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FIFTH-ORDER COMPACT FINITE DIFFERENCE SCHEME FOR BURGERS-HUXLEY EQUATION

BURGERS-HUXLEY DENKLEMİ İÇİN BEŞİNCİ MERTEBEDEN KOMPAKT SONLU FARK ŞEMASI

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Abstract

The Burgers-Huxley equation arises in several problems in science. The compact finite difference scheme (CFDS) has been developed for the Burgers-Huxley equation. This scheme has been compared different methods for the Burgers-Huxley equation. Dispersive properties are investigated for the linearized equations to examine the nonlinear dynamics after discretisation. The accuracy and computational efficiency of the compact finite differences scheme are shown in numerical test problems.

Keywords: Burgers-Huxley equation, compact finite difference scheme, dispersion analysis.

Öz

Bilimde çeşitli problemlerde Burgers-Huxley denklemi ile karşılaşılmaktadır. Bu çalışmada kompakt sonlu fark şeması, Burgers-Huxley denkleminin çözümü için uygulanmıştır. Çözümler farklı yöntemlerle elde edilen sonuçlarla karşılaştırılmıştır. Lineer olmayan denklem diskrize edildikten sonra çözümün doğruluğunu analiz etmek için dağılım analiz yapılmıştır. İncelenen problemler üzerinde kompakt sonlu fark şemasının doğruluğu ve hesaplama verimliliği gösterilmiştir.

Anahtar Kelimeler: Burgers-Huxley denklemi, kompakt sonlu fark şeması, dağılım analiz.

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1. INTRODUCTION

The aim of this study is develop a numerical solution using CFDS for the Burgers-Huxley equation

$$v_t - v_{xx} + \tilde{\alpha} v^{\delta} v_x = \bar{\beta} v\left(1 - v^{\delta}\right)\left(v^{\delta} - \tilde{\gamma}\right), \; 0 \le x \le 1, \quad t \ge 0 \tag{1}$$

The solution of (1) is given by

$$v(x,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left(w_1(x - w_2 t)\right)\right)^{\frac{1}{\delta}} \tag{2}$$

where $w_1 = \frac{-\tilde{\alpha}\delta + \delta\sqrt{\tilde{\alpha} + 4\bar{\beta}(1+\delta)}}{4(1+\delta)}$, $w_2 = \frac{\tilde{\alpha}\tilde{\gamma}}{1+\delta} - \frac{(1+\delta-\tilde{\gamma})(-\tilde{\alpha}+\sqrt{\tilde{\alpha}^2+4\bar{\beta}(1+\delta)})}{2(1+\delta)}$, the parameters $\tilde{\alpha}, \bar{\beta}, \tilde{\gamma}$ and δ are constants such that $\bar{\beta} \ge 0, \delta > 0, \tilde{\gamma} \in (0,1)$ (Satsuma, 1987) The initial and boundary conditions obtained using the solution in (1).

The Burgers-Huxley equation appears in many different areas, such as the motion of the domain wall of a ferroelectric material in an electrical field, certain ecological models, etc. (Yefimova et al.,2004) The generalized Burgers-Huxley equation was investigated by Satsuma et al. in 1987.

There are many research to get the solution of the Burgers-Huxley equation using different methods. Javidi computed the solution using the spectral collocation method and pseudospectral method (Javidi 2006, Javidi 2006). Bratsos (2011) used a fourth-order finite difference scheme in two time level recurrence relation to get the numerical solution. Also various mathematical methods such as adomain decomposition method (Ismail et al., 2004, Hashim et al., 2006), homotopy analysis method (Molabahramin et al., 2009), variational iteration method (Batiha et al., 2011), NSFD schemes (Zibaei et al., 2016), local discontinuous Galerkin method and tanh-coth method (Wazwaz 2008), the collocation method (Singh et al., 2024), Galerkin method (Chin, 2023), finite element method and lie symmetry analysis (Anjali et al, 2025), Crank Nicolson logarithmic finite difference method (Celikten et al, 2022) have been used to solve the equation.

Many researchers used the CFDS for the solution of Burgers-Huxley equation. Shenggao et al. (2011) applied compact scheme which has a fourth-order accuracy in space and second-order accuracy in time. Mohanty et al. (2015) used new two-level implicit compact operator method with on of order two in time and four in space for the solution of Burgers-Huxley equation. In addition to that modified compact finite difference method given to get numerical solution (Rusli et al., 2025).

Although there are many methods to construct compact schemes, the Pade Approximation Method and the Taylor Series Method are two of the most basic and commonly used approximations. For the first and second derivative approximations compact finite difference schemes (CFDS) were given both for the inner points and the boundary points by using the Taylor approximation (Lele, 1992). The paper is arranged as follows: In Section II, CFDS for Burgers-Huxley equation is presented. Section III deals with the dispersion relations of compact finite difference scheme for linearized

Burgers-Huxley equation. In Section IV, numerical results for different problems are presented in tables and conclusion is given in Section V.

2. COMPACT FINITE DIFFERENCES SCHEME

The spatial domain is given as $[a, b]$ with the spatial step length h=$(b-a)/(N-1)$. Here N represents spatial grid points ($x_i = h(i-1)$). Equal sub-intervals have been chosen to better compare the obtained results with those in the literatüre. Furthermore, the method is expressed for equal sub-intervals. $f_i = f(x_i)$ are the function values at the grid points and the approximation to the first derivative f_i' is expressed as in the following

$$\bar{\beta} f_{i-2}' + \tilde{\alpha} f_{i-1}' + f_i' + \tilde{\alpha} f_{i+1}' + \bar{\beta} f_{i+2}' = c\frac{f_{i+3}-f_{i-3}}{6h} + b\frac{f_{i+2}-f_{i-2}}{4h} + a\frac{f_{i+1}-f_{i-1}}{2h} \quad (3)$$

The Taylor expansion is applied to find out the coefficients. If we choose the coefficients $\bar{\beta}$= c=0, we obtain tridiagonal schemes, Although the order of accuracy for derivatives is $O(h^5)$ for inner points, it is $O(h^6)$ for the boundary points. Also, the approximation for the second derivative of the function is given as in the following

$$\bar{\beta} f_{i-2}'' + \tilde{\alpha} f_{i-1}'' + f_i'' + \tilde{\alpha} f_{i+1}'' + \bar{\beta} f_{i+2}'' = c\frac{f_{i+3}-2f_i+f_{i-3}}{9h^2} + b\frac{f_{i+2}-f_i+f_{i-2}}{4h^2} + a\frac{f_{i+1}-2f_i+f_{i-1}}{h^2} \quad (4)$$

The approximation formula are given in the equation (4) and (5) for boundary point at $i = 1$.

- For first derivative

$$f_1' + \tilde{\alpha} f_2' = \frac{1}{h}(af_1 + bf_2 + cf_3 + df_4) \quad (4)$$

- For second derivative

$$f_1'' + \tilde{\alpha} f_2'' = \frac{1}{h^2}(af_1 + bf_2 + cf_3 + df_4 + ef_5) \quad (5)$$

For the boundary point at $i = N$, we can get easily using the equations (4) and (5). The compact schemes with their calculated coefficients are obtained for inner points from $i = 3$ up to $i = N-2$ and for boundary points at $i = 2$ and $i = N-1$, respectively.

$$\frac{h}{3}(-17f_1' - 14f_2' + f_3') = f_0 + 8f_1 - 9f_2 \quad , i = 2 \quad (6)$$

$$\frac{h}{3}(f_{i-1}' + 4f_i' + f_{i+1}') = -f_{i-1} + f_{i+1} \quad , i = 3, .., N-2 \quad (7)$$

$$\frac{h}{3}\left(\frac{1}{8} f_{N-3}' - \frac{5}{8} f_{N-2}' + \frac{19}{8} f_{N-1}' + \frac{9}{8} f_N'\right) = -f_{N-1} + f_N \quad , i = N-1 \quad (8)$$

$$\frac{h^2}{12}(14f_1'' - 5f_2'' + 4f_3'' - 5f_4'') = f_0 - 2f_1 + f_2 \quad ,i = 2 \tag{9}$$
$$\frac{h^2}{12}(f_{i-1}'' + 10f_i'' + f_{i+1}'') = f_{i-1} - 2f_i + f_{i+1} \quad ,i = 3,..,N-2 \tag{10}$$
$$\frac{h^2}{12}(-f_{N-1}'' + 4f_{N-3}'' - 5f_{N-2}'' + 14f_{N-1}'') = f_{N-2} - 2f_{N-1} + f_N \quad ,i = N-1 \tag{11}$$

For example, with 7 nodes, the matrices below are constructed using the compact schemes from (6) , (7), and (8) as shown below

$$A_1 = \begin{bmatrix} \frac{-17h}{3} & \frac{-14h}{3} & \frac{h}{3} & 0 & 0 \\ \frac{h}{3} & \frac{4h}{3} & \frac{h}{3} & 0 & 0 \\ 0 & \frac{h}{3} & \frac{4h}{3} & \frac{h}{3} & 0 \\ 0 & 0 & \frac{h}{3} & \frac{4h}{3} & \frac{h}{3} \\ 0 & \frac{h}{24} & \frac{-5h}{24} & \frac{19h}{24} & \frac{9h}{24} \end{bmatrix}, \qquad V' = \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \\ f_5' \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 8 & -9 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} f_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The matrix V' having the approximation values of the first derivative at each node is calculated via the equation (12).

$$A_1V' = K_1V + H_1 \tag{12}$$

We will apply the LU decomposition technique to the known matrix A_1 and then the following calculations to obtain V':

$$(L_0V_0)V' = K_1V + H_1 \tag{13}$$
$$L_0^{-1}L_0V_0V' = L_0^{-1}K_1V + L_0^{-1}H_1$$

Let's take T_1 as $L_0^{-1}K_1$ and G_1 as $L_0^{-1}H_1$

$$T_1 = L_0^{-1}K_1 \tag{14}$$
$$G_1 = L_0^{-1}H_1 \tag{15}$$

Let's repeat the operations done so far for V_0

$$V_0V' = T_1V + G_1 \tag{16}$$
$$V_0^{-1}V_0V' = V_0^{-1}T_1V + V_0^{-1}G_1$$
$$S_1 = V_0^{-1}G_1 \tag{17}$$
$$V' = C_1V + S_1 \tag{18}$$

Using the compact schemes in (9), (10) and (11), having the approximation values at each grid point the matrices below are obtained to get the matrix V''.

$$A_2 = \begin{bmatrix} \frac{14h^2}{12} & \frac{-5h^2}{12} & \frac{4h^2}{12} & \frac{-h^2}{12} & 0 \\ \frac{h^2}{12} & \frac{10h^2}{12} & \frac{h^2}{12} & 0 & 0 \\ 0 & \frac{h^2}{12} & \frac{10h^2}{12} & \frac{h^2}{12} & 0 \\ 0 & 0 & \frac{h^2}{12} & \frac{10h^2}{12} & \frac{h^2}{12} \\ 0 & \frac{-h^2}{12} & \frac{4h^2}{12} & \frac{-5h^2}{12} & \frac{14h^2}{12} \end{bmatrix}, \quad V'' = \begin{bmatrix} f_1'' \\ f_2'' \\ f_3'' \\ f_4'' \\ f_5'' \end{bmatrix}$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}, \quad H_2 = \begin{bmatrix} f_0 \\ 0 \\ 0 \\ 0 \\ f_6 \end{bmatrix}$$

$$A_2V'' = K_2V + H_2 \tag{19}$$
$$V'' = C_2V + S_2 \tag{20}$$

2.1. Compact Finite Difference Scheme for Burgers-Huxley Equation

Equations (18) and (20) is substituted to the equation (1). We use CFDS for spatial dimension and finite differences along the time axis, lastly they are written via explicit approximation and the discretization scheme is obtained. The discrete solution at a point will be represented $V_j^n \approx V(x_j, t^n)$

$$\frac{V_j^{n+1}-V_j^n}{\Delta t} - \left(C_2V_j^n + S_2\right) + \tilde{\alpha}V^{\delta}\left(C_1V_j^n + S_1\right) = \bar{\beta}\left(1-\left(V_j^n\right)^{\delta}\right)\left(\left(V_j^n\right)^{\delta} - \tilde{\gamma}\right) \tag{21}$$

This scheme is explicit because the solution values at time t^{n+1} are calculated directly from the known values at time t^n.

3. DISPERSION RELATION

Traveling wave or soliton solutions of nonlinear PDEs can be investigated through their dispersive behavior. To understand the solution behaviour of the compact finite differences methods, linearized equations and investigate numerical dispersion relation will be considered. The linearized PDEs will be solved again by using the compact finite differences scheme in order to compare to continuous and the discrete versions of the dispersion relations of the equations. Assume that $\tilde{v}: \mathbb{R}^2 \to \mathbb{R}$ is a second order continuously differentiable function, such that $|\tilde{v}(x,t)| < 1$. Let also $v = \tilde{v} + \bar{v}$, where $\bar{v}$ and $\tilde{v}$ are solutions to (1). Hence, $v_t = \bar{v}_{t+}\tilde{v}_t$, $v_x = \bar{v}_x + \tilde{v}_x$ and $v_{xx} = \bar{v}_{xx} + \tilde{v}_{xx}$. For dispersion analysis we get $\tilde{\alpha} = \bar{\beta} = \delta = 1$, $\tilde{\gamma} = 0.001$. Substituting this into the equation (1), we get

$$\bar{v}_t + \tilde{v}_t + (\bar{v}+\tilde{v})(\tilde{v}_x + \bar{v}_x) - (\bar{v}_{xx} + \tilde{v}_{xx}) - \big(1-(\bar{v}+\tilde{v})\big)(\bar{v}+\tilde{v}-0.001) \tag{22}$$

Since $\bar{v}$ is the solution of (1)

$$\bar{v}_t + \overline{v}\overline{v}_x - \bar{v}_{xx} - (1-\bar{v})(\bar{v} - 0.001) = 0$$

When we also ignore the higher order terms and linearize it around the constant solution $\bar{v}$ the linearized equation becomes

$$\tilde{v}_t + a\tilde{v}_x - \tilde{v}_{xx} - 2.002a\tilde{v} + 3a^2\tilde{v} + 0.001\tilde{v} = 0 \quad (23)$$

where $a = \bar{v}$.

Assuming each wave mode as a solution of a linearized PDE (23), the solution takes the following form

$$\tilde{v} = \hat{v}e^{i(kx+wt)}$$

where $\hat{v} = \hat{v}(k)$ denotes the amplitude, k is the wave number, and w is the frequence. By substituting this solution into linearized equation and simplifying, we get

$$w = ik^2 - k + (3a^2 + 0.001 - 2.002a)i \quad (24)$$

Which is a dispersion relationship for the linearization of the Burgers-Huxley equation (23) around the plane wave solution.

In order to get discrete version of the dispersion relations for the equations (23), the linearized system will be solved again using the compact finite difference scheme. The compact finite differences scheme is applied by taking $a = 1$ the following discritized equation (25) is obtained

$$\tilde{v}_j^{n+1} - \tilde{v}_j^n + \Delta t C_1 \tilde{v}_j^n + \Delta t S_1^n - \Delta t C_2 \tilde{v}_j^n - \Delta t S_2^n + \Delta t 0.999 \tilde{v}_j^n = 0 \quad (25)$$

where $\tilde{v}_j^n = \tilde{v}(x_j, t_n)$ and which has a discrete general solution of the form

$$\tilde{v}_j^n = \hat{v}e^{i(kx_j + wt_n)} = \hat{v}e^{i(jk\Delta x + nw\Delta t)} = \hat{v}e^{i(j\bar{k} + n\bar{w})}$$

where $\bar{k} = k\Delta x$ is the numerical wave number and $\bar{w} = w\Delta t$ is the numerical frequency such that $-\pi \leq k \leq \pi$ and $-\pi \leq w \leq \pi$. As in the continuous case, substituting the numerical plane wave solution in (25) into the linearized equation and simplifying, the numerical dispersion relation is calculated as in (26) (Aydın et al., 2004).

$$\bar{w} = \frac{-i}{c}[\ln(\gamma_2) - \ln(1+\gamma_1)] - \frac{1}{c} j\bar{k} \quad (26)$$

where $\gamma_1 = -1 + \Delta t C_1 - \Delta t C_2 + 0.999\Delta t$, $\gamma_2 = \Delta t(-S_1^n + S_2^n)$, $c = \frac{1+n+\gamma_1}{1+\gamma_1}$.

In Figure (1) and Figure (2) the exact and numerical dispersion of the linearized Burgers-Huxley equation are shown for the continuous w, k and discrete $\bar{w}, \bar{k}$ which are frequencies and wave numbers for real and imaginary parts, respectively. In all computations, Δt and Δx are let to be 0.1. For the continuous dispersion relation and the discrete dispersion relation there are certain frequencies corresponding to each wavenumber. In Figure (1) and Figure (2) the analytical and numerical dispersion relation have been plotted. If the dispersion graphs given in Figure (1) and Figure (2) are compared with each other, it's obvious that, real continuous and real numerical dispersion curves have similar behaviours.The imaginary exact dispersion relation has the term k^2 but any similar term is not included in imaginary numerical dispersion relation. From the figures, we see that numeric dispersion relations of the compact finite difference method preserves the continous dispersion relation for small wave numbers k.

Dispersion analysis of the compact finite difference scheme for the problem studied in this paper reveals that there doesn't exist diffeomorphisms ψ_1 and ψ_2 satisfying the exact dispersion for the imaginary part

$$D_N(\overline{w},\overline{k}) = D(\psi_1(\overline{w}),\psi_2(\overline{k}))$$

where $D_N(\overline{w},\overline{k})$ is numerical dispersion relation and $D(w,k)$ is exact dispersion relation.

These dispersion relations exist for the real parts.

4. NUMERICAL RESULTS

In this part, solution for equation in (1) is obtained using CFDS. To illustrate the efficiency of the CFDS for the problem handled in this study, the maximum error which is given by the equation below

$$L_\infty = max_{1\le j\le N}|v(x_j,t) - V(x_j,t)|$$

where $v(x_j,t)$ and $V(x_j,t)$ refer to the exact solution and solution via compact finite difference scheme, respectively.

Example 1.
Consider equation (1) with $\tilde{\alpha} = \bar{\beta} = 1$, $\tilde{\gamma} = 0.001$ and $\Delta t = 0.00005$. Numerical solutions at $t = 0.2$ and $t = 1$ are presented for computational domain $[0,1]$ in Table (1) and Table (2) . For $N = 8$ and $N = 16$ numerical solutions are given. As it can be seen from the Table (1) and Table (2) the compact finite difference scheme is more accurate.

Example 2.
Obtained numerical solutions have been presented with maximum errors for $\tilde{\alpha} = 5$, $\Delta t = 0.0001$ in Table (3) and Table (4). In Table (3) and Table (4), the errors are presented for various values of $\tilde{\gamma}$ and $\bar{\beta}$ for $N = 9$.

5. CONCLUSION

In this study, compact finite difference scheme is used to solve Burgers-Huxley equation. The performance of the scheme is tested on the considered test problems, and maximum absolute errors are computed. For some constant values, maximum error values for the presented scheme are almost the same with those for the other methods (Javidi et al., 2009, Zhang et al., 2012) in literature. But the error values of presented scheme are more satisfactory in comparison with the error values in (Bratsos, 2011) and (Mohanty et al., 2015). This study focuses on analyzing the dispersive characteristics of the linearized Burger-Huxley equation and the numerical dispersive properties of the compact finite difference method. The real numerical and real exact dispersion relations are very close to each other.

Table 1. Maximum Error with $\tilde{\alpha}$=1, $\bar{\beta}$ =1, $\tilde{\gamma}$=0,001, Δt=0,00005 and N=8 for Example 1

t		**Javidi et al., 2009**	**Zhang et al., 2012**	**Presented method**
0,2	$\delta = 1$	4,0138e-8	3,7725e-8	2,6442e-8
	$\delta = 4$	1,3139e-5	1,2348e-5	9.3302e-6
	$\delta = 8$	3,5540e-5	3,3397e-5	2,6091e-5
1	$\delta = 1$	4,6849e-8	4,3914e-8	2,8443e-8
	$\delta = 4$	1,5325e-5	1,4366e-5	1,0063e-5
	$\delta = 8$	4,1407e-5	3,8818e-5	2,8157e-5

Table 2. Maximum Error with $\tilde{\alpha}$=1, $\bar{\beta}$=1, $\tilde{\gamma}$=0,001, Δt=0,00005 and N=16 for Example 1

t		**Javidi et al., 2009**	**Presented method**
0,2	$\delta = 1$	4,0138e-8	3,3402e-8
	$\delta = 4$	1,3139e-5	1,1287e-6
	$\delta = 8$	3,5540e-5	3,0979e-5
1	$\delta = 1$	4,6849e-8	3,7349e-8
	$\delta = 4$	1,5325e-5	1,2630e-5
	$\delta = 8$	4,1406e-5	3,4658e-5

Table 3. Maximum Error with $\tilde{\alpha}$=5, $\tilde{\gamma}$=0,001, Δt=0,0001 for Example 2

t		**Javidi et al., 2009**	**Bratsos 2011**	**Mohanty et al., 2015**	**Presented Method**
0,3	$\bar{\beta} = 1$	3,1616e - 8	3,1570e-8	3,0414e-8	2,3242e-8
	$\bar{\beta} = 4$	3,9742e-7	3,9668e-5	3,8439e-7	3,0600e-7
	$\bar{\beta} = 8$	5,0365e-6	5,0291e-6	4,9161e-6	4,0219e-6
0,9	$\bar{\beta} = 1$	3,3394e-8	3,3393e-8	3,2090e-8	2,3805e-8
	$\bar{\beta} = 4$	4,1977e-7	4,1976e-7	4,0977e-7	3,1351e-7
	$\bar{\beta} = 8$	5,3166e-6	5,3165e-6	5,2161e-6	4,1195e-6

Table 4. Maximum Error with $\tilde{\alpha}$=5, $\tilde{\gamma}$=0,0001, Δt=0,0001 for Example 2

t		**Javidi et al., 2009**	**Bratsos 2011**	**Mohanty et al., 2015**	**Presented Method**
0,3	$\bar{\beta} = 1$	3,1630e-10	3,1584e -10	3,0428e-10	2,3253e-10
	$\bar{\beta} = 10$	4,9760e-9	3,9702e - 9	3,8442e-9	3,0614e-9
	$\bar{\beta} = 100$	5,0389e- 8	5,0316e - 8	4,9176e-8	4,0239e-8
0,9	$\bar{\beta} = 1$	3,3409e-10	3,3408e -10	3,2099e-10	2,3816e-10
	$\bar{\beta} = 10$	4,1996e-9	4,1995e - 9	4,0983e-9	3,1365e-9
	$\beta = 100$	5,3223e-8	5,3221e - 8	5,2211e-8	4,1234e-8

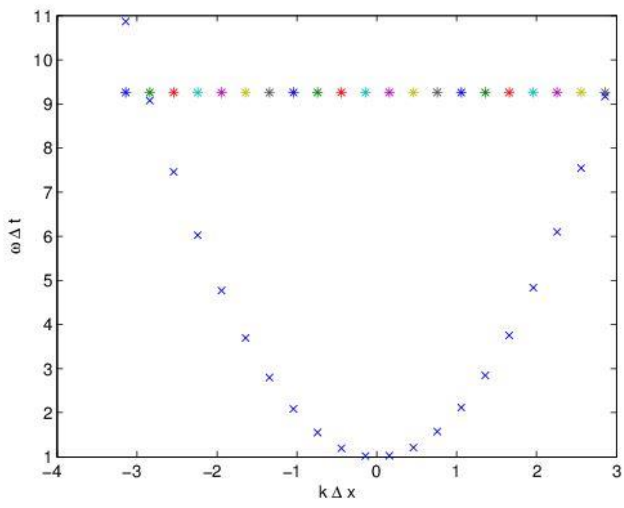

Figure 1. For Δt= Δx=0,1, Exact (dot) and Numeric (solid) Dispersion Relation for Imaginary Part of Linearized Burgers-Huxley Equation

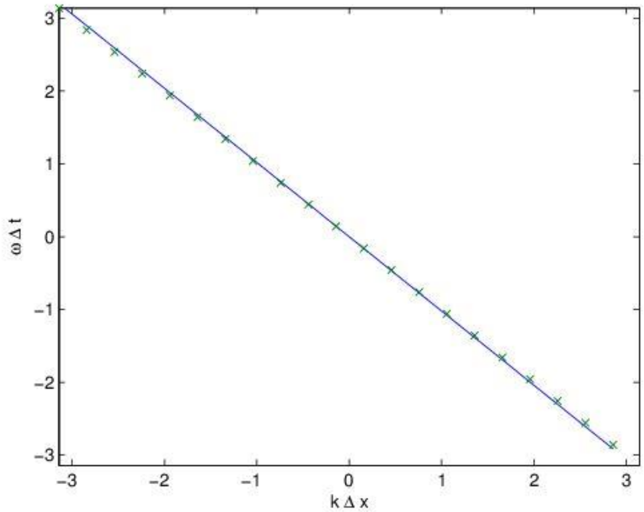

Figure 2. For Δt= Δx=0,1, Exact (dot) and Numeric (solid) Dispersion Relation for Real Part of Linearized Burgers-Huxley Equation

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REFERENCES

Anjali, K., Vivek. S. & Rajesh, K. G. (2025). Comparative Analysis of the Singularly Perturbed Generalized Burgers-Huxley Problem via Approximate Lie Symmetry and Exponentially Fitted Finite Element Method. *International Journal of Theoretical Physics*, 64, https://doi.org/10.1007/s10773-025-05882-1.

Aydın, A. & Karasozen, B. (2010). Multisymplectic box schemes for the complex modified Korteweg-de Varies equation. *Journal of Mathematical Physics*, 51, 083511.

Batiha, B., Noorani, M.S.M. & Hashim, I. (2008). Application of variational iteration method to the generalized Burgers-Huxley equation. *Chaos Solitons @ Fractals* , 36, 660-663.

Bratsos, A.G. (2011). A fourth order improved numerical scheme for the generalized Burgers-Huxley equation. *Am J Comput Math*, 1, 152-158.

Celikten, G. & Cankurt, A. (2022). A Numerical Solution of the Generalized Burgers-Huxley Equation. *AKÜ FEMUBID*, 22, 75-84.

Chin, PWM. (2023). The analysis of the solution of the Burgers-Huxley equation using the Galerkin method. *Numerical Methods For Partial Differential Equations,* 39, 2787-2807.

Hashim, I., Noorani, M.S.M. & Al-Hadidi, MRS. (2006). Solving the generalized Burgers-Huxley equation using the adomain decomposition method. *Math Comput Model*, 43, 1404-1411.

Ismail, H.N.A., Raslan, K. & Rabboh, A.A.A. (2004). Adomain decomposition method for Burgers-Huxley and Burgers-Fisher equations. *Appl Math Comput* , 159, 291-301.

Javidi, M. (2006). A numerical solution of the generalized Burgers-Huxley equation by pseudospectral method and darvishi's preconditioning. *Appl Math Comput*, 175, 1619-1628.

Javidi, M. (2006). A numerical solution of the generalized Burgers-Huxley equation by spectral collocation method. *Appl Math Comput,* 178, 338-344.

Javidi, M. & Golbabai, A. (2009). A new domain decomposition algorithm for generalized Burger's-Huxley equation based on Chebyshev polynomials and preconditioning. *Chaos Solitons and Fractals,* 39, 849-857.

Lele, S.K. (1992). Compact finite difference schemes with Spectral-like. Resolution, *Journal of Computational Physics*, 103, 16-42.

Mohanty, R.K. Weizhong, D. & Donn, L. (2015). Operator compact method of accuracy two time in time and four in space for the solution of time dependent Burgers-Huxley equation. *Numer Algor*, 70, 591-605.

Molabahramin, A. & Khani, F. (2009). The homotopy analysisi method to solved the Burgers-Huxley equation. *Nonlinear Analysis, Real World Applications*, 2, 589-600.

Rusli, R., Nasir, M. A. S., Harun, N., Rathi, S. & Zubaidah, S. (2025). A new modified compact finite difference scheme for solving the generalized Burgers-Huxley equation in teaching and learning processes. *Journal of Quality Measurement and Analysis,* 21, 195-205.

Satsuma, J. (1986). Exact Solutions of Burgers Equation with Reaction Terms. Topics in Soliton Theory and Exactly Solvable Nonlinear Equations, World Sci Pub, Singapore.

Shenggao, Z. & Xiaoliang, C. (2011). A linearly semi-implicit compact scheme for the Burgers-Huxley equation. *International Journal of Computer Mathematics,* 88, 795-804.

Singh, A., Dahiya, S. & Emdifar, H. (2024). Numerical Solution of Burgers-Huxley Equation Using a Higher Order Collocation Method. *Journal of Mathematics,* doi.org/10.1155/2024/2439343.

Yefimova, O.Y. & Kudryashov, N.A. (2004). Exact solutions of the Burgers-Huxley equation. *J Appl Math Mech.*, 3, 413-420.

Zhang, R., Yu, X. & Zhao, G. (2012). The local discontinuous Galerkin Method for Burger's-Huxley and Burger's-Fisher equations. *Applied Mathematics and Computation*, 17, 8773-8778.

Zibaei, S., Zeinadini, M. & Namjoo, M. (2016). Numerical solutions of Burgers-Huxley equation by exact finite difference and NSFD schemes. *Journal of Difference Equations and Applications*, 22, 1098-1113.

Wazwaz, A.M. (2008). Analytic study on Burgers, Fisher, Huxley equations and combined forms of these equations. *Applied Mathematics and Computation*, 2, 754-761.