Çankaya Üniversitesi Fen-Edebiyat Fakültesi,
Journal of Arts and Sciences Sayı: 9 / Mayıs 2008

Calculation of Signal Sources Coordinates In 2D And 3D Space

Yuriy Alyeksyeyenkov[1]

Abstract

Methods of calculations of coordinate of signal sources which are inaccessible and direct measurement of their properties is impossible are analyzed. Cases of 2D – 3D spaces and single – multiple sources of signal are investigated separately. Recommendations and necessary mathematical equations for practical applications are given. Advantage of sources coordinate measurement using signal parameters in frequency domain is shown. Experimental analysis of methods on the base of MATLAB modeling demonstrates full coincidence of theory and practice.

Introduction

In many practical cases analyzing of properties and parameters of system there is no access to the source of a signal arising inside of it. An example of such system is the automobile engine in which it is necessary to find out signal sources and the reasons of non-standard behavior that can be analyzed only when it works. In medicine is actual research of signals of lungs and heart of the patient for carrying out correct diagnostics without direct access to a source of these signals. Determining coordinates of signal source locations can considerably facilitate process of its research and diagnostics.

Let the signal from such an object acts on sensor-receivers and further be applied to the input of the system analyzing it. Typically, such signal can have any form and from point of occurrence extends in all directions, achieving sensor with a various time delay which is defined by speed of distribution of a signal in the environment and distance

1 Computer Engineering Department, Cankaya University, Ankara

from a source up to a sensor. Absence of well-defined borders of signals, not periodicity, complicates carrying out of measurements in time. Since the difference of time delays between various sensors is very small, these differences cannot be measured with a necessary degree of accuracy. At the same time, carrying out the analysis of these signals in frequency domain, it is possible to obtain a necessary data set for exact identification of origination point of a signal. The case of systems of the limited physical size in which there is no attenuation of a signal at its distribution from a source up to the receiver is examined. Let a signal $x_a(t)$ be registered by a sensor placed in a point *A*. A signal $x_b(t)$, accepted by a sensor in a point *B* is the copy of this signal delayed in time

$$x_b(t) = x_a(t - \Delta t_b),$$

Where Dt_b - time delay of a signal in a point *B* with respect to point *A*. By the same way the signals in other points of receiver locations concerning same point *A* can be presented.

Having applied Fourier transform [1], we shall determine spectral representation of the signal received by sensors. Connection between time and frequency representation of a signal at point *A* we shall designate as

$$X(j\omega) \overset{F}{\Leftrightarrow} x_a(t)$$

The spectrum of a signal received at point *B* in conformity with time shifting property can be written in the form of

$$X(j\omega)e^{j\omega\Delta t_b} \overset{F}{\Leftrightarrow} x_a(t - \Delta t_b),$$

where the factor $e^{j\omega\Delta t_b}$ specifies phase shift of each spectral component of a signal at point *B* concerning a signal accepted at point *A*. The solution of a problem of the signal sources coordinates determining is possible, if the phase shift satisfies $\varphi_b = \omega\Delta t_b < 2\pi$ for all spectral components.

The phase difference of signals measured between sensors is proportional to a difference of distances from a point of occurrence of a signal up to a point of its registration by sensors. Connection between phase shift and a time delay can be established in the form of equation

$$\lambda = \frac{2\pi v}{\omega},$$

where λ – length of a wave at frequency ω, *v* – speed of signal distribution in the environment. Designating a difference of distances from a signal source up to sensors *A* and *B* as Δl_{ab}, and having in mind, that this difference is less than length of a wave, it is possible to define connection of linear distance differences with a time delay of signal distribution as

$$\Delta l_{ab} = \frac{1}{2\pi}\lambda\omega\Delta t_{ab} = \frac{1}{2\pi}\lambda\Delta\varphi_{ab}, \quad (1)$$

where $\Delta\varphi_{ab}$ hase shift of a signal on frequency w and Δt_{ab} time shift of a signal at a point of measuring *B* with respect to point *A*.

Here all linear differences, time delays and phase shifts are measured concerning point *A*. Thus, measurement of time delays between channels in time domain can be replaced by measurement of phase shifts between channels in frequency domain. As points of location of sensors are fixed, also their coordinates are known, the problem is reduced to determining of a point of location of a signal source on the basis of the phase shifts analysis between sensors. We shall examine various cases of the decision of this problem.

Measuring of signal source coordinates in 2D space.

For measuring the source of a signal coordinates a in two-dimensional spaces two sensors-receivers of signals are necessary, however in case of equal remoteness of a source from sensors, phase differences at all frequencies will be equal to zero that leads to indefiniteness of the decision.

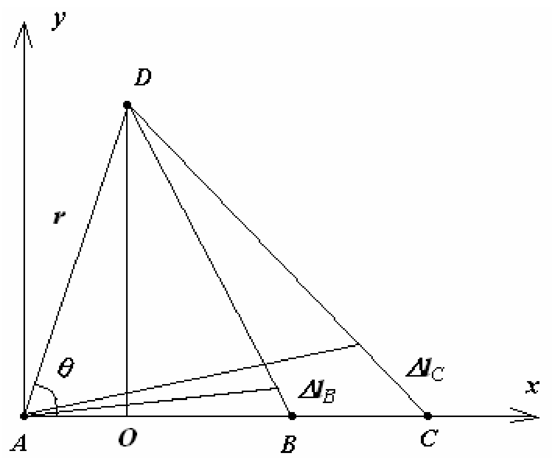

Figure 1. *Measuring of signal source coordinates in 2D space.*

Here the source of signal can be at any distance from sensors on a line of a perpendicular constructed in the middle of a line connecting sensors. Hereinafter we shall make on attention to the mathematical decisions having a positive sign that specifies location of a signal source inside the investigated environment while the solution with a negative sign defines a symmetric place of a source location, but from the external side with respect to the object. To eliminate this indefiniteness we add an additional third sensor-receiver of a signal C (Fig. 1).

The source of a signal is placed on a plane in point *D* which coordinates are necessary to determine. Three sensors are on an axis *x* and have coordinates *A* (0, 0), *B* (d_1, 0) and *C* (d_2, 0). Entering designations Dl_1=*DB-DA* and Dl_2=*DC* - *DA*, having designated distance between points *D* and *A* as *r* and a angle of an inclination of line *AD* with respect to axis *x* as *q*, it is possible to write down a number of obvious equations

$$AO2 + OD2 = AD2$$

$$OB = AB - OA = d_1 - r\cos\theta$$

$$DB^2 = (d_1 - r\cos\theta)^2 + r^2\sin^2\theta = d_1^2 + r^2 - 2rd_1\cos\theta$$

As *DB-DA* = Δl_1 and

$$\sqrt{d^2 + r^2 - 2rd\cos\theta} - r = \Delta l_1,$$

we can write

$$d^2 - 2rd\cos\theta = 2\Delta l_1 r + \Delta l_1^2$$

Or in other form for triangle ADB it is possible to write down

$$2(d_1\cos\theta + \Delta l_1)r = d_1^2 - \Delta l_1^2 \qquad (2)$$

Similar expression for triangle *ADC* looks like

$$2(d_2\cos\theta + \Delta l_2)r = d_2^2 - \Delta l_2^2 \qquad (3)$$

Carrying out division of (2) on (3) and finding the solution with respect to $\cos\theta$ we have

$$\cos\theta = \frac{\Delta l_2(d_1^2 - \Delta l_1^2) - \Delta l_1(d_2^2 - \Delta l_2^2)}{d_1(d_2^2 - \Delta l_2^2) - d_2(d_1^2 - \Delta l_1^2)} = f$$

From which, the value we need

$$\theta = \arccos f$$

To find *r*, we multiply expression (2) on d_2, expression (3) on d_1 and then we shall find their difference

$$2[(d_1 d_2\cos\theta + \Delta l_1 d_2) - (d_1 d_2\cos\theta + \Delta l_2 d_1)]r = (d_1^2 d_2 - \Delta l_1^2 d_2) - (d_2^2 d_1 - \Delta l_2^2 d_1)$$

Making obvious transformations and solving equation for *r* gives

$$r = \frac{d_2(d_1^2 - \Delta l_1^2) - d_1(d_2^2 - \Delta l_2^2)}{2(\Delta l_1 d_2 - \Delta l_2 d_1)}$$

Measuring of signal source coordinates in point D (x_0, y_0) is now reduced to calculation of

$$x_0 = r\cos\theta$$

$$y_0 = r\sin\theta$$

Measuring of signal source coordinates in 3D space.

Let four receivers of a signal are on a plane *xy* at distance *d* from each other and have coordinates in three-dimensional space (0,0,0), (*d*, 0,0,), (0, *d*, 0) and (*d*, *d*, 0) accordingly, and the source of a signal is removed from sensors in space on coordinate *z*. Let the coordinates of a source of the signal which is being at arbitrary point $A(x_0, y_0, z_0)$ be necessary to determine (Figure 2). As well as in the previous case, we consider the solution with the positive sign, specifying on an arrangement of a source of a signal inside of investigated system.

The distance between base sensor in a point of origin and point *A* is

$$\rho = \sqrt{x_0^2 + y_0^2 + z_0^2},$$

and differences of distances between point *A* and points of location of sensors with respect to base are

$$\begin{aligned}
\Delta l_1 &= \sqrt{(x_0 - d)^2 + y_0^2 + z_0^2} - \rho \\
\Delta l_2 &= \sqrt{x_0{}^2 + (y_0 - d)^2 + z_0^2} - \rho \\
\Delta l_3 &= \sqrt{(x_0 - d)^2 + (y_0 - d)^2 + z_0^2} - \rho
\end{aligned} \tag{4}$$

Coordinates of an investigated point *A* with use of spherical coordinates can be defined as

$$\begin{aligned}
x_0 &= \rho \sin\alpha \cos\theta \\
y_0 &= \rho \sin\alpha \sin\theta \\
z_0 &= \rho \cos\alpha = \sqrt{\rho^2 - x_0^2 - y_0^2}
\end{aligned} \tag{5}$$

Expressions (4) in view of (5) can be transformed to a form

$$\begin{aligned}
\sqrt{\rho^2 - 2dx_0 + d^2} &= \Delta l_1 + \rho \\
\sqrt{\rho^2 - 2dy_0 + d^2} &= \Delta l_2 + \rho \\
\sqrt{\rho^2 - 2d(x_0 + y_0) + d^2} &= \Delta l_3 + \rho
\end{aligned} \tag{6}$$

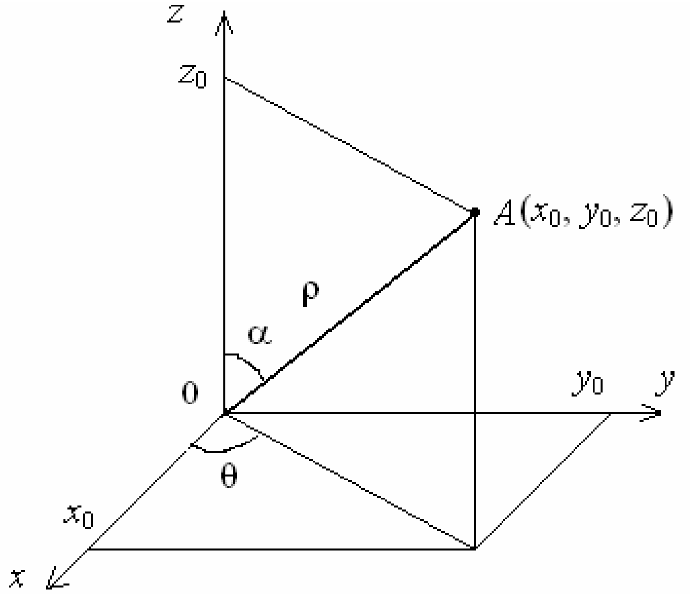

Figure 2. *A projection of arbitrary point $A(x_o, y_o, z_o)$ on coordinate axis.*

and expressions (6) can be written down as

$$d^2 - \Delta l_1^2 = 2\rho(\Delta l_1 + d\sin\alpha\cos\theta) \tag{7}$$

$$d^2 - \Delta l_2^2 = 2\rho(\Delta l_2 + d\sin\alpha\sin\theta) \tag{8}$$

$$2d^2 - \Delta l_3^2 = 2\rho(\Delta l_3 + d\sin\alpha\sin\theta + d\sin\alpha\cos\theta) \tag{9}$$

Making subtraction (7) and (8) from (9) gives

$$\Delta l_1^2 + \Delta l_2^2 - \Delta_3^2 = 2\rho(\Delta l_3 - \Delta l_1 - \Delta l_2)$$

whence we find value of the vector module *r* expressed in differences of distances

$$\rho = \frac{1}{2}\frac{(\Delta l_1^2 + \Delta l_2^2 - \Delta l_3^2)}{(l_3 - \Delta l_1 - \Delta l_2)} \tag{10}$$

Doing further transformations, we shall preliminary determine difference of expressions (9) and (7), having written down

$$d^2 + \Delta l_1^2 - \Delta l_3^2 = 2\rho(\Delta l_3 - \Delta l_1 + d\sin\alpha\sin\theta) \tag{11}$$

Now executing cross multiplication of expressions (7) and (8) and, making similar actions with expressions (7) and (11) we obtain system of equations

$$(d^2-\Delta l_1^2)(\Delta l_2+d\sin\alpha\sin\theta)=(d^2-\Delta l_2^2)(\Delta l_1+d\sin\alpha\cos\theta) \tag{12}$$

$$(d^2-\Delta l_1^2)(\Delta l_3-\Delta l_1+d\sin\alpha\sin\theta)=(d^2+\Delta l_1^2-\Delta l_3^2)(\Delta l_1+d\sin\alpha\cos\theta) \tag{13}$$

Making division (13) on (12) we get

$$\frac{\Delta l_3-\Delta l_1+d\sin\alpha\sin\theta}{\Delta l_2+d\sin\alpha\sin\theta}=\frac{d^2+\Delta l_1^2-\Delta l_3^2}{d^2-\Delta l_2^2} \tag{14}$$

After simplification and solving (14) for $\sin\alpha\sin\theta$ we obtain

$$\sin\alpha\sin\theta=\frac{d^2(\Delta l_1+\Delta l_2-\Delta l_3)+\Delta l_3\Delta l_2^2+\Delta l_2\Delta l_1^2-\Delta l_1\Delta l_2^2-\Delta_2\Delta l_3^2}{d(\Delta l_3^2-\Delta l_1^2-\Delta l_2^2)},$$

which in view of (10) can be written down in the form of

$$\sin\alpha\sin\theta=\frac{d}{2\rho}+\frac{\Delta l_3\Delta l_2^2+\Delta l_2\Delta l_1^2-\Delta l_1\Delta l_2^2-\Delta_2\Delta l_3^2}{d(\Delta l_3^2-\Delta l_1^2-\Delta l_2^2)} \tag{15}$$

By analogy to actions made above, having determined a difference of expressions (9) and (8), we get

$$d^2+\Delta l_2^2-\Delta l_3^2=2\rho(\Delta l_3-\Delta l_2+d\sin\alpha\cos\theta) \tag{16}$$

Now making cross multiplication of expressions (8) and (16), having writing down

$$(d^2-\Delta l_2^2)(\Delta l_3-\Delta l_2+d\sin\alpha\cos\theta)=(d^2+\Delta l_2^2-\Delta l_3^2)(\Delta l_2+d\sin\alpha\sin\theta) \tag{17}$$

and having carried out division of expression (17) on expression (12) gives

$$\frac{(\Delta l_3-\Delta l_2+d\sin\alpha\cos\theta)}{(\Delta l_1+d\sin\alpha\cos\theta)}=\frac{(d^2+\Delta l_2^2-\Delta l_3^2)}{(d^2-\Delta l_1^2)} \tag{18}$$

Solving (18) for product of trigonometrical functions $\sin\alpha\cos\theta$ we obtain expression

$$\sin\alpha\cos\theta=\frac{d}{2\rho}+\frac{\Delta l_3\Delta l_1^2+\Delta l_1\Delta l_2^2-\Delta l_2\Delta l_1^2-\Delta_1\Delta l_3^2}{d(\Delta l_3^2-\Delta l_1^2-\Delta l_2^2)} \tag{19}$$

Considering (10) and making substitution (15) and (19) in (5), required expressions for determining of geometrical coordinates of a signal source in three-dimensional space is obtained

$$x_0 = \frac{d}{2} + \frac{\rho}{d}\frac{(\Delta l_3 \Delta l_1^2 + \Delta l_1 \Delta l_2^2 - \Delta l_2 \Delta l_1^2 - \Delta_1 \Delta l_3^2)}{(\Delta l_3^2 - \Delta l_1^2 - \Delta l_2^2)} \quad (20)$$

$$y_0 = \frac{d}{2} + \frac{\rho}{d}\frac{(\Delta l_3 \Delta l_2^2 + \Delta l_2 \Delta l_1^2 - \Delta l_1 \Delta l_2^2 - \Delta_2 \Delta l_3^2)}{(\Delta l_3^2 - \Delta l_1^2 - \Delta l_2^2)} \quad (21)$$

$$z_0 = \sqrt{\rho^2 - x_0^2 - y_0^2} \quad (22)$$

Where all linear differences Δl_i, according to (1), can be presented as phase shifts $\Delta\varphi_i$ calculated from the spectral representation of a signal in frequency domain.

Measuring of signal sources array coordinates in 3D space.

Let the source of a signal be not a dotted, but complex object with an uncertain place of location of signal sources $y_k(t)$, placed in points A_k, where k – is a number of a source of a signal ($0 < k < M-1$) and M - quantity of signal sources. Their parameters and geometrical coordinates can be determined by receiving a signal with the array of the sensors, placed on a plane xy. In [2] the measuring system with two sources of a signal was investigated. Here we consider system with arbitrary number of sources of a signal. Let the sources of a signal be placed inside of investigated object and produce the signals that are distributed in all directions. As well as in previously considered cases we think that sensors are on insignificant distance from a source and no attenuation on amplitude of the signal received by each sensor. Let sensors array of size $N \times N$ form a square, inside of which these sensors are placed with linear step d. To denote position of each sensor we shall use a notation $S_{i,j}$, where $0 < i, j < N-1$. Geometrical coordinates of each sensor will be denoted as $S_{i,j}(id, jd, 0)$.

The signal received by a sensor $S_{i,j}$ is superposition of partial signals from all sources, and this signal can be described by equation

$$x_{i,j}(t) = \sum_{k=0}^{M-1} y_k(t - \Delta t_k^{i,j})$$

where $\Delta t_k^{i,j}$ - a time delay of signal distribution from k-th source to $S_{i,j}$ sensor.

The spectrum of a signal received by each sensor can be presented in form

$$X_{i,j}(j\omega) = \left|X_{i,j}(j\omega)\right| e^{j\varphi_{i,j}(\omega)} = \sum_{k=0}^{M-1} Y_k(j\omega) e^{j\omega\Delta t_k^{i,j}} \overset{F}{\Leftrightarrow} x_{i,j}(t)$$

Then, the module and phase of a signal spectrum can be presented in the form

$$\left|X_{i,j}(j\omega)\right| = \left|\sum_{k=0}^{M-1} Y_k(j\omega) e^{j\omega\Delta t_k^{i,j}}\right|$$

$$\varphi_{i,j}(\omega) = \arctan\left(\frac{\mathrm{Im}(\sum_{k=0}^{M-1} Y_k(j\omega) e^{j\omega\Delta t_k^{i,j}})}{\mathrm{Re}(\sum_{k=0}^{M-1} Y_k(j\omega) e^{j\omega\Delta t_k^{i,j}})}\right)$$

The phase differences between two sensors in points i, j and c, e ($0 < c, e < N-1$) at any frequency ω we shall define in the form

$$\Delta\theta_{(i,j)(c,e)}(\omega) = F(\Delta t_k^{i,j}, \Delta t_k^{c,e}) = \varphi_{i,j}(\omega) - \varphi_{c,e}(\omega)$$
$$= \arctan\left(\frac{\mathrm{Im}(\sum_{k=0}^{M-1} Y_k(j\omega)e^{j\omega\Delta t_k^{i,j}})}{\mathrm{Re}(\sum_{k=0}^{M-1} Y_k(j\omega)e^{j\omega\Delta t_k^{i,j}})}\right) - \arctan\left(\frac{\mathrm{Im}(\sum_{k=0}^{M-1} Y_k(j\omega)e^{j\omega\Delta t_k^{c,e}})}{\mathrm{Re}(\sum_{k=0}^{M-1} Y_k(j\omega)e^{j\omega\Delta t_k^{c,e}})}\right)$$

where the difference of phases of received signal $\Delta\theta_{(i,j)(c,e)}(\omega)$, measured between sensors in points i, j and c, e are components of the signals received from sources A_k. Creating system of the linear equations for all possible pairs of sensors and solving them for phases of the signals received from each source at all frequencies, we shall have necessary parameters $\Delta t_k^{i,j}$ and $\Delta t_k^{c,e}$, that will be used for determining of geometrical coordinates of signal sources. The system has solution if the relation $(N \times N) > M$ is satisfied. The subsequent definition of linear differences according to (1) between each pair of sensors at any points of their location i, j and c, e allows use of (20-22) to define the geometrical coordinates of each source of signal A_k. Another advantage of this method is that spectrum structure of each internal source of the signal is determined, that can be useful for future analysis of system behavior.

Experimental modeling of algorithms.

The algorithms of measuring of coordinates of a signal source in 2D and 3D space were investigated on a program level by using MATLAB.

Parameters of models were chosen so, that phase shifts for any of examined case did not exceed 2p. For cases of determining of coordinates in 2D and 3D space, the distance from sensors up to a source of a signal was within the limits of 1 – 10 cm, speed of distribution of a signal was in range of $300 < v < 500$ m/s, the frequencies of a modeled signals were in range from Hz up to units of kHz.

Measuring of geometrical coordinates of signal sources array was researched by using the model of sensors system of $N = 4$, at amount of sources of signals $M = 10$.

Full concurrences of the modeled and calculated geometrical coordinates were received for all variations of researched parameters.

REFERENCES

Alan V. Oppenheim and Alan S. Willsky, Signals and Systems, Prentice Hall, 1997, 957 p.

Yu. F. Alyeksyeyenkov and L. P. Dyuzhayev, Shaping the acoustic images of objects, Radioelectronics and Communications Systems, Vol.48, #11, 2005, Allerton Press.