



Modeling and LQR Feedback Control for an Active Suspension of a High-Speed Train

Bir Yüksek Hızlı Trenin Aktif Süspansiyonu için Modelleme ve LQR Geribildirim Kontrolü

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ABSTRACT

In high-speed railway vehicles, ride comfort and ride quality are important. While the train is moving, it encounters different vibrations caused by the rails. Long-term experience of these vibrations can lead to health problems in passengers. In addition, the durability of the train decreases due to vibrations, its performance decreases and maintenance costs increase. A rigid suspension system is required for the train to be used well. Passive suspension, which is a traditional suspension, establishes a balance between the ride quality of the train and passenger comfort. However, passive suspension is a fixed suspension system and cannot adjust the suspension stiffness according to changing conditions. Active suspension, on the other hand, can adjust the suspension stiffness according to changing conditions. In this research, an active suspension controlled by a Linear Quadratic Regulator is designed to control the train body. Thus, vibrations are minimized and passenger comfort is increased. Various simulations were made to compare the passive and active suspensions and superiority of active suspension was proven.

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ÖZET

Yüksek hızlı demiryolu araçlarında sürüş konforu ve yolculuğun kalitesi önemlidir. Tren hareket halindeyken rayların neden olduğu çeşitli titreşimlerle karşılaşır. Bu titreşimlerin uzun süreli yaşanması yolculara sağlık sorunlarına yol açabilir. Ayrıca trenin titreşimler nedeniyle dayanıklılığı azalır, performansı düşer ve bakım maliyetleri artar. Trenin iyi kullanılabilmesi için rijit bir süspansiyon sistemine ihtiyaç vardır. Geleneksel bir süspansiyon olan pasif süspansiyon, trenin sürüş kalitesi ile yolcu konforu arasında bir denge kurar. Ancak pasif süspansiyon sabit bir süspansiyon sistemidir ve süspansiyon sertliğini değişen koşullara göre ayarlayamaz. Aktif süspansiyon ise süspansiyon sertliğini değişen koşullara göre ayarlayabilir. Bu çalışmada tren gövdesini kontrol etmek için Doğrusal Karesel Düzenleyici tarafından kontrol edilen aktif bir süspansiyon tasarlanmıştır. Böylece titreşimler en aza indirilmiş ve yolcu konforu artırılmıştır. Pasif ve aktif süspansiyonları karşılaştırmak için çeşitli simülasyonlar yapılmış ve aktif süspansiyonun üstünlüğü kanıtlanmıştır.

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1. INTRODUCTION

Railway transport has a greater capacity to carry loads than other transport methods [1]. It is energy-efficient and economical. It consumes less fuel and has lower carbon emissions than other transport methods [2]. It is not affected by the traffic congestion seen in road transport and the adverse weather conditions seen in air transport [3]. Therefore, trains are more stable in terms of timing. In addition, the railway has a long service life of 30-35 years [4]. Considering that high-speed trains travel at speeds between 220 and 355 km per hour, it is clear that it is a much faster transportation alternative compared to road transportation [5]. Suspension design is of great importance in order to make this fast transportation alternative comfortable [6].

Suspension systems are divided into three groups: passive, semi-active, and active. Passive suspension is the classic and most preferred suspension system [7]. It consists of spring, damper, and axle mechanisms [8]. It is low-cost, completely mechanical, and cannot adjust the suspension stiffness according to changing road conditions [9]. In semi-active suspension systems, the damping coefficient or spring coefficient can be variable. They do not produce force, but they can change these coefficients with an external force or signal, allowing the system to reach the desired position in a shorter time. Their equations are like uncontrolled systems. In active suspension, there is an actuator and controller. In this way, the suspension stiffness can be adjusted according to changing road conditions [10]. A more comfortable journey is provided against sudden acceleration and sudden braking [11].

Fu et al. designed a semi-active suspension and used an LQG (Linear Quadratic Gaussian) controller for a high-speed train. They decreased body vibration and improved ride comfort [12]. Singh and Kumar modeled a passenger train with a passive suspension system. Then, they replaced the lateral passive dampers of passive suspension with magneto-rheological dampers and developed a semi-active suspension. They compared the system with passive suspension and the system with semi-active suspension. They stressed that the semi-active suspension reduces vibration and increases riding comfort [13]. Wang et al. proposed a hydraulic semi-active damper for a high-speed train. As a result of their simulations, they observed that the stability of the train increased [14]. Zhang et al. proposed an active suspension with a sliding mode controller for a high-speed train. Compared to the passive suspension system, the vibration of the train body is reduced, and the risk of derailment is significantly reduced [15]. Stichel et al. designed an active suspension for a passenger train. They have made simulations compared to the passive suspension. As a result of the simulations, they have shown that the active suspension increases riding comfort [16]. Wu et al. proposed a semi-active suspension system using a magnetorheological elastomer isolator. This semi-active suspension system reduced vertical vibration compared to the passive suspension system [17]. Zhang et al. developed an active suspension with sliding mode control to increase the running safety of a high-speed train. The developed suspension was compared with the classical passive suspension, and a significant reduction in vertical vibration was observed [18]. Zhou et al. compared the fuzzy-control method and the machine-learning control method for the control of a semi-active suspension system. In their simulations, it was observed that control with machine learning was more successful and improved comfort [19]. Yang et al. used displacement-velocity and sky-hook control techniques for a semi-active suspension of a high-speed train. In their simulations, it has been proven that the displacement velocity control technique is more successful than the sky-hook control technique and the classical passive suspension method, and it provides better riding comfort [20].

In this study, an active suspension with LQR (Linear Quadratic Regulator) feedback control was designed for a high-speed train. A comparative analysis was made with the widely used passive suspension system to show the superiority of the proposed active suspension. First, the mathematical model of suspension was created. Then, the LQR feedback controller design was carried out. MATLAB Simulink software was used for modeling and simulations. Suspension travel, sprung mass acceleration, wheel deflection, and body displacement simulations were made for active and passive suspensions. The rise time, overshoot, and settling time data of active and passive suspensions in the simulations were compared. As a result of this comparison, it was shown that the active suspension showed less oscillation and had a shorter settling time. Thus, it has been observed that the active suspension with an LQR controller is more robust than the passive suspension and provides better ride comfort. In this work, the suspension model was explained in Section 2. Full state feedback control was given in Section 3. LQR control was described in Section 4. Simulations were given in Section 5. The conclusion is defined in the last section.

2. SUSPENSION MODEL

In this part, equations, parameters, and the state space of the model are explained. Newton's 2nd law of motion is used in the creation of the suspension model. Figure 1(a) is used to examine parameters of the active suspension model.

The quarter car model is widely preferred in the analysis of vertical vibrations caused by railway disturbances due to its simple and understandable structure. The mass of the train is divided into two. The mass of the train is called the sprung mass, and the bogie of the train is called the unsprung mass. Bogie is a component of railway vehicles. Its function is to provide movement of the wheels and carry the rest of the vehicle. Suspension springs and dampers are tied with the sprung and unsprung systems and the track. The transverse and longitudinal deflections of the suspension system are considered insignificant when compared to the vertical deflections. The passive suspension

system is shown in Figure 1(b). Since there is no control element in this system, the actuator force will not be taken into account.

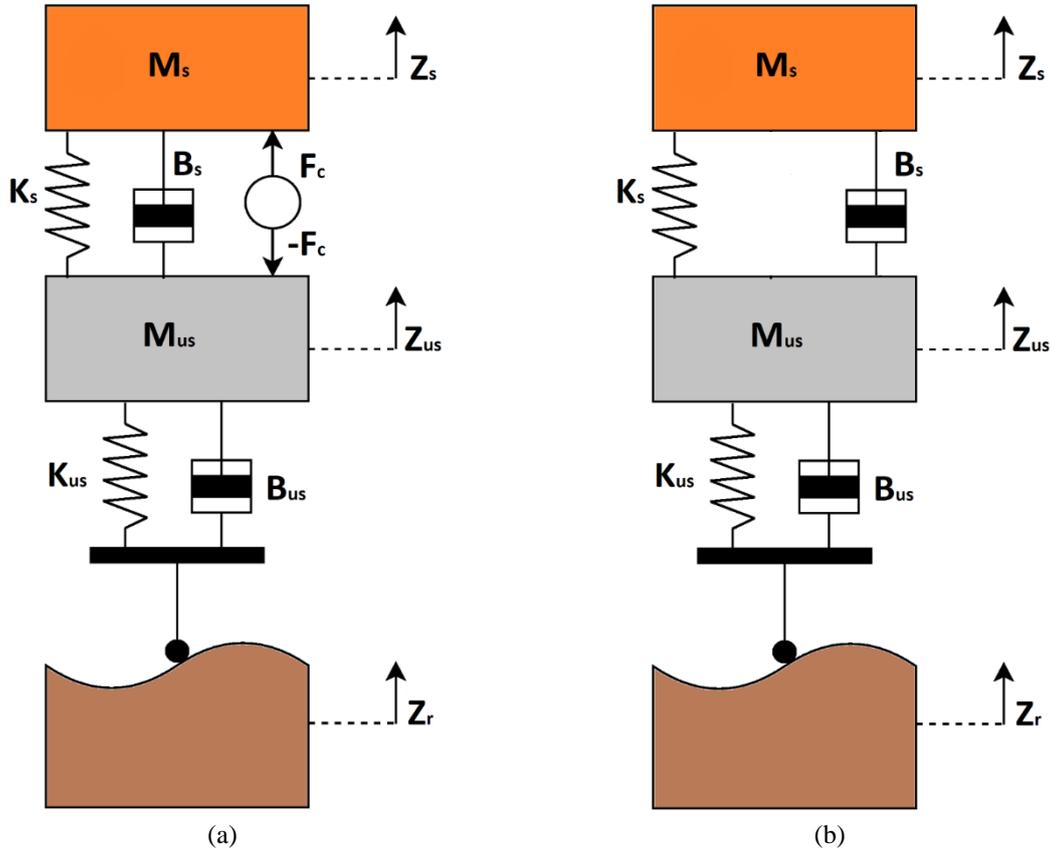


Figure 1. (a) Quarter model of the active suspension; (b) Quarter model of the passive suspension.

The definitions of the parameters of the active and passive suspensions in Figure 1 are presented in Table 1. The input parameters of the active and passive suspensions are given in Table 2.

Table 1. The definitions of suspension parameters

Parameters	Definitions
M_s	Body mass of railway vehicle (sprung mass)
M_{us}	Mass of the bogie, wheel and other parts (unsprung mass)
K_s	Spring constant of the sprung mass
K_{us}	Spring constant of the unsprung mass
B_s	Inherent damping constant of the suspension
B_{us}	Inherent damping constant of the wheel assembly
F_c	Actuator control force for active suspensions
Z_s	Body displacement of railway vehicle (sprung mass)
Z_{us}	Displacement of the unsprung mass
Z_r	Excitation due to railway disturbance

Table 2. The input parameters of the active and passive suspension systems

Parameters	Definitions
M_s	5333 kg
M_{us}	906.5 kg
K_s	430000 N/m
K_{us}	2440000 N/m
B_s	20000 sec/m
B_{us}	40000 sec/m

2.1. Equations of Motions

The motion equations of the active and passive suspensions are obtained with Newton's 2nd law of motion.

$$F = ma \tag{1}$$

When Equation (1) is translated for acceleration, Equation (2) is obtained.

$$a = F/m \tag{2}$$

Utilizing Newton's equation of motion, Equation (3) is written for the active suspension system given in Figure 1(a).

$$M_s \ddot{Z}_s = B_s \dot{Z}_{us} - B_s \dot{Z}_s - K_s(Z_s - Z_{us}) + F_c \quad (3)$$

When Equation (3) is transposed for spring-mass acceleration, it becomes:

$$\ddot{Z}_s = (B_s \dot{Z}_{us})/M_s - (B_s \dot{Z}_s)/M_s - K_s(Z_s - Z_{us})/M_s + F_c/M_s \quad (4)$$

Equation (5) gives the forces affecting the unsprung mass.

$$M_{us} \ddot{Z}_s = -B_s \dot{Z}_{us} - B_{us} \dot{Z}_{us} + B_s \dot{Z}_s + B_{us} \dot{Z}_r - K_s(Z_{us} - Z_s) - K_{us}(Z_{us} - Z_r) - F_c \quad (5)$$

When both sides of Equation (5) are divided by M_{us} , Equation (6) is obtained. Thus, the acceleration of the unsprung mass is found.

$$\ddot{Z}_s = -(B_s \dot{Z}_{us})/M_{us} - (B_{us} \dot{Z}_{us})/M_{us} + (B_s \dot{Z}_s)/M_{us} + (B_{us} \dot{Z}_r)/M_{us} - K_s(Z_{us} - Z_s)/M_{us} - K_{us}(Z_{us} - Z_r)/M_{us} - F_c/M_{us} \quad (6)$$

2.2. State Space

The state space of suspension is explained in this part. A general definition of the state space is given below.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (7)$$

The definitions of the state space variables in Equation (7) are presented in Table 3.

Table 3. The definitions of the state space variables

Parameters	Definitions
x	State variables vector
\dot{x}	Derivative of the state variables vector
y	Output vector
u	Input vector
A	System matrix
B	Input matrix
C	Output matrix
D	Feedforward matrix

The state space of the active suspension is written using the previously derived equations. The state variables are defined as in Equation (8).

$$\begin{cases} X_1 = Z_s - Z_{us} \\ X_2 = \dot{Z}_s \\ X_3 = Z_{us} - Z_r \\ X_4 = \dot{Z}_{us} \end{cases} \quad (8)$$

In Equation (8), $Z_s - Z_{us}$ symbolizes the suspension displacement. \dot{Z}_s is body velocity of railway vehicle. $Z_{us} - Z_r$ is the wheel deflection. \dot{Z}_{us} is vertical velocity of wheel.

The input u consists of railway disturbance \dot{Z}_r and actuator force F_c . Suspension displacement $Z_s - Z_{us}$ and body acceleration of the railway vehicle \ddot{Z}_s are the desired y outputs of the system.

Using the above equations, the state space could be given as in Equation (9).

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & -\frac{K_s}{M_{us}} & -\frac{B_s+B_{us}}{M_{us}} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \\ -1 & 0 \\ \frac{B_{us}}{M_{us}} & -\frac{1}{M_{us}} \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_c \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_c \end{bmatrix} \quad (9)$$

3. FULL STATE FEEDBACK CONTROL

The full-state feedback control, which is called pole placement, is the most suitable way to obtain a system's desired pole positions [21].

In the state space matrix, each state element is fed back to the controller input u through the gain K , presented by the feedback vector, and can be set to achieve the desired closed-loop poles [22]. Thus, input u is represented by Equation (10).

$$u = -Kx \quad (10)$$

Substituting the above Equation (10) into Equation (7) produces the state space in Equation (11). Figure 2 presents the full-state feedback control scheme.

$$\dot{x} = Ax - BKx \quad (11)$$

The above equation can be simplified and rewritten as:

$$\dot{x} = x(A - BK) \quad (12)$$

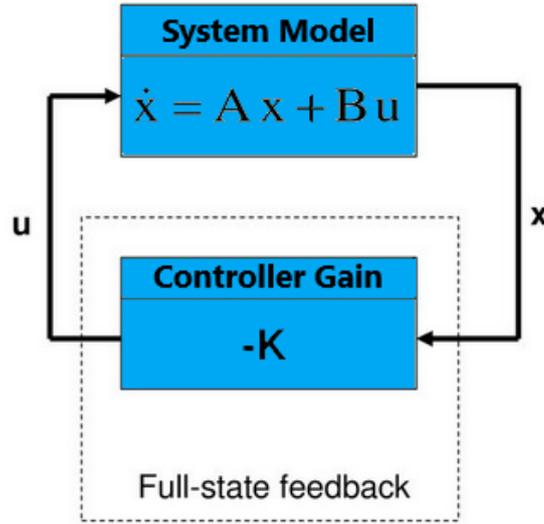


Figure 2. Full-state feedback control scheme.

4. LQR

LQR controller is a common kind of state feedback control method that systematically determines the controller gain K [23], [24]. LQR control method is used for designing the active suspension controller because it is one of the classical controller approaches for linear MIMO (Multi-Input and Multi-Output) time-invariant systems, and its design is basic. The benefit of the LQR control is that the performance parameter's elements could be weighted based on the individual's desired output. [25]. The most important purpose of the LQR controller design is to improve rail handling ability.

The main objective of LQR control is to minimize the cost equation J , which is the performance parameter given by Equation (13), and obtain the optimum gain K .

$$J = \frac{1}{2} \int_0^t (x^t Qx + u^t Ru) dt \quad (13)$$

In Equation (13), x^t is state vector and includes state variables of the system and u^t is control input. The main objective is to minimize the performance index J by determining Q and R matrices. Q is a positive diagonal definite, and R is a positive constant. The targeted closed-loop response is achieved by adjusting the weight matrices, penalizing poor performance by varying the Q matrix or penalizing actuator effort by varying the R matrix, till good results are achieved with respect to the cost function.

According to Equation (10), the performance index with feedback regulator and response is given as follows:

$$F_c = -Kx \quad (14)$$

This means matrices A and B correspond to the actuator control force in the feedback regulator and specify the following matrices.

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & -\frac{K_s}{M_{us}} & -\frac{B_s+B_{us}}{M_{us}} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\frac{1}{M_{us}} \end{bmatrix} \quad (15)$$

Since A and B are controllable, the LQR control method could be implemented using the MATLAB program. These matrices are entered into the MATLAB program. Thus, the weight matrices Q and R have become adjustable in the MATLAB program. The Q matrix is the weight matrix of the states. The R matrix is the weight matrix of the control input. Since the system has 4 states, the Q matrix is a 4x4 matrix. Since the system has a single controller input u, the R matrix is a 1x1 matrix. The Q matrix is defined diagonally. The Q and R values are determined manually. The performance measurements which have partial importance and should be focused on are the cases connected to suspension movement and train body acceleration. By changing the non-zero variables in Q matrix and the input weighting of the R matrix with trial and error, Q and R are obtained as in Equation (16).

$$Q = \begin{bmatrix} 1760 \times 10^6 & 0 & 0 & 0 \\ 0 & 11.6 \times 10^6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R = 0.01 \quad (16)$$

The state matrices A and B and the weight matrices Q and R were put into the MATLAB program. Then, the controller gain K was calculated using the $K = \text{lqr}(A, B(:,2), Q, R)$ command in MATLAB. Thus, the feedback gain K in Equation (17) is obtained.

$$K = [1.7075 \times 10^5 \quad 0.3637 \times 10^5 \quad 0.7759 \times 10^5 \quad 0.0052 \times 10^5] \quad (17)$$

5. SIMULATIONS

Active and passive suspension systems are modeled using MATLAB Simulink. Comparative simulations were performed by giving a disturbance to the designed suspension systems. Simulations were performed by selecting the disturbance in order with the help of a manual switch. The rail disturbance is a step input with a step disturbance height of 0.06 m. The step block implies a step input between two definable values at a certain time, with a step duration of 0 seconds determined with a starting value of 0 for the simulated model. Suspension travel, sprung mass acceleration, wheel deflection, and body displacement simulations were performed for active and passive suspension systems. Simulations were performed for 10 seconds, and the horizontal axis represents time. In the simulations, it was assumed that the train traveled at a constant speed for 10 seconds. Travel, displacement, and deflection values are taken in m, and the acceleration value is taken in m/s^2 . Rise time, overshoot, and settling time values of both suspension systems were obtained. The rise time is taken as the time for the system response to reach 90% of the given reference value. Settling time was taken as the settling time within $\pm 2\%$ of the given reference. The overshoot values in the simulations are calculated numerically in m or m/s^2 . The Simulink model, including the active suspension, passive suspension, and disturbance, is given in Figure 3. Figures 4 to 9 show MATLAB simulations.

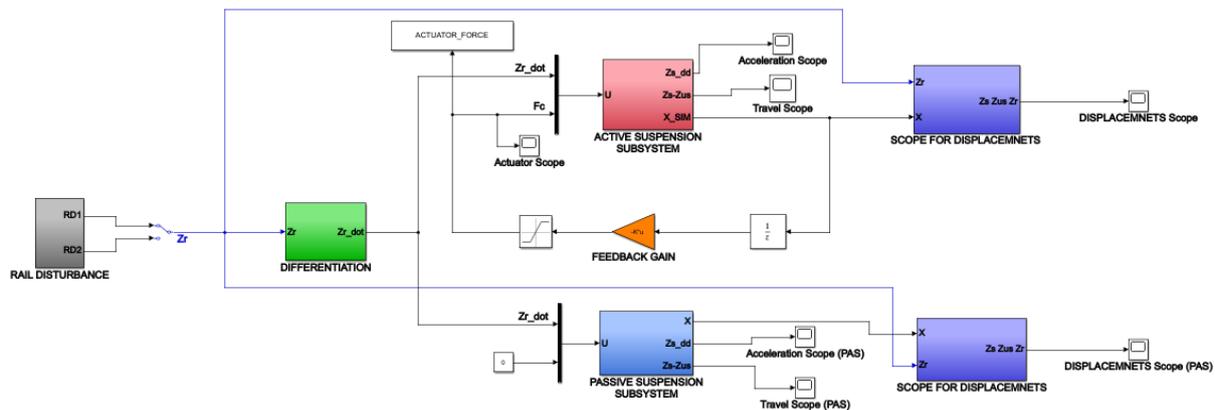


Figure 3. Active suspension, passive suspension, and disturbance.

Figure 4 presents the actuator force generated by the active suspension when a step input rail disturbance is encountered. The force generated by the actuator is applied in the reverse direction to the sprung mass and overshoots because of upward movement propensity of the sprung mass. Figure 4 represents that the actuator response is stable and shows a normal response to the disturbance.

Figure 5 presents the comparison of the suspension travel of the two suspensions. Both suspension systems show an overshoot of -0.049 m. The active suspension has a rise time of 1.20 seconds, while the passive suspension has a rise time of 1.18 seconds. The passive suspension has a slightly faster rise time. However, the active suspension system has a much shorter settling time than the passive suspension. The settling time of active suspension is 1.78 seconds, while the settling time of passive suspension is 3.29 seconds. Passive suspension oscillates too much and takes too long to settle within $\pm 2\%$ of the reference value.

Therefore, the active suspension gives a faster response, improves ride comfort, and reduces vibrations.

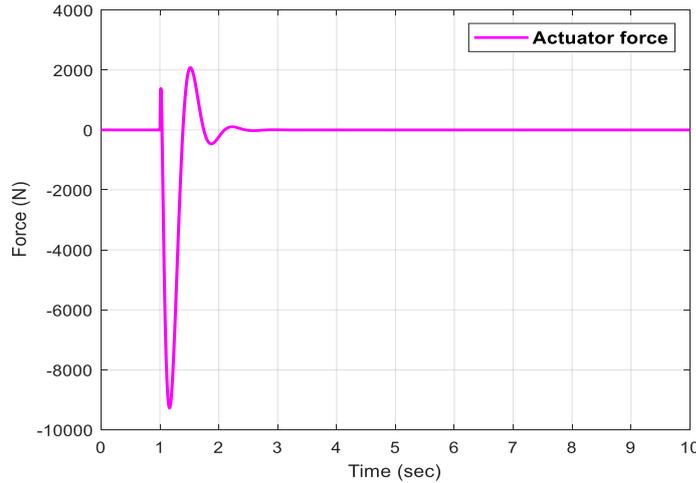


Figure 4. Actuator force.

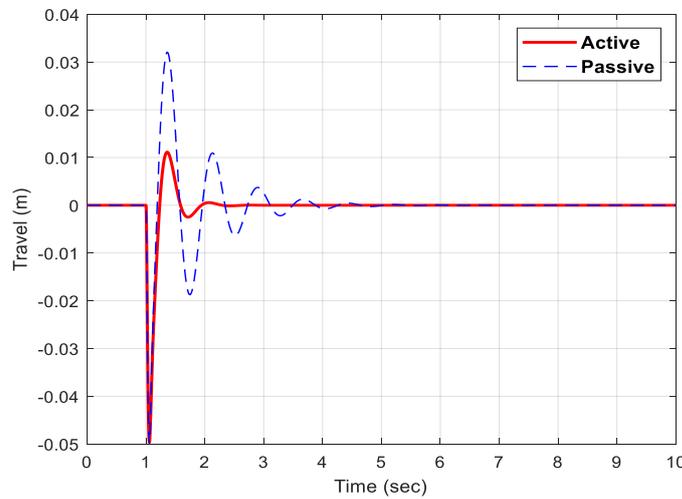


Figure 5. Suspension travel.

Table 4 gives the rise time, overshoot, and settling time data of active and passive suspension systems for suspension travel.

Table 4. The rise time, overshoot, and settling time data for suspension travel

Suspension	Rise Time(s)	Overshoot (m)	Settling Time (s)
Active	1.20 s	-0.049 m	1.78 s
Passive	1.18 s	-0.049 m	3.29 s

In Figure 6, suspensions are compared in terms of the acceleration of the sprung mass. The rise time of the active suspension is 1.08 seconds, while the rise time of the passive suspension is 1.1 seconds. The rise times of the suspensions are very close to each other. The active suspension, like the passive suspension, presents an acceleration of 7.47 m/s^2 . However, when compared in terms of settling time, the active suspension shows a 42% improvement and settles in 1.73 seconds. Passive suspension oscillates for a longer period of time and takes longer to settle within $\pm 2\%$ of the reference value. The settling time of the passive suspension is 2.96 seconds. It is clear that the shorter settling time of the active suspension improves ride comfort and railroad holding.

Table 5 gives the time response data of active and passive suspensions for sprung mass acceleration.

Table 5. The rise time, overshoot, and settling time data for sprung mass acceleration

Suspension	Rise Time (s)	Overshoot (m/s^2)	Settling Time (s)
Active	1.08 s	7.47 m/s^2	1.73 s
Passive	1.1 s	7.47 m/s^2	2.96 s

Figure 7 shows the wheel deflection simulation of the suspensions. The rise times of active and passive suspensions are the same, 1.05 seconds. Both active and passive suspension systems show an overshoot of -0.025 m .

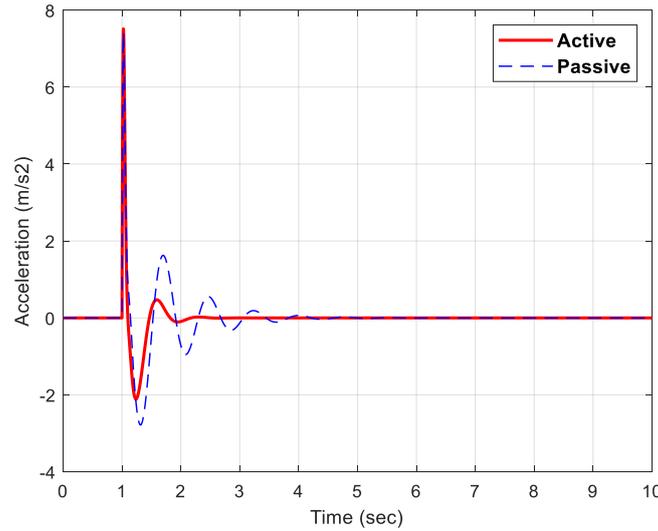


Figure 6. Sprung mass acceleration.

The passive suspension has a settling time of 2.12 seconds while the active suspension has a settling time of 1.39 seconds. This time is 35% shorter than the settling time of the passive suspension. Passive suspension oscillates for a longer period of time and takes longer to settle within $\pm 2\%$ of the reference value. In this case, the active suspension with the LQR controller has less vibration, minimizes irregular movements and wheel deflection, and provides better railroad handling.

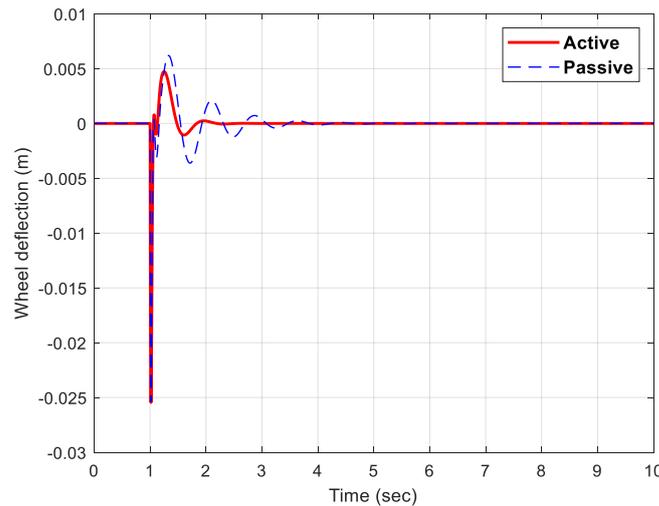


Figure 7. Wheel deflection.

Table 6 gives the time response data of active and passive suspension systems for wheel deflection.

Table 6. The rise time, overshoot and settling time data for wheel deflection

Suspension	Rise Time (s)	Overshoot (m)	Settling Time (s)
Active	1.05 s	-0.025 m	1.39 s
Passive	1.05 s	-0.025 m	2.12 s

In Figure 8, displacements of sprung and unsprung masses under rail disturbances for the passive suspension are compared. In Figure 9, the simulation for the active suspension is given. The unsprung masses have the same rise time in both suspension systems. The rise time is 1.045 seconds. In the passive suspension system, the rise time of the sprung mass is 1.172 seconds. In the active suspension system, the rise time of the sprung mass is 1.19 seconds. The rise times of the sprung masses in the active and passive suspension systems are very similar. Sprung mass shows an overshoot of -0.052 m in both of the suspensions. The overshoot of the unsprung mass is the same in both active and passive suspension systems and is -0.026 m. In the passive suspension, the unsprung mass has a settling time of 1.88 seconds. In the active suspension, the unsprung mass has a settling time of 1.42 seconds. In this case, it means a 25% shorter settling time compared to the passive suspension. The settling time of the sprung mass is 3.42 seconds in the passive suspension and 1.54 seconds in the active suspension. In this case, the active suspension system provides a 55% reduction in settling time. It is proven that by applying the proposed LQR

control method, the output response of the rail vehicle body and bogie can reach the steady state in approximately the same time as seen in Figure 9. Thus, the active suspension system with LQR control can significantly decrease the body displacement of the railway vehicles compared to the passive suspension. This ensures good riding comfort for the travelers.

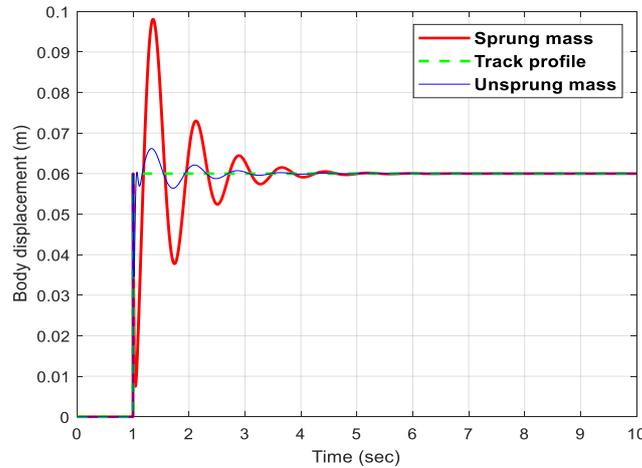


Figure 8. Body displacements for the passive suspension.

Table 7 shows the time response of the passive suspension system for body displacements.

Table 7. Time response data of passive suspension for body displacements

Body	Rise Time (s)	Overshoot (m)	Settling Time (s)
Sprung mass	1.172 s	-0.052 m	3.42 s
Unsprung mass	1.045 s	-0.052 m	1.88 s

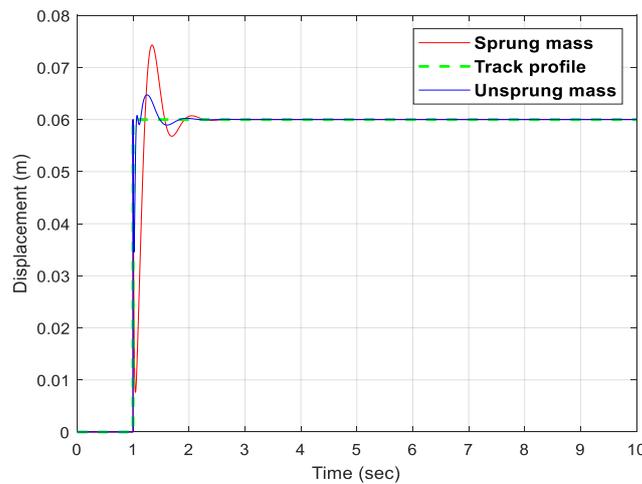


Figure 9. Body displacements for the active suspension.

Table 8 shows the time response of the active suspension system for body displacements.

Table 8. Time response data of the active suspension for body displacements

Body	Rise Time (s)	Overshoot (m)	Settling Time (s)
Sprung mass	1.19 s	-0.052 m	1.54 s
Unsprung mass	1.045 s	-0.026 m	1.42 s

6. CONCLUSION

In this work, an active suspension with LQR feedback control was developed for a high-speed train. The active suspension with LQR controller was compared with the classical passive suspension system without controller. Firstly, the schematic representation and parameters of the suspension are explained. Then, the equations of motion, state space representation and LQR feedback control are explained. MATLAB software is used for modeling and simulation. To emphasize the superior features of the developed active suspension system, comparative simulations were made with passive suspension. Suspension travel, sprung mass acceleration, wheel deflection and body displacements simulations were made under the rail disturbance. In the simulations, the rise

time, overshoot and settling time values of active and passive suspensions were obtained. These values are given in Table 4 to Table 8. This makes it possible to make a detailed mathematical analysis. Rise times are very similar in both suspension systems, although passive suspension has a slightly faster rise time than the active suspension with the LQR controller. Even though the maximum overshoot shown by active and passive suspensions is the same, the passive suspension oscillates too much. It was observed that the active suspension with the LQR controller oscillates less and significantly shortens the settling time. Simulation results showed that the active suspension with the LQR controller had a shorter settling time ranging from 25% to 42%. Thus, it was shown that the active suspension with the LQR controller provided less vibration to the passengers and provided a more comfortable journey. In this study, only the LQR controller design was used as the controller. In future studies, different controller designs for the active suspension system will be implemented and compared with the LQR controller system.

Author Contributions

The entire study was carried out by a sole author.

Conflict of Interest

The author has no conflicts of interest to declare.

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