ATATÜRK ÜNİVERSİTESİ YAYINLARI
ATATURK UNIVERSITY PUBLICATIONS

Research Article *Araştırma Makalesi* DOI: 10.16951/trendbusecon.1631861

Determining the Optimum Ticket Prices in Bus Transportation by Using Game Theory: Case of Gümüshane-İstanbul

Otobüs Taşımacılığında Optimum Bilet Fiyatlarının Oyun Teorisi ile Belirlenmesi: Gümüşhane-İstanbul Örneği

Banu BOLAYIR[1]

[1]Gümüşhane University, Faculty of Economics and Administrative Sciences, Department of Management Information Systems, Gümüşhane, Türkiye

ABSTRACT

In a competitive environment, companies must make strategic decisions aligned with their objectives to sustain their existence and advance. One of the methods used to determine the best strategy for companies is game theory. Using the game theory, this study aims to determine the optimal ticket prices for the services of two competing bus companies operating on the same route (Gümüşhane-İstanbul). For this purpose, data on ticket prices and passenger numbers set by the two bus companies between October 2023 and October 2024 were utilized. Based on the data obtained from the two bus companies, the companies' strategies were first identified, and revenue values were calculated. Subsequently, a two-player, zero-sum, and 7x7 game matrix was constructed using these revenue values. Through the maxmin and minmax methods in game matrix, the saddle point of the game was identified, and the expected values for each strategy determined for each of the two bus companies were calculated separately. Considering these calculations, it was concluded that both companies should set ticket prices between 1,250 TL and 1,350 TL to maximize their revenues.

JEL Codes: C72, D43, L11, R42

Keywords: Bus Transportation, Decision-Making, Game Theory, Optimum Ticket Price, Price Competition, Transportation Economics

ÖZ

Rekabet ortamında şirketler, varlıklarını sürdürmek ve ilerlemek için hedeflerine uygun stratejik kararlar almalıdır. Şirketler için en iyi stratejiyi belirlemede kullanılan yöntemlerden biri oyun teorisidir. Bu çalışmada oyun teorisini kullanarak iki rakip otobüs şirketinin aynı güzergâhtaki (Gümüşhane-İstanbul) seferleri için optimum bilet fiyatlarının belirlenmesi amaçlanmıştır. Bu amaçla iki otobüs şirketinin Ekim 2023 ile Ekim 2024 tarihleri arasında belirledikleri bilet fiyatları ile yolcu sayılarına ilişkin veriler kullanılmıştır. İki otobüs şirketinden alınan verilere göre ilk olarak firmaların stratejileri belirlenmiş ve kazanç değerleri hesaplanmıştır. Daha sonra bu kazanç değerleri kullanılarak iki oyunculu, sıfır toplamlı ve 7x7'lik bir oyun matrisi oluşturulmuştur. Oyun matrisinde maxmin ve minmax yöntemleri ile oyunun eyer noktası belirlenmiş ve her iki otobüs şirketinin belirlenen stratejileri için beklenen değerleri ayrı ayrı hesaplanmıştır. Bu hesaplamalar dikkate alınarak her iki şirketin en fazla kazanç sağlayabilecekleri bilet fiyatlarının 1.250 TL ile 1.350 TL arasında olması gerektiği sonucuna varılmıştır.

JEL Kodları: C72, D43, L11, R42

Anahtar Kelimeler: Otobüs Taşımacılığı, Karar Verme, Oyun Teorisi, Optimum Bilet Fiyatı, Fiyat Rekabeti, Ulaşım Ekonomisi

Geliş Tarihi/Received 02.02.2025
Kabul Tarihi/Accepted 25.12.2025
Yayın Tarihi/Publication Date 31.03.2026

Sorumlu Yazar/Corresponding author: Banu BOLAYIR
E-mail: banubolayir@gumushane.edu.tr

Cite this article: Bolayır, B. (2026). Determining the Optimum Ticket Prices in Bus Transportation by Using Game Theory: Case of Gümüşhane-İstanbul. *Trends in Business and Economics*, *40*(2), 258-271.

Introduction

In its broadest definition, transportation refers to the movement of people, goods, and services from one location to another (Atış & Çelikoğlu, 2022). Transportation activities are categorized into four main types: road, rail, air, and sea transportation. Among them, road transportation is the most widely used mode at both national and international levels. This extensive use increases the risk of accidents in road transportation, necessitating higher investments and monitoring efforts to enhance safety (Temel & Cevher, 2016). By making accurate investments and implementing effective monitoring systems in transportation, a country can utilize its resources efficiently and contribute to its overall development.

As can be seen in road transportation statistics from 2021, Türkiye, with a surface area of 780,043 km², has a total road network of 68,526 km, consisting of 3,532 km of motorways, 30,965 km of state roads, and 34,029 km of provincial roads. In 2021, a total of 336.188 million passengers per kilometer used road transportation in Türkiye, of which 84.572 million traveled by bus. Furthermore, the percentages of passenger transportation in 2021 were distributed as follows: 92.8% via roads with 336.188 million passengers per kilometer, 6.4% via airways with 23.017 million passengers per kilometer, 0.6% via railways with 2.251 million passengers per kilometer, and 0.2% via seaways with 751 million passengers per kilometer (General Directorate of Highways, 2021). Based on this data, road transportation emerges as the most preferred mode of transportation in Türkiye.

Considering the fact that the most popular way of transportation in Türkiye is the road transportation, the present study aims to determine the optimal ticket prices for competing bus companies operating on the same route. For this purpose, the present study employs game theory as its methodological approach. The data used in this study include the ticket prices and passenger numbers of two bus companies operating on the Gümüşhane-İstanbul route during the October 2023-October 2024 period. The results achieved in this study reveal that to maximize their revenues, the optimal ticket price range for the two bus companies should be between 1,250 TL and 1,350 TL. This study aims to contribute to the literature by utilizing game theory to establish the optimal ticket prices for bus companies operating on the same route.

Following the introduction, the present study is structured into several sections: the literature review, the method section where game theory is explained and application of the game theory is presented, the results section presenting the findings, and finally, the conclusion and recommendations section presenting the evaluation of the results achieved in this study and offering suggestions for future studies.

Literature Review

The studies on the applications utilizing the game theory approach in the literature were reviewed. The identified studies are as follows:

Nam and Kim (2000) modeled the diplomatic normalization negotiations between North Korea and Japan using the stag hunt game.

Özkan and Akçaöz (2002) applied game theory to field crops using data collected between 1980 and 1999 in Antalya Province, focusing on major field crops grown during this period.

Hayes (2003) utilized game theory to analyze decision-making processes and cooperation in libraries.

Mumcu and Kahramaner (2004) analyzed the decision-making process during the crisis between Türkiye and Syria for September and October 1998, applying game theory with a focus on the diplomacy of that period.

Chung (2005) applied game theory to analyze the crisis between China and Taiwan, a key issue affecting the USA.

Kiracı (2008) studied market structures and company costs using a Cournot game theory model and Rogers' Diffusion of Innovations Theory.

Reddy (2009) applied zero-sum game theory to detect malicious nodes on the positive data path in wireless sensor networks.

Madani (2010) investigated the applicability of game theory in water resource management and conflict resolution.

Şahin and Miran (2010) utilized game theory for farm planning under risky conditions. Data were collected via face-to-face surveys with 162 agricultural enterprises in Bayındır District of İzmir Province, based on their 2005 production period.

Bekmez and Çalış (2011) used game theory to determine the direction of credit use between the banking sector and various customer types under asymmetric information.

Wood (2011) investigated global cooperation for reducing greenhouse gas emissions using both cooperative and non-cooperative game theory approaches.

Liang and Xiao (2012) classified strategies in network security solutions employing game theory into two categories: attack-defense analysis and security metrics.

Neuhaus and Nel (2013) developed a game-theory-based model for civil litigation processes in South Africa.

Yalçıntaş (2015) examined the perception of fairness in ultimatum games used in economic decision-making, identifying factors influencing economic behavior.

Abdalzaher et al. (2016) modeled protection against malicious attacks in wireless sensor networks by using game theory.

Yavuz and Eren (2016) determined the returns of a portfolio created from the most commonly used financial market instruments between 2009 and 2014 using game theory.

Uysal et al. (2017) created a two-person zero-sum game matrix based on daily passenger numbers and ticket prices for flights between Antalya and Atatürk Airports in 2015, and they determined optimal pricing policies by using game theory.

Asoy (2018) developed a game-theory-based model for the glass ceiling syndrome to investigate the issue of gender inequality.

Acar (2019) calculated periodic portfolio optimizations for six different investment instruments between 2012 and 2018 using game theory.

Archetti and Pienta (2019) used game theory to study the behavior of cancerous cells.

Saberi et al. (2019) modeled a non-cooperative Stackelberg game to help retailers determine optimal strategies for product diversity and sales planning, considering competitors' strategies.

Soydan and Ünal (2020) applied game theory to evaluate decisions aiming to increase market shares of taxi and Uber companies operating in İstanbul's private urban transportation sector.

Using three different scenarios, Nagurney (2021) developed a game-theory model for a supply chain network involving multiple firms implementing the profit maximization strategy and competing without cooperation during the COVID-19 pandemic, addressing labor constraints due to social distancing.

Çalmaşur et al. (2022) developed zero-sum and non-zero-sum games based on the strategies of local, national, and international coffee businesses in competition with one another.

Herrera and López (2022) utilized game theory to analyze the dynamics within bus transportation.

Yazır et al. (2022) employed game theory to calculate the effects of economic growth on maritime freight tonnage and its subsequent impact on market surplus or scarcity in a competitive environment characterized by economic imbalances. The study used economic data from the International Monetary Fund (IMF).

Ahmad (2023) analyzed the issue of the inability of Palestine and Israel to implement a peace resolution over land in accordance with the Oslo Accords by modeling strategic behavior using game theory.

Ahmad et al. (2023) conducted an analysis on the effectiveness and limitations of various game theory-based models applied in route selection modeling.

Li and Jiang (2023) developed a tripartite evolutionary game model to study marine ecological pollution management in coastal areas. They analyzed stakeholder interests and preferences by formulating central government policies on marine pollution control, such as damage remediation and performance evaluations, and demonstrated their impact on the strategic decisions of local governments and maritime enterprises.

Özkaya and Bakkaloğlu (2023) modeled an international stalemate that escalated into war between two nations of unequal military strength, using matrix-input games within game theory to analyze material and moral losses.

Adler and Andreana (2024) designed an applied game theory model to investigate the equilibrium outcomes in the aviation transportation market, where competition may be disrupted by government mechanisms.

Fedeli et al. (2024) applied game theory to examine public investments in tourist destinations and determine optimal actions in competitive scenarios.

Fischer and Toffolo (2024) used game theory to explore interactions between a local electricity supplier and a group of independent heating consumers, hypothesizing that both parties could benefit from cooperating to reduce emissions in alignment with their economic interests.

Guerss and Ibrahimi (2024) employed game theory to develop fencing strategies for both pre-attack planning and execution during an attack.

Using Pinghu city in Zhejiang province as a case study, Wang et al. (2024) proposed a tripartite payment game model involving local governments, water supply companies, and water users to address water pricing in areas facing water scarcity.

A review of the literature reveals successful applications of game theory across various fields. However, no similar study has been conducted to determine optimal ticket prices for bus companies using game theory. Therefore, this study is expected to contribute to the body of literature on game theory applications.

Method

Game Theory

Game theory, a method used to predict how individuals behave in conflicts while pursuing their own interests and to determine the most accurate strategy among various possibilities, is the mathematical study of competition and cooperation (Madani, 2010).

In other words, game theory is an analytical approach used to examine the complex interactions between independent and rational stakeholders (Li & Jiang, 2023).

The game theory model represents a game between groups of players who aim to maximize their benefits (or revenues) through the strategies they employ, which are implemented through cumulative player actions. These strategies may involve collaborative or non-collaborative behaviors (Abdalzaher et al., 2016).

The fundamental concepts of game theory include the game, players, situation, actions, revenues, strategies, equilibrium (peak) point - where no player tends to change in a game - and Nash Equilibrium, a game point where no player can increase their revenue by unilaterally changing their strategy if other players' strategies remain unchanged (Arslan & Çetin, 2021).

The roots of game-theoretic thinking can be traced back to the Bible and the Talmud, which contains rules regulating the societal lives of the Babylonians. While several developments related to game theory occurred until the early 20th century, two pioneers, Émile Borel and John von Neumann, significantly shaped modern game theory. In 1921, Émile Borel demonstrated the first mathematical formulation of mixed strategies using the minmax method for two-player games, representing expected gains or losses of a player through a matrix. Building upon Borel's work, John von Neumann established the axiomatic foundations of game theory, discovered strategic games, and published his seminal paper proving the minmax theorem in 1928. Game theory was first introduced to the field of economics through the book Theory of Games and Economic Behavior, authored by John von Neumann and Oskar Morgenstern in 1944, which laid the groundwork for its current form. In 1950, John Forbes Nash garnered attention for his doctoral dissertation, which demonstrated that games possess an equilibrium point under certain assumptions for utility functions, earning him the Nobel Prize in 1994. Nash later expanded von Neumann's theories, introducing the "Nash Equilibrium" as a strategic solution for cooperative and non-cooperative games involving "n" players in his 1950-1953 articles. His contributions significantly advanced the development of modern game theory (Acar, 2019; Arslan & Çetin, 2021; Nash Jr., 1950; Nash, 1951; Nash, 1953; Özkaya & Bakkaloğlu, 2023; Peters, 2008; Şahin & Eren, 2012; von Neumann & Morgenstern, 1944).

Game theory has since evolved through numerous studies and has been successfully applied to solve a wide range of problems. Today, it is widely recognized as a critical tool for understanding cooperation and conflict among individuals in fields such as social sciences and natural sciences (Wang et al., 2010).

The conditions for game theory are as follows. A situation meeting these conditions is called a game (Esin, 2003):

- The number of players participating in the game and the number of strategies available to them are finite.

- Each player knows the possible strategies for themselves and their opponent but does not know which strategy their opponent will implement.

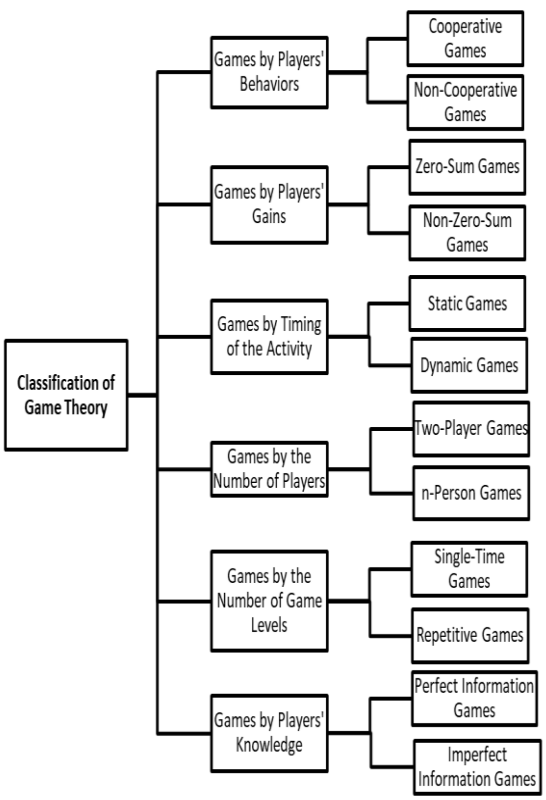
- Regardless of the strategies chosen, each player's gain or loss is bounded.

- Players' profits or losses depend not only on their own decisions but also on the strategies employed by their opponents.

- All potential behaviors must be calculable.

The classification of games is provided in Figure 1.

Figure 1.
Classification of Games



Source: Ahmad et al., 2023; Arslan & Çetin, 2021.

After each game, the matrix collectively displaying the predetermined payoffs for the respective strategies to one player by the other is referred to as the "game matrix", "payoff matrix of the game", or "reward matrix". Unless specified otherwise, the matrix is structured relative to the player listed on the left side, with positive values indicating gains for that player, negative values representing losses, and zero indicating neither gain nor loss (Esin, 2003; Öztürk, 2009). The gains and losses presented in the matrix for one player are mirrored as losses or gains, respectively, for the other player, reflecting the inverse relationship between their outcomes.

In two-player games, the matrix representation of the game is presented in Table 1. In this table, it is assumed that Player A has m strategies and Player B has n strategies (Uysal et al., 2017).

Table 1.
Payoff Matrix for a Two-Player Game

		Player B				
	Strategies	**B_1**	**B_2**	**B_3**	**.....**	**B_n**
Player A	**A_1**	a_{11}	a_{12}	a_{13}	.....	a_{1n}
	A_2	a_{21}	a_{22}	a_{23}	.....	a_{2n}
	A_3	a_{31}	a_{32}	a_{33}	.....	a_{3n}
	.....	.....	.....	.....	.....	.....
	A_m	a_{m1}	a_{m2}	a_{m3}	.....	a_{mn}

Source: Uysal et al., 2017.

A game is classified as a zero-sum game if, at the conclusion of the game, the sum of all gains and losses among the players equals zero, regardless of the strategies employed (Öztürk, 2009). In zero-sum games, one player's gain directly corresponds to the other player's loss. Conversely, in non-zero-sum games, players win or lose together (Aktan & Bahçe, 2013). In non-zero-sum games, the mathematical sum of the players' gains and losses is not zero.

The solution to two-player zero-sum games involves determining the equilibrium point of the game using maxmin (selecting the maximum value among the minimum values of each row) and minmax (selecting the minimum value among the maximum values of each column) strategies. If the maxmin and minmax values are equal, the game has an equilibrium point, and the strategy selected with a probability of "1" is referred to as a pure strategy. In some games, strategies may be chosen with certain probabilities; these are called mixed strategies (Uzunoğlu Koçer et al., 2014).

The equilibrium point of the game, also known as the saddle point, represents the value of the game. In cases where the maxmin and minmax values are not equal, matrix, algebraic, graphical, linear programming, and iterative methods are employed to determine the value of the game and the optimal strategies for the players (Esin, 2003).

In this study, an application of a two-player zero-sum game is presented.

This study was carried out in line with the rules of scientific research and publication ethics. Ethics committee approval of the research was granted by the Scientific Research and Publication Ethics Committee of Gümüşhane University (on November 15, 2024 and decree number 2024/9).

Application

This study aims to determine the optimal ticket prices for two competing bus companies operating between Gümüşhane and İstanbul. Game theory was employed for this purpose. For confidentiality purposes, the competing bus companies are referred to as Company A and Company B. Although the companies are anonymized as A and B, ethical approval was obtained from the Scientific Research and Publication Ethics Committee of Gümüşhane University (Date: November 15, 2024 and Number: 2024/9). Written informed consent was obtained from the employees of the company participating in this study.

The data used in the study, collected between October 2023 and October 2024, include ticket prices set by the companies and the corresponding passenger numbers. These data were obtained from authorized personnel of the respective companies. The strategies devised for Companies A and B, along with the calculated revenue values by multiplying the average passenger numbers and the average ticket prices, are presented in Tables 2 and 3. The price range for the strategies was determined to have an interval of 100 TL. All calculations were performed using Microsoft Excel Solver.

Table 2.

Strategies, Passenger Numbers, Ticket Prices, and Revenues for Company A (October 2023-October 2024)

Strategy	Average Number of Passengers	Average Ticket Price (TL)	Revenue (TL)
A1 (setting a price between 750 TL and 850 TL)	850	800	680,000
A2 (setting a price between 850 TL and 950 TL)	460	900	414,000
A3 (setting a price between 950 TL and 1,050 TL)	410	1,000	410,000
A4 (setting a price between 1,050 TL and 1,150 TL)	500	1,100	550,000
A5 (setting a price between 1,150 TL and 1,250 TL)	550	1,200	660,000
A6 (setting a price between 1,250 TL and 1,350 TL)	950	1,300	1,235,000
A7 (setting a price between 1,350 TL and 1,450 TL)	800	1,400	1,120,000

Table 3.

Strategies, Passenger Numbers, Ticket Prices, and Revenues for Company B (October 2023-October 2024)

Strategy	Average Number of Passengers	Average Ticket Price (TL)	Revenue (TL)
B1 (setting a price between 750 TL and 850 TL)	800	800	640,000
B2 (setting a price between 850 TL and 950 TL)	550	900	495,000
B3 (setting a price between 950 TL and 1,050 TL)	460	1,000	460,000
B4 (setting a price between 1,050 TL and 1,150 TL)	490	1,100	539,000
B5 (setting a price between 1,150 TL and 1,250 TL)	570	1,200	684,000
B6 (setting a price between 1,250 TL and 1,350 TL)	880	1,300	1,144,000
B7 (setting a price between 1,350 TL and 1,450 TL)	790	1,400	1,106,000

As seen in Tables 2 and 3, even though the increases in ticket prices initially caused a decrease in the average number of patients between October 2023 and October 2024, the number of passangers increased after that period because road transportation is preferred more, despite the increases in ticket prices. To better understand this finding, the diagrams for the average numbers of passangers and average ticket prices of Company A and Company by are illustrated in Figures 2 and 3.

Figure 2.
Passenger Numbers and Ticket Prices for Company A (October 2023-October 2024)

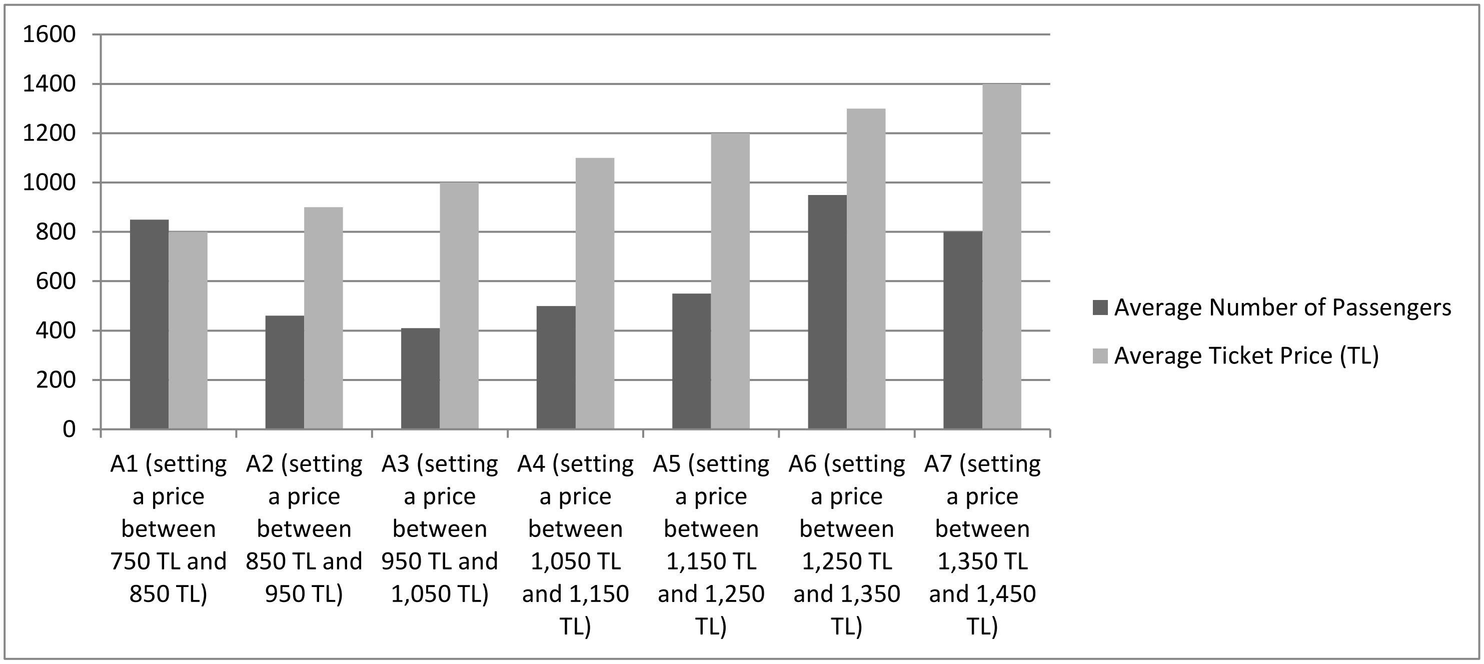
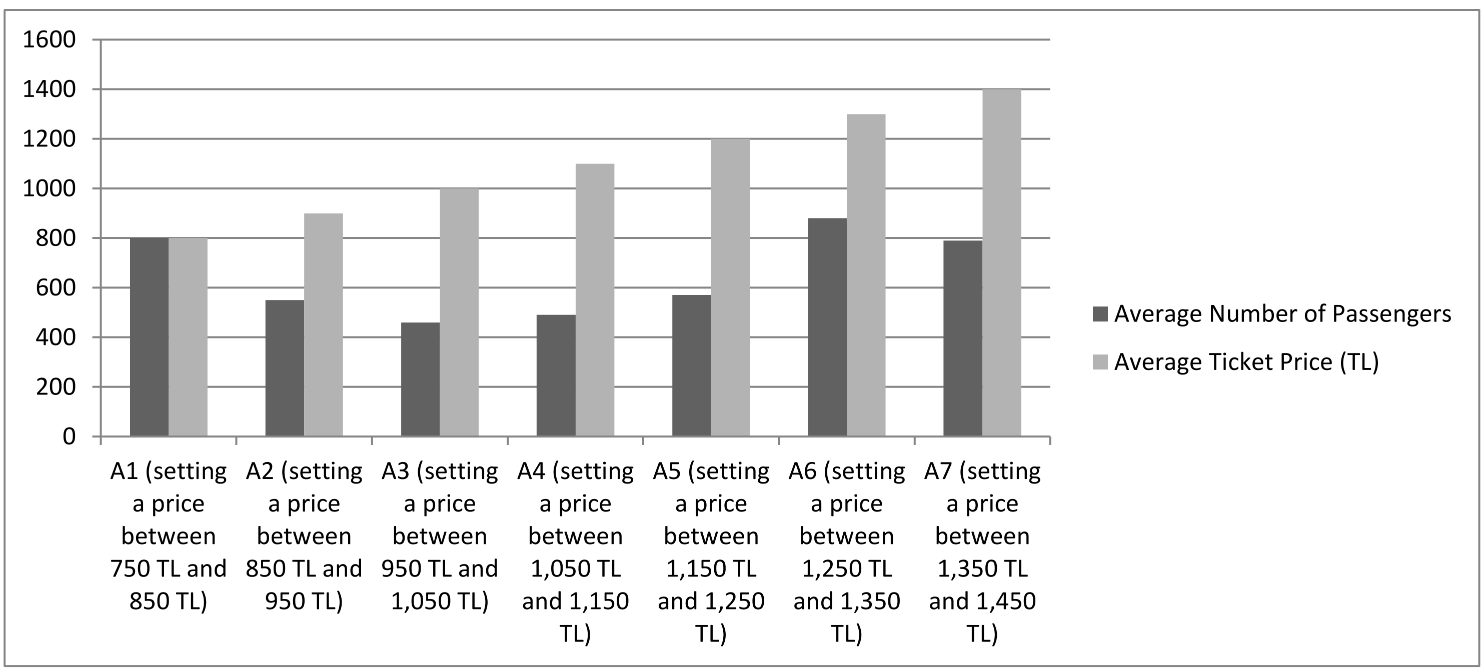


Figure 3.
Passenger Numbers and Ticket Prices for Company B (October 2023-October 2024)



Results

Based on the revenues from Tables 2 and 3, a two-player zero-sum 7×7 game matrix was constructed by dividing the revenues of Company A by those of Company B, as shown in Table 4. The zero-sum condition is satisfied when the sum of the revenues for Company A equals the sum of the revenues for Company B, highlighting the competitive nature of the two companies. Additionally,

Table 4 includes the calculated maxmin and minmax values.

Table 4.

Game Matrix Constructed for Companies A and B (Revenue A/Revenue B)

		Company B							
	Revenue A/Revenue B	**B_1**	**B_2**	**B_3**	**B_4**	**B_5**	**B_6**	**B_7**	**maxmin**
Company A	**A_1**	1.063	1.374	1.478	1.262	0.994	0.594	0.615	0.594
	A_2	0.647	0.836	0.900	0.768	0.605	0.362	0.374	0.362
	A_3	0.641	0.828	0.891	0.761	0.599	0.358	0.371	0.358
	A_4	0.859	1.111	1.196	1.020	0.804	0.481	0.497	0.481
	A_5	1.031	1.333	1.435	1.224	0.965	0.577	0.597	0.577
	A_6	1.930	2.495	2.685	2.291	1.806	1.080	1.117	**1.080**
	A_7	1.750	2.263	2.435	2.078	1.637	0.979	1.013	0.979
	minmax	1.930	2.495	2.685	2.291	1.806	**1.080**	1.117	

In Table 4, except for the maxmin column and the minmax row, the calculated values were obtained by dividing the revenues determined for strategies of Company A by the revenues determined for the corresponding strategies of Company B. The resulting values were rounded to the thousands after the decimal point. For example, the A1B1 cell was calculated by dividing the revenue value of 680,000 TL for strategy A1 by the revenue value of 640,000 TL for strategy B1. Similarly, the A2B7 cell was calculated by dividing the revenue value of 414,000 TL for strategy A2 by the revenue value of 1,106,000 TL for strategy B7. After performing these calculations, the minimum cell values (row-wise) were identified for each strategy of Company A. Subsequently, the maximum value among these minimum values (1.080) was determined. In a similar manner, the maximum cell values (column-wise) were identified for each strategy of Company B, and the minimum value among these maximum values (1.080) was determined. Since the maxmin value and the minmax value in the matrix were found to be equal, the game has a saddle (equilibrium) point, which is "1.080". Based on the equilibrium point value of 1.080, when Company A adopts strategy A6 and Company B adopts strategy B6—corresponding to ticket prices set between 1,250 TL and 1,350 TL—both companies achieve a scenario where their revenues are maximized.

Finally, to calculate the expected values for the players, it was initially assumed that both bus companies had equal probabilities of selecting their strategies. The expected values calculated below represent the gains for Company A from each strategy selection and the corresponding losses for Company B.

If the probability of Company A selecting a strategy is denoted as p_i, where i=1,2,…,7, then the probability of selecting each strategy is:

$p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = 1/7 = 0.143$

$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 = 1$

Expected values for Company A;

Expected Value A_1 = (0.143*1.063)+(0.143*1.374)+(0.143*1.478)+(0.143*1.262)+(0.143*0.994)+(0.143*0.594)+(0.143*0.615)=1.055

Expected Value A_2 = (0.143*0.647)+(0.143*0.836)+(0.143*0.900)+(0.143*0.768)+(0.143*0.605)+(0.143*0.362)+(0.143*0.374)=0.642

Expected Value A_3 = (0.143*0.641)+(0.143*0.828)+(0.143*0.891)+(0.143*0.761)+(0.143*0.599)+(0.143*0.358)+(0.143*0.371)=0.636

Expected Value A_4 = (0.143*0.859)+(0.143*1.111)+(0.143*1.196)+(0.143*1.020)+(0.143*0.804)+(0.143*0.481)+(0.143*0.497)=0.854

Expected Value A_5 = (0.143*1.031)+(0.143*1.333)+(0.143*1.435)+(0.143*1.224)+(0.143*0.965)+(0.143*0.577)+(0.143*0.597)=1.024

Expected Value A_6 = (0.143*1.930)+(0.143*2.495)+(0.143*2.685)+(0.143*2.291)+(0.143*1.806)+(0.143*1.080)+(0.143*1.117)=1.917

Expected Value A_7 = (0.143*1.750)+(0.143*2.263)+(0.143*2.435)+(0.143*2.078)+(0.143*1.637)+(0.143*0.979)+(0.143*1.013)=1.738

Considering the expected values calculated for Company A, the highest expected value is A_6.

If the probability of Company B selecting a strategy is denoted as q_i, where j=1,2,…,7, then the probability of selecting each strategy is:

$q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = q_7 = 1/7 = 0.143$

$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 = 1$

Expected values for Company B;

Expected Value B_1 = (0.143*1.063)+(0.143*0.647)+(0.143*0.641)+(0.143*0.859)+(0.143*1.031)+(0.143*1.930)+(0.143*1.750)=1.133

Expected Value B_2 = (0.143*1.374)+(0.143*0.836)+(0.143*0.828)+(0.143*1.111)+(0.143*1.333)+(0.143*2.495)+(0.143*2.263)=1.464

Expected Value B_3 = (0.143*1.478)+(0.143*0.900)+(0.143*0.891)+(0.143*1.196)+(0.143*1.435)+(0.143*2.685)+(0.143*2.435)=1.576

Expected Value B_4 = (0.143*1.262)+(0.143*0.768)+(0.143*0.761)+(0.143*1.020)+(0.143*1.224)+(0.143*2.291)+(0.143*2.078)=1.345

Expected Value B_5 = (0.143*0.994)+(0.143*0.605)+(0.143*0.599)+(0.143*0.804)+(0.143*0.965)+(0.143*1.806)+(0.143*1.637)=1.060

Expected Value B_6 = (0.143*0.594)+(0.143*0.362)+(0.143*0.358)+(0.143*0.481)+(0.143*0.577)+(0.143*1.080)+(0.143*0.979)=0.634

Expected Value B_7 = (0.143*0.615)+(0.143*0.374)+(0.143*0.371)+(0.143*0.497)+(0.143*0.597)+(0.143*1.117)+(0.143*1.013)=0.655

Considering the expected values calculated for Company B, the lowest expected value is B_6.

It can be seen in the calculations above that the equilibrium point of the game matrix - A6 strategy for Company A and B6 strategy for Company B - represent the highest expected value for the A company and the lowest expected value for the B company, respectively.

Considering these calculations, both A and B bus companies would achieve the highest revenue if they reduce their current ticket prices and set the ticket prices in the range of 1,250 TL to 1,350 TL.

Reliability and Validity

The reliability and validity of this study are assessed based on the methods used, data sources, and analytical processes.

Reliability

Reliability pertains to the replicability of the study and the consistency of the obtained results. The data used in this study were directly collected from bus companies, and the calculations were performed using mathematical methods commonly applied in game theory. All calculations were conducted using Microsoft Excel Solver, and independent replications of the analysis yielded similar results.

When compared to previous studies, the ticket price range proposed in this study (1,250 TL-1,350 TL) aligns with the results reported in similar optimization studies employing game theory. This consistency indicates that the results are generalizable and applicable to different transportation types.

However, a limitation in terms of reliability is that the data were not validated against financial reports of the companies or independent institutions. However, the alignment of the results with those reported in previous studies (Herrera & López, 2022; Uysal et al., 2017) supports the reliability of the analysis.

Validity

Validity refers to the extent to which a study accurately measures what it intends to measure. In this study, a game theory approach was employed to determine the optimum ticket prices for two bus companies. The data used in this study were directly obtained from the two bus companies between October 2023 and October 2024, which enhances the internal validity of this study.

Moreover, the game theory framework applied in this study has been successfully utilized in various sectors in previous studies (Ahmad et al., 2023; Fischer & Toffolo, 2024). Particularly in the transportation sector, studies utilizing game theory for pricing strategies support the validity of this methodology (Herrera & López, 2022; Uysal et al., 2017).

Furthermore, the equilibrium point of the formulated game matrix was determined using maxmin and minmax strategies. The expected values for both bus companies were subsequently calculated to verify the results, thereby enhancing the accuracy of the computations (Nash, 1951; von Neumann & Morgenstern, 1944).

Discussion

This study analyzes the price competition between two bus companies using a game theory and based on data from a specific time period. The analysis is built upon data collected between October 2023 and October 2024, providing a structured framework for understanding the firms' decision-making processes. In this context, radical market shifts—such as sudden increases in fuel prices, government subsidies, or economic crises—are beyond the scope of this study. Instead, the study focuses on understanding decision-making mechanisms based on data obtained within a defined timeframe, leading to the determination of an optimal ticket price range for the two bus companies using maxmin and minmax strategies.

Reviewing the studies on determining optimal ticket prices using game theory, one study similar to the present study could be found. This study, carried out by Uysal et al. (2017), aimed to determine the ideal pricing strategies for two airline companies. For this purpose, game theory was applied using data on the daily number of passengers and ticket prices for flights between Antalya and Atatürk Airports in 2015. The similarity between the study carried out by Uysal et al. (2017) and the present study is the use of game theory to determine optimal ticket prices, the difference lies in the transportation types analyzed. On the other hand, game theory is a method widely used for competitive sectors to determine the best decisions under uncertainty and risk, enabling them to achieve their targets, increase profits, and maintain sustainability in the industry. In this context, the study carried out by Uysal et al. (2017) and the present study demonstrated that game theory can be used to determine optimal ticket prices across different transportation types.

Conclusion and Recommendations

In the real world, every manager makes decisions under uncertainty or risk by relying on specific information while delivering a service. In competitive sectors, choosing an optimal strategy is very important for a company's survival, growth, and profitability. Game theory is one of the most preferred methods for making optimal decisions in uncertain or risky environments.

This study applied game theory to determine the optimal ticket prices for two competing passenger bus companies operating on the same route. For this analysis, the two competing companies were named A and B. Data on ticket prices and passenger numbers for the Gümüşhane-İstanbul route during the period between October 2023 and October 2024 were utilized. The results of the game theory application indicate that both companies should set their ticket prices in the range of 1,250 TL to 1,350 TL to maximize their revenue. Tables 2 and 3 also reveal that the strategies yielding the highest revenue for both companies are the A6 and B6 strategies, corresponding to ticket prices within the 1,250 TL to 1,350 TL range.

The current high inflation rate in Türkiye has significantly affected fuel prices, which in turn has affected ticket prices in the transportation sector. Despite the increases in ticket prices due to rising fuel costs, Tables 2 and 3 show an increase in the number of passengers for both companies. This increase in the number of passengers in road transportation originates from personal reasons such as service quality and transportation safety. It makes the road transportation the most popular method of transportation in Türkiye. However, each company must consider its profitability. Under current conditions, both companies should collaborate to set their ticket prices in the range of 1,250 TL to 1,350 TL, where they achieve the highest revenue.

This study examines the ticket pricing strategies of two competing bus companies operating on the same route within the framework of game theory. Due to the inherently competitive nature of the market, the present study adopts a zero-sum game approach, in which the gain of one company corresponds to the loss of the other. Zero-sum games provide an appropriate framework for analyzing price competition in the bus transportation sector. Consequently, instead of alternative competition models such as Cournot or Stackelberg, this study focuses on a two-player zero-sum game matrix, employing maxmin and minmax strategies. These methods offer insights into how firms adopt risk-averse pricing strategies and select the optimal course of action in worst-case scenarios.

Future studies could use game theory to determine optimal ticket prices for different modes of transportation over a specific time frame. Moreover, future research could combine various transportation types to determine optimal ticket prices and compare these modes in terms of their optimal price ranges under different conditions. Future studies could expand on this study by incorporating factors such as demand elasticity and customer loyalty. Additionally, an analysis considering the effects of fuel prices, macroeconomic variables, and time-based pricing (e.g., holiday seasons and regular periods) could provide further insights into pricing strategies. In this study, since there is an equilibrium point in the game matrix, methods such as matrix, algebraic, graphical, linear programming, and iterative methods were not employed. For game matrices without an equilibrium point, one or more of these methods can be utilized to determine the value of the game and identify the players' optimal strategies.

Ethics Committee Approval: This study was carried out in line with the rules of scientific research and publication ethics. Ethics committee approval of the research was granted by the Scientific Research and Publication Ethics Committee of Gümüşhane University (on November 15, 2024 and decree number 2024/9).
Informed Consent: Written informed consent was obtained from the employees of the company participating in this study.
Peer-review: Externally peer-reviewed.
Conflict of Interest: The author have no conflicts of interest to declare.
Financial Disclosure: The author declared that this study has received no financial support.
Use of Artificial Intelligence: No artificial intelligence-based tools were used at any stage of this study.

Etik Komite Onayı: Bu çalışma bilimsel araştırma ve yayın etiği kurallarına uygun olarak hazırlanmıştır. Araştırmanın etik kurul onayı Gümüşhane Üniversitesi Bilimsel Araştırma ve Yayın Etiği Kurulu tarafından verilmiştir (15 Kasım 2024 tarih ve 2024/9 sayılı karar).
Katılımcı Onamı: Bu çalışmaya katılan firmadaki çalışanlardan yazılı bilgilendirilmiş onam alınmıştır.
Hakem Değerlendirmesi: Dış bağımsız.
Çıkar Çatışması: Yazar, çıkar çatışması olmadığını beyan etmiştir.
Finansal Destek: Yazar, bu çalışma için finansal destek almadığını beyan etmiştir.
Yapay Zeka Kullanımı: Bu çalışmanın herhangi bir aşamasında yapay zeka tabanlı araçlar kullanılmamıştır.

References

Abdalzaher, M. S., Seddik, K., Elsabrouty, M., Muta, O., Furukawa, H., & Abdel-Rahman, A. (2016). Game theory meets wireless sensor networks security requirements and threats mitigation: A survey. *Sensors*, *16*(7), 1003. [CrossRef]

Acar, E. (2019). Oyun teorisi ile bireysel yatırım kararı: Minimax yaklaşımıyla portföy optimizasyonu. *International Social Sciences Studies Journal*, *5*(34), 2286-2297. [CrossRef]

Adler, N., & Andreana, G. (2024). Aiding airlines for the benefit of whom? An applied game-theoretic approach. *European Journal of Operational Research*, *314*(2), 552-564. [CrossRef]

Ahmad, A. (2023). Land for peace? Game theory and the strategic impediments to a resolution in Israel-Palestine. *Defence and Peace Economics*, *34*(4), 385-409. [CrossRef]

Ahmad, F., Almarri, O., Shah, Z., & Al-Fagih, L. (2023). Game theory applications in traffic management: A review of authority-based travel modelling. *Travel Behaviour and Society*, *32*, 100585. [CrossRef]

Aktan, C. C., & Bahçe, A. B. (2013). Kamu tercihi perspektifinden oyun teorisi. *Hukuk ve İktisat Araştırmaları Dergisi*, *5*(2), 93-117. [CrossRef]

Archetti, M., & Pienta, K. J. (2019). Cooperation among cancer cells: Applying game theory to cancer. *Nature Reviews Cancer*, *19*(2), 110-117. [CrossRef]

Arslan, B., & Çetin, E. (2021). Vekâlet teorisinde fırsatçılık kavramı ve oyun teorisi arasındaki ilişki. *Uluslararası Yönetim Akademisi Dergisi*, *4*(2), 439-454. [CrossRef]

Asoy, E. (2018). Toplumsal cinsiyet ayrımı ve cam tavan sendromu üzerine oyun teorisi modeli: Cam tavanın kırıldığı oyun dengesi. *Marmara Üniversitesi Kadın ve Toplumsal Cinsiyet Araştırmaları Dergisi*, *2*(2), 85-95. [CrossRef]

Atış, E., & Çelıkoğlu, Ş. (2022). Ulaşım coğrafyası bakımından Zonguldak Havalimanı. *Premium E-Journal of Social Science (PEJOSS)*, *6*(25), 474-488. [CrossRef]

Bekmez, S., & Çalış, F. (2011). Oyun teorisi çerçevesinde Türk bankacılık sistemi ve asimetrik bilgi problemi. *Süleyman Demirel Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi*, *16*(2), 79-96. [CrossRef]

Chung, N. (2005). The Sino-Taiwanese crisis: A game theoretic analysis. *Sigma: Journal of Political and International Studies*, *23*(1), 111-135. [CrossRef]

Çalmaşur, G., Daştan, H., Aysin, M., & Oral, F. (2022). Firmalar arasındaki rekabetin oyun teorisi aracılığıyla incelenmesi. In *8th International Congress on Economy Administration and Market Surveys* (pp. 87-96), İstanbul: Güven Plus Grup A.Ş. Yayınları. [CrossRef]

Esin, A. (2003). *Yöneylem araştırmalarında yararlanılan karar yöntemleri* (4nd ed.). Ankara: Gazi Publishing.

Fedeli, G., Tavares, J. M., & Kumail, T. (2024). Bridging information gaps: Public sector intervention in tourism promotion during crises-a game theory approach. *Zeitschrift für Tourismuswissenschaft*, *16*(1), 1-14. [CrossRef]

Fischer, R., & Toffolo, A. (2024). Game theory-based analysis of policy instrument consequences on energy system actors in a Nordic municipality. *Heliyon*, *10*(4). [CrossRef]

General Directorate of Highways (2021). *Karayolu ulaşım istatistikleri*. https://www.kgm.gov.tr/SiteCollectionDocuments/KGMdocuments/Yayinlar/YayinPdf/KarayoluUlasimIstatistikleri2021.pdf Access Date: 19.10.2024.

Guerss, F. Z., & Ibrahimi, K. (2024). Fencing sport strategy using game theory. In *2024 11th International Conference on Wireless Networks and Mobile Communications (WINCOM)* (pp. 1-6). IEEE. [CrossRef]

Hayes, R. M. (2003). Cooperative game theoretic models for decision-making in contexts of library cooperation. *Library Trends*, *51*(3), 441-461. [CrossRef]

Herrera, F., & López, S. I. (2022). Bus drivers in competition: A directed location approach. *Research in Transportation Economics*, *95*, 101220. [CrossRef]

Kiracı, Y. D. D. A. (2008). Küreselleşme ve yeni ekonomik düzende piyasa yapısı ve şirketlerin uzun vadeli maliyetleri üzerine bir oyun teorisi modeli. *Bilgi Ekonomisi ve Yönetimi Dergisi*, *3*(2), 61-75. [CrossRef]

Li, J. M., & Jiang, S. S. (2023). How can governance strategies be developed for marine ecological environment pollution caused by sea-using enterprises?-A study based on evolutionary game theory. *Ocean & Coastal Management*, *232*, 106447. [CrossRef]

Liang, X., & Xiao, Y. (2012). Game theory for network security. *IEEE Communications Surveys & Tutorials*, *15*(1), 472-486. [CrossRef]

Madani, K. (2010). Game theory and water resources. *Journal of Hydrology*, *381*(3-4), 225-238. [CrossRef]

Mumcu, C., & Kahramaner, Y. (2004). Oyun teorik yaklaşımla 1998 Türkiye-Suriye krizinin analizi. *İstanbul Ticaret Üniversitesi Dergisi*, 117-149. [CrossRef]

Nagurney, A. (2021). Supply chain game theory network modeling under labor constraints: Applications to the Covid-19 pandemic. *European Journal of Operational Research*, *293*(3), 880-891. [CrossRef]

Nam, C. H., & Kim, W. (2000). North Korea-Japan negotiations for diplomatic normalization: A game-theoretic analysis. *The Korean Journal of Defense Analysis*, *12*(1), 109-130. [CrossRef]

Nash Jr., J. F. (1950). Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences*, *36*(1), 48-49. [CrossRef]

Nash, J. (1951). Non-cooperative games. *Annals of Mathematics*, *54*(2), 286-295. Stable [CrossRef]

Nash, J. (1953). Two-person cooperative games. *Econometrica*, *21*(1), 128-140. [CrossRef]

Neuhaus, T., & Nel, A. L. (2013). Modeling the civil litigation process in South Africa using game theory. In *International Conference on Education, Law and Humanities (ELH'2013)* (pp. 106-113), 27-28 November 2013, Johannesburg, South Africa. [CrossRef]

Özkan, B., & Akçaöz, H. V. (2002). Game theory and its application to field crops in Antalya province. *Turkish Journal of Agriculture and Forestry*, *26*(5), 303-309. [CrossRef]

Özkaya, M., & Bakkaloğlu, A. (2023). Askeri açıdan denk olmayan iki ülke savaşının oyun teorisi ile modellenmesi. *Journal of Advanced Research in Natural and Applied Sciences*, *9*(2), 268-275. [CrossRef]

Öztürk, A. (2009). *Yöneylem araştırması* (12nd ed.). Bursa: Ekin Publishing.

Peters, H. (2008). *Game theory: A multi-leveled approach*. Berlin Heidelberg: Springer. [CrossRef]

Reddy, Y. B. (2009). A game theory approach to detect malicious nodes in wireless sensor networks. In *2009 Third International Conference on Sensor Technologies and Applications* (pp. 462-468), IEEE. [CrossRef]

Saberi, Z., Saberi, M., Hussain, O., & Chang, E. (2019). Stackelberg model based game theory approach for assortment and selling price planning for small scale online retailers. *Future Generation Computer Systems, 100*, 1088-1102. [CrossRef]

Soydan, N. T. Y., & Ünal, R. (2020). Game theory approach to inner city private transportation companies. In A. Özekin & N. T. Y. Soydan (Eds.), *New Approaches in Operational Research* (pp. 161-177). Bursa: Ekin Publishing. [CrossRef]

Şahin, A., & Miran, B. (2010). Risk koşullarında tarım işletmelerinin planlanması: Oyun teorisi yaklaşımı. *Hayvansal Üretim*, *51*(1), 31-39. [CrossRef]

Şahin, S., & Eren, E. (2012). Oyun teorisinin gelişimi ve günümüz iktisat paradigmasının oluşumuna etkileri. *Hukuk ve İktisat Araştırmaları Dergisi*, *4*(1), 265-274. [CrossRef]

Temel, F., & Cevher, Ş. C. (2016). Karayolunda güvenli radyoaktif madde taşımacılığı. In *Sürdürülebilir Ulaşım İçin Yol Trafik Güvenliği Ulusal Kongresi Bildiri Kitabı* (pp. 78-80), Kutem. [CrossRef]

Uysal, F., Gülmez, M., & Çubukcu, H. A. (2017). Turizmde havayolu şirketlerinin fiyat belirleme politikaları ve oyun teorisi uygulaması. *Uluslararası İktisadi ve İdari Bilimler Dergisi*, *3*(1), 5-19. **[CrossRef]**

Uzunoğlu Koçer, U., Albayrak, O., Kumaç, S., & Uzun, V. (2014). Dizüstü bilgisayar tercihinde öğrenci beğenilerinin firmaların stratejilerine etkisi: Oyun kuramı yaklaşımı. *Dokuz Eylül Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, *16*(1), 159-178. **[CrossRef]**

von Neumann, J., & Morgenstern, O. (1944). *Theory of games and economic behavior*. Princeton, New Jersey: Princeton University Press.

Wang, B., Wu, Y., & Liu, K. R. (2010). Game theory for cognitive radio networks: An overview. *Computer Networks*, *54*(14), 2537-2561. **[CrossRef]**

Wang, S., Liu, S., Yao, S., Guo, X., Soomro, S. E. H., Niu, C., Quan, L., & Hu, C. (2024). Game theory applications in equilibrium water pricing of multiple regional sources and users. *Water*, *16*(13), 1845. **[CrossRef]**

Wood, P. J. (2011). Climate change and game theory. *Annals of the New York Academy of Sciences*, *1219*(1), 153-170. **[CrossRef]**

Yalçıntaş, M. (2015). Ekonomik karar almada adalet ve oyun teorisi. *Maliye ve Finans Yazıları*, (103), 247-273. **[CrossRef]**

Yavuz, M. & Eren, T. (2016). Finansal araçların oyun teorisi ile analiz edilmesi. *Bartın Üniversitesi İ.İ.B.F. Dergisi*, *7*(13), 122-139. **[CrossRef]**

Yazır, D., Şahin, B., & Yıp, T. (2022). Ship tonnages determined by the world economic growth with game theory. *Turkish Journal of Engineering*, *6*(4), 300-305. **[CrossRef]**

Genişletilmiş Özet

Amaç: Türkiye'de en çok tercih edilen ulaşım türünün karayolu olması göz önünde bulundurularak bu çalışmada, rekabet halinde olan otobüs firmalarının aynı güzergâhtaki seferleri için optimum bilet fiyatlarının belirlenmesi amaçlanmıştır.

Yöntem: Çalışmada belirlenen amaca ulaşmak için oyun teorisi yöntemi kullanılmıştır. Çalışmada kullanılan veriler, Gümüşhane ile İstanbul arasında yolcu taşıyan iki otobüs firmasının 2023 Ekim-2024 Ekim döneminde belirledikleri bilet fiyatları ve taşıdıkları yolcu sayılarıdır. Bu veriler kullanılarak, öncelikle firmaların stratejileri belirlenmiş ve kazanç değerleri hesaplanmış, daha sonra hesaplanan kazanç değerlerine göre iki kişilik, sıfır toplamlı ve 7x7'lik oyun matrisi kurulmuştur. Kurulan oyun matrisinde maxmin ve minmax yöntemleri ile oyunun eyer noktası bulunmuştur. Bulunan oyunun eyer noktası ile iki firmanın belirlenen stratejileri için ayrı ayrı beklenen değerleri hesaplanmıştır. Çalışmadaki tüm hesaplamalar, Microsoft Excel Solver programında yapılmıştır.

Bulgular: Yapılan hesaplamalardan, 2023 Ekim-2024 Ekim döneminde her iki firmanın da en fazla kazancı elde edebilmesi için bilet fiyatı aralığını 1.250 TL ile 1.350 TL arasında belirlemeleri gerektiği tespit edilmiştir.

Özgünlük: Otobüs firmaları için optimum bilet fiyatlarını oyun teorisi ile belirlemek amacıyla yapılan bu çalışmaya benzer bir çalışma bulunmadığından bu çalışmanın literatürdeki oyun teorisi uygulamalarına katkısının olacağı düşünülmektedir.