İstanbul İktisat Dergisi
Istanbul Journal of Economics,
IJE 2025, 75 (1): 207–221

https://doi.org/10.26650/ISTJECON2024-1643134

İstanbul İktisat Dergisi
Istanbul Journal of Economics

Submitted: 19.02.2025
Revision Requested: 29.05.2025
Last Revision Received: 03.06.2025
Accepted: 11.06.2025

Research Article

Open Access

Portfolio Optimisation in the Cryptocurrency Market: Hybrid Integration of Markowitz and Ridge Methods

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Abstract

Constructing an effective asset allocation strategy requires building well-diversified portfolios that maintain robust performance beyond the sample data. The classical Markowitz portfolio optimisation, while widely used, is known to suffer from issues such as estimation errors and sensitivity to multicollinearity, which can significantly distort the allocation process and reduce performance reliability. In order to surmount the aforementioned challenges, the incorporation of Machine Learning echniques, specifically Ridge regression, into the portfolio creation process has been effected. This has resulted in the provision of a hybrid model that combines the strengths of Markowitz optimisation and Ridge regression. The integration of these approaches within the hybrid model serves to mitigate the prediction risks while maintaining the diversification benefits inherent to the Markowitz framework. The model was trained using an 80/20 split and cross-validation was employed to prevent overfitting. The findings indicate that this integrated approach attains the maximum Sharpe ratio, thereby significantly enhancing risk-adjusted returns and portfolio stability when applied to cryptoasset returns. The findings emphasise the merits of integrating classical optimisation methodologies with machine learning to develop more robust and adaptive asset allocation strategies. By analysing the impact of high-volatility cryptoassets on portfolio performance, it makes important contributions to both the literature and practical portfolio strategies for investors.

Keywords Hybrid method · Crypto assets · Markowitz optimisation · Ridge method

Jel Codes G11, G15, O16

Citation: Kaplan Yıldırım, R., Münyas, T. & Kadooğlu Aydın, G. (2025). Portfolio optimisation in the cryptocurrency market: hybrid integration of markowitz and ridge methods. *İstanbul İktisat Dergisi–Istanbul Journal of Economics*, *75*(1), 207-221. https://doi.org/10.26650/ISTJECON2024-1643134

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Portfolio Optimisation in the Cryptocurrency Market: Hybrid Integration of Markowitz and Ridge Methods

The relationship between risk and return in Harry Markowitz's article titled "Portfolio Selection" published in 1952, which is the basis of modern portfolio theory, is recognised as an important turning point in the investment world (Yiğiter, & Akkaynak, 2017). Markowitz (1952) explained the desire of investors to obtain maximum return with minimum risk in the modern finance literature by emphasising the importance of the balance between risk and return with the optimal portfolio. Modern Portfolio Theory mathematically assumes that the higher the risk, the greater the return potential of the investment on the basis of the risk-return balance (Abensur, & Carvalho, 2022). Investors may choose high-risk investment instruments when seeking higher returns, which may lead to higher losses. However, when two assets with different risk levels are preferred, the return from one asset is expected to compensate for the loss from the other asset. Therefore, it is possible to balance risk and return by investing in more than one asset (diversification) rather than investing in only one asset and assuming the risk.

Markowitz's portfolio theory not only emphasises the importance of diversifying investments in order to reduce overall portfolio risk but also specifies effective diversification methods (Kamil, et al. 2006). Diversification has become an increasingly important strategy at both the individual and macroeconomic levels. In order to mitigate the effects of economic shocks and instabilities, as well as to reduce vulnerabilities, investment diversification is supported by strategies that aim to optimise risk-return expectations for different asset classes (Sisay, 2024). Diversification is critical not only to optimise risk management but also to ensure sustainable investments as it increases the potential for expected returns (Asih et al., 2024). When choosing financial investment instruments, an investor who wants to build a good portfolio should also consider global assets. This is because including global investment instruments in the investment portfolio not only provides protection against country-specific risks but also contributes to increasing the long-term return potential due to the effects of macroeconomic variables such as economic growth, inflation or interest rates in different countries. In this context, diversification, which stands out as an important tool in achieving a risk-return balance, can only achieve its purpose when the right strategy is implemented. The right strategy is to form a portfolio by considering the risk and return performance of all assets in national and global financial markets. In recent years, crypto assets, which are accepted as a new asset class in financial markets because of technological development, have been frequently discussed by both investors and academics. It is observed that crypto assets are one of the main dynamics of economic, technological and social transformations beyond being just an investment instrument, as they have innovative infrastructure such as blockchain and are far from centralisation and authoritarian structure (Liu, & Tsyvinski, 2021; Yıldırım, 2019). These digital assets have high volatility as their value is determined by factors such as supply-demand balance, technological infrastructure, investor perception and speculative dynamics (Brauneis, & Mestel, 2019; Corbet et al., 2018). For this reason, crypto asset investments carry both potential gains and serious risks. In the literature, there are ongoing debates on the classification of cryptoassets as money, commodity, or a completely different financial instrument and their use for portfolio diversification or hedging (Bouri et al., 2017; Feng et al., 2018). Bitcoin has been intensively researched in the context of financial diversification and as a safe haven instrument, especially due to its leading position in the crypto market and its wide usage area (Konuşkan et al., 2019). Accordingly, the role of cryptocurrencies as an alternative investment instrument and its relationship with other asset classes offer a critical research area for future financial innovations and market regulations (Saggu, A., Ante, L., & Kopiec, K., 2025). The aim of this study is to investigate the effect of cryptoassets on the risk and return of the investment portfolio. For this purpose, the risk and return of 15 crypto assets (BTC, ETH, USDT, BNB, USDC, XRP, DOGE, TRX, ADA, BCH, LITE, DAI, LTC, XMR, and XLM), which are included in the portfolio and have the highest trading volume, were analysed. In order to determine the portfolio performance measures of crypto assets, Markowitz optimisation was applied and then the Ridged regression method (Arı, & Önder, 2013), which is used when there is a high correlation between more than one explanatory variable and minimises the weight of variables and obtains parameter estimates with smaller variance compared to the least squares method in the presence of multiple linear connections. The data were analysed using a hybrid approach that combines Markowitz optimisation and Ridge regression method because the regulatory power of the Ridge method reduces the Markowitz over-sensitivity problem and the risk minimisation mechanism in the Markowitz method balances the tendency of the Ridge method to exclude low-importance assets from the portfolio.

In the second part of the study, the studies on the subject in the literature are included, the methodology and the analysis of the data obtained as a result of the methods used are mentioned in the third part and the findings are discussed in the last part.

Literature Review

Markowitz shows us how to use modern portfolio theory to de-risk all capitals/securities in an environment of high volatility (uncertainty) and to construct a theory of asset pricing. The Markowitz model and modern portfolio theory are concepts that have attracted attention in both national and international studies. In their study, Škrinjarić and Šostarić (2014) wanted to show how the Markov chains methodology and Markowitz portfolio optimisation model can lead to more effective results on investment decisions. They conclude that this integration leads to higher efficiency and lower risk for investors. Chambers, Hamzacebi, and Bayramoğlu (2016), who use grey system theory to support Markowitz portfolio optimisation during periods of high volatility, reveal that models developed with the modern approach can be optimised during periods of high volatility and offer effective strategies. They also argued that the grey system approach can improve portfolio performance and enables the development of new strategies to minimise investor risks. Another study that uses Markowitz's portfolio theory to obtain the best portfolio in the stock market is Chao, Tao and Zeng (2019). In the study, the risk returns of various stocks are analysed and the ways to form an optimal portfolio are shown. Similar to other studies, it is stated that the Markowitz theory is applicable for investors and effective results can be obtained. Janková (2019) compares the Markowitz theory and low-risk portfolio theories and finds that the downside risk approach in particular better serves the goal of minimising investors' losses. Raisa and Cristian (2021), aiming to determine the stock portfolio return and a minimum risk portfolio model, emphasised that Markowitz theory is an effective tool in financial portfolio management, that is, it will contribute to the development of strategies that can help investors adopt a more informed and systematic approach. Blay (2024) emphasised that Markowitz is not only an academic concept but also plays an important role in investors' portfolio construction processes. Finally, Savage and Ball (2024) developed a model called "Markowitzatron" to adapt the traditional Modern Portfolio Theory (MPT) to the oil industry. As a result, it is emphasised that integrating MPT into energy markets can provide significant advantages compared to traditional financial instruments.

Traditional currency is a store of value, a unit of account and, most importantly, a medium of exchange. Cryptocurrencies, especially Bitcoin accounts, are used as speculative investment instruments rather than as alternative currencies or a medium of exchange. Bitcoin also offers diversification advantages over various other financial assets. Since cryptocurrencies are highly volatile, when portfolio theory is applied to cryptocurrency portfolios, it has been pointed out that the parameters that will cause problems have higher potential estimation errors. Platanakis and Urquhart (2019). In his study, Mazanec (2021) stated that the potential performance of portfolios using digital currencies together with traditional assets can be increased. Yermack (2015) compared the basic functions and features of Bitcoin and traditional currencies and argued that Bitcoin falls short of traditional currencies in many respects due to its volatility and limited acceptance. Kristoufek (2015) investigated Bitcoin price formation and price drivers. He argues that Bitcoin is not only a speculative asset; fundamental factors such as money supply, price level and trade use play a role in price formation. In contrast, Baek and Elbeck (2015), who argued that Bitcoin is a speculative asset, argued that the price of Bitcoin is driven by buyers and sellers. Dyhrberg (2016), who examines whether Bitcoin has the same capabilities as other hedging factors in the financial market, argues that, especially in times of uncertainty, Bircoin can function as a short-term hedge against the dollar like gold. And also by Dyhrberg (2016), he wanted to determine how Bitcon, US dollar and gold prices change and the existence of a relationship between them. As a result of the study, he stated that Bircoin has higher volatility than the dollar and gold and it behaves like gold during periods of uncertainty. Al-Yahyaee et al. (2019) found that for oil and S&P GSCI investors, Bitcoin and gold offer diversification benefits and there is evidence of hedging effectiveness and downside risk mitigation. Recently, the cryptocurrency market has attracted a lot of attention among investors. The main reasons why the cryptocurrency market is in the limelight are the diversification advantages and hedging capabilities that cryptocurrencies offer. The relationship between cryptocurrency and portfolio diversification emerges as investors combine cryptocurrencies with traditional asset classes to optimise returns and manage risk. High volatility and low correlation are the hallmarks of cryptocurrencies. Thanks to this feature, cryptocurrencies can reduce the overall risk profile of a portfolio or increase potential returns. Studies show that when digital assets such as Bitcoin, the most popular cryptocurrency, are used with traditional investment instruments, there is an increase in the overall performance of portfolios. Grujić and Šoja (2022) constructed 2 portfolios with and without Bitcoin to determine how desirable it is to invest in Bitcoin compared to other financial instruments. At the end of the study, it was found that the rational behaviour of institutional investors requires investing in Bitcoin using the Markowitz model and that Bitcoin is a good source of diversification in a portfolio of traditional financial instruments for both risk-averse investors and investors with a low risk appetite. Similarly, Šoja and Chamil (2019), who investigated whether Bitcoin is a useful diversification tool for investors, emphasised that Bitcoin can be a useful diversification tool for both risk-averse and risk-prone investors in a portfolio that includes traditional assets such as gold and stocks. Guesmi, Saadi, Abid, and Ftiti (2019), who argued that Bitcoin provides significant hedging advantages for portfolio diversification for investors, examined the conditional cross-effects and volatility between bitcoin and financial indicators (gold, oil, stocks). They emphasise that hedging strategies that include gold, oil, emerging equity markets and Bitcoin significantly reduce portfolio risk. Platanakis and Urquhart (2019) compared the performance of Markowitz diversification and the advanced Black-Litterman model with the VBC that controls the forecast errors in the cryptocurrency portfolio. They concluded that complex portfolio techniques that control for forecast errors are preferred when managing cryptocurrency portfolios.

Methodology

This section presents the methodology of the unit root tests and optimisation techniques used in the study to analyse the time series properties of the cryptocurrency assets.

Unit root test

In applied macroeconomic or financial time series research, the data under analysis often experience various shocks over the long term, such as economic recessions or financial crises. These shocks, if they exert a permanent influence on the time series, disrupt the stationarity of the series and result in a non-stationary process. Stationarity is a critical property as it determines whether the regression results reflect a valid relationship (Granger, & Newbold, 1974; Gujarati, 1995). Numerous unit root tests are available in the literature to examine the stationarity of the time series. This study utilises the Augmented Dickey-Fuller (ADF) unit root test as the reference method. The ADF test is frequently employed to assess the presence of a unit root in the time series data. Dickey and Fuller (1981) extended the Dickey-Fuller (DF) test, which is based on the AR(1) process, by accounting for higher-order autocorrelations. To address serial correlation issues, the ADF test relies on the AR(p) process and incorporates the lagged difference terms into the regression equation (Gujarati, 1995). The ADF test includes three model specifications, based on the inclusion of an intercept (μ) and deterministic trend (t), as detailed below:

$$\Delta y_t = \delta y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_i \tag{1}$$

$$\Delta y_t = \mu + \delta y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_i \tag{2}$$

$$\Delta y_t = \mu + \beta t + \delta y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_i \tag{3}$$

The null hypothesis for all three model strategies states the presence of a unit root, indicating a non-stationary process (Dickey, & Fuller, 1979; 1981).

Optimisation techniques

To develop an asset allocation strategy, the study begins with the global minimum variance portfolio optimisation framework. The problem is defined as follows:

$$\begin{gathered} w^* = argmin_{w \in \mathbb{R}^K} \left\{ w' \hat{\Sigma} w \right\} \\ \text{s.t.} \quad \mathbf{1}'_K w = 1 \end{gathered} \tag{4}$$

where w^* denotes the optimal weight vector, $\hat{\Sigma}$ is the estimated covariance matrix, and K is the number of assets. The analytical solution, assuming that the covariance matrix is non-singular, is expressed as

$$w^* = \left(\mathbf{1}'_K \hat{\Sigma}^{-1} \mathbf{1}_K \right)^{-1} \hat{\Sigma}^{-1} \mathbf{1}_K \tag{5}$$

However, the classic Markowitz model is prone to several issues, including sensitivity to estimation errors and the influence of multicollinearity among asset returns. These problems lead to unstable weights and, in turn, poor out-of-sample performance. To overcome these limitations, this study integrates Ridge regression into the portfolio optimisation process. Ridge regression introduces a regularisation penalty to shrink the weight estimates, reducing the adverse effects of multicollinearity while ensuring greater stability. In the Ridge regression, the coefficients are determined by minimising the sum of squared errors (SSE) while incorporating a penalty term applied to the coefficients (Hoerl, & Kennard, 1970). This approach represents the L2 regularisation, where λ corresponds a tuning parameter. The mathematical formulation of the Ridge regression is presented in Equation (6), (Tibshirani, 1996):

$$SSE_{L_2} = \sum_{i=1}^{n}(y_i - \grave{y}_i)^2 + \lambda \sum_{j=1}^{P}\beta_J^2 \tag{6}$$

The regularised problem is expressed as

$$w^* = argmin_{w \in \mathbb{R}^K}\left\{w'\hat{\Sigma}w + \lambda \sum_{i=1}^{K}\rho(w_i)\right\}$$
$$\text{s.t.} \quad \mathbf{1}_K' w = 1 \tag{7}$$

Here, λ is the regularisation parameter, which determines the intensity of the shrinkage applied to the asset weights. Unlike classical Markowitz optimisation, the Ridge approach stabilises the solution without imposing sparsity, thereby producing more consistent weight vectors.

Finally, in order to compare portfolio optimisation and composition obtained with and without regularisation methods, the following metrics are used. To further enhance portfolio performance, this study develops a hybrid model that integrates the strengths of both Markowitz optimisation and Ridge regression. By leveraging Ridge's ability to handle multicollinearity and Markowitz's diversification benefits, the hybrid model provides improved stability and accuracy in asset weight estimation. This approach achieves the maximum sharpe ratio, outperforming each individual technique in terms of risk-adjusted returns. The regularisation parameter λ is selected through a 10-fold CV process, where λ is determined as the value that minimises out-of-sample variance. To systematically compare the performance and structure of portfolios constructed with and without regularisation techniques, a set of well-defined evaluation metrics is employed. To compare the performance and structure of portfolio optimisation with and without regularisation methods, the analysis uses the following evaluation metrics:

Out-of-sample variance:

$$\sigma^2 = \frac{1}{T - \tau - 1}\sum_{t=\tau+1}^{T}(r_t - r^{(}))^2 \tag{8}$$

Sharpe ratio:

$$SR = r^{(}\frac{)}{\sqrt{\sigma^2}} \tag{9}$$

Data, optimisation structures and empirical results

Table 1 provides an overview of the abbreviations, units of measurement, and database of the variables analysed in this paper. The variables are derived from the price series of 15 cryptocurrency assets, and to compute the return series, the price data were first transformed by taking their natural logarithms, followed by the application of first differencing.

Table 1
Data Information

Variables	Abbrevation	Unit	Data Transformation	Database
Bitcoin	BTC	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Ethereum	ETH	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Tether	USDT	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Binance Coin	BNB	Price (USD); return	Logarithmic difference	https://tr.investing.com/
USD Coin	USDC	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Ripple	XRP	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Dogecoin	DOGE	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Tron	TRX	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Cardano	ADA	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Bitcoincash	BCH	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Chainlink	LINK	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Dai	DAI	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Litecoin	LTC	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Monero	XMR	Price (USD); return	Logarithmic difference	https://tr.investing.com/
Stellar	XLM	Price (USD); return	Logarithmic difference	https://tr.investing.com/

In the first stage of the study, descriptive statistics are employed to provide an overview of the variables. Descriptive statistics serve as essential tools to summarise variables within a dataset and highlight their key characteristics. The descriptive statistics for cryptocurrency returns are presented in Table 2.

Table 2
Summary Statistics

Variables	Mean	Median	Std. Dev.	Maximum	Max. Date	Minimum	Min. Date	JB Stats.	p-Value
BTC	0.000897	0.000524	0.035766	0.177424	2021-02-08	-0.49728	2020-03-12	574.75***	0.0000
ETH	0.001539	0.001311	0.045026	0.230772	2020-10-19	-0.58964	2019-08-08	2441.61***	0.0000
USDT	0.000008	0.0	0.001466	0.019797	2018-09-24	-0.01514	2018-09-20	36817.57***	0.0000
BNB	0.002224	0.001508	0.047359	0.530574	2020-07-17	-0.58116	2019-08-08	14072.06***	0.0000
USDC	0.000007	0.0	0.007561	0.124631	2018-06-01	-0.22808	2018-05-31	14150431.30***	0.0000
XRP	0.000195	0.000437	0.0526	0.548118	2022-12-09	-0.54101	2020-05-21	30248.30***	0.0000
DOGE	0.001953	-0.000069	0.144433	4.141786	2019-06-12	-4.15589	2019-06-13	5816.47***	0.0000
TRX	0.001186	0.002167	0.045625	0.340529	2020-08-26	-0.57085	2019-08-08	19795.23***	0.0000
ADA	0.001117	0.000123	0.051216	0.286973	2020-07-08	-0.5372	2019-08-08	1002.86***	0.0000
BCH	0.000545	-0.000036	0.054757	0.459116	2023-07-29	-0.59772	2019-08-08	14102.52***	0.0000
LINK	0.001817	0.001286	0.059884	0.475424	2018-11-08	-0.63715	2019-08-08	3114.78***	0.0000
DAI	0.000002	0.0	0.005202	0.057088	2019-08-08	-0.05895	2019-08-09	3526449.81***	0.0000
LTC	0.000453	0.000932	0.048732	0.258175	2018-07-06	-0.48678	2019-08-08	7847.48***	0.0000
XMR	0.000583	0.002111	0.04636	0.342203	2020-10-15	-0.53539	2020-10-14	53940.69***	0.0000
XLM	-0.00011	0.000369	0.050151	0.553585	2020-06-03	-0.44031	2020-03-12	21052.73***	0.0000

Note: **** 1% significance level.*

Table 2 provides summary statistics for the returns of the selected cryptocurrency assets, offering valuable insights into their distribution and temporal characteristics. Among the cryptocurrencies analysed, DOGE exhibits the highest mean return aproximately 0.002, indicating a relatively strong average performance over the period studied. Conversely, XLM demonstrates the lowest mean return aproximately −0.0001, the only asset with a negative mean. When examining extreme values, the highest maximum return is observed for DOGE (approximately 4.1418 on 2019-06-12), while the lowest minimum return is recorded for the same cryptocurrency (aproximately −4.1559 on 2019-06-13), underscoring DOGE's extraordinary price fluctuations within a short period. The dates of maximum returns vary widely and are not clustered. Peak performance is asset-specific and not driven by a common market event. On the other hand, the dates of minimum returns frequently occur in 2019 or 2020, possibly reflecting market-wide downturns during these periods. Overall, most cryptocurrencies exhibit positive mean returns, with XLM as the exception, and the descriptive statistics indicate substantial heterogeneity in their risk and return profiles.

Table 3

Covariance analysis

Variables	BTC	ETH	USDT	BNB	USDC	XRP	DOGE	TRX	ADA	BCH	LINK	DAI	LITE	XMR	XLM
BTC	0.001279														
ETH	9.84E-06	0.002027													
USDT	−4.4E-07	−5.4E-07	2.1484E-06												
BNB	−3.6E-06	0.001524	−2.4521E-06	0.002243											
USDC	−6E-06	1.9E-05	−4.0659E-07	1.48E-05	5.72E-05										
XRP	2.98E-05	−0.0002	−1.8211E-06	−0.00017	−7.2E-06	0.002767									
DOGE	5.08E-05	0.001473	−4.1531E-07	0.001367	6.83E-06	8.85E-05	0.020861								
TRX	1.84E-05	0.00142	−1.9148E-06	0.001321	1.14E-05	−0.00024	0.001275	0.002082							
ADA	1.83E-05	0.001755	−5.6401E-07	0.001562	1.9E-05	−0.00022	0.001724	0.001524	0.002623						
BCH	−9.3E-06	0.001852	−6.4599E-07	0.00159	9.05E-06	−0.00015	0.001775	0.00159	0.0019	0.002998					
LINK	−6E-05	0.001868	−4.6367E-06	0.001684	−4.3E-06	−0.00021	0.001655	0.001513	0.001967	0.001998	0.003586				
DAI	1.38E-06	−2.4E-05	8.51689E-07	−1.8E-05	2.34E-06	1.93E-05	1.02E-05	−1.9E-05	−2.4E-05	−2.7E-05	−2.9E-05	2.71E-05			
LITE	−1.5E-05	0.001769	−6.741E-07	0.001562	1.78E-05	−0.00018	0.0016	0.001473	0.001831	0.002119	0.001884	−2.5E-05	0.002375		
XMR	−2.4E-05	0.001427	−4.0251E-07	0.001381	1.78E-05	−0.00024	0.001487	0.001298	0.001472	0.001599	0.001516	−2.2E-05	0.001512	0.002149	
XLM	1.63E-05	0.001586	1.59433E-08	0.001433	1.01E-05	−0.00021	0.001664	0.001488	0.001944	0.001831	0.001847	−2.2E-05	0.001704	0.001369	0.002515

At the core of Markowitz's portfolio theory lies the objective of minimising portfolio risk while maximising a given level of return. In this context, the covariance matrix is employed to calculate the total risk of a portfolio and assess the relationships between assets. The diagonal elements of the covariance matrix represent the variance of each asset and provide information about their individual levels of volatility. For instance, the variance of BTC is calculated as 0.00128, while the variance of ETH is 0.00203. This indicates that ETH is more volatile and therefore a riskier asset compared to BTC. Since volatility reflects the magnitude of fluctuations in an asset's returns, more volatile assets often have their portfolio weights reduced based on risk tolerance in Markowitz optimisation.

Additionally, the off-diagonal elements of the covariance matrix reflect the magnitude of the co-movement between the two assets. For example, the covariance between the BTC and ETH returns is calculated as approximately 0.0001. This positive value indicates that these two assets tend to produce similar returns. In contrast, the covariance between BTC and USDC is approximately −0.00001. This negative value indicates that the returns of these assets generally move in opposite directions.

Markowitz optimisation evaluates both individual risks (variance) and the relationships between assets (covariance) to determine the portfolio weights. The total risk of the portfolio is calculated as the interaction of the asset weights with the covariance matrix. In this framework, portfolio diversification allows the losses of one asset to be offset by the gains of another. The negative covariance between BTC and USDC demonstrate that including these two assets in the same portfolio could reduce the total risk (as is the case in the portfolio). In contrast, the positive covariance between BTC and ETH indicates that the diversification effect would be more limited.

Before proceeding to optimisation processes, we performed the ADF unit root test to evaluate the time series properties of the variables. The ADF unit root test determines whether the mean, variance, and covariance of the variables remain constant over time, indicating their stationarity. Stationarity in time series serves as a critical prerequisite in both theoretical and empirical analyses, as non-stationary variables reduce the predictive power of models and produce misleading results (Granger and Newbold, 1974). Accurate computation of the var-cov matrix, fundamental to optimisation models based on the Markowitz portfolio theory, depends on the stationarity of the variables. Non-stationary variables distort portfolio risk assessments, leading to erroneous calculations. Similarly, regularisation techniques such as Ridge regression require stationarity to ensure reliable parameter estimates and maintain the overall model accuracy.

Table 4

Results of the ADF unit root test

	Test strategies		
Variables	**Intercept (τ Stat.)**	**Intercept & trend (τ Stat.)**	**Results**
BTC	−49.7205***	−49.7248***	**I(0)**
ETH	−50.8747***	−50.8984***	**I(0)**
USDT	−19.2762***	−19.2716***	**I(0)**
BNB	−31.2256***	−31.2592***	**I(0)**
USDC	−19.3368***	−19.3327***	**I(0)**
XRP	−49.4755***	−49.4644***	**I(0)**
DOGE	−35.0646***	−35.0624***	**I(0)**
TRX	−51.5012***	−51.4896***	**I(0)**
ADA	−50.4248***	−50.4655***	**I(0)**
BCH	−49.6276***	−49.621***	**I(0)**
LINK	−50.6368***	−50.7221***	**I(0)**
DAI	−37.9376***	−37.9287***	**I(0)**
LTC	−49.9045***	−49.9171***	**I(0)**
XMR	−55.4164***	−55.4148***	**I(0)**
XLM	−49.5032***	−49.4932***	**I(0)**

Note: **,**,*** denote 10%, 5%, and 1% significance levels, respectively.*

Table 4 presents the results of the ADF unit root test conducted on the returns of fifteen cryptocurrencies. The test statistics indicate that the null hypothesis of a unit root is rejected for all return series at significance levels, indicate that the returns are stationary. This finding is inindicate line with our expectations, as the return series are essentially the first-diff. transformations of the price series, which are generally stationary. This finding implies that cryptocurrency returns do not exhibit persistent trends or long-term memory, making them suitable for further econometric modelling and analysis. The stationarity of the return series is a crucial prerequisite for employing time series techniques that assume mean-reverting behaviour, such as volatility modelling or forecasting frameworks.

Table 5
Optimisation results

Performance criteria	Markowitz	Ridge	Hibrit (Markowitz and ridge)
Portfoy returns	0.2468	0.3664	**0.2609**
Portfoy volatility	0.3834	0.5989	**0.3158**
Sharpe ratio	0.5655	0.5618	**0.7311**
Weights			
Cryptoasset return; weights (%)	Btc: 13.93%	Btc: 9.85%	Btc: 21.01%
	Eth: 11.99%	Eth: 11.08%	Eth: 7.44%
	Usdt: 0.89%	Usdt: 0.07%	Usdt: 10.34%
	Bnb: 13.33%	Bnb: 19.24%	Bnb: 23.94%
	Usdc: 8.57%	Usdc: 0.02%	Usdc: 9.07%
	Xrp: 11.20%	Xrp: 4.51%	Xrp: 6.74%
	Doge: 5.70%	Doge: 8.41%	Doge: 1.59%
	Trx: 4.47%	Tron: 8.99%	Tron: 2.86%
	Ada: 6.48%	Ada: 7.71%	
	Bch: 2.05%	Bch: 2.46%	
	Link: 4.99%	Link: 18.34%	
	Dai: 11.99%	Dai: 0.04%	
	Ltc: 0.53%	Ltc: 4.66%	
	Xmr: 2.11%	Xmr: 5.66%	
	Xlm: 1.75%	Xlm: 0.78%	

Note: *The dataset was split into 80% for the training set and 20% for the test set for use in ML and ML-based techniques.*

Table 5 shows the portfolio performance measures obtained using Markowitz optimisation, Ridge regression and the Hybrid (Markowitz+Ridge) approach, which is a combination of both methods. Performance measures include portfolio return, volatility, and the Sharpe ratio. The annual portfolio return is calculated as 0.2468 with the Markowitz method, 0.3664 with the Ridge method, and 0.2609 with the Hybrid approach. While the Ridge method targets a higher return thanks to its regulatory power, the Hybrid model optimises this return and preserves the balanced structure of Markowitz. The hybrid model provides a more realistic structure by controlling Ridge's risk-taking tendency. Portfolio volatility was measured as 0.3834 in the Markowitz method, 0.5989 in the Ridge method and 0.3158 in the Hybrid model. In this context, Ridge accepts higher risk, but this leads to increased volatility. The hybrid model reduces Ridge's volatility and improves the risk-oriented structure of Markowitz. The Sharpe ratio shows the performance of a portfolio in optimising returns per unit of risk. It is calculated as 0.5655 for the Markowitz method, 0.5618 for the Ridge method and 0.7311 for the Hybrid (Markowitz+Ridge) approach. These results show that the Hybrid approach combines the strengths of both methods and optimises the risk-return trade-off of the portfolio by significantly increasing the Sharpe ratio. Ridge's regularisation power and Markowitz's risk minimisation contributed to the higher performance of the Hybrid model.

Figure 1

Portfolio return and risk

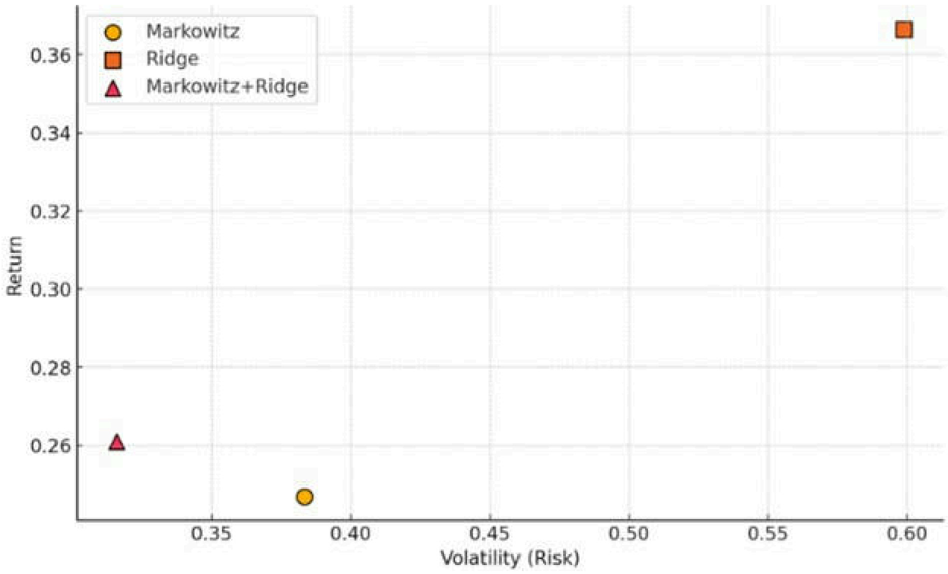
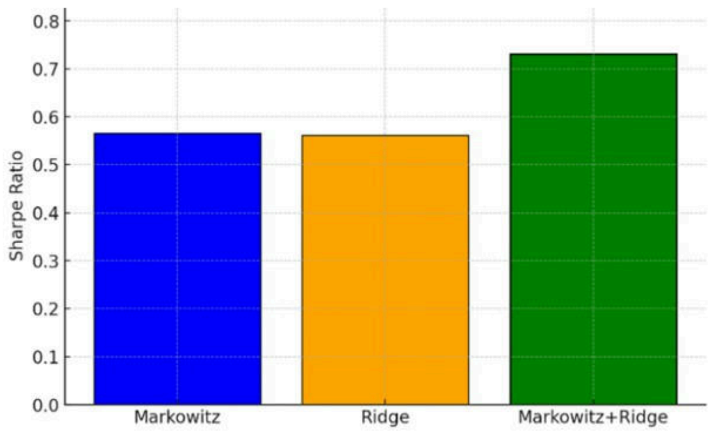


Figure 1 shows more clearly the balance achieved by the hybrid method. The combination of the Markowitz and Ridge methods provides a solution that improves the weaknesses of both algorithms. This is because Markowitz optimisation is overly dependent on the sensitivity of the data. This can lead to errors in predicting future returns based on historical data. However, the Hybrid approach reduces Markowitz's over-sensitivity problem by adding the regularisation power of Ridge. On the other hand, instead of excluding low-importance assets from the portfolio, the Ridge regression tries to artificially balance these assets. This may lead to unrealistic weights. The hybrid model balances this tendency of Ridge with Markowitz's risk minimisation mechanism.

Figure 2

Sharpe ratio comparison



As can be seen from Figures 1 and 2, the hybrid model has a higher sharpe ratio and lower volatility. Therefore, it shows that the portfolio obtained using the hybrid model provides higher risk-adjusted returns and manages volatility effectively.

Figure 3

Portfolio weight distribution

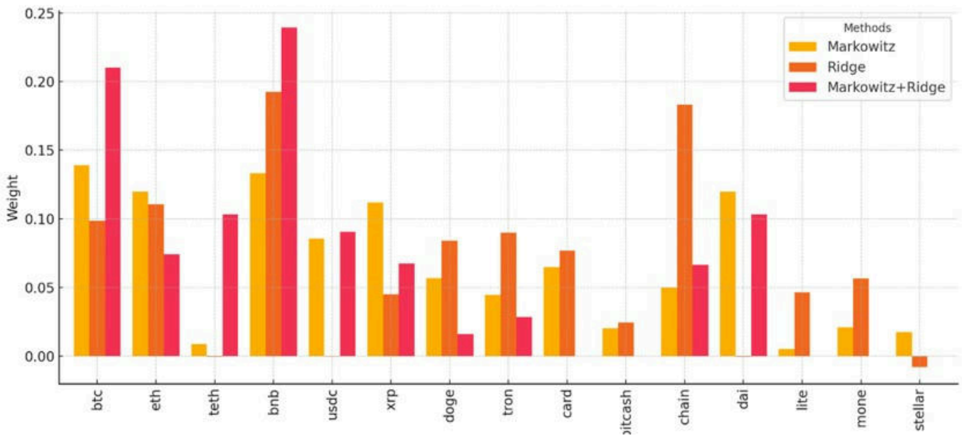


Table 6

Algorithm's parameters

Parameter	Markowitz	Ridge	Hybrid (Markowitz+Ridge)
Expected returns (R)	Historical returns	Returns predicted by Ridge regression	Returns predicted by Ridge regression
Covariance matrix (Σ)	Computed based on historical data	Computed based on Ridge-predicted returns	Computed based on Ridge-predicted returns
Risk (volatility)	Derived from the cov. matrix	Variance of the predicted returns	Hybrid approach (using covariance matrix)
Regularisation coefficient (λ)	Not used	Used in Ridge ($\lambda > 0$)	Used for Ridge-predicted returns
Cross validation (fold)	Not used	10 fold CV	10 fold CV
Sharpe ratio (SR)	Optimisation target	Optimisation target	Optimisation target

Conclusion

This study explores the integration of Markowitz optimisation and Ridge regression to address key limitations in traditional portfolio allocation, such as sensitivity to estimation errors and multicollinearity. Focusing on crypto asset returns, we implement a methodology where the dataset is split into 80% training and 20% testing subsets. To prevent overfitting and ensure model generalizability, CV is performed during the optimisation process. The results reveal that the standalone Markowitz and Ridge approaches yield lower sharpe ratios when applied individually. While Markowitz optimisation benefits from diversification, its sensitivity to estimation errors reduces its effectiveness. On the other hand, Ridge regression successfully mitigates multicollinearity but fails to achieve optimal risk-adjusted returns on its own. To handle these issues, by combining the strengths of both techniques into a hybrid model, we achieved the highest Sharpe ratio, significantly outperforming the individual methods. This integrated approach not only enhances portfolio stability but also improves risk-adjusted performance, demonstrating its practical utility in constructing robust cryptoasset portfolios. As a result, the findings we obtained underscore the importance of blending classical optimisation frameworks with ML techniques to address inherent weaknesses and improve portfolio efficiency.

The findings indicate that this integrated approach attains the maximum Sharpe ratio and substantially enhances risk-adjusted returns and portfolio stability when applied to cryptoasset returns. These findings underscore the merits of integrating classical optimisation methodologies with machine learning to develop more robust and adaptable asset allocation strategies. By analysing the impact of high-volatility cryptoassets on portfolio performance, significant contributions are made to both the existing literature and practical portfolio strategies for investors.

Features such as high volatility and low correlation of cryptoassets have increased the importance of the hybrid model. The ability of the hybrid model to produce consistent results under different market conditions shows that it is a powerful tool that can overcome the shortcomings of traditional methods. In this framework, the findings shows that this hybrid approach has several applications for both investors and academic researchers.

High volatility and low correlation are the hallmarks of the cryptoasset market. The performance of the hybrid model is evaluated on the basis of these characteristics. The applied model has provided effective results in the cryptomarket. However, the model's reliance on the Ridge penalty parameter, which is sensitive to data structure, may limit its robustness in low-volatility or highly correlated traditional markets. How the Ridge regression adjustment parameter, which is critical to the performance of the model, will adapt to different market dynamics is an important uncertainty. In addition, the fact that the data set of the study is limited to the crypto asset market relationship, which is the main subject of the study, may limit the generalizability of the Hybrid model for financial markets. In this framework, testing whether the model is applicable to different markets and asset classes may contribute to filling the gap in the literature.

For future studies, research areas such as the application of the hybrid model to other asset classes and the integration of macroeconomic variables into the model are recommended. This study concludes that combining traditional and modern approaches is a powerful and feasible method for portfolio optimisation in financial markets. The hybrid model may be especially beneficial for institutional investors managing high-volatility assets where traditional methods underperform.

Ethics Committee Approval: This study does not require ethics committee approval.
Peer Review: Externally peer-reviewed.
Author Contributions: Conception/Design of Study- R.K.Y., T.M., G.K.A.; Data Acquisition- T.M.; Data Analysis/ Interpretation- R.K.Y.; Drafting Manuscript- R.K.Y., T.M., G.K.A.; Critical Revision of Manuscript- R.K.Y., T.M., G.K.A.; Final Approval and Accountability- R.K.Y., T.M., G.K.A.
Conflict of Interest: The authors have no conflict of interest to declare.
Grant Support: The authors declared that this study has received no financial support.

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