Akademik Araştırmalar ve Çalışmalar Dergisi 2025, 17(32), 447-463
Journal of Academic Researches and Studies 2025, 17(32), 447-463
https://doi.org/10.20990/kilisiibfakademik.1641024

Makale Türü: Araştırma Makalesi
Paper Type: Research Paper

Geliş Tarihi/Received Date: 16.02.2025
Kabul Tarihi/Accepted Date: 14.05.2025

Government Support and SME Growth in Türkiye: A Romer Model Approach with Ridge Regression

Türkiye'de Devlet Desteği ve KOBİ Büyümesi: Ridge Regresyon ile Bir Romer Modeli Yaklaşımı

Serkan ŞENGÜL[1], Pınar KARAHAN DURSUN[2]

Abstract

Purpose: This study examines the growth performance of Small and Medium Enterprises (SMEs) in Türkiye through the lens of Romer's (1990) endogenous growth model, focusing on the effects of government support, technological progress, and human capital. The research addresses the empirical gap in applying Romer's framework at the microeconomic level and introduces Ridge regression as a methodological innovation to address multicollinearity challenges common in SME data.

Design/methodology: Using a unique national dataset covering 2015–2022, the study applies Ridge regression analysis to estimate the effects of government subsidies, R&D expenditures, patent applications, human capital, and imports on SME production performance. Ridge regression over traditional methods is motivated by its ability to deliver stable coefficient estimates in the presence of high correlations between predictors.

Findings: The empirical results reveal that government support ($\beta = 0.152$, $p < 0.01$), R&D expenditures ($\beta = 0.166$, $p < 0.01$), and imports ($\beta = 0.150$, $p < 0.05$) have significant positive effects on SME production value. Human capital, measured by the number of employees ($\beta = 0.140$, $p<0.05$), also positively impacts performance. Interestingly, patent applications show an adverse short-term effect ($\beta = -0.020$, $p<0.05$), suggesting potential lags in innovation commercialization.

Limitations: The analysis is constrained by the short timeframe (2015–2022), limiting the ability to assess long-term dynamics or lagged effects. Future research could incorporate more extended data periods and explore alternative modeling approaches.

Originality/Value: This study is among the first to empirically test Romer's growth model at the SME level in Türkiye. It combines micro-level data with Ridge regression to address multicollinearity issues. The findings offer theoretical and policy insights into how government intervention and innovation drive SME growth.

Keywords: SMEs, Romer Growth Model, Innovation, Government Support, Ridge Regression.

Öz

Amaç: Bu çalışma, Türkiye'de Küçük ve Orta Büyüklükteki İşletmelerin (KOBİ) büyüme performansını Romer'in (1990) endojen büyüme modeli çerçevesinde incelemekte ve devlet destekleri, teknolojik ilerleme ve beşeri sermayenin KOBİ büyümesi üzerindeki etkilerine odaklanmaktadır. Çalışma, Romer modeli uygulamasındaki ampirik boşluğu doldurmayı ve KOBİ verilerinde yaygın olan çoklu doğrusal bağlantı sorunlarını ele almak üzere Ridge regresyonunu metodolojik bir yenilik olarak sunmayı amaçlamaktadır.

Tasarım/Yöntem: 2015–2022 dönemini kapsayan ulusal veri seti kullanılarak, KOBİ üretim performansı üzerinde devlet destekleri, Ar-Ge harcamaları, patent başvuruları, beşeri sermaye ve ithalatın etkileri Ridge regresyonu ile tahmin edilmiştir. Ridge regresyonu, yüksek korelasyon içeren tahmin değişkenlerinde istikrarlı katsayı tahminleri sunması nedeniyle tercih edilmiştir.

Bulgular: Ampirik sonuçlar, devlet desteklerinin ($\beta = 0{,}152$, $p < 0{,}01$), Ar-Ge harcamalarının ($\beta = 0{,}166$, $p < 0{,}01$) ve ithalatın ($\beta = 0{,}150$, $p < 0{,}05$) KOBİ üretim değeri üzerinde anlamlı ve olumlu etkileri olduğunu göstermektedir. Çalışan sayısı ile ölçülen beşerî sermaye ($\beta = 0{,}140$, $p<0{,}05$) da performansı olumlu yönde etkilemektedir. İlginç bir şekilde, patent başvuruları kısa vadede negatif bir etki göstermiştir ($\beta = -0{,}020$, $p< 0{,}05$), bu da yeniliklerin ticarileşmesinde bir gecikme olabileceğine işaret etmektedir.

Sınırlılıklar: Analiz, kısa zaman dilimi (2015–2022) ile sınırlıdır, bu da uzun vadeli dinamiklerin veya gecikmeli etkilerin değerlendirilmesini kısıtlamaktadır. Gelecekteki çalışmalar, daha uzun veri setlerini ve alternatif modelleme yaklaşımlarını içerebilir.

Özgünlük/Değer: Bu çalışma, Romer'in büyüme modelini Türkiye'de KOBİ düzeyinde ampirik olarak test eden ilk çalışmalardan biri olup, çoklu doğrusal bağlantı sorunlarını ele almak için Ridge regresyonunu mikro düzeyde veriyle birleştirmektedir. Bulgular, devlet müdahalesi ve inovasyon yoluyla KOBİ büyümesinin nasıl şekillendiğine dair teorik ve politika düzeyinde değerli içgörüler sunmaktadır.

Anahtar Kelimeler: KOBİ, Romer Büyüme Modeli, İnovasyon, Devlet Destekleri, Ridge Regresyonu

[1]Dr. Öğr. Üyesi, Mudanya Üniversitesi, Sanat ve Sosyal Bilimler Fakültesi, Ekonomi ve Finans, serkan.sengul@mudanya.edu.tr, ORCID: 0000-0001-9891-9477

[2]Dr. Öğr. Üyesi, Mudanya Üniversitesi, Sanat ve Sosyal Bilimler Fakültesi, Ekonomi ve Finans, pinarkarahan.dursun@mudanya.edu.tr, ORCID: 0000-0002-8289-6570

1. INTRODUCTION

Until the late 1980s, growth theories were primarily shaped by the Solow (1956) model, which assumed that technological progress was exogenous and predicted that growth rates across countries would converge. However, empirical evidence failed to support this prediction, raising doubts about the model's core assumptions. In response, endogenous growth models pioneered by Romer (1986) emerged, proposing that technology is an endogenous outcome shaped by a country's investments in research and development (R&D) and innovation (Acemoğlu, 2009; Özer & Çiftçi, 2009). Romer's framework emphasizes the importance of knowledge accumulation, technological progress, and human capital formation as drivers of sustained economic growth. In this view, growth is not merely a result of capital accumulation or labor expansion but can be continuously fueled by internal processes such as innovation and learning spillovers (Romer, 1990). These models also highlight the crucial role of government in enabling development, particularly in emerging economies, where proactive state interventions are needed to stimulate human capital and support innovation.

The relevance of the Romer model extends notably to small and medium-sized enterprises (SMEs), widely seen as engines of economic growth, innovation, and employment creation (Batrancea et al., 2022). As Romer's theory suggests, sustained growth depends on enhancing technological capabilities and knowledge resources, and SMEs are key actors in this process. Strengthening SMEs' innovation ability improves their performance and contributes to broader macroeconomic development. However, SMEs often face critical barriers in financing technological and innovative activities, underscoring government support's necessity. Prior research has shown that government financial assistance boosts SMEs' innovation capacity, increases patenting activity, and helps overcome typical constraints such as limited access to training and technology transfer (Doh & Kim, 2014; Oum et al., 2014). Recent empirical studies have increasingly explored the relationship between government support, SME performance, and economic growth (Batrancea et al., 2012; Xiang & Worthington, 2017; Kim et al., 2020; Anvar & Li, 2021; Peçe, 2024).

Despite this growing literature, important gaps remain. While numerous studies have examined factors shaping SME performance, few have systematically applied the Romer endogenous growth framework at the microeconomic level, especially in the context of developing countries such as Türkiye. Even more rare is the combination of this theoretical approach with advanced econometric techniques like Ridge regression, which can effectively address multicollinearity among variables such as government subsidies, R&D expenditures, and imports. This study seeks to fill these gaps by analyzing the production performance of Turkish SMEs through the lens of Romer's theory, using Ridge regression to enhance the robustness and precision of the empirical analysis. Specifically, the study investigates how government incentives, R&D expenditures, human capital, and import expenditures shape SME production outcomes in Türkiye between 2015 and 2022, thereby testing the key theoretical predictions of the Romer model in a real-world SME context.

The study hypothesizes that government incentives positively influence SME production performance, that R&D expenditures foster innovation-driven output gains, that human capital—measured through the number of employees—strengthens production capacity, and that import expenditures improve access to intermediate goods and technologies, further enhancing productivity. By analyzing these relationships, the study not only evaluates the empirical validity of the Romer framework but also provides new insights into the mechanisms through which policy interventions and firm-level factors jointly affect SME performance.

As of 2022, SMEs in Türkiye accounted for 99.7% of total enterprises, 70.6% of employment, 47.5% of personnel costs, 42.5% of turnover, and 36.3% of production value (TUIK, 2023), highlighting their critical role in the national economy. Methodologically, this study employs Ridge regression, a technique particularly well-suited to handling the multicollinearity that often arises when modeling interrelated predictors. Recent empirical research has widely endorsed Ridge regression for improving economic model estimation stability and predictive accuracy (Hoerl & Kennard, 1970; Zou & Hastie, 2005; Hastie et al., 2009). By integrating Romer's theoretical framework with Ridge regression, this study makes a conceptual and methodological contribution to the literature.

The study aims to enrich the understanding of SME growth dynamics in Türkiye by bridging theoretical insights from endogenous growth theory with empirical evidence drawn from a unique national dataset. It offers original contributions by applying the Romer framework at the SME level, using modern econometric techniques, and providing evidence-based recommendations for policymakers seeking to strengthen the role of SMEs in promoting sustained economic growth.

The remainder of the paper is organized as follows: Section 2 presents the literature review, Section 3 outlines the theoretical framework, Sections 4 and 5 describe the data and methodology, Section 6 reports the empirical results, and Section 7 concludes.

2. LITERATURE REVIEW

A substantial body of research has examined the impact of government financial support on SME performance and economic growth, highlighting both the potential and the limitations of such interventions across different contexts. Batrancea et al. (2012) analyzed the role of various financing sources for SMEs across 28 EU countries, demonstrating that instruments such as interest rates, business angels, bank loans, and public subsidies significantly promote economic growth. Similarly, Seo (2017), employing panel logit regression for OECD countries, identified financial pro-cyclicality reduction policies as particularly effective in enhancing employment and economic performance. Doh and Kim (2014) and Xiang and Worthington (2017) offered complementary evidence, showing that government assistance enhances technological innovation and income among SMEs in South Korea and Australia, respectively. Kim et al. (2020) extended these findings to the climate-tech sector, showing that government credit guarantees increase sales volumes. Anwar and Li (2021) emphasized that financial and non-financial government support improves financial performance among Pakistani SMEs.

In Türkiye, empirical studies have largely confirmed the positive effects of government support, though often within narrower analytical frameworks. For instance, Olcay and Bulu (2015) reported that financial support positively affects SMEs' net sales, while Arslan (2016), using data envelopment analysis, found that loan interest subsidies did not enhance financial performance. Yıldız (2018) highlighted that financial support significantly boosts SMEs' brand value, and Peçe (2024) demonstrated that KOSGEB credit support increases firm profitability in the TR81 region. While these studies offer important insights, they often fail to situate Türkiye's experience within the broader international context. Research by Beck, Demirgüç-Kunt, and Maksimovic (2005) provides a critical perspective, illustrating that financial and legal constraints disproportionately limit SME growth globally—an angle seldom addressed in the Turkish literature. Accordingly, integrating institutional and structural dimensions offers a valuable extension to existing national studies, which this paper aims to address.

The nexus between R&D expenditure and economic growth has also received considerable scholarly attention. In the case of Türkiye, Bozkurt (2015) identified a one-way causal relationship between economic growth and R&D expenditure, whereas Dereli and Salğar (2019) reported bidirectional causality. Öztürk and Çınar (2021), employing FMOLS and Granger causality tests, found that R&D investment fosters economic growth, while Erzurum (2023) offered a more critical perspective, suggesting that the absence of effect may be due to Türkiye's low R&D-to-GDP ratio. Internationally, Sokolov-Mladenovic et al. (2016) demonstrated the positive impact of R&D on growth within EU countries, while Güneş (2019) found one-way causality from growth to R&D in OECD nations. Bayraktar et al. (2022) further enriched this debate by establishing bidirectional causality in BRICS-T countries. Ersin et al. (2022) highlighted that even countries with relatively low R&D intensity can benefit from innovative investment.

Critically, Hall and Lerner (2009) advanced the discussion by addressing the financing challenges specific to R&D activities, particularly for small firms, noting that market failures and underdeveloped capital markets often lead to underinvestment despite the substantial social benefits of innovation. These empirical findings align with influential theoretical frameworks such as Romer's (1990) endogenous growth theory and Aghion and Howitt's (1992) creative destruction model, which both underscore the pivotal role of knowledge accumulation and innovation as engines of long-term growth. Additionally, Porter, Stern, and Furman (2002) contributed to the concept of national innovative capacity, emphasizing the complex interplay between policy, institutions, and industrial clusters in sustaining innovation performance. Integrating these perspectives enables this study to move beyond narrowly framed national analyses and position its theoretical foundation within a broader international and interdisciplinary context.

Human capital is another fundamental determinant of economic growth, widely acknowledged in literature. In Türkiye, Boztosun et al. (2016) provided evidence of a robust long-term relationship and bidirectional causality between human capital and economic growth. Complementing these findings, Pelinescu et al. (2019) confirmed the positive impact of human capital and R&D on economic performance across EU countries, while Mwatu (2023) observed similar results in Austria. Ogbeifun and Shobande (2020) underscored the importance of human capital in fostering economic growth among OECD nations. Barro's (1991) seminal cross-country study revealed that initial human capital levels significantly shape long-term growth trajectories, influencing economic performance, fertility patterns, physical investment, and political stability.

Despite these extensive empirical and theoretical contributions, the literature remains fragmented, often examining government support, R&D investment, and human capital as isolated drivers of economic performance. Few studies explore their dynamic interplay or examine how these factors collectively shape SME growth and broader macroeconomic outcomes. Addressing this critical gap, the present study integrates financial support, R&D, and human capital into a unified endogenous growth framework operationalized through ridge regression to address multicollinearity concerns. Combining insights from Turkish and international contexts, this paper contributes novel empirical evidence and theoretical refinement, offering a more holistic understanding of the mechanisms driving SME growth and economic development.

3. THEORETICAL FRAMEWORK: ROMER GROWTH MODEL

This study draws upon the Romer (1990) endogenous growth model, which emphasizes the role of knowledge accumulation, innovation, and human capital in driving long-term economic growth. The Romer framework assumes that technological progress results from purposeful investment decisions by profit-maximizing agents, particularly firms engaged in R&D activities. A distinguishing feature of the model is the no-rivalrous and partially excludable nature of knowledge, which generates positive externalities and knowledge spillovers that enhance the productivity of other firms and the broader economy.

In the specific context of SMEs, the Romer model offers important theoretical insights. Due to their limited access to external finance, SMEs often underinvest in R&D and human capital despite the substantial social benefits of their innovative activities. Government support, therefore, plays a critical role in alleviating market failures, enhancing SMEs' capacity to invest in innovation, and enabling them to contribute to aggregate knowledge accumulation and productivity growth. This creates a virtuous cycle whereby public intervention strengthens firm-level outcomes and amplifies economy-wide innovation dynamics.

The central relationships relevant to our empirical analysis can be summarized using the following simplified production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where Y_t is total output, A_t represents the stock of knowledge, K_t is physical capital, L_t is labor, and α is the capital share. Knowledge accumulation is governed by:

$$\frac{dA_t}{dt} = \delta A_t L_{A,t} \tag{2}$$

where $L_{A,t}$ is labor allocated to R&D, and δ measures the productivity of R&D effort. On the balanced growth path, long-run output growth depends critically on the growth of knowledge, which is fueled by R&D investments and human capital formation.

To ensure a clear connection between theory and empirical application, we explicitly map the theoretical constructions to the empirical variables used in this study. Specifically, the stock of knowledge A_t is proxied by R&D expenditure and patent applications; physical capital K_t is represented by capital investment data; labor L_t is measured using employment figures; government financial support is proxied by KOSGEB support programs; and human capital is captured using education indicators such as tertiary enrollment or workforce skills data. This mapping is summarized schematically in Appendix A, where the complete mathematical derivation of the model is also provided.

By integrating these elements into the empirical framework, the study aims to capture the multifaceted drivers of SME growth. The Romer model predicts that when firms engage in R&D and human capital development, they enhance their productivity and contribute to the broader technological frontier, benefiting other firms through spillovers. For SMEs, government support is particularly vital as it mitigates financing constraints and enables firms to reach innovation thresholds that would otherwise be unattainable. This theoretical foundation justifies the inclusion of financial support, R&D, and human capital variables in the empirical analysis and informs the interpretation of the results.

4. DATA

This study uses a national-level dataset covering 2015–2022 to analyze the production performance of small and medium-sized enterprises (SMEs) in Türkiye. The data obtained from KOSGEB and the Turkish Statistical Institute (TÜİK) includes six key variables: KOSGEB subsidies, SME production values, R&D expenditures, number of patent applications, number of employees, and import expenditures. These variables are integrated into the empirical model to explain the growth dynamics of SMEs within the framework of the Romer endogenous growth model.

Specifically, SME production value (denoted as *Value*) is modeled as the dependent variable. This variable represents the total economic output produced by SMEs in a given year and serves as a direct measure of SME growth. The study's main objective is to examine how production value responds to government incentives, innovation efforts, human capital, and changes in import activity.

The state subsidies provided by the Small and Medium Enterprises Development Organization (KOSGEB), denoted as *Subsidies*, are treated as proxies for physical capital in the model. In line with Romer's theoretical framework, these government incentives are expected to enhance SMEs' production capacity by facilitating capital accumulation and stimulating innovation and technological upgrading.

R&D expenditures (*R&D*) and the number of patent applications (*Patents*) represent innovation and technological development concepts. While R&D expenditures capture the input dimension of innovative activity, the number of patent applications reflects the tangible output of these efforts. Both variables are included as independent predictors to assess the effects of innovation on SME performance.

The number of employees (*Labor*) is incorporated as an indicator of human capital, which, alongside physical capital, is a fundamental engine of growth in the Romer model. This variable captures the labor capacity of SMEs and their contribution to production.

Import expenditures (*Imports*) are a proxy for access to intermediate goods and international inputs. Intermediate imports enable SMEs to access higher-quality materials and advanced production inputs, potentially enhancing productivity and competitiveness.

Logarithmic transformations of all variables are applied to reduce heteroscedasticity, account for scale differences, and minimize the influence of outliers. Table 1 presents descriptive statistics summarizing the primary variables, including means, standard deviations, minimums, and maximums.

Table 1: Descriptive Statistics

	Mean	Std. Dev.	Min	Max
ln(Value)	28.472	0.63276	27.7671	29.7085
ln(Subsidy)	21.07021	1.03644	19.6729	23.03292
ln(R&D)	22.82267	0.818049	21.81409	24.27063
ln(Patent)	7.090097	0.229572	6.784457	7.354362
ln(Labor)	16.27099	0.06452	16.20966	16.39329
ln(Import)	19.23892	0.74464	18.40166	20.59494

Table 1 presents the descriptive statistics for the log-transformed variables used in the empirical analysis. The results indicate substantial variation across key dimensions of SME activity in Türkiye over the 2015–2022 period. The production value, subsidies, and R&D expenditures exhibit relatively high dispersion, reflecting the diverse scale and intensity of economic activities among SMEs. Patent applications and labor show more stable patterns, while import expenditures display moderate variability. These summary statistics highlight the heterogeneity across SMEs and provide a solid foundation for investigating the relationships between government support, innovation, human capital, international inputs, and SME growth within the Romer growth framework.

Importantly, we clarify that the dataset consists of national-level aggregate data, not firm-level microdata. While this allows for analyzing broad sectoral dynamics and policy impacts, we acknowledge that aggregation and omitted variable bias may limit the ability to capture firm-level heterogeneity. We address this limitation in the conclusion section.

Given the relatively short period (2015–2022), formal unit root and stationarity tests were not conducted, as such tests tend to have limited statistical power in small samples. Instead, we relied on logarithmic transformation to mitigate nonstationary concerns, and we encourage future research with longer time series to explore formal unit root diagnostics.

5. METHODOLOGY

Ridge regression is a widely used method to overcome the difficulties in multiple linear regression analysis cases. When there is a high level of correlation between independent variables (multicollinearity), the reliability of classical regression analysis decreases, and coefficient estimates become unstable. In this case, Ridge regression makes the estimates more stable by adding an L2 penalty term (Hoerl & Kennard, 1970). It also reduces the risk of overfitting by controlling the complexity of the model, which is an important advantage when the training data set size is small or the number of independent variables is large (Tikhonov, 1963). Ridge regression offers flexibility in variable selection by setting some coefficients close to zero, allowing analysts to control variables' contribution (Bishop, 2006). For these reasons, Ridge regression is frequently preferred in data analysis and machine learning applications. To summarize, this method, which ensures that all necessary variables are included in the model in case of multicollinearity, aims to obtain parameter estimates with more minor variance than least squares estimates and to exclude unnecessary variables in the model (Marquardt & Snee, 1975).To describe the Ridge regression model in matrix format, we can use the general formula for the model. Ridge regression is an extension of normal regression and can be expressed as follows:

$$\hat{y} = X\beta + \varepsilon \tag{3}$$

In this equation, $\hat{y}$ is the dependent variable (SME Production Value), X is the independent variable matrix (nxp), β is the regression coefficient vector $(px1)$ and ε is the error term.

Ridge regression is used when variables have high multiplicity relationships, especially in the case of multiple linear regression. The objective of Ridge regression is to minimize the following loss function when estimating the coefficients β:

$$\min_{\beta}(\|y - X\beta\|^2 + \lambda\|\beta\|^2) \tag{4}$$

In this equation, $\|y - X\beta\|^2$ is the sum of squared errors (RSS), λ is a positive constant that controls the ridge penalty term and $\|\beta\|^2$ is the L2 norm of the β coefficients. The L2 norm is added as a penalty term to reduce the model's complexity and prevent overfitting, making it more general and stable.

As a result, the closed form solution of Ridge regression can be expressed as follows:

$$\hat{\beta}_{ridge} = (X^TX + \lambda I)^{-1}X^Ty \tag{5}$$

Adding positive constants to the diagonal elements of the explanatory variables correlation matrix significantly minimizes the matrix's conditional number. Since the Ridge solution for $\lambda = 0$ is equivalent to the least squares solution, the Ridge estimation can also be expressed as a linear transformation of the least squares estimation (Sakallıoğlu & Kaçıranlar, 2008). The most important point in Ridge regression is correctly determining the λ value.

While Hoerld and Kennard (1970) proposed the Ridge Trace method to determine λ in their article, this study will use the variance inflation factor (VIF) proposed by Marquardt and Snee (1975). VIF is one of the most popular ways in literature to identify the multicollinearity problem. Accordingly, VIF is used to measure the correlation between independent variables. High VIF values indicate that a particular independent variable is highly correlated with other independent variables. This may affect the model's reliability and reduce the coefficients' predictability. Generally, variables with a VIF value of 10 and above carry a high risk of multicollinearity. The VIF value is calculated for each independent variable using the following formula:

$$VIF_i = \frac{1}{1-R_i^2} \tag{6}$$

where R_i^2 is the coefficient of determination value obtained from the regression model of independent variable i with other independent variables.

In addition to Ridge, alternative penalized regression methods such as LASSO (Least Absolute Shrinkage and Selection Operator; Tibshirani, 1996) and Elastic Net (Zou & Hastie, 2005) are commonly used. LASSO performs coefficient shrinkage and variable selection by setting some coefficients exactly to zero, which can be helpful when a sparse model is desired. Elastic Net combines the strengths of Ridge and LASSO by applying both L1 and L2 penalties, making it suitable for situations with highly correlated variables and a need for automatic variable selection. However, Ridge regression was preferred in this study because the main goal was to stabilize coefficient estimates under multicollinearity without eliminating variables from the model. Ridge ensures that all theoretically justified variables remain in the model, which aligns with the research aim of evaluating their collective impact on SME performance. Additionally, Ridge is less sensitive to small sample sizes and multicollinearity, offering more stable and interpretable estimates in this context (Hoerl & Kennard, 1970; Marquardt & Snee, 1975; Hastie, Tibshirani & Friedman, 2009).

While cross-validation is typically recommended to select the optimal λ, this study used VIF to assess the degree of multicollinearity among predictors. VIF was chosen over other diagnostic tools (such as condition index or eigenvalue decomposition) because of its simplicity, widespread use, and intuitive interpretability. Nevertheless, we acknowledge the reviewer's suggestion, and in the empirical results section, we report model comparisons using OLS, Ridge, and LASSO and apply cross-validation to determine the optimal λ.

6. EMPIRICAL RESULTS

To examine the model, it is necessary first to examine the relationship between the variables. The table below (Table 2) shows the correlation coefficients between the independent variables in the data set and the *Variance Inflation Factor (VIF)* values calculated for each variable. This analysis is essential for diagnosing multicollinearity and understanding its potential impact on the reliability and interpretability of the model results. While correlation coefficients reflect the strength of linear relationships between pairs of variables, VIF values quantify how much the variance of a regression coefficient is inflated due to multicollinearity. As a rule of thumb, a correlation coefficient exceeding 0.7 or a VIF value above 10 indicates a strong linear dependence and raises concerns about coefficient stability and predictive accuracy.

Table 2: Correlation Matrix and VIF Values Between Independent Variables

	ln(Subsidy)	ln(R&D)	ln(Patent)	ln(Labor)	ln(Import)	VIF
ln(Subsidy)	1					40.707542
ln(R&D)	0.983584	1				101.81215
ln(Patent)	0.783171	0.839498	1			9.048543
ln(Labor)	0.943675	0.927551	0.643195	1		15.00791
ln(Import)	0.968408	0.990013	0.871805	0.908742	1	75.99186

The table reveals high correlations between government subsidies, R&D expenditures, and import expenditures, indicating that these variables often move together and are highly interdependent.

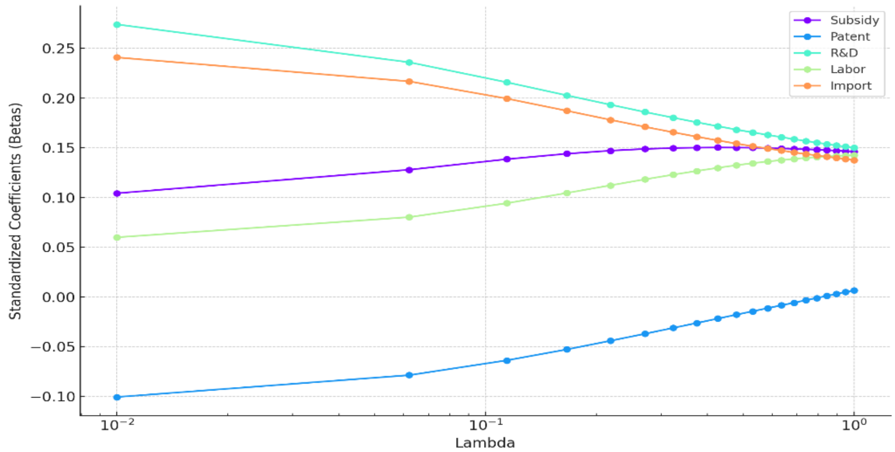
Such interrelations can pose significant risks in regression analysis because they reduce the model's ability to isolate the unique effect of each predictor on the dependent variable. In economic terms, this means that policy recommendations drawn from such models might be misleading, as the actual contribution of each factor to SME growth would be obscured.

Moreover, the VIF values confirm this concern: several variables show values far exceeding the critical threshold of 10, with R&D reaching over 100 and import expenditures over 75. This level of multicollinearity can severely inflate standard errors, reduce statistical power, and distort hypothesis testing, ultimately weakening the robustness of economic inferences.

Given this context, Ridge regression provides an appropriate and practical solution. By adding L2 regularization, Ridge regression shrinks the magnitude of the coefficients, balancing bias and variance to improve model generalizability. Importantly, it allows all predictors to remain in the model while controlling their influence, thus retaining the whole economic story without sacrificing statistical validity. This approach enables policymakers and researchers to derive more reliable insights, especially when highly interrelated innovation, labor, and subsidy variables are involved.

Figure 1: Ridge Trace Plot

The Ridge Trace plot (Figure 1) and the Ridge coefficient table (Table 3) display how the



standardized coefficients of the independent variables change across different λ (regularization parameter) values. As λ increases, the coefficients progressively shrink toward zero, the fundamental mechanism that allows Ridge regression to mitigate multicollinearity and reduce model overfitting.

At minimal λ values (around 10^{-2}), the coefficients are relatively large, reflecting the raw strength of the relationships between predictors and the dependent variable. However, as λ increases, the shrinkage effect reduces the variability of the coefficients, resulting in a more stable and generalizable model. Notably, the shrinkage rates differ across variables: while the coefficients of Subsidy and Patent shrink rapidly, the coefficients of R&D and Import decline more gradually. This pattern suggests that R&D and Import are particularly influential drivers of SME production performance, even under penalization, while Subsidy and Patent are more sensitive to regularization.

Another noteworthy point in the graph is the difference in the rate at which the coefficients shrink across variables. For example, the coefficients of variables such as *Subsidy* and *Patent* shrink rapidly as λ increases. On the other hand, the coefficients of *R&D* and *Import* variables decrease at a slower rate. This suggests that variables such as R&D and Import have a more dominant effect in the model, so the Ridge regression is less affected by the penalization effect. However, as the penalty increases, the differences between the variables start to decrease, and the coefficients become closer to each other, especially when λ is high. This implies that the model becomes more balanced, and regression can produce more stable results.

Table 3: Ridge Coefficients

λ	*Subsidy*	*Patent*	*R&D*	*Labor*	*Import*
0.01	0.1042	-0.1008	0.2741	0.0599	0.2409
0.0621	0.1279	-0.0788	0.2360	0.0802	0.2168
0.1142	0.1385	-0.0639	0.2159	0.0942	0.1995
0.1663	0.1440	-0.0529	0.2027	0.1045	0.1872
0.2184	0.1471	-0.0442	0.1933	0.1122	0.1781
0.2705	0.1491	-0.0376	0.1862	0.1185	0.1707
0.3226	0.1504	-0.0323	0.1804	0.1242	0.1648
0.3747	0.1513	-0.0282	0.1756	0.1290	0.1599
0.4268	0.1518	-0.0249	0.1717	0.1331	0.1559
0.4789	0.1521	-0.0223	0.1684	0.1367	0.1526
0.5310	0.1523	-0.0201	0.1657	0.1399	0.1497
0.5831	0.1524	-0.0183	0.1634	0.1427	0.1472
0.6352	0.1525	-0.0168	0.1614	0.1452	0.1450
0.6873	0.1526	-0.0155	0.1597	0.1474	0.1430
0.7394	0.1526	-0.0144	0.1582	0.1494	0.1412
0.7915	0.1527	-0.0134	0.1569	0.1511	0.1396
0.8436	0.1527	-0.0115	0.1557	0.1526	0.1381
0.8957	0.1527	0.0018	0.1547	0.1540	0.1368
0.9478	0.1527	0.0021	0.1538	0.1552	0.1356
1.000	0.1527	0.0035	0.1530	0.1563	0.1344

Economically, the magnitude of these coefficients can be interpreted as elasticities, given that the model is specified in logarithmic form. For example, at the optimal λ = 0.531, a 1% increase in Subsidy is associated with an approximate 0.15% increase in SME production, holding other factors constant. Similarly, a 1% increase in R&D spending yields about a 0.17% increase in output, and a 1% rise in Import spending contributes about 0.15% to SME production. Labor shows a robust effect with a 0.14% elasticity. Importantly, the Patent coefficient is slightly negative (-0.02), suggesting that short-term patenting activity may not yet translate into production gains. This result may be explained by the well-documented lag between innovation and its economic returns, consistent with the literature on innovation diffusion (Hall et al., 2005). Future models incorporating lagged patent variables may help clarify this relationship.

Regarding model performance, the Ridge regression with λ = 0.531 yields an R^2 of 0.941 and an MSE of 0.0041, indicating excellent predictive accuracy. These performance metrics highlight Ridge regression's ability to address multicollinearity without sacrificing explanatory power. From a policy perspective, these findings suggest that public support (subsidies), R&D investment, and access to global markets (imports) play central roles in enhancing SME productivity. However, innovation policies should consider the temporal lag between patenting and productivity to capture the returns to technological advancement fully.

Table 4: Model Comparison

Model	ln(Subsidy)	ln(R&D)	ln(Patent)	ln(Labor)	ln(Import)	R^2	RMSE
OLS	0.0998	0.3750	-0.4971	0.8929	0.3538	0.9972	0.0501
Ridge (λ = 0.531)	0.1523	0.1657	-0.0201	0.1399	0.1497	0.9410	0.0641
LASSO (λ = 0.531)	0.0759	0.3894	-0.0547	0.7653	0.2861	0.9834	0.0722

The model comparison table (Table 4) provides valuable insights into the differences between the OLS, Ridge, and LASSO regression results. First, looking at the R^2 values, OLS yields the highest explanatory power (0.9972), followed by LASSO (0.9834) and Ridge (0.9410). However, it is important to note that the slightly lower R^2 of Ridge and LASSO is compensated by their ability to handle multicollinearity and reduce overfitting, which is particularly crucial given the high correlation and VIF values observed in the dataset.

When examining the RMSE (Root Mean Squared Error), Ridge shows a slightly higher error (0.0641) compared to OLS (0.0501), but it outperforms LASSO (0.0722). This suggests that Ridge achieves a good trade-off between model complexity and predictive accuracy, offering a more stable estimation framework under multicollinearity.

Regarding the coefficients, the differences are notable and meaningful from an economic perspective. For example, under OLS, the coefficient for ln(Patent) is negative (-0.4971), which may appear counterintuitive, as patents are generally expected to stimulate growth. However, Ridge reduces this adverse effect significantly (-0.0201), and LASSO almost eliminates it (-0.0547), indicating that the penalization mechanisms help mitigate potential distortions caused by multicollinearity or small-sample effects. This finding points to further robustness checks, such as adding lag structures or exploring nonlinear specifications, especially for the patent variable.

From an elasticity interpretation, the Ridge model suggests that a 1% increase in subsidies increases production value by approximately 0.15%, while a 1% increase in R&D boosts it by around 0.16%. Notably, labor (0.14%) and imports (0.15%) also exert a positive and relatively balanced effect, highlighting the multi-faceted drivers of SME growth. LASSO shows similar patterns but zeroes out less significant variables more aggressively, as seen with ln(Patent), which is effectively shrunk to zero.

Overall, the comparison highlights that while OLS offers the best fit in purely statistical terms, Ridge provides a more reliable framework in multicollinearity and maintains economic interpretability. LASSO further improves variable selection but at the cost of slightly higher prediction error. Together, these models paint a comprehensive picture of the determinants of SME performance in Türkiye, and the Ridge regression emerges as an exceptionally robust choice for balancing accuracy and stability.

7. CONCLUSION AND DISCUSSION

This study aims to analyze the production performance of Small and Medium-Sized Enterprises (SMEs) in Türkiye within the framework of the Romer Growth Model and Ridge regression methods. Romer's (1990) endogenous growth theory emphasizes the role of innovation, human capital, and technological development in sustaining long-term economic growth. The data set used in the study consists of production values, state supports (by KOSGEB), R&D expenditures, patent applications, number of employees, and import expenditures of Turkish SMEs covering the years 2015–2022. The production value of SMEs is used as the dependent variable. Independent variables are determined in line with the assumptions of the Romer model, including innovation, physical capital accumulation, human capital, and intermediate goods imports.

The Ridge regression method is preferred in this study, primarily to address the multicollinearity problem. Multicollinearity can distort the model's predictive power when there is a high correlation between independent variables (Hoerl & Kennard, 1970). The VIF analysis of the data shows that KOSGEB supports R&D expenditure, and import expenditures are highly correlated. Therefore, Ridge regression alleviated this issue by shrinking the coefficients of the variables, thus improving the overall validity of the model (Marquardt & Snee, 1975).

The results demonstrate that KOSGEB incentives significantly enhance the production performance of SMEs. Government incentives expand firms' production capacities through physical capital accumulation. Anchoring this finding more explicitly in Romer's framework, physical capital investments increase immediate output and indirectly stimulate knowledge accumulation and innovation, amplifying long-term growth effects (Romer, 1990). The positive effect of incentives aligns with Romer's view that policy can actively accelerate technological change.

The positive impact of R&D expenditures on production value reinforces the central claim of Romer's theory that technological progress is a key driver of growth. R&D activities foster the creation of new products and processes, boosting SME competitiveness in domestic and international markets (Jones & Vollrath, 2013). Notably, the Ridge regression shows that R&D coefficients are relatively stable under penalization, suggesting their dominant and resilient role in explaining SME performance, mirroring Romer's emphasis on the self-reinforcing nature of innovation.

The positive effect of import expenditures is consistent with the theoretical expectation that intermediate goods imports enhance technological diffusion and production efficiency (Acemoğlu, 2009). This aligns with empirical studies showing that access to imported inputs improves firm productivity, particularly in emerging economies.

However, patent applications show an unexpected adverse effect, suggesting innovative efforts may have delayed economic payoffs. This result is not entirely inconsistent with Romer's framework, which acknowledges that the transformation of knowledge into productivity gains can be gradual. The negative short-term coefficient may reflect adjustment costs or the time lag between innovation and commercialization. Future research should test this relationship using lagged variables, which we could not incorporate here due to the short period of the dataset.

As a proxy for human capital, the positive effect of the number of employees on output is consistent with Romer's argument that skilled labor enhances both production and innovation capacity. Firms with more human resources can simultaneously increase production and contribute to developing new ideas (Lucas, 1988). This underscores the importance of policies targeting both the quantity and quality of labor within SMEs.

Compared to other empirical studies, our findings align with research highlighting the growth-enhancing effects of government support and R&D in SME contexts (e.g., Hall & Lerner, 2010; Aghion & Howitt, 1992). The negative patent result contrasts with some studies documenting immediate innovation gains. This discrepancy suggests the need for more nuanced research on the commercialization process and its timing in SMEs.

The study contributes to the literature by integrating Romer's endogenous growth framework with a rigorous empirical approach using Ridge regression to address multicollinearity—an innovation rarely applied in the SME context. While existing studies often focus on macroeconomic indicators, our micro-level analysis offers detailed insights into the specific channels through which innovation, human capital, government support, and imports affect SME growth. This integrated approach strengthens the theoretical and empirical bridge between endogenous growth theory and real-world SME dynamics.

Based on our results, several policy recommendations can be made. Given the positive impact of KOSGEB incentives, sustaining such support while strategically directing it toward innovation-intensive activities is crucial and consistent with the theoretical insights of Romer's model. However, these recommendations must be framed within our model's limits, particularly recognizing our data's short-term focus and the unexplored lag effects. For R&D and patenting, it is essential to develop programs that support not only invention but also the commercialization phase, helping SMEs translate knowledge into marketable products. Regarding human capital, investment in training programs and efforts to retain skilled labor will strengthen the innovation capacity of SMEs. Finally, policies facilitating the import of intermediate goods, such as tax breaks or streamlined regulations, can further enhance productivity.

Despite the robustness of our findings, several limitations should be acknowledged. First, the dataset only spans eight years, restricting our ability to explore long-term dynamics and lag effects, particularly for variables like patents. Second, while Ridge regression effectively addressed multicollinearity, future research should compare results with alternative methods (e.g., LASSO, Elastic Net) and expand diagnostic testing. Lastly, measurement issues in variables like import expenditure may have limited our ability to capture their full effect.

In conclusion, while this study offers valuable insights into the production dynamics of SMEs in Türkiye, it also opens avenues for future research to deepen the empirical and theoretical understanding of the innovation-growth nexus in small firms.

Etik Beyan: *Bu çalışmada "Etik Kurul" izini alınmasını gerektiren bir yöntem kullanılmamıştır.*
Yazar Katkı Beyanı*:1. Yazarın katkı oranı %50, 2. Yazarın katkı oranı ise %50'dir.*
Çıkar Beyanı*: Yazarlar arasında çıkar çatışması yoktur.*

Ethics Statement: *In this study, no method requiring the permission of the "Ethics Committee" was used.*
Author Contributions Statement*: 1st author's contribution rate is 50%, 2nd author's contribution rate is 50%.*
Conflict of Interest**:** *There is no conflict of interest among the authors.*

REFERENCES

Acemoğlu, D. (2009). Introduction to Modern Economic Growth, Princeton University Press, 1-977.

Aghion, P., & Howitt, P. (1992). A model of growth through creative destruction. *Econometrica, 60*(2), 323–351. https://doi.org/10.2307/2951599

Anwar, M., & Li, S. (2021). Spurring competitiveness and financial and environmental performance of SMEs through government financial and non-financial support. *Environment, Development and Sustainability*, *23*(5), 7860-7882. https://doi.org/10.1007/s10668-020-00951-3

Arslan, B. (2016). The Impact of KOSGEB's financial support on financial efficiency of SME's: An Application in Düzce Province. *Journal of Management Economics and Business, ICAFR 16 Special Issue*, *12*(12), 27-39.

Barro, R. J. (1991). Economic growth in a cross section of countries. *The Quarterly Journal of Economics, 106*(2), 407-443. https://doi.org/10.2307/2937943

Batrancea, L. M., Balcı, M. A., Chermezan, L., Akgüller, Ö., Masca, E. S., & Gaban, L. (2022). Sources of SMEs financing and their impact on economic growth across the European Union: Insights from a panel data study spanning sixteen years. *Sustainability*, *14*(22), 15318, 1-18. https://doi.org/10.3390/su142215318

Bayraktar, Y., Dündar, N., & Özyılmaz, A. (2022). The relationship between R&D expenditures and economic growth in BRICS-T countries. *Eskişehir Osmangazi Üniversitesi İktisadi ve İdari Bilimler Dergisi*, *17*(3), 893-910. https://doi.org/10.17153/oguiibf.1151022

Beck, T., Demirgüç-Kunt, A., & Maksimovic, V. (2005). Financial and legal constraints to growth: Does firm size matter? *The Journal of Finance, 60*(1), 137–177. https://doi.org/10.1111/j.1540-6261.2005.00727.x

Bishop, C. M. (2006). *Pattern Recognition and Machine Learning*. Springer.

Bozkurt, C. (2015). R&D expenditures and economic growth relationship in Turkey. *International Journal of Economics and Financial Issues*, *5*(1), 188-198.

Boztosun, D., Aksoylu, S., & Ulucak, Z. Ş. (2016). The role of human capital in economic growth. *Economics World*, *4*(3), 101-110. doi: 10.17265/2328-7144/2016.03.001

Dereli, D. D., & Salğar, U. (2019). Relationship between R&D expenditures and economic growth: An evaluation on Turkey. *Journal of Life Economics*, *6*(3), 345-360. https://doi.org/10.15637/jlecon.6.021

Doh, S., & Kim, B. (2014). Government support for SME innovations in the regional industries: The case of government financial support program in South Korea. *Research Policy*, *43*(9), 1557-1569. https://doi.org/10.1016/j.respol.2014.05.001

Ersin, Ö., Ustabaş, A., & Acar, T. (2022). The nonlinear effects of high technology exports, R&D and patents on economic growth: A panel threshold approach to 35 OECD countries. *Romanian Journal of Economic Forecasting*, *25*(1), 26-44.

Erzurum, R. (2023). The effect of R&D expenditures and innovation on economic growth in Turkey. Uludağ University, Social Science Institution, Economics, Master Thesis.

Güneş, H. (2019). The relationship between R-D expenditures and economic growth: panel data analysis for the OECD countries. *The Sakarya Journal of Economics*, *8*(2), 160-176.

Hall, B. H., & Lerner, J. (2009). The financing of R&D and innovation. *NBER Working Paper No. 15325*. National Bureau of Economic Research, 609-639. https://doi.org/10.3386/w15325

Hastie, T., Tibshirani, R., & Friedman, J. (2009). *The elements of statistical learning: Data mining, inference, and prediction* (2nd ed.). Springer, 1-745. https://doi.org/10.1007/978-0-387-84858-7

Hoerl, A. E., & Kennard, R. W. (1970). Ridge regression: Biased estimation for nonorthogonal problems. *Technometrics*, 12(1), 55-67.

Jones, C. I. and Vollrath, D. (2013). Introduction to economic growth. W. W. Norton & Co., 3rd edition.

Kim, D., Yeom, S., & Ko, M. C. (2020). The interactive effect of government financial support and firms' innovative efforts on company growth: a focus on climate-tech SMEs in Korea. *Sustainability*, *12*(22), 9666. 1-12. https://doi.org/10.3390/su12229666

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22, 3-22.

Marquardt, D.W and Snee, R.D. (1975). Ridge regression in practice. *The American Statistician*, 29 (1), 3-20.

Mwatu, Shadrack, (2023). Effect of human capital on economic growth in Austria: A time series analysis, 1-10. http://dx.doi.org/10.2139/ssrn.4362459

Ogbeifun, L., & Shobande, O. A. (2022). A reevaluation of human capital accumulation and economic growth in OECD. *Journal of Public Affairs*, *22*(4), e2602. 1-12. https://doi.org/10.1002/pa.2602

Olcay, G. A., & Bulu, M. (2015). Who should really get government support: an analysis of Turkish SME cases. *International Journal of Knowledge-Based Development*, *6*(1), 34-49. https://doi.org/10.1504/IJKBD.2015.069445

Öztürk, S., & Çınar, U. (2021). The relationship between R&D expenditures and economic growth within the scope of endogenous growth theories: Time series analysis for Turkey. *International Journal on Social Sciences*, *6*(2), 54-71. DOI: 10.46291/Al-Farabi.060204

Özer, M., & Çiftçi, N. (2009). AR-GE tabanlı içsel büyüme modelleri ve AR-GE harcamalarının ekonomik büyüme üzerine etkisi: OECD ülkeleri panel veri analizi. *The Journal of Social Economic Research*, *8*(16), 219-240.

Pelinescu, E., Pauna, C., Saman, C., & Diaconescu, T. (2019). Human capital, innovation, and economic growth in the EU countries. *Romanian Journal of Economic Forecasting*, *22*(4), 160-173.

Peçe, M. A. The effect of KOSGEB supports on the profitability of enterprises in TR81 region: Panel time series analysis. *Journal of Emerging Economies and Policy*, *9*(2), 69-75.

Porter, M. E., Stern, S., & Furman, J. L. (2002). The determinants of national innovative capacity. *Research Policy, 31*(6), 899–933. https://doi.org/10.1016/S0048-7333(01)00152-4

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94, 1002-1037.

Romer, P. M. (1990). Endogenous technological change. *Journal of political economy*, 98, 71-102.

Romer, P. M. (1994). The origins of endogenous growth, the journal of economic perspectives,.8(1), 3-22.

Sakallıoğlu, S. & Kaçıranlar, S. (2008). A new biased estimator based on ridge estimation. *Statistical Papers*. 49(4): 669-89.

Seo, J. Y. (2017). A study of effective financial support for SMEs to improve economic and employment conditions: Evidence from OECD countries. *Managerial and decision economics*, *38*(3), 432-442. https://doi.org/10.1002/mde.2838

Sokolov-Mladenović, S., Cvetanović, S., & Mladenović, I. (2016). R&D expenditure and economic growth: EU28 evidence for the period 2002–2012. *Economic Research-Ekonomska Istraživanja*, *29*(1), 1005-1020. http://dx.doi.org/10.1080/1331677X.2016.1211948

Tibshirani, R. (1996). Regression shrinkage and selection via the Lasso. *Journal of the Royal Statistical Society: Series B (Methodological), 58*(1), 267–288. https://doi.org/10.1111/j.2517-6161.1996.tb02080.x

Tikhonov, A. N. (1963). Solution of Incorrectly Posed Problems and the Regularization Method. *Soviet Mathematics Doklady*, 4, 1624-1627.

TUİK (2023). Small and Medium Sized Enterprises Statistics, 2022. https://data.tuik.gov.tr/Bulten/Index?p=49438&dil=2 (10.10.2024)

Tung, L. T., & Hoang, L. N. (2024). Impact of R&D expenditure on economic growth: evidence from emerging economies. *Journal of Science and Technology Policy Management*, *15*(3), 636-654. https://doi.org/10.1108/JSTPM-08-2022-0129

Xiang, D., & Worthington, A. C. (2017). The impact of government financial assistance on the performance and financing of Australian SMEs. *Accounting Research Journal*, *30*(4), 447-464. https://doi.org/10.1108/ARJ-04-2014-0034

Yıldız, Ş. (2018). The effect of government incentives on the brand value of the firm: A KOSGEB application in Antalya. Social Science Institution, MBA.

Zou, H., & Hastie, T. (2005). Regularization and variable selection via the Elastic Net. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *67*(2), 301–320. https://doi.org/10.1111/j.1467-9868.2005.00503.x

APPENDIX A: A: Full Mathematical Derivation of the Romer Model

Romer (1990) defines the final good production function as follows:

$$Y = H_Y^{1-\alpha} \int_0^{A(t)} X_i^{\alpha}\, di \tag{1}$$

According to equation (1), two factors of production are required to produce final goods: human capital H_Y and intermediate goods $X_i, i \in A(t)$. In the equation, $A(t)$ denotes the number of patents. It is assumed that as long as there is R&D, the number of patents increases and therefore each patent obtained/developed leads to the production of a new intermediate good. The production function assumes that the variety of intermediate goods is continuous. In this way, the production function can be written as follows:

$$Y = H_Y^{1-\alpha} \sum_0^{A(t)} X_i^{\alpha} \tag{2a}$$

$$Y = H_Y^{1-\alpha}(X_1^{\alpha} + X_2^{\alpha} + X_3^{\alpha} + \cdots + X_A^{\alpha}) \tag{2b}$$

Looking at the market solution, the profit equation of the representative firm:

$$\pi_Y = H_Y^{1-\alpha} \int_0^{A(t)} X_i^{\alpha}\, di - w_Y H_Y - \int_0^{A(t)} p_i X_i\, di \tag{3}$$

In this equation, w_Y is the cost of labor and p_i is the cost of the intermediate good.

Since the final good sector is a perfectly competitive market, the representative firm will maximize profits. The first-order conditions for profit maximization are as follows:

$$\frac{\partial \pi_Y}{\partial H_Y} = (1-\alpha) H_Y^{-\alpha} \int_0^{A(t)} X_i^{\alpha}\, di - w_Y = 0 \tag{3a}$$

$$\frac{\partial \pi_Y}{\partial X_i} = \alpha H_Y^{1-\alpha} X_i^{\alpha-1} - p_i = 0 \qquad i \in [0, A] \tag{3b}$$

These equations are also called *inverse factor-demand functions.* For example, the inverse factor-demand function in (3b) determines the demand function of the intermediate-goods sector. This is because X_i, which is the ith factor of production for the final-goods sector, expresses the output to be produced for the intermediate-goods sector. The production function of the intermediate-goods sector is defined in Romer (1990) as follows:

$$X_i = \frac{1}{\eta} k_i \qquad \eta > 0 \tag{4}$$

According to equation (4), $\frac{1}{\eta}$ represents the productivity of the intermediate-goods sector. If the only factor used in the production of intermediate goods is physical capital, then the profit equation will be as follows:

$$\pi_i = p_i X_i - r k_i \tag{5a}$$

$$\pi_i = p_i X_i - r\eta X_i \tag{5b}$$

In equation (5), it is the real interest rate, and since the capital depreciation rate is assumed to be zero in the Romer (1990) model, the capital rental rate and the real interest rate are the same value. Since the ith intermediate-good producer produces with the patent purchased from the R&D company, its product (even if it is similar to other products in the market) is unique, i.e., a monopoly. Therefore, p_i is not a fixed value but a value that varies with demand. In other words, it is of the form $p_i = p_i(X_i)$. In fact, this relationship between p_i and X_i is clear from equation (3b).

When the first-order condition for profit maximization is examined, the basic relationship between price and cost in monopolistic structures is found:

$$X_i\left(1 + \frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} - r\eta \frac{\partial X_i}{\partial p_i}\frac{1}{X_i}\right) = 0 \implies X_i\left(1 + \varepsilon_p - r\eta\varepsilon_p \frac{1}{p_i}\right) = 0 \tag{6}$$

The ε_p in this equation is nothing but the price elasticity of demand. Since the product above is equal to zero, and since the first term being zero is a *trivial solution*, the second term is expected to be zero. Hence;

$$\left(1 + \varepsilon_p - r\eta\varepsilon_p \frac{1}{p_i}\right) = 0 \implies p_i = \frac{r\eta}{\frac{1+\varepsilon_p}{\varepsilon_p}} \tag{7}$$

The price elasticity of demand can be easily obtained from equation (3b). For this, it is sufficient to take the logarithmic differential of the equation:

$$\ln(\alpha) + (1-\alpha)\ln(H_Y) + (\alpha - 1)\ln(X_i) = \ln(p_i) \Rightarrow \varepsilon_p = \frac{\partial \ln(X_i)}{\partial \ln(p_i)} = -\frac{1}{1-\alpha} \tag{8}$$

If the value of ε_p is substituted into the price, $p_i = \frac{r\eta}{\alpha}$ is found[3]. Based on this result, when the profit equation is re-examined, it will be seen that $r\eta X_i$ is the *total cost function*. Then *the marginal cost function* will be $r\eta$. Therefore, the monopoly producer of intermediate good i sells it by adding a profit-margin to its marginal cost ($\frac{1}{\alpha}$>1 since α<1). This is precisely the *mark-up pricing* strategy.

In Romer's (1990) model, the R&D sector produces patents (new knowledge) using only human capital. According to Romer, previously discovered patents (stock of knowledge) are positive externalities for patents currently being worked on/developed. According to Romer (1990), the patent (knowledge) production function is as follows:

$$\dot{A} = a_{R\&D} H_{R\&D} A \tag{9}$$

In equation (9), $\dot{A}$ is the number of patents produced at any point in time, $a_{R\&D}$ is the productivity of the R&D sector, $H_{R\&D}$ is the number of human capital employed in the R&D sector and A is the patent stock. A represents the positive externality in the equation. Romer (1990) assumes total human capital as $H_{R\&D} + H_Y = \bar{H}$ as constant. The reason for this assumption is that endogenous growth in the model is already provided by endogenous technological change. There is no need for a second source of endogenous growth. Romer (1990) assumed that the R&D sector is perfectly competitive. In this case, the profit equation will be as follows:

[3]According to this result, the price is the same for all $i \in [0, A]$ in the intermediate goods market. The fact that the price is the same does not mean that these intermediate goods are the same intermediate good. The price is constant across intermediate goods, but not over time. Since r is time-dependent, p_i also changes over time. If there is a steady state of the model and r reaches a constant value in the steady state, then p_i will also reach a constant value over time. Since the prices of all intermediate goods are identical, it is not necessary to relate price to i, i.e. all values are independent of i. Therefore, since r and H_Y are the same for all intermediate goods, the demand for intermediate goods is constant across sectors.

$$\pi_{R\&D} = P_A\dot{A} - w_{R\&D}H_{R\&D} \tag{10}$$

In equation (10), P_A denotes the price of a single patent. If $\dot{A}$ number of patents are produced at time t, the real sales revenue of the R&D sector will be $P_A\dot{A}$. The cost of the R&D sector consists only of wages paid to human capital. As mentioned above, in the patent production function, the patent stock is a positive externality. Therefore, there is no cost input. The *equilibrium process* in the R&D sector requires the following condition:

$$P_A a_{R\&D} A = w_{R\&D} \tag{11}$$

The following equation is used to derive the P_A value:

$$P_A(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}\pi(\tau)d\tau \qquad \tau \in [t,\infty) \qquad (12)^4$$

The derivative of the $P_A(t)$ term on the left side of equation (12) with respect to t is $\dot{P}_A$. Therefore, the derivative of this equation with respect to t is $\dot{P}_A = -\pi(t) + r(t)P_A(t)$. Romer (1990) noted that P_A is constant at steady state, hence it is $\dot{P}_{A,ss} = 0$. Based precisely on this property of P_A, Romer stated that the following equation is true:

$$P_{A,ss} = \frac{\pi_{ss}}{r_{ss}} \tag{13}$$

When this equation is substituted in equation (11), the new equation to be obtained is as follows:

$$w_{R\&D,ss} = \frac{\pi_{ss}}{r_{ss}} a_{R\&D} A_{ss} \tag{14}$$

Recall that human capital has two alternative uses: the final-goods production sector and the R&D sector. If $w_{R\&D,ss} > w_{Y,ss}$, all human capital would want to work in the R&D sector. Conversely, all human capital would want to be employed in the final-goods sector. If an equilibrium exists, $w_{R\&D,ss} = w_{Y,ss}$ must necessarily exist.

Since the parameter $w_{R\&D,ss}$ was found in equation (14), it is necessary to use equation (3a) to find the value of the parameter $w_{Y,ss}$.

$$(1-\alpha)H_Y^{-\alpha}\int_0^{A(t)} X_i^{\alpha}\,di = w_Y \implies (1-\alpha)H_Y^{-\alpha}X^{\alpha}A = w_Y$$

$$\implies (1-\alpha)H_{Y,ss}^{-\alpha}X_{ss}^{\alpha}A_{ss} = w_{Y,ss}$$

Now $w_{R\&D,ss} = w_{Y,ss}$ is able to use the equation. When the equations in this equation are substituted[5]:

$$(1-\alpha)H_{Y,ss}^{-\alpha}X_{ss}^{\alpha}A_{ss} = \frac{\pi_{ss}}{r_{ss}}a_{R\&D}A_{ss} \implies$$

$$(1-\alpha)H_{Y,ss}^{-\alpha}X_{ss}^{\alpha} = \frac{\left(\frac{1-\alpha}{\alpha}\right)r_{ss}\eta X_{ss}}{r_{ss}}a_{R\&D} \implies$$

$$H_{Y,ss}^{-\alpha} = \left(\frac{\eta a_{R\&D}}{\alpha}\right)X_{ss}^{1-\alpha} \implies$$

$$H_{Y,ss}^{-\alpha} = \left(\frac{\eta a_{R\&D}}{\alpha}\right)\left(\frac{\alpha^2}{r_{ss}\eta}\right)H_{Y,ss}^{1-\alpha} \implies$$

$$H_{Y,ss} = \frac{r_{ss}}{\alpha a_{R\&D}}$$

[4]To solve this integral, it is possible to find an expression for P_A that can be used to solve the model by taking the derivative of both sides of the equation with respect to the initial time t. However, in order to find this, it is necessary to apply Leibniz's rule of the derivative of the integral.

[5]Equations $X = \left(\frac{\alpha^2}{r\eta}\right)^{\frac{1}{1-\alpha}}H_Y$ and $\pi = \left(\frac{1-\alpha}{\alpha}\right)r\eta X$ are used in the solution here.

Finally, from equation (9), the information production function is expected to be as follows at steady state:

$$\hat{A}_{ss} = a_{R\&D} H_{R\&D,ss} \implies$$

$$\hat{A}_{ss} = a_{R\&D} (\bar{H} - H_{Y,ss}) \implies$$

$$\hat{A}_{ss} = a_{R\&D} \bar{H} - \frac{r_{ss}}{\alpha}$$

In order to find the endogenous growth rate of the Romer growth model, it is necessary to maximize the utility obtained by households. This solution is the same as the household utility maximization developed by Ramsey (1928).

$$Max\, U(C_t) = \int_0^{\infty} e^{-\rho t} u(c_t) dt$$

$$s.t.\, (\dot{FA}) = r_t(FA) + w_{R\&D} H_{R\&D} + w_Y H_Y - P_A \dot{A} + A\pi_t - C_t \quad (15)$$

In this equation, one of the forms of the instantaneous utility function $u(c_t) = \frac{c^{1-\theta}-1}{1-\theta}$ shown in the Ramsey model, which yields a steady state result, is used. Moreover, U is the *overall utility*, c is the *per capita consumption*, *u(c)* is the *momentary utility* and ρ is the *subjective rate of discount*. Moreover, *FA* represents the value of financial assets owned by households, which is a dynamic parameter.

Thanks to the previously obtained $w_{R\&D} = w_Y$ equations, $w_{R\&D} H_{R\&D} + w_Y H_Y = w\bar{H}$ can be written. Hence, when the Hamiltonian equation of the total utility maximization problem is written:

$$H = e^{-\rho t} \frac{c^{1-\theta}-1}{1-\theta} + \lambda\{r_t(FA) + w_t \bar{H} - P_A \dot{A} + A\pi_t - C_t\} \quad (16)$$

When the mathematical solution of the first order maximizing conditions is done, the steady state result $\hat{\lambda}_{ss} = -r_{ss}$ will be obtained. Its derivative with respect to time will also take the value $\dot{r}_{ss} = 0$ and r_{ss} will be constant at steady state. Together with the other first order maximizing conditions, the *Keynes-Ramsey Rule* will be obtained $\left(\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho)\right)$.

Since there will be $\hat{C}_{ss} = \frac{1}{\theta}(r_{ss} - \rho)$ in the steady state and r_{ss} is constant, the parameter $\hat{C}_{ss}$ is also constant. On the other hand, the macroeconomic budget constraint is obtained with the equations $r_t = MPP_K$ and $w_t = MPP_L$ to be obtained by solving the Hamiltonian equation[6] $(K = \dot{Y} - C)$.

In the light of all the information, it can now be shown that r_{ss} is a fixed number:

$$a_{R\&D} \bar{H} - \frac{r_{ss}}{\alpha} = \frac{1}{\theta}(r_{ss} - \rho) \implies$$

$$r_{ss} = [a_{R\&D} \theta \bar{H} + \rho] \left(\frac{\alpha}{\alpha + \theta}\right)$$

Plugging this equation into $\hat{C}_{ss}$ makes it easy to find the endogenous growth rate g:

$$g = \frac{1}{\theta}\left([a_{R\&D} \theta \bar{H} + \rho]\left(\frac{\alpha}{\alpha + \theta}\right) - \rho\right) \Rightarrow$$

$$g = \frac{\alpha a_{R\&D} \theta \bar{H} - \rho}{\alpha + \theta} \quad (17)$$

It will be positive as long as $\alpha a_{R\&D} \bar{H} > \rho$ holds. According to the Romer (1990) model, the productivity of the R&D sector and the total stock of human capital affect the growth rate positively, while the subjective discount rate and the elasticity of marginal utility affect the growth rate negatively. Finally, as α, which indicates monopolistic price-setting power in the intermediate-goods market, increases (i.e. as monopolistic price-setting power decreases), the growth rate increases.

[6] In the Romer model, the margin of erosion is not defined. The $\hat{Y}_{ss} = \hat{K}_{ss} = \hat{C}_{ss} = \hat{A}_{ss}$ equations can be easily found together with the budget constraint equation.