Sakarya University Journal of Science, 22 (6), 1631-1637, 2018.

SAKARYA ÜNİVERSİTESİ FEN BİLİMLERİ ENSTİTÜSÜ DERGİSİ
SAKARYA UNIVERSITY JOURNAL OF SCIENCE

e-ISSN: 2147-835X
Dergi sayfası: http://www.saujs.sakarya.edu.tr

Received
11-01-2018
Accepted
26-03-2018

Doi
10.16984/saufenbilder.377423

Evaluation of Two Stage Modified Ridge Estimator and Its Performance

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ABSTRACT

Biased estimation methods are more desirable than two stage least squares estimation for simultaneous equations model suffering from the problem of multicollinearity. This problem can also be handled by using some prior information. Taking account of this knowledge, we recommend two stage modified ridge estimator in this article. The new estimator can also be evaluated as an alternative to the previously proposed two stage ridge estimator. A widespread performance criterion, mean square error, is taken into consideration to compare the two stage modified ridge, two stage ridge and two stage least squares estimators. A real life data analysis is investigated to support the theoretical results in practice. In addition, the intervals of the biasing parameter which provide the superiority of the two stage modified ridge estimator are determined with the help of figures. The researchers who deal with simultaneous systems with multicollinearity can opt for the two stage modified ridge estimator.

Keywords: modified ridge estimator, multicollinearity, simultaneous equations model, two stage least squares

1. INTRODUCTION

The matrix form of the simultaneous equations model is as follows

$$Y\Gamma + XB = U, \tag{1}$$

where $Y_{T\times M}$ and $X_{T\times K}$ are matrices of observations, $\Gamma_{M\times M}$ and $B_{K\times M}$ are the matrices of structural coefficients and $U_{T\times M}$ is the matrix of structural disturbances. The elements of X are nonstochastic and fixed with $rank(X) = K \leq T$ and the structural disturbances have zero mean and they are homoscedastic.

The model (1) can be written as

$$Y = X\Pi + V, \tag{2}$$

which is the reduced form. The reduced form coefficients are

$$\Pi = -B\Gamma^{-1} \tag{3}$$

and

$$V = U\Gamma^{-1}. \tag{4}$$

With the help of zero restrictions criterion the the equation below is the first equation of the system

$$y_1 = Y_1\gamma_1 + X_1\beta_1 + u_1. \tag{5}$$

There are $m_1 + 1$ included and $m_1^* = M - m_1 - 1$ excluded jointly dependent variables and K_1 included and $K_1^* = K - K_1$ excluded predetermined variables. $Y = [y_1 \quad Y_1 \quad Y_1^*]$ and $X = [X_1 \quad X_1^*]$ are variables with the size of $T \times m_1$, $T \times m_1^*$, $T \times K_1$ and $T \times K_1^*$ corresponding to Y_1, Y_1^*, X_1 and X_1^*. $\gamma_{.1} = [1 \quad -\gamma_1 \quad 0\,]'$ and $\beta_{.1} = [-\beta_1 \quad 0\,]'$ are variables

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with the size of $m_1 \times 1$ and $K_1 \times 1$ corresponding to γ_1 and β_1 and u_1 is the first column of U.

The partition of the reduced form equation (2) can be arranged as follows

$$[y_1 \quad Y_1 \quad Y_1^*] = [X_1 \quad X_1^*]\begin{bmatrix}\pi_{11} & \Pi_{11} & \Pi_{11}^* \\ \pi_{21} & \Pi_{21} & \Pi_{21}^*\end{bmatrix} + [v_1 \quad V_1 \quad V_1^*], \tag{6}$$

where

$$y_1 = X\pi_1 + v_1 \tag{7}$$

and

$$Y_1 = X\Pi_1 + V_1. \tag{8}$$

In two preceding equations, $\pi_1 = [\pi_{11} \quad \pi_{21}]'$ and $\Pi_1 = [\Pi_{11} \quad \Pi_{21}]'$ are variables with the size of $K_1 \times 1$, $K_1^* \times 1$, $K_1 \times m_1$, $K_1^* \times m_1$, $T \times 1$ and $T \times m_1$ corresponding to π_{11}, π_{21}, Π_{11}, Π_{21}, v_1 and V_1.

Considering only the first column of Γ, B and U in the reduced form coefficients (3) and (4), identifiability relationship between the structural parameters and the reduced form parameters for the first equation are obtained respectively as follows:

$$\pi_{11} = \Pi_{11}\gamma_1 + \beta_1, \tag{9}$$

$$\pi_{21} = \Pi_{21}\gamma_1 \tag{10}$$

and

$$v_1 = V_1\gamma_1 + u_1. \tag{11}$$

Reconsidering the first equation of the system (5),

$$y_1 = Z_1\delta_1 + u_1 \tag{12}$$

is obtained where

$$Z_1 = [\,Y_1 \quad X_1]_{T\times p_1}, \tag{13}$$

$$\delta_1 = [\,\gamma_1 \quad \beta_1\,]'_{p_1\times 1} \tag{14}$$

and $p_1 = m_1 + K_1$.

Thus, the structural equation (12) can be rewritten in the form of the following equation

$$y_1 = [\,X\Pi_1 \quad X_1\,]\begin{bmatrix}\gamma_1 \\ \beta_1\end{bmatrix} + v_1 \tag{15}$$

by replacing the equations (8) and (11) so as to reach the final form of the equation (15):

$$y_1 = \bar{Z}_1\delta_1 + v_1, \tag{16}$$

where $\bar{Z}_1 = E(Z_1) = [\,X\Pi_1 \quad X_1\,]$, $E(v_1) = 0$ and $E(v_1v_1') = \sigma^2 I$.

Two stage least squares (TSLS) estimation is commonly used for estimating the structural parameters of the equation (16) depending on its ease of computation.

Explanatory endogenous variables are replaced by their instrumental variables which are ordinary least squares (OLS) estimates that are obtained by using the exogenous variables to apply the first stage of TSLS estimation. Then, for the second stage, OLS estimation is used again to obtain the regression coefficients.

TSLS estimator is defined as follows

$$\delta_1^{LS} = (\bar{Z}_1'\bar{Z}_1)^{-1}\bar{Z}_1'y_1. \tag{17}$$

Since $\bar{Z}_1$ is unknown,

$$\widehat{\Pi}_1 = (X'X)^{-1}X'Y_1 \tag{18}$$

is used at the first stage to generate

$$\hat{\bar{Z}}_1 = [\,X\widehat{\Pi}_1 \quad X_1]. \tag{19}$$

By doing so, the operational form of the TSLS estimator

$$\hat{\delta}_1^{LS} = \left(\hat{\bar{Z}}_1'\hat{\bar{Z}}_1\right)^{-1}\hat{\bar{Z}}_1'y_1 \tag{20}$$

is yielded.

In the presence of multicollinearity, TSLS does not give sensitive estimates anymore. So, alternative estimation methods to TSLS are required to deal with multicollinearity. In this context, the most popular estimator is ridge estimator (RE) of Hoerl and Kennard [1] which is recommended for estimating parameters in simultaneous equations model by Vinod and Ullah [2]. Two stage ridge regression yields estimates having smaller variance than the TSLS estimation.

When researchers confront with the problem of multicollinearity, biased estimation methods seem to be more attractive than the TSLS estimation. Such an alternative method is two stage RE which is given by Vinod and Ullah [2]. Ordinary and operational forms of the two stage RE are

$$\delta_1^{RE} = (\bar{Z}_1'\bar{Z}_1 + kI)^{-1}\bar{Z}_1'y_1 \tag{21}$$

and

$$\hat{\delta}_1^{RE} = \left(\hat{\bar{Z}}_1'\hat{\bar{Z}}_1 + kI\right)^{-1}\hat{\bar{Z}}_1'y_1, \tag{22}$$

where $k > 0$.

Sometimes additional prior information can be come across in simultaneous equations model and this can help to overcome the problem of multicollinearity, as well. By this consideration we define a new two stage estimator for the structural coefficients based on the idea of modified ridge estimation of Swindel [3] subsequently.

In a summary, the organization of this article is as follows: Section 2 includes the new estimator for the simultaneous equations model; performance discussion of this new estimator is given in Section 3; Section 4 deals with the numerical example; Section 5 is for concluding remarks.

2. THE SUGGESTION OF NEW ESTIMATOR

There is a considerable interest in the existence of prior information. In this point of view, Swindel [3] offered a modified ridge estimator (MRE), which consists of a RE family, based on this information in linear regression model. By getting inspired from this idea, we propose two stage MRE based on prior information as

$$\delta_1(k,\delta_1^0) = (\bar{Z}_1'\bar{Z}_1 + kI)^{-1}(\bar{Z}_1'y_1 + k\delta_1^0), \quad (23)$$

where $k > 0$ and δ_1^0 is an arbitrary point in the parameter space which acts like the role of the origin.

Instead of this current form, to consider the prior information as a random variable seems more applicable as suggested by Swindel. Within this contex, we replace δ_1^0 with δ_1^{RE} in equation (23) and two stage MRE is derived to be:

$$\delta_1^{MRE} = (\bar{Z}_1'\bar{Z}_1 + kI)^{-1}(\bar{Z}_1'y_1 + k\delta_1^{RE}), \quad (24)$$

where $k > 0$ and δ_1^{RE} is as in the equation (21). To simplify the expression (24), $\bar{Z}_1(k) = (\bar{Z}_1'\bar{Z}_1 + kI)^{-1}\bar{Z}_1'\bar{Z}_1$ is used so that

$$\delta_1^{MRE} = \bar{Z}_1(k)\delta_1^{LS} + \left(I - \bar{Z}_1(k)\right)\delta_1^{RE} \quad (25)$$

is obtained.

Let $\hat{\bar{Z}}_1(k) = \left(\hat{\bar{Z}}_1'\hat{\bar{Z}}_1 + kI\right)^{-1}\hat{\bar{Z}}_1'\hat{\bar{Z}}_1$, thus,

$$\hat{\delta}_1^{MRE} = \hat{\bar{Z}}_1(k)\hat{\delta}_1^{LS} + \left(I - \hat{\bar{Z}}_1(k)\right)\hat{\delta}_1^{RE} \quad (26)$$

is used in practice.

Notice that a convex combination of TSLS estimator and two stage RE reveals by suggesting the new estimator in the equation (25). This convex combination unifies the advantages of included estimators.

δ_1^{MRE} reduces to δ_1^{LS} as $k \to 0$ and δ_1^{RE} as $k \to \infty$. As k increases, δ_1^{MRE} follows a way through the parameter space from δ_1^{LS} to δ_1^{RE}. Therefore, we expect that the deficiencies that are arised from multicollinearity with the use of TSLS estimator will be eliminated.

3. MSE PERFORMANCE OF THE NEW ESTIMATOR

The measure of the matrix mean square error (MSE) for any particular estimator $\bar{\delta}_1$ of δ_1, is

$$MSE(\bar{\delta}_1) = V(\bar{\delta}_1) + Bias(\bar{\delta}_1)Bias(\bar{\delta}_1)', \quad (27)$$

where the first part is the variance function and the second part is the squared bias function.

The model (16) can be written in a canonical form as follows

$$y_1 = Z\alpha_1 + v_1, \quad (28)$$

where $Z = \bar{Z}_1P$, $\alpha_1 = P'\delta_1$ and P is an orthogonal matrix such that $Z'Z = P'\bar{Z}_1'\bar{Z}_1P = \Lambda_1 = diag(\lambda_{11}, \dots, \lambda_{1p_1})$ where λ_{1i} are the eigenvalues of $\bar{Z}_1'\bar{Z}_1$.

By using this canonical form, the TSLS estimator, the two stage RE and the two stage MRE can be written as

$$\alpha_1^{LS} = \Lambda_1^{-1}Z'y_1 = A_1y_1, \quad (29)$$

$$\alpha_1^{RE} = (\Lambda_1 + kI)^{-1}Z'y_1 = H_kZ'y_1 = A_2y_1, \quad (30)$$

and

$$\alpha_1^{MRE} = (\Lambda_1 + kI)^{-1}(Z'y_1 + k\alpha_1^{RE}) = H_k\Lambda_1\alpha_1^{LS} + kH_k\alpha_1^{RE} = (H_k + kH_k^2)Z'y_1 = A_3y_1, \quad (31)$$

where $H_k = (\Lambda_1 + kI)^{-1}$, $A_1 = \Lambda_1^{-1}Z'$, $A_2 = H_kZ'$ and $A_3 = (H_k + kH_k^2)Z'$.

The $MSEs$ of the foregoing estimators are

$$MSE(\alpha_1^{LS}) = \sigma^2\Lambda_1^{-1}, \quad (32)$$

$$MSE(\alpha_1^{RE}) = \sigma^2(I - kH_k)H_k + k^2H_k\alpha_1\alpha_1'H_k', \quad (33)$$

and

$$MSE(\alpha_1^{MRE}) = \sigma^2H_k(I - kH_k)(I + kH_k)^2 + k^4H_k^2\alpha_1\alpha_1'H_k^2. \quad (34)$$

We refer the following lemmas that are to be used in theoretical comparisons.

Lemma 1. (Trenkler, [4]). Let $\bar{\delta}_1$ and $\bar{\delta}_2$ be two homogeneous linear estimators of δ_1 such that $D = V(\bar{\delta}_1) - V(\bar{\delta}_2) > 0$.

If $Bias(\bar{\delta}_2)' D^{-1} Bias(\bar{\delta}_2) < \sigma^2$ then $MSE(\bar{\delta}_1) - MSE(\bar{\delta}_2) > 0$.

Lemma 2. (Pliskin, [5]). A prior mean δ_1^0 is said to be good if $MSE(\delta_1^{RE}) - MSE(\delta_1(k, \delta_1^0))$ positive semidefinite for all positive values of k when both δ_1^{RE} and $\delta_1(k, \delta_1^0)$ are computed using the same value of k.

Firstly, we choose the superior one from two stage MRE and the TSLS estimator.

Theorem 1. Let k be fixed.

If $\alpha_1' H_k^2 (A_1 A_1' - A_3 A_3')^{-1} H_k^2 \alpha_1 < \frac{\sigma^2}{k^4}$ then $MSE(\alpha_1^{LS}) - MSE(\alpha_1^{MRE}) > 0$,

where $H_k = (\Lambda_1 + kI)^{-1}$, $A_1 = \Lambda_1^{-1} Z'$ and $A_3 = (H_k + kH_k^2) Z'$.

Proof:

$$\begin{aligned} V(\alpha_1^{LS}) - V(\alpha_1^{MRE}) &= \sigma^2 \Lambda_1^{-1} \\ &\quad - \sigma^2 H_k (I - kH_k)(I + kH_k)^2 \\ &= \sigma^2 k^2 H_k (H_k + \Lambda_1^{-1} + \mathrm{k} H_k^2) H_k \\ &= \sigma^2 (A_1 A_1' - A_3 A_3'). \end{aligned}$$

Since $H_k + \Lambda_1^{-1} + \mathrm{k} H_k^2 > 0$, $V(\alpha_1^{LS}) - V(\alpha_1^{MRE}) > 0$. From the Lemma 1 the proof is completed.

Secondly, we discuss the superiority of the two stage MRE to the two stage RE.

Theorem 2. Let k be fixed.

If $\alpha_1' H_k^2 (A_2 A_2' - A_3 A_3')^{-1} H_k^2 \alpha_1 < \frac{\sigma^2}{k^4}$ then $MSE(\alpha_1^{RE}) - MSE(\alpha_1^{MRE}) > 0$,

where $H_k = (\Lambda_1 + kI)^{-1}$, $A_2 = H_k Z'$ and $A_3 = (H_k + kH_k^2) Z'$.

Proof:

$$\begin{aligned} V(\alpha_1^{RE}) - V(\alpha_1^{MRE}) &= \sigma^2 (I - kH_k) H_k \\ &\quad - \sigma^2 H_k (I - kH_k)(I + kH_k)^2 \\ &= \sigma^2 H_k \Lambda_1 H_k (2kH_k + k^2 H_k^2) \\ &= \sigma^2 (A_2 A_2' - A_3 A_3'). \end{aligned}$$

$2kH_k + k^2 H_k^2 > 0$ thus $V(\alpha_1^{RE}) - V(\alpha_1^{MRE}) > 0$. From the Lemma 1 the proof is completed.

Through Theorem 1 and Theorem 2, we derive sufficient conditions for the superiority of the two stage MRE to the two stage RE and the TSLS estimator, as well. From the intuition behind Lemma 2 leads us to conclude that the prior mean δ_1^{RE} is a good information.

The current topic is to take account of the choice of the biasing parameter. Since the biasing parameter acts a prominent role in the performance of the mentioned estimators, the selection of this parameter is crucial. In this paper, we mainly determine the intervals of the biasing parameter with ridge trace so as to give the best results in the sense of mean square error for our new estimator. In addition, to estimate the biasing parameter we use some of the existing methods which are defined by Hoerl and Kennard [1], Hoerl et al. [6], Lawless and Wang [7] and Kibria [8]. These are defined to be as follows:

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\sum_{i=1}^{p_1} \hat{\alpha}_{1i}^2}, \quad \text{(Hoerl and Kennard, [1])} \tag{35}$$

$$\hat{k}_{HKB} = \frac{p_1 \hat{\sigma}^2}{\sum_{i=1}^{p_1} \hat{\alpha}_{1i}^2}, \quad \text{(Hoerl et al., [6])} \tag{36}$$

$$\hat{k}_{LW} = \frac{p_1 \hat{\sigma}^2}{\sum_{i=1}^{p_1} \lambda_{1i} \hat{\alpha}_{1i}^2}, \quad \text{(Lawless and Wang, [7])} \tag{37}$$

$$\hat{k}_{AM} = \frac{1}{p_1} \sum_{i=1}^{p_1} \frac{\hat{\sigma}^2}{\hat{\alpha}_{1i}^2}, \quad \text{(Kibria, [8])} \tag{38}$$

$$\hat{k}_{GM} = \frac{\hat{\sigma}^2}{(\prod_{i=1}^{p_1} \hat{\alpha}_{1i}^2)^{\frac{1}{p_1}}}, \quad \text{(Kibria, [8])} \tag{39}$$

$$\hat{k}_{M} = Median \left\{ \frac{\hat{\sigma}^2}{\hat{\alpha}_{1i}^2} \right\}_{i=1}^{p_1}, \quad \text{(Kibria, [8])} \tag{40}$$

where $\hat{\alpha}_{1i}$ and $\hat{\sigma}^2$ are the TSLS estimates of α_{1i} and σ^2.

4. APPLICATION

We consider a constructed model given in Griffiths et. al [9] to illustrate the theoretical results. This aggregate econometric model of the U.S. economy is as follows

Equation 1: $c_t = \gamma_{12} y_t + \beta_{11} + \beta_{12} c_{t-1} + e_{1t}$,

Equation 2: $i_t = \gamma_{22} y_t + \beta_{21} + \beta_{22} r_t + e_{2t}$,

Identity: $y_t = c_t + i_t + g_t$,

where

c_t is private consumption expenditure in year t,

i_t is private investment expenditure in year t,

y_t is gross national expenditure in year t,

g_t is government expenditure in year t,

r_t is a weighted average of interest rates in year t.

While c_t, i_t and y_t are used as endogenous variables, g_t and r_t are used as exogenous ones in this model.

The suggested data from Griffiths et al. [9] (p. 611) is used while doing the numerical example.

Since it is more convenient for application estimated scalar mean square error (mse) values are utilized. These estimated mse values for the foregoing estimators and estimates of the biasing parameter in canonical form are computed and shown in the Table 1.

Table 1. Estimated mse values for the estimators

	$\hat{k}$	$\widehat{mse}(\hat{\alpha}_1^{LS})$	$\widehat{mse}(\hat{\alpha}_1^{RE})$	$\widehat{mse}(\hat{\alpha}_1^{MRE})$
Equation 1	$\hat{k}_{LW} = 0.0030$	254.84	**252.25**	254.83
	$\hat{k} = 1.5$	254.84	**20.95**	60.99
	$\hat{k} = 3$	254.84	**7.34**	23.90
	$\hat{k} = 5$	254.84	**3.22**	10.71
	$\hat{k}_{HK} = 151.87$	254.84	**0.24**	0.32
	$\hat{k} = 360$	254.84	0.25	**0.25**
	$\hat{k}_{HKB} = 455.62$	254.84	0.27	**0.24**
	$\hat{k}_M = 583.28$	254.84	0.30	**0.23**
	$\hat{k} = 800$	254.84	0.34	**0.24**
	$\hat{k}_{GM} = 1281.60$	254.84	0.42	**0.29**
	$\hat{k} = 3000$	254.84	0.56	**0.44**
	$\hat{k}_{AM} = 6052.86$	254.84	0.64	**0.56**
Equation 2	$\hat{k}_{LW}$ =0.0039	24.67	**24.58**	24.67
	$\hat{k}_{HK}$ =0.7219	24.67	**18.38**	21.89
	$\hat{k} = 1$	24.67	**18.80**	20.62
	$\hat{k} = 1.5$	24.67	20.84	**19.05**
	$\hat{k} = 2$	24.67	23.51	**18.42**
	$\hat{k}_{HKB}$ =2.16	24.67	24.44	**18.38**
	$\hat{k} = 3$	24.67	28.97	**19.14**
	$\hat{k} = 4$	24.67	33.74	**21.25**
	$\hat{k} = 5$	24.67	37.71	**23.86**
	$\hat{k}_{GM}$ =42.60	**24.67**	65.40	59.51
	$\hat{k}_M$ =186.61	**24.67**	70.40	68.84
	$\hat{k}_{AM}$ =252.76	**24.67**	70.81	69.65

Figures 1-2 named as ridge trace are drawn to determine the intervals for the biasing parameter and demonstrate the estimated mse performance of the estimators.

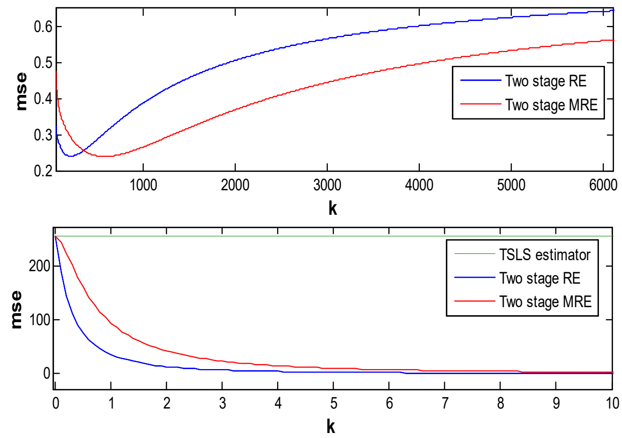

Figure 1. Estimated mse values for Equation 1

Figure 1 is drawn in two parts corresponding to Equation 1. The first part below illustrates estimated mse behaviors of the TSLS estimator, two stage RE and the two stage MRE for the smaller values of the biasing parameter. In the meantime, the second part above is the plot for a wide range of k values. The two stage MRE outperforms the TSLS estimator and the two stage RE for the k values approximately greater than 360 whereas two stage RE is the best estimator for the smaller values of the biasing parameter. Based on the results obtained by means of ridge trace in Figure 1, for some chosen k values the estimated mse values are indicated in the Table 1. Besides, by using some existing methods in the previous section, estimated k values are computed for the Equation 1 and demonstrated in the Table 1. For example, for $\hat{k}_{LW} = 0.0030$ and $\hat{k}_{HK} = 151.87$ the two stage RE gives smaller estimated mse values than the TSLS estimator and the two stage MRE since these k values are rather small. On the other hand, the two stage MRE with $\hat{k}_{HKB} = 455.62$, $\hat{k}_M = 583.28$, $\hat{k}_{GM} = 1281.60$ and $\hat{k}_{AM} = 6052.86$ values is superior to the others. Thus, this result becomes compatible with the findings from the ridge trace.

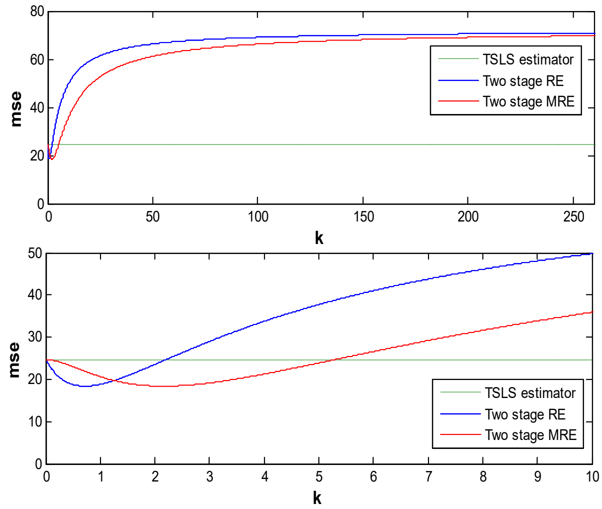

Figure 2. Estimated mse values for Equation 2

Similar to the Figure 1, Figure 2 is drawn in two parts to show estimated mse performances of the foregoing estimators, corresponding to Equation 2. The first part below is plotted for the smaller values of the biasing parameter while the second part above is the plotted for an extended range of k values. Till the magnitude of the k is nearly 1.5, the two stage RE is the best in comparison to the TSLS estimator and the two stage MRE. In the case that k lies between 1.5 and 5, the two stage MRE reaches its minimum in the sense of mse. When k is approximately greater than 5, the line for the two stage MRE is still below the line for two stage RE but it is above the line for the TSLS estimator. By this way, the Figure 2 plays a role in choosing some of the k values shown in the Table 1 for the Equation 2. In addition some computed k values for the existing estimation methods are also included in the Table 1. While $\hat{k}_{LW}$ =0.0039 and $\hat{k}_{HK}$ =0.7219, the two stage RE is preferable and this agrees with the ridge trace result obtained from Figure 2. Since $\hat{k}_{HKB}$ =2.16, the two stage MRE has the smallest estimated mse values at this point of estimate. As for $\hat{k}_{GM}$ =42.60, $\hat{k}_{M}$ =186.61 and $\hat{k}_{AM}$ =252.76, these estimation methods are useless for our new estimator since those are greater than 5.

5. CONCLUDING REMARKS

This article recommends the two stage MRE which is assigned to reduce the effect of multicollinearity in the simultaneous systems. This estimator is such a convex combined estimator that is resulted in unifying the advantages of the TSLS estimator and two stage RE. Taking two stage RE as prior information, the two stage MRE becomes desirable with regard to dispelling multicollinearty. Within this framework, the new estimator is preferable to the two stage RE and the TSLS estimator.

We succeed in demonstrating the superiority of the two stage MRE over the two stage RE and the TSLS estimator with the help of theorems. The problem of choosing the biasing parameter of the two stage MRE is settled by the technique of ridge trace as well as some specific estimation methods.

The conclusion that two stage MRE outperforms the two stage RE and the TSLS estimator is drawn from data analysis based on the data set Griffiths [9]. Furthermore, graphical representation is accomplished to observe the estimated mse performances. By means of the graphs, it is observed that for greater values of the biasing parameter the two stage MRE is better than the two stage RE and the reverse is valid for smaller values of the biasing parameter.Likewise to the theoretical results, the numerical results are in favor of the two stage MRE.

ACKNOWLEDGMENTS

This paper is supported by Çukurova University Scientific Research Projects Unit Project Number: FBA-2018-9770.

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