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Continuous Dependence For Benjamin-Bona-Mahony-Burger Equation

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ABSTRACT

In this work, it is proved that the solutions of Benjamin-Bona-Mahony-Burger equation depends continuously on the coefficients.

Keywords: BBMB equation, continuous dependence.

1. INTRODUCTION

In this study, it is considered the following problem for the BBMB equation:

$$u_t - u_{xxt} - \alpha u_{xx} + \gamma u_x + f(u) = 0 \qquad (1)$$

$$u(x,0) = u_0(x), \qquad x \in \Omega \qquad (2)$$

$$u(x,t) = 0, \qquad x \in \partial\Omega, \qquad t > 0 \qquad (3)$$

where $u(x,t)$ states the velocity of fluid, α is a positive number, γ is an arbitrary real number, $\Omega \subset \mathbb{R}^n$ is a bounded domain whose boundary $\partial\Omega$ and $f(u)$ is a C^2-smooth nonlinear function which states

$$f(u)u \geq F(u) \geq 0 \qquad (4)$$

$$|f(u) - f(v)| \leq K|u - v| \qquad (5)$$

where $F(u) = \int_0^u f(s)ds$, K is a positive number, $f(u)$ provides Lipschitz inequality.

The models proposed for the mathematical expression of the basic laws of nature are often not linear. Most of these models are based on nonlinear partial differential equations.

Pseudoparabolic problems emerge in the many areas of physics and mathematics such as consolidation of clay, long waves propagation with small amplitude, fluid flow of fissured rock and thermodynamics[1-5]. BBMB equation(1) is a special case of pseudoparabolic-type equations.

Continuous dependence on coefficients of solutions of partial differential equations is a kind of structural stability that reflects the influence of small changing on coefficient of the solutions of equations. In the recent years, many results of this type can be found in the literature[6-13].

In this paper the authors carried out on the continuous dependence of the coefficients on the BBMB equation solutions.

Throughout in paper, $\|.\|$ and $(\,,)$ state the norm and inner product $L^2(\Omega)$. $H_0^1(\Omega)$ is a Hilbert space.

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2. A PRIORI ESTIMATE

In this chapter, it is got a priori estimate of (1)-(3).

Theorem 1. Let $u_0 \in H_0^1(\Omega)$. Under the assumption (4), if u is the solution of (1)-(3) problem, then the following estimate holds:

$$\|u_x\|^2 \leq D \tag{6}$$

where $D > 0$ which depending on the parameters of (1)-(3) problem.

Proof. If (1) is multiplied by u and integrated over Ω, then it is got

$$\frac{d}{dt}[\,E(t)] + \alpha\|u_x\|^2 + \int_\Omega uf(u)dx = 0 \tag{7}$$

where $E(t) = \frac{1}{2}\|u\|^2 + \frac{1}{2}\|u_x\|^2$. If (7) is integrated on the interval $(0,t)$ and from (4) it is got

$$\frac{1}{2}\|u\|^2 + \frac{1}{2}\|u_x\|^2 \leq E(0). \tag{8}$$

Hence (6) follow from (8).

3. CONTINUOUS DEPENDENCE ON PARAMETERS

In this chapter, it is proved that (1)-(3) problem solution is continuous dependence on the coefficient α and γ in $H_0^1(\Omega)$.

Now, it is supposed that u and v are respectively (9)-(10) problems solutions:

$$\begin{cases} u_t - u_{xxt} - \alpha_1 u_{xx} + \gamma u_x + f(u) = 0, \\ \quad u(x,0 = u_0(x), \qquad x \in \Omega, \\ \quad u(x,t) = 0, \qquad x \in \partial\Omega, t > 0, \end{cases} \tag{9}$$

$$\begin{cases} v_t - v_{xxt} - \alpha_2 v_{xx} + \gamma v_x + f(v) = 0 \\ \quad v(x,0) = u_0(x), \qquad x \in \Omega, \\ \quad v(x,t) = 0 \qquad x \in \partial\Omega, t > 0. \end{cases} \tag{10}$$

Let $u - v = w$ and $\alpha_1 - \alpha_2 = \alpha$. Then w is a solution of the problem

$$\begin{cases} w_t - w_{xxt} - \alpha_1 w_{xx} - \alpha v_{xx} + \gamma w_x + f(u) - f(v) = 0, \\ \qquad w(x,0) = 0, \qquad x \in \Omega, \\ \qquad w(x,t) = 0, \qquad x \in \partial\Omega, t > 0. \end{cases} \tag{11}$$

The following theorem states that (1)-(3) problem solution depends continuously on the coefficient α in $H_0^1(\Omega)$.

Theorem 2. Suppose that (5) holds. Let w be (11) problem solutions. Then w provides the inequality

$$\frac{1}{2}\|w\|^2 + \frac{1}{2}\|w_x\|^2 \leq \frac{D}{2}|\alpha_1 - \alpha_2|^2 e^{M_1 t} \tag{12}$$

where D and M_1 are positive constants.

Proof. If (11) is multiplied by w and integrated over Ω, it is got

$$\frac{d}{dt}\left[\frac{1}{2}\|w\|^2 + \frac{1}{2}\|w_x\|^2\right] + \alpha_1\|w_x\|^2 + \alpha(w_x, v_x) + \int_\Omega \big(f(u) - f(v)\big)w dx = 0. \tag{13}$$

If (5) and Cauchy-Schwarz inequality are used in (13), it is got

$$\frac{d}{dt}\left[\frac{1}{2}\|w\|^2 + \frac{1}{2}\|w_x\|^2\right] \leq |\alpha|\|w_x\|\|v_x\| + K\|w\|^2. \tag{14}$$

If arithmetic-geometric mean inequality is used in (14), it is had

$$\frac{d}{dt}E_1(t) \leq M_1 E_1(t) + \frac{|\alpha|^2}{2}\|v_x\|^2 \tag{15}$$

where $E_1(t) = \frac{1}{2}\|w\|^2 + \frac{1}{2}\|w_x\|^2$ and $M_1 = maks\{1, 2K\}$. If Gronwall inequality and (6) are used in (15), it is obtained

$$E_1(t) \leq \frac{D_1}{2}|\alpha|^2 e^{M_1 t}. \tag{16}$$

Hence the proof is completed.

Finally, it is proved that the (1)-(3) problem solution depends continuously on the coefficient γ.

Now, it is supposed that u and v are respectively (17)-(18) problems solutions:

$$\begin{cases} u_t - u_{xxt} - \alpha u_{xx} + \gamma_1 u_x + f(u) = 0, \\ \quad u(x,0) = u_0(x), \qquad x \in \Omega, \\ \quad u(x,t) = 0, \qquad x \in \partial\Omega,\ t > 0, \end{cases} \tag{17}$$

$$\begin{cases} v_t - v_{xxt} - \alpha v_{xx} + \gamma_2 v_x + f(v) = 0, \\ \quad v(x,0) = u_0(x), \qquad x \in \Omega, \\ \quad v(x,t) = 0, \qquad x \in \partial\Omega,\ t > 0. \end{cases} \tag{18}$$

Let $u - v = w$ and $\gamma_1 - \gamma_2 = \gamma$. Then w is a solution of the problem

$$\begin{cases} w_t - w_{xxt} - \alpha w_{xx} + \gamma_1 w_x + \gamma v_x + f(u) - f(v) = 0, \\ \qquad w(x,0) = 0, \quad x \in \Omega, \\ \qquad w(x,t) = 0, x \in \partial\Omega, \qquad t > 0. \end{cases} \tag{19}$$

The following theorem is the main result of this section.

Theorem 3. Suppose that (5) holds. Let w be (19) problem solution. Then w provides the inequality

$$\frac{1}{2}\|w\|^2+\frac{1}{2}\|w_x\|^2\le\frac{D}{2}|\gamma_1-\gamma_2|^2e^{M_2t} \qquad (20)$$

where D and M_2 are positive constants.

Proof. If (19) is multiplied by w and integrated over Ω, we have

$$\frac{d}{dt}\left[\frac{1}{2}\|w\|^2+\frac{1}{2}\|w_x\|^2\right]+\alpha\|w_x\|^2-\gamma(v,w_x)$$
$$+\int_{\Omega}\big(f(u)-f(v)\big)w dx=0. \qquad (21)$$

If (5) and Cauchy-Schwarz inequality are used in (21), it is got

$$\frac{d}{dt}\left[\frac{1}{2}\|w\|^2+\frac{1}{2}\|w_x\|^2\right]$$
$$\le|\gamma|\|v\|\|w_x\|+K\|w\|^2. \qquad (22)$$

If arithmetic-geometric mean inequality is used in (22), it is had

$$\frac{d}{dt}E_2(t)\le ME_2(t)+\frac{|\gamma|^2}{2}\|v\|^2 \qquad (23)$$

where $E_2(t)=\frac{1}{2}\|w\|^2+\frac{1}{2}\|w_x\|^2$ and

$M_2=maks\{1,2K\}$. If Gronwall inequality and (6) are used in (23), it is obtained

$$E_2(t)\le\frac{D}{2}|\gamma|^2e^{M_2t}. \qquad (24)$$

Hence the proof is completed.

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