MULTIVARIATE MARKOV CHAIN MODEL: AN APPLICATION TO S&P500 AND FTSE-100 STOCK EXCHANGES

ÇOK DEĞİŞKENLİ MARKOV ZİNCİRİ MODELİ: S&P500 VE FTSE-100 BORSALARINA UYGULAMASI

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Abstract

Markov chains are the stochastic processes that have many application areas. The data that belong to the system being analyzed in the Markov chains stem from a single source. The multivariate Markov chain model is a model that is used for the purpose of showing the behavior of multivariate categorical data sequences produced from the same source or a similar source. In this study we explain the multivariate Markov chain model that is based on the Markov chains from a theoretical standpoint in detail. As for an application, we take on the daily changes that occur in the S&P-500 Index in which the shares of the 500 greatest companies of the United States of America are traded and the daily changes that occur in the UK FTSE 100 Index as two categorical sequences. And we display the proportions that show how much they influence each other via a multivariate Markov chain model.

Keywords: Markov Chain, Stock Exchange, Multivariate Markov Chain.

JEL Classification: C02, C19, C44, F31

Öz

Markov zincirleri pek çok uygulama alanına sahip olan stokastik süreçlerdir. Markov zincirlerinde incelenen sisteme ait veriler tek bir kaynaktan gelmektedir. Çok değişkenli Markov modeli ise aynı kaynak veya benzer kaynaktan üretilen çoklu kategorik veri dizilerinin davranışlarını göstermek amacı ile kullanılan bir modeldir. Bu çalışmada Markov zincirleri üzerine kurulu olan çok değişkenli Markov zinciri modeli teorik açıdan detaylı bir biçimde anlatılmıştır. Uygulama olarak ise Amerika Birleşik Devletlerinin en büyük 500 şirketinin işlem gördüğü S&P-500 borsa endeksi değerlerinden oluşan günlük değişimler ile İngiltere borsası FTSE-100 endeksi değerlerinden oluşan günlük değişimler iki kategorik dizi olarak ele alınıp bu dizilerin birbirlerini ne oranda etkiledikleri çok değişkenli Markov zinciri modeli ile ortaya konmuştur.

Anahtar Kelimeler: Markov Zinciri, Borsa, Çok Değişkenli Markov Zinciri.

JEL Sınıflaması: C02, C19, C44, F31

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1. Introduction

Standard & Poor's 500 Index consists of the stock shares of the 500 companies with the highest value operating in the United States of America. The companies that form the S&P 500 Index are selected from amongst the most powerful companies of their corresponding sector according to the criteria such as capital, liquidity, market share and profitability. S&P 500 Index, which represents the stocks of the corporations like Xerox, Bank of America, Johnson & Johnson, Amazon, Southwest Airlines and Facebook represents approximately 75 percent of the US stock exchange market. While the index was initially established in an attempt to measure the performance of the American economy, later on it ranked among the most popular indices of the Forex stock market. The profit or loss gathered in the transactions made in the S&P 500 Index of the Forex market is paid in US dollars. Over the last 10 years, S&P 500 Index registered 665 dollars as an all-time record low in 2009, whereas it hit 2133 dollars as an all-time record high in May 2015.

The FTSE-100 Index, on the other hand, consists of the 100 companies in UK with the highest market value. The index began on 3 January 1984 at the base level of 1000, and it reached its highest closing value (7103.98) to date on 27 April 2015. The FTSE-100 Index, consisting of the shares of corporations such as Vodafone, BP, HSBC, Rolls-Royce, Aviva and Marks&Spencer, represents about 81% of the UK Stock Exchange market. [1] Markov processes, the essence of this study, are the processes in which the probability of the conditions that are likely to appear in future are calculated using current data rather than the past ones (Levin et al., 1982). A Markov process is a stochastic process whose conditional probability function satisfies the Markov property (Sheldon, 2009).

The general theory of Markov chains was developed during the 1930s and the 1940s. Markov chains have been applied and used extensively in many areas, including law (Pettiway et al., 1994), marketing Yang et al., 2010), healthcare (Chinnaratha et al., 2017), income distribution (Rey, 2014), finance (Feng, 2002), software reliability (Singh et al., 2016), data mining (Cheng et al., 2012), production (Gillespie and Fulton, 2001), migration (Constant and Zimmermann, 2012), meteorology (Gill et al., 2015).

In all of the applications where Markov chain models are used, depending on the data type and the established model, results related to the probabilities of the future state sequences (long term equilibrium), analyses of absorbing Markov chains or company preference probabilities have been obtained. For example, in a study that deals with the consumers' fruit juice preferences, only the juice brands they (consumers) prefer now and the juice brands they will prefer next have been taken into account. And in the multivariate Markov chain model, data sequences related to each other have been taken into account. That is to say, data sequences that are correlated with each other have been used. In this way, closer-to-the-truth estimations have emerged. For the above example, it is obvious that if other refreshments like soda, ayran or lemonade have been

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1   Investing web page, http://tr.investing.com.

taken into consideration in the customers' fruit juice preferences, more realistic results have been achieved. For this reason researchers may be obliged to use several categorical data sequences together at the same time. At this point, the use of multivariate Markov chain model has formed a different point of view.

The most important advantage of the Multivariate Markov model allows the next state of a category to depend not only on its current state (inter-transition) but also on the current state (intra-transitions) of other series. Therefore, Multivariate Markov Chain plays a key role and is a valuable instrument for focusing on different topics in applied sciences.

Ching and et al. (2002) have applied Multivariate Markov chains to the categorical data related to the demands. Siu and et al. (2005) have used Multivariate Markov chains for credit risk management. Ching and et al. (2005) have applied Multivariate Markov chains to genetics.

The purpose of this study is to reveal how much S&P-500 and FTSE-100 indices, two of the leading stock exchange markets in the world, influence each other by using a first-order multivariate Markov chain.

In the second part of this work, we primarily define Markov chain and approach the Multivariate Markov chain model from a theoretical viewpoint in detail. In the third part, using the daily changes that occur in both S&P-500 and FTSE-100 indices as a categorical sequence, we establish the multivariate Markov chain model. On the assumption that daily-followed change amounts of the two sequences that form the model influence each other at some rate, we find how much they have an impact on the other. In the last part, we make comments on the model obtained.

2. Multivariate Markov Chain

2.1. Markov Chain

In a random experiment, the set of all possible outcomes of this experiment is called the sample space (Can, 2006). The function that is defined from the set of all events in the sample space to the real numbers is called a random variable (Sheldon, 1996).

A discrete time stochastic process is a family of random variables $\{X_t, t \in \mathbb{N}\}$ that are indexed by the parameters t$\in$ N=$\{0, 1, 2, ...\}$ and defined on a given probability space. A Markov process is a stochastic process and its conditional probability distribution function verifies the fact named as "Markov property".

If the state-space of a Markov process is discontinuous, then the Markov process becomes a discrete-time stochastic process and is called a Markov chain (MC). In general a categorical data sequence is represented by a sequence of vectors. $x_1, x_2, x_3, ... x_T$ The indices T in this expression is the length of the sequence. If the process is in the state k, then we write $x_i = e_k$ (Here e_k

is the unit vector for the input). A first order discrete-time Markov chain with an m number of discrete states satisfies the following conditions:

$$P(x_{t+1} = e_{t+1} | x_0 = e_0, x_1 = e_1, ..., x_t = e_t) = P(x_{t+1} = e_{t+1} | x_t = e_t) \qquad (1)$$

$$x_i \in M$$

$$P(x_{t+1} = e_{t+1} | x_t = e_t) \qquad (2)$$

The conditional probabilities in (2) are named as single-step transition probabilities of the Markov chain. These probabilities give the conditional probability of transitioning from the state i to state j provided that time parameter increments from n to $n+1$. Moreover these probabilities are independent of the n time parameters and are written as follows:

$$p_{ij} = P(x_{n+1} = e_i | x_n = e_j), \forall i, j \in M. \qquad (3)$$

The matrix P which consists of the elements p_{ij} is called the matrix of transition probabilities and satisfies both of the following conditions:

i. $0 \le p_{ij} \le 1$, $\qquad\qquad$ ∀i,j∈M $\qquad\qquad\qquad\qquad\qquad (4)$

ii. $\sum_{i=1}^{m} p_{ij} = 1$, $\qquad\qquad$ ∀i,j∈M $\qquad\qquad\qquad\qquad\qquad (5)$

Lemma 1. The Matrix P has an eigenvalue that is equal to 1 and all of the eigenvalues of P are smaller than or equal to 1 (Ching et al., 2002).

Lemma 2. Let A be an irreducible square matrix of order m. Then the following hold:

i. The matrix A has a positive real eigenvalue $\lambda = \max_k |\lambda_k(A)|$. Here $\lambda_k(A)$ denotes the k th eigenvalue of A.

ii. There exists a vector z, whose entries are real and positive, such that Az=λz.

iii. λ is a simple eigenvalue of A.

Using the two lemmas above, one can show that there exists a positive vector z such that Pz=z. Here the vector z is named as the stable probability vector.

$$\sum_{i=1}^{m} z_i = 1 \qquad (6)$$

Here z_i is the stable state probability of the system in the state i.

2.2. The Structure of a Multivariate Markov Chain Model

A multivariate Markov chain model is a model that is used to depict the behavior of the multiple categorical sequences that are being produced by the same source or by similar sources (Maskawa,

2003). Assume that we have s number of categorical sequences each of which has m different states. Let $x_n^{(k)}$ be state vector of the k th sequence at the time n. If the k th sequence is in the j th state at the time n, then the following equality can be written (Ching et al., 2007).

$$x_n^{(k)} = e_j = (0, \dots, 0, 1, 0, \dots, 0)^T \tag{7}$$

In a multivariate Markov chain model the following conditions hold:

$$x_{n+1}^{(j)} = \sum_{k=1}^{s} \lambda_{jk} P^{(jk)} x_n^{(k)}, \; j = 1, 2, \dots, s \tag{8}$$

$$\lambda_{jk} \geq 0, 1 \leq j, k \leq s \; ve \; \sum_{k=1}^{s} \lambda_{jk} = 1, \; j = 1, 2, \dots, s. \tag{9}$$

The state probability distribution of the k th sequence in the $n+1$ th step depends on the weighted mean of $P^{(jk)} x^{(k)}$. Here $P^{(jk)}$ are the transition probabilities from the states of k th sequence to the states of j th sequence, and $x_n^{(k)}$ is the state probability distribution of the k th sequence at the n th step (Zhang et al., 2008). In the matrix notation the following is true:

or

$$x_{n+1} \equiv \begin{bmatrix} x_{n+1}^{(1)} \\ x_{n+1}^{(2)} \\ \vdots \\ x_{n+1}^{(s)} \end{bmatrix} = \begin{bmatrix} \lambda_{11} P^{(11)} & \lambda_{12} P^{(12)} & \cdots & \lambda_{1s} P^{(1s)} \\ \lambda_{21} P^{(21)} & \lambda_{22} P^{(22)} & \cdots & \lambda_{2s} P^{(2s)} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{s1} P^{(s1)} & \lambda_{s2} P^{(s2)} & \cdots & \lambda_{ss} P^{(ss)} \end{bmatrix} \begin{bmatrix} x_n^{(1)} \\ x_n^{(2)} \\ \vdots \\ x_n^{(s)} \end{bmatrix} \equiv Q x_n \tag{10}$$

$$x_{n+1} = Q x_n x_{n+1} = Q x_n.$$

Lemma 3. For $1 \leq j, k \leq s$ if $\lambda_{jk} \geq 0$, then the matrix Q has an eigenvalue that is equal to 1, and the eigenvalues of Q are smaller than or equal to 1.

Lemma 4. For $1 \leq j, k \leq s$ assume that $\lambda_{jk} \geq 0$ and $P^{(jk)}$ is irreducible. Then there exists a vector $x = [x^{(1)}, x^{(2)}, \dots, x^{(s)}]^T$ such that $x = Qx$ and

$$\sum_{i=1}^{m} [x^{(j)}]_i = 1, \quad 1 \leq j \leq s \tag{11}$$

2.3. Estimation of Model Parameters

In order to calculate the model parameters, one must determine the transition probability matrix of each data sequence. For this, in the corresponding data sequences transition frequency from the states in the k th sequence to the states in the j th sequence are calculated. Hence transition frequency matrix for the data sequence is formed. And after normalizing the transition frequency matrix, one obtains the matrix of transition probabilities (Zhang et al., 2008).

Let $f_{i_j i_k}^{(jk)}$ represent the transition matrix from the state i_k in the sequence $\{x_n^{(k)}\}$ to the state i_j in the sequence $\{x_n^{(j)}\}$. Then the transition frequency matrix can be written as follows:

$$Fr^{(jk)} = \begin{bmatrix} f_{11}^{(jk)} & f_{21}^{(jk)} & \cdots & f_{m1}^{(jk)} \\ f_{12}^{(jk)} & f_{22}^{(jk)} & \cdots & f_{m2}^{(jk)} \\ \vdots & \vdots & \vdots & \vdots \\ f_{1m}^{(jk)} & f_{2m}^{(jk)} & \cdots & f_{mm}^{(jk)} \end{bmatrix} \tag{12}$$

Using this transition frequency matrix $Fr^{(jk)}$, one obtains the matrix of transition probabilities $P^{(jk)}$:

$$\hat{P}^{(jk)} = \begin{bmatrix} \hat{p}_{11}^{(jk)} & \hat{p}_{21}^{(jk)} & \cdots & \hat{p}_{m1}^{(jk)} \\ \hat{p}_{12}^{(jk)} & \hat{p}_{22}^{(jk)} & \cdots & \hat{p}_{m2}^{(jk)} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{p}_{1m}^{(jk)} & \hat{p}_{2m}^{(jk)} & \cdots & \hat{p}_{mm}^{(jk)} \end{bmatrix} \tag{13}$$

In order to calculate the value of $\hat{p}_{i_j i_k}^{(jk)}$, one uses the following rule:

$$\hat{p}_{i_j i_k}^{(jk)} = \begin{cases} \dfrac{f_{i_j i_k}^{(jk)}}{\sum_{i_k=1}^{m} f_{i_j i_k}^{(jk)}}, & \sum_{i_k=1}^{m} f_{i_j i_k}^{(jk)} \neq 0 \\[2em] 0, & in\ other\ cases \end{cases} \tag{14}$$

After we make estimations for $\hat{P}^{(jk)}$, we need to calculate the values of the parameters λ_{jk}. A multivariate Markov chain has a stable probability vector shown as x. This vector x can be estimated via calculating the probability of the occurrence of each state in each sequence, and is denoted as below:

$$x = \left[\hat{x}^{(1)}, \hat{x}^{(2)}, \ldots, \hat{x}^{(s)}\right]^T. \tag{15}$$

Moreover one can establish a relation between the parameters λ_{jk}, matrices $P^{(jk)}$, the stable probability vector x and the long term equilibrium vector as follows:

$$\begin{bmatrix} \lambda_{11}P^{(11)} & \lambda_{12}P^{(12)} & \cdots & \lambda_{1s}P^{(1s)} \\ \lambda_{21}P^{(21)} & \lambda_{22}P^{(22)} & \cdots & \lambda_{2s}P^{(2s)} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{s1}P^{(s1)} & \lambda_{s2}P^{(s2)} & \cdots & \lambda_{ss}P^{(ss)} \end{bmatrix} \hat{x} \approx \hat{x} \tag{16}$$

The solution of the following optimization problem can be a used as a method in determining the parameters $\lambda = \{\lambda_{jk}\}$ from (16) (Dongmei and Ching, 2010):

$$\begin{cases} \min_{\lambda} \max_{i} \left| \left[\sum_{k=1}^{m} \lambda_{jk} \hat{P}^{(jk)} \hat{x}^{(k)} - \hat{x}^{(j)}\right]_i \right| \\[1.5em] subject\ to\ \sum_{k=1}^{s} \lambda_{jk} = 1, and\ \lambda_{jk} \geq 0, \ \forall k. \end{cases} \tag{17}$$

Then for each j, the problem expressed in (17) is transformed into s different linear programming problems. On the condition that,

$B = \left[\hat{P}^{(J1)} \hat{x}^{(1)} \middle| \hat{P}^{(J2)} \hat{x}^{(2)} \middle| \cdots \middle| \hat{P}^{(Js)} \hat{x}^{(s)} \right]$ the model can be written as follows (Ching and Ng, 2006):

Objective function:

min w_j (18)

$$\begin{cases} \begin{pmatrix} w_j \\ w_j \\ \vdots \\ w_j \end{pmatrix} \geq \hat{x}^{(J)} - B \begin{pmatrix} \lambda_{j1} \\ \lambda_{j2} \\ \vdots \\ \lambda_{js} \end{pmatrix}, \begin{pmatrix} w_j \\ w_j \\ \vdots \\ w_j \end{pmatrix} \geq -\hat{x}^{(J)} + B \begin{pmatrix} \lambda_{j1} \\ \lambda_{j2} \\ \vdots \\ \lambda_{js} \end{pmatrix} \\ w_j \geq 0 \\ \sum_{k=1}^{s} \lambda_{jk} = 1, \quad \lambda_{jk} \geq 0, \quad \forall k. \end{cases}$$

Constraints: (19)

3. Application of Multivariate Markov Chain For FTSE-100 and S&P-500

In this section the net daily changes that occur in the closing prices of S&P-500 index and in the closing prices of FTSE-100 index have been used as data. For the S&P-500 index the net daily changes that occurred between January 4, 2006 and November 16, 2015 have been used. And for the FTSE-100 index the net daily changes that occurred between January 4, 2006 and August 7, 2015 have been used [2]. While determining the daily changes, the data sequences have been formed by considering in only the data of the previous day.

Figure 1: Closing Values of FTSE-100 and S&P-500 Stock Exchanges
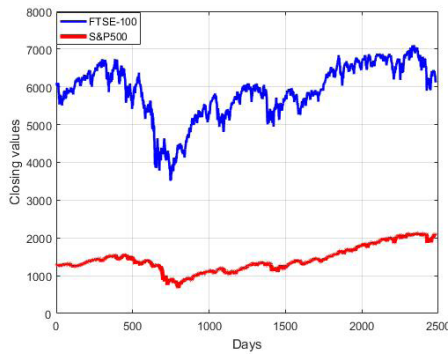
Source: Investing web page (2015)

2   Investing web page, http://tr.investing.com, Access Date: 19.12.2015.

Figure-1 shows closing values of FTSE-100 and S&P500 index in between 2015 and 2006. S&P500 had taken values roughly range of 2100-700 when FTSE-100 took values range of 7000-3500. Taking into account the coefficient of variation of the stock exchanges (CV_{FTSE}=0.126; $CV_{S\&P500}$=0.235) the risk of the stock exchange S&P500 is higher than that of the stock exchange FTSE-100. Moreover, looking at the distribution of the stock exchanges, it is seen that the S&P500 stock market is positive skewness (0.456) and the FTSE-100 stock market is to the negative skewness (-0.837). This means that it is more difficult to obtain a return above the average in the S&P500 market than to the FTSE-100 stock market.

In order to form the data sequence, daily amounts of increase and decrease for S&P-500 and FTSE-100 indices have been determined. The changes in the interval $0 \leq E_1(E_2) \leq 3\sigma$ $0 \leq E_1(E_2) \leq 3\sigma$ have been notated by M_1 and it signifies an increase; on the other hand, the changes in the interval $E_1(E_2) \geq 3\sigma$ have been notated by M_2 and it signifies a high increase. The changes in the interval $E_1(E_2) \leq -3\sigma$ have been represented by M_3 and it means a high decrease, while the changes in the interval $-3\sigma < E_1(E_2) < 0$ have been represented by M_4 it means a decrease. This way we have obtained two categorical data sequences and in both sequences n = 2483 entries have been used. It is easy to see that these sequences are finite discrete-time.

$$E_1 = \{M_4, M_1, M_1, M_4, M_3, M_4, M_2, \dots, M_1, M_1, M_4\}$$

$$E_2 = \{M_1, M_1, M_1, M_2, M_1, M_4, M_4, \dots, M_4, M_3, M_1\}$$

Here the data sequence E_1 represents the changes in the S&P-500 index and the data sequence E_2 represents the changes in the FTSE-100 index. With respect to these sequences, one can say that the Markov chain model has 4 states. According to these data sequences transition frequency matrices have been calculated as follows:

$$Fr^{(11)} = \begin{bmatrix} 750 & 5 & 7 & 612 \\ 2 & 2 & 9 & 10 \\ 11 & 3 & 2 & 10 \\ 611 & 13 & 8 & 428 \end{bmatrix},$$

$$Fr^{(22)} = \begin{bmatrix} 701 & 5 & 10 & 597 \\ 6 & 1 & 2 & 8 \\ 7 & 0 & 1 & 12 \\ 598 & 11 & 7 & 517 \end{bmatrix}$$

$$Fr^{(12)} = \begin{bmatrix} 736 & 12 & 6 & 620 \\ 9 & 2 & 4 & 8 \\ 16 & 0 & 1 & 9 \\ 551 & 3 & 9 & 497 \end{bmatrix},$$

$$Fr^{(21)} = \begin{bmatrix} 738 & 9 & 9 & 557 \\ 9 & 1 & 2 & 5 \\ 12 & 2 & 1 & 5 \\ 615 & 11 & 14 & 493 \end{bmatrix}.$$

Of the transition frequency matrices above, $Fr^{(11)}$ represents the transition frequencies obtained through the transitions between the states of sequence E_1, whereas $Fr^{(12)}$ represents the transition frequencies obtained through the transitions from the states of sequence E_2 to the states of sequence E_1. Similarly, $Fr^{(22)}$ stands for the transition frequencies obtained through the transitions between the states of sequence E_2, and $Fr^{(21)}$ stands for the transition frequencies obtained through the transitions from the states of sequence to the states E_1 of sequence E_2.

The matrices of transition probabilities have been created through the normalization of transition frequency matrices as follows:

$$\hat{P}^{(11)} = \begin{bmatrix} 0.54585 & 0.21739 & 0.26923 & 0.57736 \\ 0.00146 & 0.08686 & 0.34615 & 0.00943 \\ 0.00801 & 0.13043 & 0.07692 & 0.00943 \\ 0.44469 & 0.56522 & 0.30769 & 0.40377 \end{bmatrix}$$

$$P^{(22)} = \begin{bmatrix} 0.53430 & 0.29412 & 0.50000 & 0.52646 \\ 0.00457 & 0.05882 & 0.10000 & 0.00705 \\ 0.00534 & 0.00000 & 0.05000 & 0.01058 \\ 0.45579 & 0.64706 & 0.35000 & 0.45591 \end{bmatrix}$$

$$P^{(12)} = \begin{bmatrix} 0.56098 & 0.70588 & 0.30000 & 0.54674 \\ 0.00686 & 0.11765 & 0.20000 & 0.00705 \\ 0.01220 & 0.00000 & 0.05000 & 0.00794 \\ 0.41997 & 0.17647 & 0.45000 & 0.43827 \end{bmatrix}$$

$$P^{(21)} = \begin{bmatrix} 0.53712 & 0.39130 & 0.34615 & 0.52547 \\ 0.00655 & 0.04348 & 0.07693 & 0.00472 \\ 0.00873 & 0.08696 & 0.03846 & 0.00472 \\ 0.44760 & 0.47826 & 0.53846 & 0.46509 \end{bmatrix}$$

The vectors x given below have been obtained using the occurrence rates of the states in the data sequences E_1 and E_2 :

$$\hat{x}_1 = (0.55354, 0.00926, 0.01047, 0.42673),$$

$$\hat{x}_2 = (0.52858, 0.00684, 0.00805, 0.45652).$$

The stable distribution shows us that stock markets are balanced in both the increase and the decrease. However, the possibility of an increase in S&P500 is somewhat higher than FTSE-100.

Theoretically the variable $m = 4$ represents the number of states and hence $m = 4$; the variable s represents the number of categorical sequences and hence $s = 2$. Therefore we need to make individual solutions for the cases $j = 1$ and $j = 2$. For the solution of the linear programming problem given by (18) and (19), the procedure below has been realized.

For $j = 1$:

Objective function: min w_1

$$\begin{pmatrix} w_1 \\ w_1 \\ w_1 \\ w_1 \end{pmatrix} \geq \hat{x}^{(1)} - B \begin{pmatrix} \lambda_{11} \\ \lambda_{12} \end{pmatrix}, \quad \begin{pmatrix} w_1 \\ w_1 \\ w_1 \\ w_1 \end{pmatrix} \geq -\hat{x}^{(1)} + B \begin{pmatrix} \lambda_{11} \\ \lambda_{12} \end{pmatrix}$$

$$w_1 \geq 0$$

Constraints: $\sum_{k=1}^{2} \lambda_{1k} = 1 \Longrightarrow \lambda_{11} + \lambda_{12} = 1, \quad \lambda_{11}, \lambda_{12} \geq 0$

and the matrix B above is calculated as

$$B = \left[\hat{P}^{(11)} \hat{x}^{(1)} \middle| \hat{P}^{(12)} \hat{x}^{(2)} \right] = \begin{bmatrix} 0.55336 & 0.55337 \\ 0.00926 & 0.00926 \\ 0.01047 & 0.01047 \\ 0.42691 & 0.42690 \end{bmatrix},$$

for $j = 2$:

Objective function: min w_2

$$\begin{pmatrix} w_2 \\ w_2 \\ w_2 \\ w_2 \end{pmatrix} \geq \hat{x}^{(2)} - B \begin{pmatrix} \lambda_{21} \\ \lambda_{22} \end{pmatrix}, \quad \begin{pmatrix} w_2 \\ w_2 \\ w_2 \\ w_2 \end{pmatrix} \geq -\hat{x}^{(2)} + B \begin{pmatrix} \lambda_{21} \\ \lambda_{22} \end{pmatrix}$$

$$w_2 \geq 0$$

Constraints: $\sum_{k=1}^{2} \lambda_{2k} = 1 \Longrightarrow \lambda_{21} + \lambda_{22} = 1, \quad \lambda_{21}, \lambda_{22} \geq 0$

Here the matrix B above is calculated as

$$B = \left[\hat{P}^{(21)} \hat{x}^{(1)} \middle| \hat{P}^{(22)} \hat{x}^{(2)} \right] = \begin{bmatrix} 0.52837 & 0.52927 \\ 0.00713 & 0.00652 \\ 0.00799 & 0.00767 \\ 0.45651 & 0.45653 \end{bmatrix}.$$

The values of λ are calculated via the solutions of the linear programming problems by taking j = 1 and j = 2 in (18) and (19):

$$\lambda = \begin{pmatrix} 0.69971 & 0.30029 \\ 0.76087 & 0.23913 \end{pmatrix}$$

If the entries of the matrix λ are substituted for in (10), the multivariate Markov chain model for the categorical data sequences E_1 and E_2 is given as follows:

$$x_{n+1}^{(1)} = 0.69971 \hat{P}^{(11)} x_n^{(1)} + 0.30029 \hat{P}^{(12)} x_n^{(2)}$$

4. Discussion and Conclusion

Because data sequences which are correlated with each other have been utilized in the multivariate Markov chain model, multivariate Markov model can be used to model the dependency of multivariate stochastic process in many events. By this means, different points of view and remarks come into existence in accordance with the analysis of the single-source being considered in Markov chain model

There are very few studies in the field of application made with multivariate Markov model and there are few studies on this topic in our country. In this study we have considered the daily changes in the prices of S&P-500 index and the daily changes in the prices of FTSE-100 index as two categorical sequences, and we have determined how much these two sequences affect each other. The findings are presented below:

Here the net changes in S&P 500 form the elements of the first sequence, and the net Daily changes in FTSE-100 form the second sequence. In the $n+1$ th step, the variable $x_{n+1}^{(1)}$ influences $\left(x_n^{(1)} \right)$ (the effect of the daily changes in S&P – 500 on itself) with an impact factor 0.69971 and influences ($x_n^{(2)}$) (the effect of the daily changes in S&P-500 on FTSE-100) with an impact factor 0.30029.

And the variable $x_{n+1}^{(2)}$ influences $x_n^{(2)}$ (the effect of the daily changes in FTSE-100 on itself) with an impact factor 0.23913, while it influences $x_n^{(1)}$ (the effect of the daily changes in FTSE-100 on S&P-500) with an impact factor 0.76087.

In this work modelling of the multivariate Markov chain is expressed theoretically as clearly as possible, and through an implementation, the theoretical structure of this model are made more understandable.

In the future possible works, by increasing the number of categorical sequences one can attain more distinct results and more clear estimations about the sequences that affect each other. Furthermore, it is contemplated that if we increase the number of states that compose the sequence and thus obtain more data to observe, the model will be more truthful. In addition, it can be compared higher-order multivariate Markov models with first-order Markov models for prediction of categorical sequences. Multivariate Markov model is that the model is easy to computationally efficient and implement. Therefore, taking into account the number of parameters and constraints, it can create new multivariate Markov models.

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