| 9 | Private Sector External Debt Instruments |
| 10 | Takasbank Money Market |
| 11 | Public Lease Certificates (TL) |
| 12 | Public Lease Certificates (FX) |
| 13 | Private Sector Lease Certificates |
| 14 | Public Overseas Lease Certificates |
| 15 | Ö.S. Overseas Lease Certificates |
| 16 | Deposit (TL) |
| 17 | Deposit (FX) |
| 18 | Participation Account (TL) |
| 19 | Participation Account (Foreign Exchange) |
| 20 | Participation Account (Gold) |
| 21 | Repo |
| 22 | Reverse-Repo |
| 23 | Precious Metals |
| 24 | BYF in Precious Metals |
| 25 | Precious Metals Public B. A. |
| 26 | Precious Metals Public Lease Certificate. |
| 27 | Foreign Public Debt Instruments |
| 28 | Foreign Private Sector B. A. |
| 29 | Foreign Shares |

| | |
|---|---|
| 30 | Foreign Stock Investment Funds |
| 31 | Participation Funds Participation Shares |
| 32 | BYF Participation Shares |
| 33 | Real Estate Investment Fund Participation Shares |
| 34 | Girişim S. Investment Fund Participation Shares |
| 35 | Futures Cash Collateral |
| 36 | Other |

It is possible to create complex and diverse financial portfolios with the money and capital market investment instruments in Turkey, which have a high variety of 36. While this variety includes many combination possibilities, it can be said that the portfolios of these funds will also vary structurally in an investment environment where there are many fund types and types. This increases the investment options that individual, corporate and institutional investors who invest in investment funds will prefer.

***Index Generation from Fund Sub-type Classification***

Table 1 and Table 2 show the number of investment funds included in the sub-groups for each fund classification type. The price time series specific to each fund type is provided from the share price and total portfolio value time series of investment funds within fund sub-classes, which are announced once a day. The total fund value of each individual investment fund is considered here in the context of market value and the total consolidated portfolio value of all associated investment funds reflects the internal market size of the relevant index, while the ratio of the total portfolio values (in TRY-₺) of each fund to the consolidated -global- total portfolio value reflects the market share, i.e. weight, of each individual investment fund within the relevant index. The price time series of each investment fund sub-class is created daily based on this price and weight data. Since each investment fund class contains at least more than 1 investment fund, the index to be created must be in the Composite Index structure. Since the return, which is the performance indicator of each individual investment fund, is calculated based on the price change, the relevant index will be a price index type. Due to the importance of the depth effect of each investment fund on the overall market, this index also has the feature of a weighted price index since it is the market share ratio considered over the fund’s total portfolio value amount information.

***Sample Healthcare Sector Fund Index Design***

A list of market-traded all health sector themed investment funds is provided.

**Table 4.** Sample Health Sector Fund Index Investment Fund Content

| No. | Fund Code | Fund Name | Total Fund Value TRY | Market share % | 31/8/2024 Price |
|---|---|---|---|---|---|
| 1 | AFS | Ak Portfolio MC Health Sector Foreign Stock Fund | 837,701,695.82 | 63.6% | 0.249136 |
| 2 | GZG | Garanti Portfolio MC Health Sector Variable Fund | 53,715,987.69 | 4.1% | 4.300620 |
| 3 | IKL | İş Portfolio MC Health Companies Mixed Fund | 392,394,302.10 | 29.8% | 6.506967 |
| 4 | TBE | TEB Portfolio MC Health and Biotechnology Variable F. | 32,758,004.66 | 2.5% | 2.071478 |
| | | TOTAL | 1,316,569,990.27 | 100.0% | |

**Source:** Takasbank (*İstanbul Takas ve Saklama Bankası A.Ş.*) (n.d.). Turkey Electronic Fund Trading Platform – TEFAS (*Türkiye elektronik fon alım satım sistemi*). https://www.takasbank.com.tr/

The share price and total portfolio value time series of these funds between 29.08.2023 and 29.08.2024 -252 business days in this example- are created and indexed according to the Laspeyres price index approach.

The index, which started from 100 based on 29.08.2023, is calculated based on the market share of the fund total value. The price-time graph of the IKL fund is shown in Figure 1. Other, GZG, AFS and TBE funds are also included, and the entire composite index of health sector investment funds observed in Figure 2 is obtained.

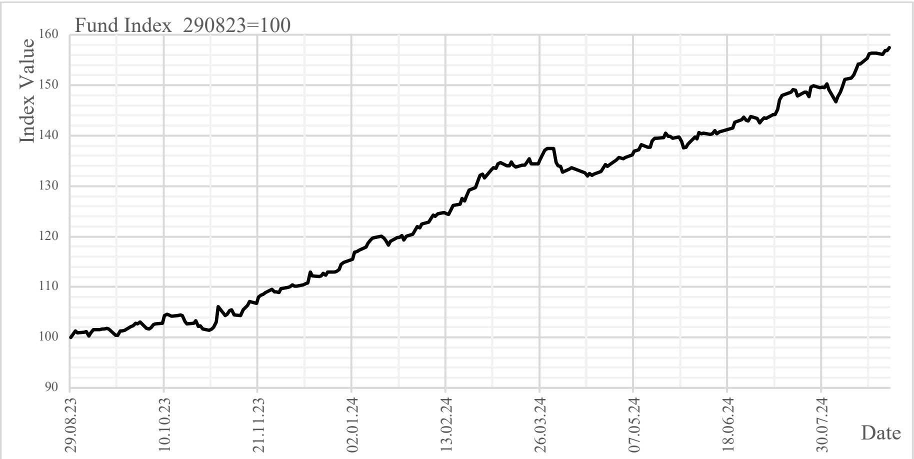


**Figure 1.** IKL İş Portfolio Health Companies Mixed Fund Price Chart

With the addition of all other funds, it becomes possible to see the general course of interest rate investment funds traded on TEFAS and the health sector theme, making investment decisions specific to the health sector, the opportunity to analyze sectoral trends from a broader perspective beyond the price performance of a single investment fund, and, as in this study, using this relevant data in portfolio optimization to guide both investment funds and sector-based investments.

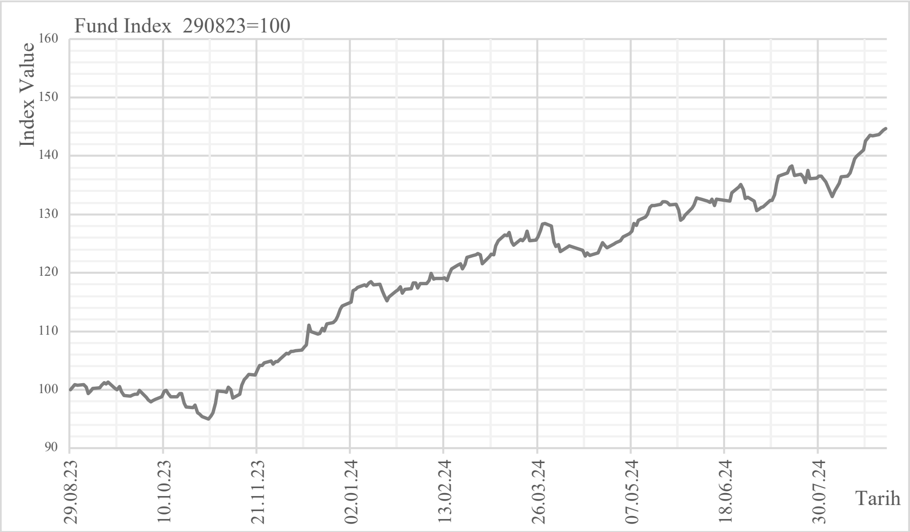


**Figure 2.** Price Index Graph of All Health Sector Funds

**Application of The Black Litterman Model to The Mutual Fund Industry**

In this research study, it is assumed that the financial investment portfolios to be optimized consist entirely of investment fund participation shares. In other words, an investment fund basket will be optimized. The approach put forward for this is that instead of official market operator indices (such as BIST 100 index for the stock market) as in traditional applications, the investment fund market is separated and modeled according to its types and types, and the composite investment fund indices that are created as a result of the optimization process are subjected to an investment fund basket portfolio, an application example where fund types and types replace the main asset classes, and its relative differences from studies conducted on traditional indices are revealed.

### Black Litterman Model Application Stages

For example, to create a portfolio according to risk perception based on the mutual fund index representing the '5' asset classes selected, the time series of these indexes covering 252 business days are created with the methods explained in the previous section. In addition, a variable named Fund Market is created to express the entire mutual fund market in Turkey as of 29.08.2024, and a 6th variable is obtained from the price-time series of all funds.

**Table 5.** Black Litterman Model Sample Mutual Fund Index Data Set

| Mutual Fund Composite Index Time Series | | | | | | |
|---|---|---|---|---|---|---|
| No. | Date | AIE Debt Instruments-Short Term | AIE Debt Instruments-Long Term | AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | AIE Stock-Share | AIE Precious Metals-Gold |
| 1 | 29.08.23 | 285.76 | 207.88 | 1124.71 | 1772.88 | 1433.20 |
| 2 | 31.08.23 | 285.95 | 207.94 | 1128.74 | 1784.75 | 1450.83 |
| 3 | 01.09.23 | 286.51 | 206.26 | 1128.55 | 1812.31 | 1452.68 |
| 4 | 04.09.23 | 286.72 | 205.87 | 1130.20 | 1824.46 | 1445.01 |
| 5 | 05.09.23 | 286.92 | 205.30 | 1130.91 | 1843.76 | 1441.51 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 248 | 23.08.24 | 419.87 | 233.53 | 1608.92 | 2477.46 | 2355.35 |
| 249 | 26.08.24 | 420.40 | 233.71 | 1612.71 | 2452.83 | 2386.64 |
| 250 | 27.08.24 | 420.83 | 233.44 | 1614.14 | 2491.95 | 2398.61 |
| 251 | 28.08.24 | 421.37 | 233.32 | 1614.74 | 2491.47 | 2378.26 |
| 252 | 29.08.24 | 423.40 | 234.09 | 1613.32 | 2513.55 | 2405.24 |

In the first stage, the daily change (return) calculation of the price time series is applied on a logarithmic (ln) basis. The calculation of the portfolio's basic statistical parameters will be carried out on the daily price change (return) time series data. Then, the mean-adjusted (de-meaned) returns, where the average change parameter of each variable is subtracted from the logarithmic change values, are continued.

The correlation matrix, where the number of rows and columns for each index (or asset) is equal - in this example, a 5 to 5 scale - is calculated on the data in the price logarithmic change table. In the case where the market's expected return and risk are given, if the market is in a mean-variance efficient state, the lambda $\lambda$ parameter can be calculated and the market's unit risk cost, i.e. the entire market risk premium, can be measured.

**Table 6.** Fund Indices Total Market Values (Total Fund Value) Share

| Fund Index | Market Value (TRY) | Market Value Share |
|---|---|---|
| AIE Debt Instruments-Short Term | 45,134,407,430.16 | 17.8% |
| AIE Debt Instruments-Long Term | 575,409,342.24 | 0.2% |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | 17,960,409,388.72 | 7.1% |
| AIE Stock-Share | 119,575,295,240.30 | 47.3% |
| AIE Precious Metals-Gold | 69,608,593,877.25 | 27.5% |
| | 252,854,115,278.67 | 100.0% |

The variance and covariance matrix is calculated to calculate the return (change/yield) and variance of the entire portfolio, that is, to calculate the co-variance of all assets in the portfolio. The variance of the portfolio will also be calculated over the same variance and covariance matrix.

**Table 7.** Variance Covariance Table (Matrix) Among Fund Indices

| S Variance - Covariance Matrix | AIE Debt Instruments-Short Term | AIE Debt Instruments-Long Term | AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | AIE Stock-Share | AIE Precious Metals-Gold |
|---|---|---|---|---|---|
| AIE Debt Instruments-Short Term | 0.000001 | 0.000001 | 0.000000 | 0.000000 | 0.000000 |
| AIE Debt Instruments-Long Term | 0.000001 | 0.000042 | -0.000001 | 0.000000 | -0.000004 |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | 0.000000 | -0.000001 | 0.000007 | 0.000009 | 0.000009 |
| AIE Stock-Share | 0.000000 | 0.000000 | 0.000009 | 0.000235 | 0.000015 |
| AIE Precious Metals-Gold | 0.000000 | -0.000004 | 0.000009 | 0.000015 | 0.000149 |

Thus, all the necessary variables are obtained to calculate the implied equilibrium excess returns. From this stage onwards, the equilibrium returns specific to the Black Litterman model can now be calculated.

**Table 8.** Black Litterman Implied Equilibrium Excess Returns ($\mu$-r_f=2λSw)

| Fund Index | Implied Equilibrium Excess Returns |
|---|---|
| AIE Debt Instruments-Short Term | 0.000004 |
| AIE Debt Instruments-Long Term | -0.000009 |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | 0.000065 |
| AIE Stock-Share | 0.001058 |
| AIE Precious Metals-Gold | 0.000447 |

The $Q$ notation written in the Black Litterman model equation includes 2 manager opinions in the example here. Accordingly, the 1st opinion indicates that the stock investment fund index will perform over 0.001% in annual periodic change (return) of the precious metals-gold fund index and that the debt instruments-short-term fund index and the debt instruments-long-term fund index are expected to create a change (return) over 0.001% on an annual periodic basis. In the P Link matrix, these preferences are processed into the matrix with the values [-1, 0, +1]. The τ Tau notation is accepted as '1'. In the Link matrix, the rows represent opinions, and the columns represent assets (indices). The row total should be equal to zero (0). (+) sign reflects the opinion in favor of the asset/index, (-) sign reflects the opinion against the asset/index, and (0) zero means that there is no opinion.

**Table 9.** Fund Manager Opinions Matrix

| **Opinion No.** | **Market expectations (Q)** | **Link Matrix, P** | | | | |
|---|---|---|---|---|---|---|
| | | **AIE Debt Instruments -Short Term** | **AIE Debt Instruments -Long Term** | **AIE Debt Instruments -Foreign Debt (Eurobond) Instruments** | **AIE Stock - Share** | **AIE Precious Metals-Gold** |
| 1 | 0,00001 | 1 | -1 | 0 | 0 | 0 |
| 2 | 0,00001 | 0 | 0 | 0 | 1 | -1 |

| **Tau =** | 1 |
|---|---|
| **Omega** | $\Omega = \tau PSP^T$ |
| 0,0000 | 0,0000 |
| 0,0000 | 0,0004 |

Combining the implied returns with the views that reflect the expectations of the investor/manager within the model. The *S* notation in the first part of the Black Litterman model equation expresses the variance-covariance matrix, and the expression shown with the *P* notation expresses the investor views. The Ω Omega notation contains the reliability coefficient of these views.

**Table 10.** Black Litterman Formula Expected Return Part 1 ( $[(\tau S)^{-1} + P^T \Omega^{-1} P]^{-1}$)

| Expected return formula part 1 | AIE Debt Instruments-Short Term | AIE Debt Instruments-Long Term | AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | AIE Stock-Share | AIE Precious Metals-Gold |
|---|---|---|---|---|---|
| AIE Debt Instruments-Short Term | 0.000001 | 0.000001 | 0.000000 | 0.000000 | 0.000000 |
| AIE Debt Instruments-Long Term | 0.000001 | 0.000022 | -0.000001 | 0.000000 | -0.000002 |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | 0.000000 | -0.000001 | 0.000007 | 0.000009 | 0.000009 |
| AIE Stock-Share | 0.000000 | 0.000000 | 0.000009 | 0.000167 | 0.000057 |
| AIE Precious Metals-Gold | 0.000000 | -0.000002 | 0.000009 | 0.000057 | 0.000124 |

The Π *Pie* notation in the second part of the Black Litterman main equation refers to implied equilibrium excess returns, while the *Q* notation includes investor estimates in terms of value. Here, these two variables are combined. Thus, a simple weighted average calculation is performed.

**Table 11.** Black Litterman Formula Part 2 ($(\tau S)^{-1}\Pi + P^T \Omega^{-1} Q$)

| Expected return formula part 2 | |
|---|---|
| AIE Debt Instruments-Short Term | 1.8736 |
| AIE Debt Instruments-Long Term | -0.2269 |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | 0.6470 |
| AIE Stock-Share | 4.3386 |
| AIE Precious Metals-Gold | 2.4770 |

The expected excess return column in Table 13 represents the expected excess return obtained as a result of the combination of the 1st and 2nd parts of the Black Litterman formula and includes investor views, while the implied equilibrium excess return column shows the averages formed by the product of the S' variance-covariance matrix between assets, market value weights and the market risk premium $\lambda$ lambda.

**Table 12.** Expected and Implied Equilibrium Excess Returns

| Fund index | Expected excess return | Implied equilibrium excess return |
|---|---|---|
| AIE Debt Instruments-Short Term | 0.000004 | 0.000004 |
| AIE Debt Instruments-Long Term | -0.000008 | -0.000009 |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | 0.000066 | 0.000065 |
| AIE Stock-Share | 0.000870 | 0.001058 |
| AIE Precious Metals-Gold | 0.000560 | 0.000447 |

The next step in the calculation of the optimum portfolio is to find the $Z$ value. The transpose of the variance-covariance matrix, that is, the inverse, is performed by multiplying the matrix of historical and CAPM risk-free interest rate difference returns.

**Table 13.** Portfolio Calculations. Historical: $\mu - r_f$ Historical: $Z = S^{-1}(\mu - r_f)$

| Portfolio calculations | Historical | CAPM | BL | Historical | CAPM | BL |
|---|---|---|---|---|---|---|
| AIE Debt Instruments-Short Term | 0.00008 | 0.0000 | 0.0000 | 95.4730 | 5.1861 | 1.5187 |
| AIE Debt Instruments-Long Term | -0.00102 | 0.0000 | 0.0000 | -26.7726 | 0.2194 | 0.1280 |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | -0.00005 | 0.0000 | 0.0001 | -15.0645 | 2.5238 | 0.6470 |
| AIE Stock-Share | -0.00010 | 0.0005 | 0.0009 | -0.2658 | 2.1388 | 3.4566 |
| AIE Precious Metals-Gold | 0.00057 | 0.0002 | 0.0006 | 3.9400 | 0.9681 | 3.3590 |
| Total | | | | 57.3101 | 11.0363 | 9.1094 |

The optimization process continues with the calculation of the portfolio expected excess return, variance and Sharpe ratio.

The last step is to create a configuration with different weight constraints and no short buying or selling. For this, the portfolio weights are on the *w* parameter.

$$\sum_{i=1}^{N} w_i = 1 \quad ve \quad w_i \geq 0 \quad i = 1, 2, 3, \ldots, N \tag{8}$$

It is necessary to provide the conditions.

In the final stage, it will be solved with portfolio weights that maximize this ratio over the criteria that determine the Sharpe ratio will be maximized and there will be no relevant restrictions, i.e. no short buying and selling.

In the Black Litterman model, the portfolio weights observed in the 'BL' column in Table 15 are obtained by reverse optimization, that is, by first determining the weights based on the market values of the relevant assets and calculating the implied expected returns accordingly.

Under the assumption that no restrictions are used in the optimization process and that short buying and selling are possible, the portfolio weightings predicted by the Black Litterman model are as follows; 16.7% of the portfolio will be from investment funds based on the Debt Securities-Short Term fund index. 1.4% of the portfolio will be from investment funds included in the Debt Securities-Long Term investment fund index. 7.1% of the portfolio will be from investment funds included in the Debt Securities-Foreign Debt (Eurobond) investment fund index. While 37.9% of the investment funds linked to the Stocks investment fund index can be included in the portfolio, it is envisaged according to the result of this calculation that 38.9% of the fund participation shares from the investment funds included in the Precious Metals-Gold investment fund index should also be included in the portfolio.

**Table 14.** Portfolio Allocation Calculations and Optimization Process ($W=z/(1^Tz)$)

| Portfolio allocation output | Unlimited weights | | | No short selling weights | | |
|---|---|---|---|---|---|---|
| | Historical | CAPM | BL | Historical | CAPM | BL |
| AIE Debt Instruments-Short Term | 1.6659 | 0.4699 | 0.1667 | 0.2000 | 0.2000 | 0.1667 |
| AIE Debt Instruments-Long Term | -0.4672 | 0.0199 | 0.0141 | 0.2000 | 0.2000 | 0.0141 |
| AIE Debt Instruments-Foreign Debt (Eurobond) Instruments | -0.2629 | 0.2287 | 0.0710 | 0.2000 | 0.2000 | 0.0710 |
| AIE Stock-Share | -0.0046 | 0.1938 | 0.3795 | 0.2000 | 0.2000 | 0.3795 |
| AIE Precious Metals-Gold | 0.0687 | 0.0877 | 0.3687 | 0.2000 | 0.2000 | 0.3687 |
| Total | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | | | | | | |
| Expected excess portfolio return | 0.0007 | 0.0001 | 0.0005 | -0.0001 | 0.0002 | 0.0005 |
| Portfolio variance | 0.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0001 |
| Expected Sharpe ratio | 0.1941 | 0.0387 | 0.0703 | -0.0231 | 0.0361 | 0.0703 |

**Table 15.** Performance Comparison of Official Market Indices and Fund Indices

| Market Index | Unconstrained BL Optimized Weight | Fund Index | Unconstrained BL Optimized Weight | Difference | |
|---|---|---|---|---|---|
| DİBS Short | 0.1819 | AIE Debt Sec.-Short Term | 0.1667 | -0.02 | |
| DİBS Long | 0.0000 | AIE Debt Sec.-Long Term | 0.0141 | 0.01 | |
| Eurobond USDTL | 0.0706 | AIE Debt Eurobond Inst. | 0.0710 | 0.00 | |
| XU100 | 0.3977 | AIE Stocks-Shares | 0.3795 | -0.02 | |
| Gold Weighted Average | 0.3497 | AIE Precious Metals-Gold | 0.3687 | 0.02 | |
| Risk Adjusted | Markte Index | Risk Adjusted | Fund Index | Absolute | Positive / |
| Performance Parameters | Result | Performance Parameters | Result | Difference | Negative |
| n | 251 | n | 251 | 0 | |
| Arithmetic mean | 0.13 | Arithmetic mean | 0.16 | 0.03 | Positive |
| Standard deviation | 0.67 | Standard deviation | 0.77 | 0.11 | Negative |
| Skewness | -0.41 | Skewness | 0.01 | 0.42 | Positive |
| Kurtosis | 1.87 | Kurtosis | 0.30 | -1.57 | Positive |
| Jensen's alpha | -0.04 | Jensen's alpha | -0.11 | -0.07 | Negative |
| Beta | 1.01 | Beta | 1.65 | 0.63 | |
| Sharpe ratio | 0.20 | Sharpe ratio | 0.21 | 0.02 | Positive |
| Sortino ratio | 0.30 | Sortino ratio | 0.36 | 0.05 | Positive |
| Omega | 1.67 | Omega | 1.71 | 0.04 | Negative |
| CPTCE | -0.09 | CPTCE | -0.08 | 0.01 | |
| Maximum drawdown | 4.88 | Maximum drawdown | 4.23 | -0.65 | Positive |
| Information ratio | -0.07 | Information ratio | -0.01 | 0.07 | Negative |
| Treynor ratio | 0.13 | Treynor ratio | 0.10 | -0.03 | Negative |
| M2 | -0.08 | M2 | -0.07 | 0.00 | |

| | | | | | |
|---|---|---|---|---|---|
| T2 | -0.04 | T2 | -0.07 | -0.03 | Negative |
| Stutzer index | 0.00 | Stutzer index | 0.14 | 0.14 | Positive |
| Upside potential ratio | 0.76 | Upside potential ratio | 0.86 | 0.11 | Positive |
| Calmar ratio | 0.03 | Calmar ratio | 0.04 | 0.01 | Positive |
| Value at Risk (VaR) 10 Days | 0.0468 | Value at Risk (VaR) 10 Days | 0.0570 | 0.0102 | Negative |
| End-of-period cumul. return | 0.3996 | End-of-period cumul. return | 0.5044 | 0.1048 | Positive |

Table 16 contains the result evaluation statistics of this research study. Accordingly, under the green column heading of the table, the performance criteria results adjusted according to the risk of a financial investment portfolio created according to the portfolio weights share obtained as a result of the optimization processed in the relevant section of this article are given below the table.

Similarly, when another optimization application carried out with the main asset indices of official market operator institutions was calculated as in the Black Litterman model optimization process we explained above, the portfolio weight ratios seen under the blue heading (at the top of the table) were obtained. Again, risk-adjusted performance measurement values are given below the table.

As shown in this study, it is possible to create very different and meaningful index combinations in the context of investment fund types and types. However, since it is preferred to create two separate comparable examples by keeping the correlation between traditional indices in the markets and investment fund indices high and thus to observe the differences between them, the index types preferred for comparison were selected from the simplest and plainest ones.

**Results**

When the total result of the Black Litterman optimization application performed on two separate composite index groups is examined in Table 16, the following points are seen:

1. The final weights of the optimization process performed on both indices are close to each other. This situation confirms that both groups can be substitutes for each other in the optimization processes. In other words, investment fund indices naturally reflect the trend of the financial markets they are related to.
2. In the comparison of traditional market indices and investment fund indices, it is seen that the investment portfolio optimized on investment fund indices stands out positively in basic performance parameters such as sharpe, sortino, maximum drawdown and stutzer index.
3. In the same comparison process, very limited negative values were obtained as absolute value in performance criteria such as standard deviation, Jensen's alpha, information ratio, Treynor and t2.

4. The fact that the numerical values in the comparison table of both results and performances are close to each other shows that we are faced with two different index types that are comparable.
5. Despite the limited change in the negatively developing performance criteria, the portfolio's risk-exposed value also increased very limitedly, whereas in the end-of-period cumulative return, a portfolio optimized over an investment fund index increased by 10.01% annually due to the minor improvement in the distribution ratios compared to a portfolio optimized over traditional indices and the indices' own internal performance. While a portfolio based on traditional main asset indices achieved a return of 39.96%, a fund based on a similar logic but based on investment funds achieved a performance of 49.97%.

**Discussion**

The aim of this study is to expand the current application area of the Black Litterman method over the investment fund market. An optimization approach is presented for portfolios investing in investment funds. The indices used for this are investment funds indices.

In this approach, which is presented for a portfolio in the fund basket structure, i.e. portfolios managed through fund participation shares investment vehicles, the need to address the end-to-end process from an investment fund perspective arises. In other words, investment funds can be purchased according to expectations regarding the main asset classes. However, when the internal dynamics of investment funds, i.e. their unique performances, are also considered in an optimization process, it is necessary to address the current financial market expectations and the dynamics of the investment fund market together in the process of building a portfolio structure appropriate for the targeted risk and expected return.

This joint consideration mechanism is also possible by modeling and indexing the investment fund market as the main assets. Thus, it becomes possible to create portfolios appropriate for the desired risk and return structure in an investment fund portfolio. Because, to reiterate, since investment funds do not always have a single investment instrument in their portfolios, apart from their own performance dynamics, it becomes difficult to estimate the performance of the relevant fund.

The selection of investment funds to be included in the portfolio according to the portfolio weights obtained after an optimization process carried out with a selected group of investment fund indices will be carried out after this stage. The important point here is that the investment funds to be included in the portfolio are of course included in these relevant indices. However, since the participation shares of all related investment funds cannot be included in the fund, the selection of investment funds to be included in the portfolio in terms of performance is also important here, and it is the point that investment funds with an expected return lower than the average of the index should not be included in the portfolio. In addition, there is a possibility that a portfolio in which funds with an expected return above the average of the index will be included will perform above the expected value of the optimization result.

On the other hand, the production of a composite index, customized and detailed according to the type, theme and type of investment funds emphasized in the application section of this study, is shown through the example of the health sector fund index. Similarly, for example, indexes can be created based on investment themes such as electric vehicles, clean energy, sustainability, and these can be used to optimize portfolios when necessary. Thus, it will be possible to develop different portfolio derivatives focused on investment themes in robot funds and autonomously managed fund basket portfolios or pension funds.

**Declaration of Interest**

The authors declare that they have no known financial interests or personal relationships that could have influenced the work reported in this paper.

**Ethical Approval**

Ethics committee approval is not required for this study, as the research did not involve data collection from human participants or animals, did not include any experimental intervention, and was conducted using publicly available secondary sources.

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