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SOME INEQUALITIES RELATED TO A NEW TYPE OF σ CONVERGENCE

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Abstract: For a non-decreasing sequence $\lambda = (\lambda_n)$ of positive integers tending to infinity such that $\lambda_{n+1} - \lambda_n \le 1$, $\lambda_1 = 1$; (V, λ)-summability was defined as the limit of the generalized de la Vallée-Pousin mean of a sequence, [10]. In this note, we have defined a new type of σ-convergence of a sequence by using the generalized de la Valée-Pousin mean and also investigated some inequalities related to this type of σ-convergence like to those that studied in [2, 3, 4, 5, 7].

Keywords : *Statistically convergence, invariant means, core theorems and matrix transformations.*

YENİ BİR σ –YAKINSAKLIK TİPİ İLE İLGİLİ BAZI EŞİTSİZLİKLER

Özet: Azalan olmayan doğal sayıların sonsuza giden ve $\lambda_{n+1} - \lambda_n \le 1$, $\lambda_1 = 1$ şartlarını sağlayan $\lambda = (\lambda_n)$ dizisi için; (V, λ)-toplanabilme, bir dizinin de la Vallée-Pousin ortalaması olarak tanımlandı, [10]. Bu çalışmada, de la Vallée-Pousin ortalaması ile tanımlanan yeni bir σ –yakınsaklık tanımladık ve bu σ –yakınsaklık için, [2, 3, 4, 5, 7]' daki benzer olan bazı eşitsizlikleri inceledik.

Anahtar kelimeler : *İstatistiksel yakınsaklık, değişmez ortalamalar, çekirdek teoremleri ve matris dönüşümleri.*

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1. INTRODUCTION

Let $A = (a_{nk})$ $(n,\ k = 1, 2,...)$ be an infinite matrix of real numbers and $x=(x_k)$ be a real number sequence. We write $Ax = ((Ax)_n)$ if $A_n(x)=\left\{\sum_k a_{nk}x_k\right\}$ converges for each n. Let X and Y be any two non-empty sequence spaces. If $x\in X$ implies that $Ax\in Y$, then we say that the matrix A maps X into Y. By (X,Y) we denote the class of matrices A which maps X into Y. If X and Y are equipped with the limits X - lim and Y - lim, respectively, $A\in (X, Y)$ and $Y - lim\, Ax = X - lim\, x$ for all $x\in X$, then we write $A\in (X, Y)_{reg}$.

Let K be a subset of N, the set of positive integers. Natural density δ of K is defined by

$$\delta(K)=\lim_n \frac{1}{n}\left|\{k\le n : k\in K\}\right| ,$$

where the vertical bars indicate the number of elements in the enclosed set. The number sequence $x=(x_k)$ is said to be statistically convergent to the number l if for every ε, $\delta\{k : |x_k - l|\geq \varepsilon\} = 0$, [8]. In this case, we write ***st-lim x=l***. We shall also write ***st*** and st_0 to denote the sets of all statistically convergent sequences and sequences of statistically convergent to zero. Fridy and Orhan [9] have introduced the notions of the statistically boundedness, statistical-limit superior (*st-limsup*) and inferior (*st-liminf*).

Let l_∞ and c be the Banach spaces of bounded and convergent sequences with the usual supremum norm. Let σ be a one-to-one mapping from N into itself and T be an operator on l_∞ defined by $Tx = x_{\sigma(k)}$. A continuous linear functional ϕ on l_∞ is said to be an invariant mean or a σ-mean if and only if ,

(i) $\phi(x) \geq 0$ when the sequenc $x = (x_k)$ has $x_k \geq 0$ for all k ,

(ii) $\phi(e) = 1$,where $e = (1, 1, 1, . . .)$,

(iii) $\phi(x) = \phi(Tx)$ for all $x\in l_\infty$.

It can be shown [12] that

$V_\sigma = \{x\in l_\infty : lim_p\, t_{pn}(x) = s$ uniformly in n , $s = \sigma - lim\, x\}$,

where

$$t_{pn}(x)=\frac{x_n+Tx_n+...+T^p x_n}{p+1} ,$$

$t_{-1,n}(x)=0$.

We say that a bounded sequence $x = (x_k)$ is σ-convergent if and only if $x\in V_\sigma$ .We denote by Z the subset of V_σ consisting of all sequences with σ-limit zero. It is well-known [12] that $x\in l_\infty$ if and only *if*

$$(T x - x) \in Z \text{ and } V_\sigma = Z \oplus Re .$$

In this paper we shall deal with the following functionals defined on l_∞ :

$l(x) = lim\, inf\, x$, $L(x) = lim\, sup x$, $V(x) = \sup_n \limsup_p t_{pn}(x)$, $W(x) = \inf_{z\in Z} L(x + z)$

$\beta(x) = st - limsup x$, $\alpha(x) = st - lim\, inf\, x$

Let $\lambda=(\lambda_n)$ be a non-decreasing sequence of positive integers tending to ∞ such that $\lambda_1 = 1$, $\lambda_{n+1} \leq \lambda_n + 1$. The generalized de la Vallée-Pousin mean is given by

$$t_n(x)=\frac{1}{\lambda_n}\sum_{k\in I_n} x_k ,\ I_n=[n-\lambda_n+1,n]$$

and (V,λ) -summability was defined in [10] as follows: A sequence x is said to be (V,λ)-convergent to a number l if $\lim_n t_n(x) = l$ and (V, λ) is the set of all (V, λ) -summable sequences.

Next, we shall quote some lemmas which will be useful to our proof.

Lemma 1.1. [7, Lemma 1] *Let* $A = (a_{nk}(i))$ *be conservative and* $\lambda \geq 0$. *Then,*

$$\lim_n \sup \sup_i \sum_k \left| a_{nk}(i) - a_k \right| \leq \lambda ,$$

if and only if

$$\lim_n \sup \sup_i \sum_k \left(a_{nk}(i) - a_k \right)^+ \leq \frac{\lambda + x}{2}$$

and

$$\lim_n \sup \sup_i \sum_k \left(a_{nk}(i) - a_k \right)^- \leq \frac{\lambda - x}{2} ,$$

where x is the characteristic of A *and for any* $t \in \mathbf{R}$, $t^+ = max\{0, t\}$ and $t^- = max\{-t, 0\}$.

Lemma 1.2. [7, Lemma 2] *Let* $\|A\| < \infty$ *and* $lim_n \, sup_i a_{nk}(i) = 0$. *Then, there exists a* $y \in l_\infty$ *with* $\|y\| \leq 1$ *such that*

(1.1)
$$\lim_p \sup \sup_i \sum_k a_{pk}(i) y_k = \lim_p \sup \sup_i \sum_k \left| c_{pk}(i) \right| .$$

In this paper, we have introduced a new type of σ-convergence by using the generalized de la Vallée-Pousin mean and studied some inequalities related to this new type of σ-convergence like to those that given in [2,3,4,5,7].

2. THE MAIN RESULTS

Definition 2.1. A bounded sequence $x = (x_k)$ is said to be σ_λ-convergent to a number *s* if

$$\lim_p t_{pn}(\lambda, x) = s \quad \text{uniformly in } \boldsymbol{n} ,$$

where

$$t_{pn}(\lambda, x) = \frac{1}{\lambda_p} \sum_{i \in Ip} x_{\sigma^i(n)} , \quad t_{-1,n}(\lambda, x) = 0$$

To illustrate this new type of convergence, we may give some examples:

Let us choose a sequence $x = (x_n)$ such that

$$x_n = \begin{cases} 1, & n = 3k, k = 1,2,.. \\ 0, & otherwise \end{cases}$$

and let $\sigma(n) = n + 2$. Now, if we choose the sequence (λ_p) such that

$$(\lambda_p) = (1, 1, 1, 2, 2, 2, 3, 3, 3, \ldots)$$

then, $\sigma_\lambda - lim\, x = 1$. But, if

(2.1) $\lambda_p = (1, 2, 2, 3, 3, 4, 4, \ldots)$

then σ_λ –limx does not exist.

By $V_\sigma(\lambda)$ and Z_λ we respectively denote the set of all σ_λ –convergent and σ_λ-convergent to zero sequences. It is clear that

$(V, \lambda) \subset V_\sigma(\lambda)$. Further, in the case $\lambda_p = p + 1$, $V_\sigma(\lambda) = V_\sigma$. Also, since $\lambda_p/(p+1)$ is bounded by 1, clearly $V_\sigma(\lambda) \subset V_\sigma$. Note that this connection is strictly with respect to the choosen sequence (λ_p). For example let $\sigma(n) = n + 1$ and $x = (x_n)$ be given by

$$x_n = \begin{cases} 1, & if \;\; n \;\; is \;\; odd \\ 0, & if \;\; n \;\; is \;\; even \end{cases}$$

Then, clearly $x \in V_\sigma$ with $\sigma - lim\, x = 1$. If we choose (λ_p) such that
$(\lambda_p) = (1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, \ldots)$
then, $\sigma_\lambda - lim\, x = 1$. But; if we choose (λ_p) as in (2.1), then $\sigma_\lambda - lim\, x$ does not exist.

Lemma 2.2. *Let X be any sequence space. Then, A∈ (X, $V_\sigma(\lambda)$) if and only if* **D** *∈ (X, c), where* **D** *is defined as in the proof.*

Proof. For any $x \in X$, let us write

$$\frac{1}{\lambda_p}\sum_{i\in Ip}\sum_{k=0}^{m} a_{\sigma^i(n),k}x_k = \sum_{k=0}^{m}\frac{1}{\lambda_p}\sum_{i\in Ip} a_{\sigma^i(n),k}x_k \quad .$$

Letting $m \to \infty$, we have

$$\frac{1}{\lambda_p}\sum_{i\in Ip}(Ax)_{\sigma^i(n)} = (Dx)_n \; ; \quad (n\in D\,) ,$$

Where **D** $=(d_{pk}(n))$ is defined by

$$d_{pk}(n) = \frac{1}{\lambda_p}\sum_{i\in Ip} a_{\sigma^i(n),k}$$

for all k, n, $p \in N$. Therefore, one can easily see that A∈ (X, $V_\sigma(\lambda)$) if and only if **D** ∈ (X, c) . This completes the proof.

One can deduce from Lemma 2.2 that A∈ (c, $V_\sigma(\lambda)$) if and only if $sup_p\Sigma_k\ |d_{pk}(n)| < \infty$, $lim_p\ d_{pk}(n) = \alpha_k$ uniformly in ***n*** and $lim_p\Sigma_k\ d_{pk}(n) = \alpha$ uniformly in ***n***. In the case A∈ (c, $V_\sigma(\lambda)$),the number $\Gamma_\lambda = \Gamma_\lambda(A) = \alpha - \Sigma_k\alpha_k$ is defined and it is said to be characteristic number of ***A*** with respect to $\boldsymbol{\lambda}$. Note that $\boldsymbol{\Gamma_\lambda}$ is a generalization of the characteristic of an infinite matrix ***A*** , (see [1, p. 46]).

Now, we may give our main results.

Theorem 2.3. *Let A ∈ (c, $V_\sigma(\lambda)$). Then, for some constant $\gamma \geq |\Gamma_\lambda|$ and for all $x\in l_\infty$,*

$$\limsup_p \sup_n \sum_k \left(d_{pk}(n) - \alpha_k\right)x_k \leq \frac{\gamma+\Gamma_\lambda}{2}L(x) - \frac{\gamma-\Gamma_\lambda}{2}l(x) \tag{2.2}$$

if and only if

$$\limsup_p \sup_n \sum_k \left|d_{pk}(n) - \alpha_k\right| \leq \gamma \tag{2.3}$$

Proof. Firstly, let (2.2) holds. Define a matrix **C** = $(c_{pk}(n))$ by

$$c_{pk}(n) = d_{pk}(n) - \alpha_k \tag{2.4}$$

for all k, n, $p\in N$. Then, the matrix **C** satisfies the conditions of Lemma 1.2. So, we have (1.1) for **C**. Hence, by (2.2), we can write

$$\limsup_p \sup_n \sum_k \left|c_{pk}(n)\right| = \limsup_p \sup_n \sum_k \; c_{pk}(n)\,y_k$$

$$\leq \frac{\gamma+\Gamma_\gamma}{2}L(y) - \frac{\gamma-\Gamma_\gamma}{2}l(y)$$

$$\leq \left(\frac{\gamma+\Gamma_\lambda}{2} + \frac{\gamma-\Gamma_\lambda}{2}\right)\!.\|y\|$$

$$\leq \gamma$$

which is the condition (2.3).

Conversely, let (2.3) holds and $x\in l_\infty$.

Then, for any given $\varepsilon > 0$, we can write

$$l(x) - \varepsilon \leq x_k \leq L(x) + \varepsilon$$

whenever $k \geq k_0$ for some $k_0 \in \mathbb{N}$. Now,

we can write

$$\sum_k c_{pk}(n)x_k = \sum_{k<k_0} c_{pk}(n)x_k + \sum_{k\geq k_0} c_{pk}(n)^+ x_k - \sum_{k\geq k_0} c_{pk}(n)^- x_k \,.$$

Hence, from Lemma 1.1 and the fact that $A \in (c, V_\sigma(\lambda))$, we get that

(2.5)
$$\limsup_p \sup_n \sum_k c_{pk}(n)x_k \le \frac{\gamma+\Gamma_\lambda}{2}(L(x)+\varepsilon) - \frac{\gamma-\Gamma_\lambda}{2}(l(x)-\varepsilon) = \frac{\gamma+\Gamma_\lambda}{2}L(x) - \frac{\gamma-\Gamma_\lambda}{2}l(x) + \gamma\varepsilon .$$

Since ε is arbitrary, the proof is completed.

In the case $\Gamma_\lambda > 0$ and $\gamma = \Gamma_\lambda$, we have the following result.

Theorem 2.4. *Let* $A \in (c, V_\sigma(\lambda.))$ *and* $x \in l_\infty$.

Then,
$$\limsup_p \sup_n \sum_k c_{pk}(n)x_k \le \Gamma_\lambda L(x)$$

if and only if

(2.6)
$$\limsup_p \sup_n \sum_k \left|c_{pk}(n)\right| = \Gamma_\lambda$$

where $c_{pk}(n)$ *is defined by (2.4).*

Also, we should note that when $A \in (c, V_\sigma(\lambda))_{reg}$ and $\lambda_p = p + 1$, Theorem 2.4 is same as Theorem 2 of [11].

Theorem 2.5. *Let* $A \in (c, V_\sigma(\lambda))$. *Then, for some constant* $\gamma \ge |\Gamma_\lambda|$ *and for all* $x \in l_\infty$,

(2.7)
$$\limsup_p \sup_n \sum_k c_{pk}(n)x_k \le \frac{\gamma+\Gamma_\lambda}{2}\beta(x) + \frac{\gamma+\Gamma_\lambda}{2}\alpha(-x)$$

if and only if (2.3) holds and

(2.8)
$$\lim_p \sum_{k\in E}\left|c_{pk}(n)\right| = 0$$

uniformly in $\boldsymbol{n}$ for every $\boldsymbol{E} \subset \boldsymbol{N}$ *with* $\delta(E) = 0$; *where* $c_{pk}(n)$ *is defined by (2.4).*

Proof. Let (2.7) holds. Then, since $\beta(x) \le L(x)$ and $\alpha(-x) \le -l(x)$, the necessity of the condition (2.3) follows from Theorem 2.3.

To show the necessity of the condition (2.8), for any $\boldsymbol{E} \subset \boldsymbol{N}$ with $\delta(E) = 0$, define a matrix $B = (b_{pk}(n))$ by

$$b_{pk}(n) = \begin{cases} c_{pk}(n), & k \in E \\ 0, & k \notin E \end{cases}$$

Then, since $A \in (c, V_\sigma(\lambda))$, we can write (1.1) for B. Now; for the same E, let us choose the sequence (y_k) as
$$y_k = \begin{cases} 1, & k \in E \\ 0, & k \notin E \end{cases}$$
Then, clearly $y \in st_0$ and so,

$$\beta(y) = \alpha(y) = st - \lim y = 0.$$

Hence, by the assumption and (1.1), we get that
$$\limsup_p \sup_n \sum_{k\in E}\left|b_{pk}(n)\right| \le \frac{\gamma+\Gamma_\lambda}{2}\beta(y) + \frac{\gamma-\Gamma_\lambda}{2}\alpha(-y) = 0$$

which implies (2.8).

Conversely; suppose that (2.3) and (2.8) hold. For any $x \in l_\infty$, let us define $E_1 = \{k : x_k > \beta(x) + \varepsilon\}$ and $E_2 = \{k : x_k < \alpha(x) - \varepsilon\}$. Then $\delta(E_1) = \delta(E_2) = 0$, [9]. Hence, the set $E = E_1 \cap E_2$ has also zero density and

(2,9) $\alpha(x) - \varepsilon \le x_k \le \beta(x) + \varepsilon$

whenever $k \notin E$. Now; it can be written that

$$\sum_k c_{pk}(n)x_k = \sum_{k\in E} c_{pk}(n)x_k + \sum_{k\notin E} c_{pk}(n)^+ x_k - \sum_{k\notin E} c_{pk}(n)^- x_k$$

Thus, since (2.8) implies that the first sum on the right hand-side is zero, by Lemma 1.1 and from

(2.9) , we get

$$\limsup_p \sup_n \sum_k c_{pk}(n)x_k$$

$$\le \frac{\gamma+\Gamma_\lambda}{2}(\beta(x)+\varepsilon) + \frac{\gamma-\Gamma_\lambda}{2}(\alpha(-x)-\varepsilon)$$
$$= \frac{\gamma+\Gamma_\lambda}{2}\beta(x) + \frac{\gamma-\Gamma_\lambda}{2}\alpha(-x)+\gamma\varepsilon .$$

Since ε is arbitrary, this completes the proof. In the case $\Gamma_\lambda > 0$ and $\gamma = \Gamma_\lambda$,

we have

Theorem 2.6. *Let $A\in (c, V_\sigma(\lambda))$ and $x\in l_\infty$. Then,*

$$\limsup_p \sup_n \sum_k c_{pk}(n)x_k \le \Gamma_\lambda \beta(x)$$

if and only if (2.6) and (2.8) hold.

In the case $A \in (c, V_\sigma(\lambda))_{\mathrm{reg}}$ and $\lambda_p = p+1$, Theorem 2.6 is reduced to Theorem 2.3 of [6].

Theorem 2.7. *Let $A \in (c, V_\sigma(\lambda))$. Then, for some constant $\gamma \ge |\Gamma_\lambda|$ and for all $x\in l_\infty$,*

(2.10) $$\limsup_p \sup_n \sum_k c_{pk}(n)x_k \le \frac{\gamma+\Gamma_\lambda}{2}V(x) + \frac{\gamma+\Gamma_\lambda}{2}V(-x)$$

if and only if (2.3) holds and

(2.11) $$\lim_p \sum_k \left|c_{pk}(n) - c_{p,\sigma(k)}(n)\right| = 0$$

*uniformly in **n** where $c_{pk}(n)$ is defined by (2.4).*

Proof. Firstly, suppose that (2.10) holds. Then, since $V(x) \le L(x)$ and $V(-x) \le -l(x)$ for all $x\in l_\infty$, the necessity of (2.3) follows from

Theorem 2.3. Define

$\mathcal{R} = (r_{pk}(n))$ by $r_{pk}(n) = c_{pk}(n) - c_{p,\sigma(k)}(n)$. Then, we have (1.1) for $\mathcal{R}$.

Let us choose y such that $y_k = 0$, $k\notin\sigma(N)$. Hence, since $(y_k - y_{\sigma(k)})\in Z$, (2.10) implies that

$$\limsup_p \sup_n \sum_k \left|r_{pk}(n)\right| = \limsup_p \sup_n \sum_k r_{pk}(n)y_{\sigma(k)}$$
$$= \limsup_p \sup_n \sum_k c_{pk}(n)(y_k - y_{\sigma(k)})$$
$$\le \frac{\gamma+\Gamma_\lambda}{2}V(y_k - y_{\sigma(k)}) + \frac{\gamma-\Gamma_\lambda}{2}V(y_{\sigma(k)} - y_k)$$
$$= 0$$

which is (2.11).

Conversely, let the conditions (2.3) and (2.11) hold. By the same argument as in Theorem 23 of [12], one can easily see that for any $x\in l_\infty$

$$\sum_k c_{pk}(n)(x_k - x_{\sigma(k)}) = \sum_k r_{pk}(n)x_{\sigma(k)}$$

where the matrices $\mathcal{C}$ and $\mathcal{R}$ are as above.

Hence, since $(x_k - x_{\sigma(k)}) \in Z$, (2.11) implies that $\mathcal{C} \in (Z, Z_\lambda)$. We also see from the assumption that (2.2) holds. Thus, taking infimum over $z\in Z$ in (2.2) we get that

$$\inf_{z\in Z}\left(\limsup_{p}\sup_{n}\sum_{k}c_{pk}(n)(x_k+z_k)\right)$$
$$\leq\frac{\gamma+\Gamma_\lambda}{2}L(x+z)\;-\frac{\gamma-\Gamma_\lambda}{2}l(x+z)$$
$$=\frac{\gamma+\Gamma_\lambda}{2}W(x)+\;\frac{\gamma-\Gamma_\lambda}{2}W(-x)\,.$$

On the other hand, since $\sigma_\lambda - lim\ \mathbf{C}\,z = 0$ for $z\in Z$,

$$\inf_{z\in Z}\left(\limsup_{p}\sup_{n}\sum_{k}c_{pk}(n)(x_k+z_k)\right)$$
$$\geq\;\limsup_{p}\sup_{n}\sum_{k}c_{pk}(n)x_k$$
$$+\;\inf_{z\in Z}\left(\limsup_{p}\sup_{n}\sum_{k}c_{pk}(n)z_k\right)$$
$$=\;\limsup_{p}\sup_{n}\sum_{k}c_{pk}(n)x_k\;.$$

Since $W(x) = V(x)$ for all $x\in\ l_\infty$ [11] , we conclude that (2.10) holds and the proof is completed.

In the case $\Gamma_\lambda > 0$ and $\gamma = \Gamma_\lambda$,

we have

Theorem 2.8. *Let* $A\in\ (c,\ V_\sigma(\lambda))$ *and* $x\in\ l_\infty$. *Then,*

$$\limsup_{p}\sup_{n}\sum_{k}c_{pk}(n)x_k\;\leq\;\Gamma_\lambda\ V(x)$$

if and only if (2.6) and (2.11) holds.

Finally, we should note that when $A\in\ (c,\ V_\sigma(\lambda))_{\mathrm{reg}}$ and $\lambda_p = p + 1$, Theorem 2.8 is same as Theorem 3 of [11].

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