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A dynamic instability study of shallow shell panels with simply supported edges

Basitçe desteklenen kenarlara sahip sığ kabuk panellerin dinamik kararsızlık çalışması

***Yazar(lar) (Author(s))**: Chitaranjan PANY* [1]

ORCID[1]: 0000-0001-8617-2134

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A Dynamic Instability Study of Shallow Shell Panels with Simply Supported Edges

Highlights

- *The evaluation of dynamic instability feature cylindrical shallow shell panelis is crucial in the design of aerospace structures in supersonic regime. When a shallow shells thin panel is impacted by supersonic flow on one of its sides, the combined effect of panel inertia forces, elastic restorative forces and supersonic airflow causes some self-excited dynamical instabilities in a particular dynamic pressure. The critical air flow at which this instability arises is influenced by the virgin curvature and panel stiffness, the density ratio of air/fluid to panel, panel's sizes, and force that plate's edge supports exerted.*
- *Dynamic instability of flat and shallow shells curved plates with simply-supported edge boundary conditions(SSSS) is investigated using efficient, high-precision triangular shallow shell finite elements(FE). Dynamic instability grows with increasing curvature parameter (in the lower range) for the SSSS panels.*
- *It has been demonstrated that dynamic instability limits are sensitivity to in-plane edge constraints and panel configurations.*

Graphical Abstract

In this study the dynamic instability of cylindrical shallow shell panel(Figure A) subjected to supersonic flow along generator is carried out for SSSS edge boundary conditions and various curvature parameters.

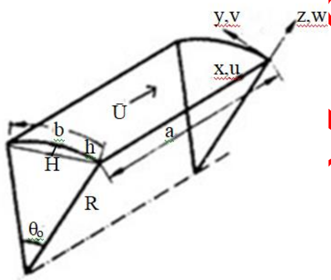

Figure. Curved panel subjected to supersonic flow (h is thickness, H is maximum shell rise height from base plane, a is length, b is curved length, Radius of curved panel is R).

Aim

It aims to find the dynamic instability results (critical dynamic pressure parameter) of flat and curved plates with different edge boundary conditions subjected to supersonic flow.

Design &Methodology

Efficient, high-precision triangular shallow shell finite elements of Cowper has been used and coded to find the dynamic instability results of flat and curved plates with SSSS edge boundary conditions.

Originality

The proposed shallow-shell FE formulation is applied to cylindrical curved plates with SSSS edges subjected to supersonic flow(along generator) to demonstrate the limited dynamic instability results, corresponding to different curvature parameters available in the literature for different FE formulations.

Findings

It is found that the dynamic instability limit of the panels grows with an increase in the curvature parameter (in the lower range) of the panels with SSSS edge constraint. It was figured out that in-plane edge constraints had an impact on dynamic instability limits.

Conclusion

The dynamic instability limits are evaluated corresponding to various curvature parameters and found it grows with increasing curvature parameter (in the lower range) for the SSSS panels. It has been demonstrated that dynamic instability limits are sensitivity to in-plane edge constraints and panel configuration, indicating important design considerations for improving panel stability, which is crucial for applications in aerospace engineering and related fields. The existing literature and recent findings demonstrate a comparable trend and align effectively.

Declaration of Ethical Standards

The materials and methods used in this study do not require ethical declaration.

A Dynamic Instability Study of Shallow Shell Panels with Simply Supported Edges

Araştırma Makalesi / Research Article

Chitaranjan PANY[1*]

[1]Structural Engineering Entity, Vikram Sarabhai Space Center, Trivandrum, India

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ABSTRACT

An efficient, general finite element (FE) of triangular shape is employed in investigating the panels (flat and shallow shell curved panels) dynamic instability subjected to supersonic air flow. The all edges of panel are simply-supported (SSSS). The fluid that's on the plate's bottom is not in motion. Linearized piston theory is used to determine aerodynamic loads. Hamilton's principle is used to generate the dynamic instability motion equations, which characterize panels instability during their interaction with the supersonic the airflow. The eigenvalues of Lagrange's motion equation are obtained by conventional methods. Here, aerodynamic damping is excluded. In panels, the basis of thin, very small deformation shells is considered. Critical dynamic pressure was determined. The FE code is corroborated by examining outcomes of a flat square panel (radius of curvature R of shallow shell tends to ∞) with the literature data. In addition, the proposed shallow-shell FE is applied to cylindrical curved plates with SSSS boundary conditions to demonstrate the limited dynamic instability results, corresponding to different curvature parameters available in the literature for different FE formulations. Results and data from the literature compare fairly. It is clear that the dynamic instability limit of the panels grows with an increase in the curvature parameter (in the lower range) of the panels with SSSS edge constraint. It was figured out that in-plane edge constraints had an impact on dynamic instability limits.

Keywords: Cylindrical Curved Panel, Dynamic instability, Finite Element Method

Basitçe Desteklenen Kenarlara Sahip Sığ Kabuk Panellerin Dinamik Kararsızlık Çalışması

ÖZ

Süpersonik hava akışına maruz kalan panellerin (düz ve sığ eğri kabuk paneller) dinamik kararsızlığının araştırılmasında üçgen şekilli, etkin ve genel bir sonlu eleman (SE) kullanılmıştır. Panelin tüm kenarları basit mesnetlidir (SSSS). Panel altındaki akışkan hareket halinde değildir. Aerodinamik yüklerin belirlenmesinde lineerleştirilmiş piston teorisi kullanılmıştır. Süpersonik hava akışıyla etkileşim sırasında panellerin kararsızlık davranışını tanımlayan dinamik kararsızlık hareket denklemleri, Hamilton prensibi kullanılarak türetilmiştir. Lagrange hareket denkleminin özdeğerleri geleneksel yöntemlerle elde edilmiştir. Bu çalışmada aerodinamik sönümleme dikkate alınmamıştır. Paneller için ince, çok küçük şekil değiştiren kabuklar esas alınmıştır. Kritik dinamik basınç belirlenmiştir. SE kodu, düz kare panelin (sığ kabuğun eğrilik yarıçapı R sonsuza yaklaştığında) sonuçları ile literatürdeki veriler karşılaştırılarak doğrulanmıştır. Ayrıca, önerilen sığ kabuk SE yöntemi, SSSS sınır şartlarına sahip silindirik eğrilikli plakalara uygulanmış ve farklı SE formülasyonları için literatürde mevcut olan çeşitli eğrilik parametrelerine karşılık gelen sınırlı dinamik kararsızlık sonuçları ortaya konmuştur. Elde edilen sonuçlar, literatürle oldukça uyumludur. SSSS kenar kısıtına sahip panellerde, eğrilik parametresinin (düşük aralıkta) artmasıyla birlikte dinamik kararsızlık sınırının da arttığı açıkça görülmüştür. Ayrıca, panel kenarlarındaki düzlem içi sınır şartlarının dinamik kararsızlık sınırları üzerinde etkili olduğu belirlenmiştir.

Anahtar Kelimeler: Silindirik Eğri Panel, Dinamik istikrarsızlık, Sonlu Elemanlar Yöntemi

1. INTRODUCTION

The dynamic instability feature evaluation of shallow shells is crucial in the design of rocket and supersonic airplane. When a thin panel is impacted by supersonic flow on one of its sides, the combined effect of panel inertia forces, elastic restorative forces and supersonic airflow causes some self-excited dynamical instabilities in a particular dynamic pressure. The critical air flow at which this instability arises is influenced by the virgin curvature and panel stiffness, the density ratio of air/fluid to panel, panel's sizes, and force that plate's edge supports exerted.

Chai et al.[1] have presented a comprehensive review of aeroelastic analysis and flutter (dynamic instability) domination to improve the stability of aeroplane structures. Amirzadegan S and Dowell EH [2] calculated the flutter (dynamic instability) and later-stabilized oscillatory behavior of a resilient shallow shells in the presence of supersonic gas flow. Galerkin method and piston theory are used to study the nonlinear oscillatory behavior of a thin isotropic curved panel (rectangular) in order to determine the flutter velocity and the limit-cycle oscillatory behavior beyond flutter limit. The panel vibration frequency and flutter dynamic pressure are presented for several key parameters (aspect ratio,

**Sorumlu Yazar (Corresponding Author)*
e-posta :c_pany@yahoo.com

different edge constraints, pressure due to fluid under rest and in-plane structural load). These parameters are dominant for flutter(dynamic instability) and limit-cycle oscillatory behavior of curved plates. The experimental and numerical results are examined.

In-plane constraints on edges have a significant effect on the flutter limits. The Panel flutter, buckling and vibration intensity in plate elasticity at supersonic speed investigated[3]. Numerical studies illustrate that the initial in-plane forces have a considerable impact on the plate's flutter speed. Stretching tensile stress in normal to the flow direction will reduce the dynamic instability of the plate/panel's whereas compressive longitudinal stress will be enhancing it. Hamid and Mohammadi[4] have studied the nonlinear dynamic instability characteristics for uniform simply-supported curved panel/plate subjected to thermal loads and supersonic aerodynamic flow based on piston theory at the top of the plate assuming cylindrical bending. They demonstrate that the plate's instability boundary under supersonic aerodynamic force is significantly influenced by the plate's curvature. The flutter boundary increases with increasing curvature and decreases with increasing thermal load. Bimarck-Nasr [5] provided the finite element (FE) dynamic instability study of the shallow plate(cylindrical) exposed to fluid flow (supersonic)using Reissner two field variational approach. Utilizing FSDT and DQM with linearized piston theory, Hassan and Yusef [6] examined the aeroelasticity behavior (dynamic instability) of CNT polymeric conical shells expose to fluid flow (supersonic).

The utilization of PS(periodic structure) approach using the FE method (FEM) highlights the latest advancements in structural mechanics as well as theoretical and more practical research[7].The problem of vibration and free wave propagation have been studied and generated dispersion relations in the periodic flat panels [8], axially line-supported infinite curved panels [9], orthogonal line-supported curved panels[10] using the PS theory[11] and Cowper's high precision triangular FEM[12]. PS theory has been applied to un-supported cylindrical shell[13]-[14]and circular ring[15] with exact approach. The aforementioned theory has also been applied to circumferentially line-supported periodic shell using beam function with Rayleigh-Ritz method[16]. Natural vibration frequencies of panel(curved) having different edge constraints have been estimated [17] using FEM[12]. Flutter analysis of isolated flat and curved panels are presented using shallow shell FEM[12] for different edge constraints[18]. Utilising FEM [12] and PS theory[9], free wave propagation of an axially periodic supported curved shell subjected to supersonic flow along generator is performed [19]. The 2D viscous gas flows (supersonic) with boundary layers, the flutter of various curved surfaces are computationally simulated [20]. This situation might adversely affect the flutter of an elastic/flexible plate. Non-linear aero-thermo-elastic flutter instability analysis using piston theory has been given by Hamid and Mohammadi[21] for the two-dimensional(2D) curved panels under the influence of in-plane load and thermal load. The non-linear structural frequency changes exhibit complex behavior as the panel curvature increases. Complex oscillatory behavior is seen when in-plane compressive stress increases. The stability of a rectangular panel in a fluid flow (supersonic) with an uneven temperature field across its thickness is investigated [22]. In order to address the instability (flutter) of the electromagnetic-thermoelastic FG panels/plates expose to airflow (supersonic) with yawed angles, Zhong et al.[23] developed an exhaustive method based on FSDT and physical spring technology.They endorsed piston theory and the Rayleigh-Ritz technique. Next, a few tests and numerical examples are used to demonstrate the stability. Bolotin's analytical technique has been proposed to be extended to the examination of composite multi-layered panels for the likely incoming flow by Muc and Flis [24]. It is shown how easy it is to apply that methodology to the optimal laminated plate analysis in the pursuit of the maximum aerodynamic pressure. Boundary conditions, stacking sequences, orthotropic modulus ratio, in-plane mechanical and thermal loads are all investigated analytically. Using comparable single layer and layer-wise FE models with variable-order shear deformation theories, Moreira et al.[25] examined panel flutter and buckling of variables stiffness laminated pre-stressed panels with simply-supported (SSSS) edges. It is explained how to develop multilayered composite structures (flat and cylindrical curved plates) utilizing semi-analytical and FEA to examine the aforementioned characteristics. It was found that the flutter (dynamic instability) limit greatly dependent on the edge constraints [26]. Xie F et al. [27] presented a general high-order zigzag theory with shear deformation to predict the nonlinear (structural nonlinearity using von Karman strain) aero-thermo-elastic properties for the laminated-composite panels exposed to supersonic air flow (first-order piston theory). Varying sizes of composite panels, thermal ramps, and filament angles are investigated to assess their critical buckling and limit-cycle oscillatory behavior. Using a NACA 0015 airfoil, the aeroelastic analysis with nonlinear energy sinks (NES) is carried out experimentally [28].Panel flutter is suppressed and the intensity of limit-cycle oscillatory behaviour is decreased [29] through dissipating energy using the NES approach. Flutter analysis of beams and panels built of FG materials was presented by Song et al. [30]. Large amplitude fundamental frequencies of uniform and tapered cantilever beam are evaluated[31]. Literature review on the best design of plate and shell structures subject to flutter boundaries has been summarized by Muc et al. [32]. Myrella et al.[33] investigated the non-linear aero-elastic behavior of multi-bay curved composite panels/plates exposed to cross-fluid flow (supersonic). Using an eight-noded shell FE, Cagdas and Adali[34] investigated the impact of fiber orientation on the failure load of a laminated curved panel under uniaxial compression with a mix of simply-supported and clamped boundary conditions. It is noted that the rotational constraints at the curved edges significantly influence the failure load.Cagdas and Adali[35] presented the optimal design of a simply- supported variable curvature laminated angle-ply composite panel subjected to uniaxial compression by employing a shear deformable degenerated shell FE.The findings indicate that the load carrying capability of thicker panels is significantly diminished when the first-ply failure (FPF) constraint is considered.Cagdas[36] provided optimal designs for laminated composite truncated cones with variable stiffness subjected to lateral external pressure by utilizing a semi-analytical degenerated shell FE based on FSDT. The numerical findings shown indicate that restricting the large end from rotation greatly boosts the failure load.The aim of [35,36] is to maximize the failure load, which is defined as the lesser of the buckling load and the FPF load.

It has been seen that aerospace industry uses cylindrical shallow shells with a square or rectangular planform in addition to flat plates. Cowper et al. [12] developed a shell element formulation that is specifically tailored for shallow shell structures. Therefore, in this work Cowper's[12] shallow shell FE (triangular element with 12 dof (degrees of freedom) per node) used to determine the dynamic instability characteristics for the flat and cylindrical shallow panels (isotropic) with SSSS edges in the supersonic air flow. Here, the linearized piston theory-based supersonic flow is integrated to the present shell finite element. The novelty of the work is as follows. The proposed shallow-shell FE[12] formulation is applied to SSSS edges cylindrical curved plates subjected to supersonic flow(along generator) to demonstrate the limited dynamic instability results, corresponding to different curvature parameters available in the literature for different FE formulations.

The dynamic instability boundary is evaluated corresponding to various curvature parameters. Furthermore, it has been demonstrated that dynamic instability limits are sensitivity to in-plane edge constraints and panel configuration, indicating important design considerations for improving panel stability, which is crucial for applications in aerospace engineering and related fields. The numerical findings were compared with published research.

2. MATHEMATICAL FORMULATION

Assume, a cylinder-shaped panel (thin and isotropic) has a supersonic airflow that passes on top of surface parallel to x-axis (generator) as illustrated in Fig. 1. The angle of attack is zero. The properties of the material are $\nu = 0.3$,$\rho = 7800$Kg/m^3,and $E = 210$N/m^2. The airflow on panel bottom side is apprehending stationary. The piston theory (linearized) is applied to top of surface. Panel will begin to move perturbly in the lateral direction(x, y, t) in the presence of disturbance. This study attempts to determine the curved plate's dynamic instability under such motion. Using Hamilton's law, it is possible to derive the dynamic instability equations of motion. This would lead to the variational form, which describes the dynamics of a non-conservative system.

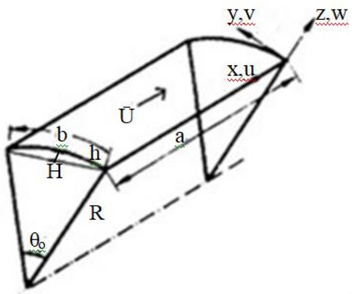

Figure 1. Curved panel subjected to supersonic flow (h is thickness, H is maximum shell rise height from base plane, a is length in axial (x) direction, b is curved length in circumferential (y) direction, Radius of curved panel is R).

$$\int_{t_1}^{t_2} [\delta L + \delta W]dt = 0 \qquad (1)$$

where L is the Langrange's functional, defined as L=T-U, and δ is the variational operator. The system's entire kinetic energy is represented by T, its total strain energy by U, its initial and end times of motion are represented by t_1 and t_2, and the virtual work produced by the non-conservative aerodynamics force fields is represented by δW.

The conforming higher order arbitrary triangular shaped (Fig. 2) is the basic Cowper's shallow shell FE, which is detailed in refs.[8],[12]. As the present problem related to shallow shell, the aforementioned shell element is used for present work. From kinetic energy and potential energy, one can get the mass matrix and stiffness matrix respectively, using stationary concept. The 36x36 stiffness and consistent mass matrices are provided[8],[12] elsewhere.

The aerodynamic load-related virtual work done W can be obtained by

$$\delta W = \int \bar{p}(x, y, t) w \, dx \, dy \qquad (2)$$

By applying the first order approximations to the 2D quasi steady supersonic piston theory[18],[24], the aerodynamic pressure intensity Eq.(3) that acting on the panel's top surface at any given location (x,y) can be expressed as follows.

$$\bar{p}(x, y, t) = -\bar{\lambda}[\frac{\partial w}{\partial x} + g \frac{\partial w}{\partial t}] \qquad (3)$$

where, $q = \frac{1}{2}\rho \bar{U}_\infty^2$, $\bar{\lambda} = \frac{2q}{\sqrt{\bar{M}_\infty^2 - 1}}$, and $g = \frac{\bar{\lambda}}{\bar{U}_\infty} \frac{\bar{M}_\infty^2 - 2}{\bar{M}_\infty^2 - 1}$

The variables w, $\bar{M}_{\infty}, \bar{U}_{\infty}$, and q represent the displacement, Mach number, free stream flow velocity of air, and dynamic-pressure the free stream air respectively. $\bar{\lambda}$ is DP (dynamic pressure) parameter and g represents aerodynamic damping term. First term in the Eq.(3) represents the flow inclination's steady contribution, and other term represents unsteady damping due to aerodynamic.

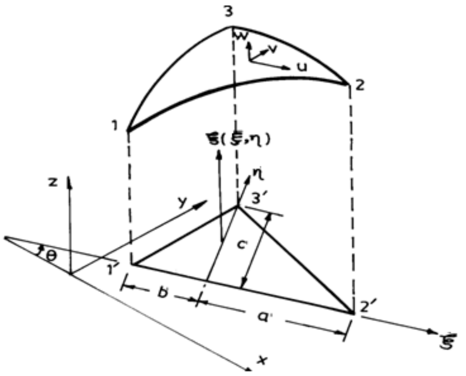

Figure 2. Geometry of Cowper's arbitrary triangular shallow shell element [12].

Since the incorporation of aerodynamic damping results in marginally higher critical dynamic instability speeds, its addition is ignored [37]. Therefore, by setting $g = 0$, more prevailing dynamic instability assessments can be assured. In local coordinates, the $\bar{p}(\xi, \eta, t)$, Eq.(3) will be

$$\bar{p} = -\bar{\lambda}[\frac{\partial w}{\partial \xi} cos\,(\theta) + \frac{\partial w}{\partial \eta} sin\,(\theta)] \quad (4)$$

Given below is the aerodynamic virtual work performed by the aerodynamic forces.

$$\delta W = -\bar{\lambda} \int (\frac{\partial w}{\partial \xi} cos\,(\theta) + \frac{\partial w}{\partial \eta} sin\,(\theta))w\, d\xi\, d\eta \quad (5)$$

The element nodal vector displacements can be used to express the deflection w as

$$\delta W = -\bar{\lambda}\delta\{\bar{q}\}^T[AM]\{\bar{q}\} \quad (6)$$

The contributions from each constituent(structure and aerodynamic) are combined using virtual work concept[33]. It is leading to the discrete form of the virtual work principle. Then the fundamental equation that describes motion for the non-damped elastic system of structure experiencing infinitesimal displacements is obtained. The panel's dynamic FE Eq.(7) is presented[38] below.

$$\delta\{\bar{q}\}^T([\bar{M}]\{\ddot{\bar{q}}\} + [\bar{K}]\{\bar{q}\}) = \delta W \quad (7)$$

Olson [39] presented the idea of an aerodynamic matrix and used FEM to address supersonic panel dynamic instability. Eq.(8), the aerodynamic force matrix is obtained by using the variational principle to calculate the work performed by the aerodynamic forces.

$$[AM] = -\bar{\lambda} \int (\{f\}\frac{\partial \{f\}^T}{\partial \xi} cos(\theta) + \{f\}\frac{\partial \{f\}^T}{\partial \eta} sin(\theta))\partial\xi\partial\eta \quad (8)$$

[AM] is the non-symmetric aerodynamic load matrix

The generalized element displacements are as follows

$$\{\bar{q}\}^T = \begin{pmatrix} u_1, u_{,\xi 1}, u_{,\eta 1}, v_1, v_{,\xi 1}, v_{,\eta 1}, w_1, w_{,\xi 1}, w_{,\eta 1}, w_{,\xi\xi 1}, \\ w_{,\xi\eta 1}, w_{,\eta\eta 1}, u_2, \ldots, u_{3,}, u_c, v_c \end{pmatrix} (9)$$

where

$$u_{,\xi} = \frac{\partial u}{\partial \xi}\,; u_{,\eta} = \frac{\partial u}{\partial \eta}; w_{,\xi\xi} = \frac{\partial w}{\partial \xi^2}; w_{,\xi\eta} = \frac{\partial w}{\partial \xi \partial \eta}$$

The corners of the element are indicated by the subscripts 1, 2, and 3, while its centriod (c) is shown by the letter c. $\{\bar{q}\}^T$denotes nodal vector. The matrix $[AM]$ is known as the aerodynamic matrix due to the non-conservative aerodynamic forces. Using notations of [8],[12], the elements of aerodynamic matrix in terms of polynomial displacement functions u, v and w are

$$[AM]_{r,s} = 0; (r, s = 1\ to\ 20) \quad (10a)$$

$$[AM]_{r,s} = -\bar{\lambda}[m_s F(m_r + m_s - 1, n_r + n_s)\,cos(\theta) - n_s F(m_r + m_s, n_r + n_s - 1)\,sin(\theta)]\,; \quad (r, s = 1\ to\ 20) \quad (10b)$$

The above equations expressed in the form of Eq.(11)

$$[AM]_{r,s} = -\bar{\lambda}[\bar{a}]_{r,s} \quad (11)$$

$$[AA]_{r,s} = [\overline{RM}]^T[\overline{TM}]^T[\bar{a}]_{r,s}[\overline{TM}][\overline{RM}] \quad (12)$$

$[\overline{TM}]$ and $[\overline{RM}]$ are transformation matrix and rotation matrix respectively[12]. The aerodynamic matrix $[AA]_{r,s}$ is in global coordinates of size 38x38. Condensing the centriodal displacements u_c and v_c (refer to Eq.(9)) is beneficial since it lowers the quantity of degrees of freedom for each element. Second, when the element is linked together, the final results won't change because these displacements are contained within the element. Finally, 38×38 consistent matrix is condensed to 36×36 matrix like stiffness $[\bar{K}]$ and mass matrixes $[\bar{M}]$ available in [8],[12] to get the aerodynamic stiffness $[K_a]$ matrix for the triangular element.

$$[K_a]_{r,s} = -\bar{\lambda}[AA]_{r,s} \quad (13)$$

2.1. Equation of Motion

For an isotropic panel with small displacements, the dynamic equilibrium of an undamped elastic structural system is derived according to Hamilton's principle. A linear assumption is examined to explain the instability of the shallow shell when it interacts with the supersonic air flow, which applies a transverse pressure of $\bar{p}$ on top plane. In linear systems, self-oscillation appears as an instability. Therefore, self-vibrations are expressed in terms of natural forms of linear vibrations of the structure. By replacing the stiffness resulting from the aerodynamic force Eq. (13) with the structural dynamic Eq. (7), one can determine the system's equation of motion[1].

$$\delta\{\bar{q}\}^T([\bar{M}]\{\ddot{\bar{q}}\} + [\bar{K}]\{\bar{q}\}) = \delta\{\bar{q}\}^T[K_a]\{\bar{q}\} \qquad (14)$$

$[K_a]$ is aerodynamic stiffness matrix. Eq. (14) must hold true for any compatible $\delta\bar{q}$, according to the concept of virtual work. Thus, one can express Eq. (14) as follows[1]:

$$[\bar{M}]\{\ddot{\bar{q}}\} + ([\bar{K}] + \bar{\lambda}[AA])\{\bar{q}\}) = \{0\} \qquad (15)$$

where the global aerodynamic stiffness matrixes are represented by $\bar{\lambda}[AA]$, and the global structural mass and structural stiffness are denoted by $[\bar{M}]$, $[\bar{K}]$, respectively. These matrices are produced during the process of assembling the each matrix of the element for entire structure. The vectors $\{\ddot{\bar{q}}\}$ and $\{\bar{q}\}$, represent acceleration and displacement, respectively.

Considering the following: a form of solving

$$\{\bar{q}\} = \{q\}e^{\omega t} \qquad (16)$$

Putting Eq. (16) in Eq. (15) becomes

$$\omega^2[\bar{M}] + ([\bar{K}] + \bar{\lambda}[AA])\{q\}) = \{0\} \qquad (17)$$

The aerodynamic stiffness matrix $\bar{\lambda}[AA]$ is asymmetric. Eq. (17) is an eigen value problem. ω is frequency in Hz.

2.2. Imposition of Boundary Condition

This indicates if there are displacements (Eq.(9)) at different nodes along the edge supports or not. All of the aforementioned displacements are presumptively present at every nodes except edges of panel. Mathematically, the constraining of a certain support is imposed by supposing that the associated displacement does not exist. The simply supported boundary condition applied on panel edges.

2.3. Solving Process

A specific DP($\bar{\lambda}$)magnitude are provided for solving Eq.(17), which is an eigenvalue problem. When $\bar{\lambda} = 0$, one of the problems is figuring out the panel's free vibration frequency. The non-symmetric aerodynamic matrix is visible when DPP (dynamic pressure parameter) $\bar{\lambda} > 0$ over certain ranges of the $\bar{\lambda}$, and some of the eigenvalues become complex. It is rather apparent that the stability of a structure is decided by real components of the complex frequencies. Lowest value is known as the critical dynamic pressure ($\bar{\lambda}_{cr}$), which is associated with two complex conjugate eigenvalues. In the absence of aerodynamic damping, the dynamic instability boundary simply corresponds to $\bar{\lambda}_{cr}$ at the time of initial coalescence.

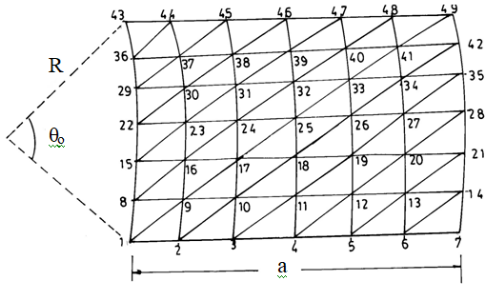

Figure 3. Triangular FE model mesh (6×6) for curved panel

3. RESULTS AND DISCUSSION

3.1. Present FE Method

The curvature impact on dynamic instability of shallow curved plates (panels) using the high precision Cowper's triangular FE are investigated as described in section 2. A 6x6 mesh is sufficient, according to a convergence assessment, to obtain reasonably accurate dynamic instability results. Hence, all of the panels are modeled using the 6 x 6 mesh as shown in Fig. 3, to study SSSS boundary conditions. As seen in Fig. 1, airflow is considered parallel to x-axis in this work. Regarding non-dimensional dynamic instability parameters, Λ and Ω, the following definitions are applicable. The dimensionless DPP (dynamic pressure parameter) is $\Lambda = \bar{\lambda}\frac{a^3}{\bar{D}}$, plate stiffness is $\bar{D} = \frac{Eh^3}{12(1-\upsilon^2)}$, ρ is plate density. The dimensionless frequency of plate is indicated as follows: $\Omega = \omega a^2\sqrt{\frac{\rho h}{\bar{D}}}$.

3.2. Validation of Present FE Code

The FE code is validated by comparing flat square panel (radius of curvature R of shallow shell tends to ∝) results with the literature data. The dimensionless DPP (Λ) for flat square panels with simply supported edges are plotted against dimensionless frequency (Ω) in Figure 4. The piston theory has been employed to understand coupled mode dynamic instability. The first and second modes' frequencies meet and overlap together when dynamic instability takes place. The current outcome, which is the dimensionless critical DPP $\Lambda_{cr} = 512.651$, agrees well with the value of 512.58 found in the literature [26]. This confirms with the current FE code.

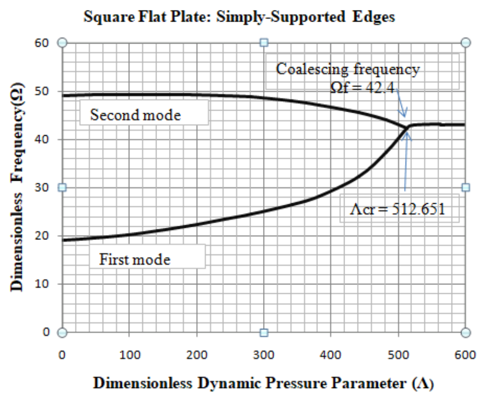

Figure 4. Dimensionless DPP (dynamic pressure parameter) versus dimensionless frequency for SSSS square panel.

3.3. Shallow Curved Panels

The curved panels have the following geometries: thickness $(h = 1.016\, x\, 10^{-3})$ in meter, radius$(R = 0.508)$ in meter, and length $(a = 1.016)$ in meter. Middle curved panel's base plane has a maximum rise of H. The findings are displayed in relation to the dimensionless critical DPP, or Λ_{cr}, in relation to the maximum shell rise and shell thickness, or $(\kappa = \frac{H}{h})$. Constant curvature geometry of a shallow shell can be roughly expressed as $H = \frac{a^2}{8R}$. For a range of curvature parameter κ, the critical pressure parameter is found for a simply supported curved panel on edges. When curvature parameter $\kappa = 0$, the Λ_{cr} value is found same value as that of a flat square panel. Flat square panel's current Λ_{cr} is 512.651; this value is in close comparison to references [23], [27], and [39].

The ratio of the shallow curved panel's dimensionless critical DPP to that of the flat square panel appears in Table 1 along with the curvature parameter (κ) for SSSS edges. Table 1 shows that as curvature grows, so does the panels' dynamic instability limit. The current results compare reasonably well to the reference [5] and exhibit the similar trend. This comparison does not prove the correctness or incorrectness of either approach; rather, it shows how sensitive the in-plane edge constraints are to the results for curved plates. For the sake of completeness, the following statements are provided to clarify. A useful solution for shallow shell FE problems is provided by ref.[5], which applies Reissner's two-field concept variables (independent approximation of stress and displacement fields) i.e. (i) transverse displacement (w), and (ii) Airy's stress function(stress resultants). Airy's stress function class I (stress resultants field variables) $F_{,xz}$/or $F_{,yz}=0$ at corner and transverse displacement w=0 on edge is referred to as a simply supported edge[5]. Airy's stress function class II (stress resultants field variables) F= 0 on the edge and transverse displacement w=0 on edge is referred to as a freely supported edge [5]. Freely supported edge did not imply that it is free (no support). The explanation is as follows. Both previously discussed cases have a transverse displacement of w=0; however, the stress resultant field variables associated with Airy's stress function are distinct. Thus, one might consider a freely supported edge to be a kind of simply supported edge boundary condition. Because stress resultants (Airy's stress function class II) field variables and the transverse displacement w are both zero on the edge. Whereas Cowper's [12] arbitrary triangular shallow shell FE (12 dof per node) based on shell theory formulations of Novozhilov's is exercised in current work. Strain energy is obtained using the membrane and bending strain in accordance with Novozhilov's shallow shell theory. Novozhilov and Reissner's shallow shell theory strain energy formulations are not identical. The two FE formulations' in-plane dof are not the same. Thus, it follows that dynamic instability limits are influenced by the in-plane edge constraints.

Table 1. Ratio of the shallow curved panel's dimensionless critical DPP $(\Lambda_{cr})$to that of the flat square panel versus the curvature parameter (κ)

κ	Present	Ref.[5]*	Ref.[5]**
0 (Flat plate)	1.00	1.00	1.00
1	1.12	1.18	1.12
2	1.72	1.75	1.17
3	2.17	2.77	1.36
≈4	2.70	3.10	1.80

*Freely supported edge(ref.[5]): Airy's stress function class II (stress resultants field variable) F= 0 on the edge and transverse displacement w=0 on the edge.

One might consider a freely supported edge to be a kind of simply supported edge boundary condition as stress resultants field variables and the transverse displacement w are both zero on the edge.

**Simply supported edge (ref.[5]):Airy's stress function class I (stress resultants field variable) $F_{,xz}$/or $F_{,yz}=0$ at corner and transverse displacement w=0 on the edge.

For flat plate and curvature parameter κ =1 (Table 1), the present results are very close to ref.[5]. The critical dynamic pressure ratios corresponding to various curvature parameters κ ≥2 are shown in Table 1 for present FE formulation and two cases of ref.[5] FE formulation. As demonstrated in the findings, for the shallow shell panel problem examined here, and based on the shell's curvature parameter (κ ≥2), the different edge conditions exhibit distinct behaviors. The distinct behavior is observed due to different in- plane edge constraints. Further, the present FE formulation critical DPP ratio results (κ ≥2) are in-between the results corresponding different edge supports of ref.[5] that discussed earlier.

4. CONCLUSION

Aiming at the supersonic panel dynamic instability study in the present work, a very efficient Cowper's triangular shallow shell FEM is proposed. The developed shallow-shell FE is applied to cylindrical curved plates with sides simply-supported(SSSS) to demonstrate the limited dynamic instability results, corresponding to different curvature parameters available in the literature for different FE formulations. Present FE code is validated by comparing square flat panel dynamic instability results with literature data. Dynamic instability boundaries for different curvatures parameters are predicted on panel. The existing literature and recent findings demonstrate a comparable trend and align effectively. It is found that dynamic instability boundaries are sensitive to in-plane edge constraints. The dynamic instability boundary of the panels grows with increasing curvature parameter (in the lower range) for panels with simply supported edges. However, there are still some limitations in the study, such as further consideration of structural nonlinearity in the panels subjected to aerodynamic loads using von Karman strain in the present FEM. Future research will continue to explore and improve the dynamic instability structural parameter prediction in order to improve the prediction accuracy and practicality considering damping, and aerodynamic heating.

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DECLARATION OF ETHICAL STANDARDS

The materials and methods used in this study do not require ethical declaration.

AUTHORS' CONTRIBUTIONS

Author has formulated the problem, performed analysis, written the paper and revised based on reviewers / editorial board comments.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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