İstatistikçiler Dergisi: İstatistik & Aktüerya

Journal of Statisticians: Statistics and Actuarial Sciences

**IDIA 10**, 2017, 1, 23-39

Geliş/Received:24.03.2017, Kabul/Accepted: 12.06.2017

www.istatistikciler.org

**Araştırma Makalesi / Research Article**

# Estimation of population mean under systematic random sampling in absence and presence non-response

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**Abstract**

In this article, we propose two new classes of estimator using transformed auxiliary variables for the estimation of systematic sample mean in absence and presence of non-response. The mathematical expressions of biases and mean square errors are determined. A numerical illustration is also performed to illustrate the performance of the proposed estimators. Based on the numerical study, the proposed estimator performs better than the usual sample estimator, regression estimator, Singh and Solanki [8], Singh et al. [9], Swain [11] and Koyuncu [3] estimators in systematic sampling.
***Keywords:*** *Systematic random sampling, Supplementary information, Bias, Relative efficiency.*

**Öz**

***Cevapsızlık olması ve olmaması durumlarında sistematik örneklemede ortalama tahmini***

*Bu çalışmada cevapsızlık varlığında ve yokluğunda sistematik örneklem ortalamasının tahmini için dönüşüm yapılmış yardımcı değişkenler kullanılarak iki yeni tahmin edici sınıfı önerildi. Yan ve hata karaler ortalamalarının matematiksel formülleri bulundu. Ayrıca önerilen tahmin edicilerin performansını göstermek için sayısal bir örnek yapılmıştır.Sayısal çalışma sonucunda, önerilen tahmin ediciler sistematik örneklemede genel örneklem ortalamasından, regresyon tahmin edicisinden, Singh and Solanki [8], Singh et al. [9], Swain [11] ve Koyuncu [3] tahmin edicilerinden daha iyi sonuç vermiştir.*
**Anahtar sözcükler:** *Sistematik rasgele örnekleme; Yardımcı bilgi, Yan, Göreli etkinlik*

## 1. Introduction

Supplementary information is used at many stages to enhance the effectiveness of an estimator of population mean $\bar{Y}$. The regression, product and ratio estimators are broadly used in numerous circumstances of survey sampling, when supplementary information is utilized at the estimation stage. Mohanty and Sahoo [4] used transformed supplementary variable for the estimation of population mean $\bar{Y}$. Their transformed supplementary variables are as given below

$$u = \frac{x+X_{min}}{X_{max}+X_{min}} \quad (1)$$

$$z = \frac{x+X_{max}}{X_{min}+X_{max}} \quad (2)$$

Systematic random sampling is a strategy of selecting sample from a bigger population through initial random start. Typically, each nth element is chosen from the whole population for inclusion in the required sample. Nowadays, systematic random sampling is becoming more famous than simple random sampling due to its simplicity, see Cochran [1], Singh and Solanki [8], Singh et al. [9] and Verma [12] have developed some enhanced estimators for the estimation of population mean $\bar{Y}$ utilizing auxiliary information under systematic random sampling scheme. Riaz et al. [6] have defined classes of estimator in circular systematic sampling.

In this study, taking inspiration from Mohanty and Sahoo [4], and Koyuncu [3], two new families of estimators are constructed for the estimation of population mean $\bar{Y}$, utilizing the supplementary information under systematic random sampling scheme. We also discuss suggested families of estimators when non-response present in the variate of interest Y only.

## 2. Preliminaries and existing estimators

Let Y be the variate of interest and X be the auxiliary variate defined on a finite population $\xi$ containing 1 to N units. Unless specified else, we consider N = nk, where n and k are positive integers. In this manner, there will be k groups (samples) each of size n. Let M be the random variable having range $\{1,2,3,\ldots,k\}$. The systematic random sample is then selected by the following random sequence as

$$\{\xi_M, \xi_{M+k}, \ldots, \xi_{M+(n-1)k}\}.$$

Let $(y_{ij}, x_{ij})$ for $(i = 1,2,\ldots,k)$ and $(j = 1,2,\ldots,n)$, denote $j^{th}$ unit in the $i^{th}$ sample. The corresponding linear systematic sample means of Y and X are $\bar{y}_{ss} = \frac{\sum_{j=1}^{n} y_{ij}}{n}$ $and$ $\bar{x}_{ss} = \frac{\sum_{j=1}^{n} x_{ij}}{n}$. Let us define following error terms:

$$\varepsilon_0 = \frac{\bar{y}_{ss} - \bar{Y}}{\bar{Y}}, \varepsilon_1 = \frac{\bar{x}_{ss} - \bar{X}}{\bar{X}}, \varepsilon_u = \frac{\bar{u}_{ss} - \bar{U}}{\bar{U}}, \varepsilon_z = \frac{\bar{z}_{ss} - \bar{Z}}{\bar{Z}}$$

where $\bar{u}_{ss} = \frac{\bar{x}_{ss}+X_{min}}{X_{min}+X_{max}}, \bar{z}_{ss} = \frac{\bar{x}_{ss}+X_{max}}{X_{max}+X_{min}}$ and $\bar{U}$ and $\bar{Z}$ denote the population means of u and z respectively. The expectations of error terms can be written as:

$$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_u) = E(\varepsilon_z) = 0,$$

$$E(\varepsilon_0^2) = \left(\frac{N-1}{Nn}\right)\{1 + \rho_y(n-1)\}C_y^2 = v_0,$$

$$E(\varepsilon_1^2) = \left(\frac{N-1}{Nn}\right)\{1+\rho_x(n-1)\}C_x^2 = v_1,$$

$$E(\varepsilon_u^2) = \left(\frac{N-1}{Nn}\right)\{1+\rho_x(n-1)\}\left(\frac{S_x}{\bar{X}+X_{min}}\right)^2 = v_u,$$

$$E(\varepsilon_z^2) = \left(\frac{N-1}{Nn}\right)\{1+\rho_x(n-1)\}\left(\frac{S_x}{\bar{X}+X_{max}}\right)^2 = v_z,$$

$$E(\varepsilon_0\varepsilon_1) = \left(\frac{N-1}{Nn}\right)\{1+\rho_y(n-1)\}^{\frac{1}{2}}\{1+\rho_x(n-1)\}^{\frac{1}{2}}\rho C_y C_x = v_{O1},$$

$$E(\varepsilon_0\varepsilon_u) = \left(\frac{N-1}{Nn}\right)\{1+\rho_y(n-1)\}^{\frac{1}{2}}\{1+\rho_x(n-1)\}^{\frac{1}{2}}\rho C_y\left(\frac{S_x}{\bar{X}+X_{min}}\right) = v_{Ou},$$

$$E(\varepsilon_0\varepsilon_1) = \left(\frac{N-1}{Nn}\right)\{1+\rho_y(n-1)\}^{\frac{1}{2}}\{1+\rho_x(n-1)\}^{\frac{1}{2}}\rho C_y\left(\frac{S_x}{\bar{X}+X_{max}}\right) = v_{Oz},$$

where $\rho_x$ and $\rho_y$ are the interclass correlations of X and Y respectively. While ρ is the coefficient of correlation between X and Y. The variance of the traditional systematic sample mean $\bar{y}_{ss}$ is

$$V(\bar{y}_{ss}) = \bar{Y}^2 v_O. \quad (3)$$

The traditional ratio and product estimators (due to Swain [10] and Shukla [7]) are as given below

$$\hat{\bar{a}}_r = \bar{y}_{ss}\left[\frac{\bar{X}}{\bar{x}_{ss}}\right], \quad (4)$$

$$\hat{\bar{a}}_p = \bar{y}_{ss}\left[\frac{\bar{x}_{ss}}{\bar{X}}\right]. \quad (5)$$

The MSEs of $\hat{\bar{a}}_r$ *and* $\hat{\bar{a}}_p$ are

$$MSE(\hat{\bar{a}}_r) = \bar{Y}^2[v_O + v_1 - 2v_O v_1], \quad (6)$$

$$MSE(\hat{\bar{a}}_p) = \bar{Y}^2[v_O + v_1 + 2v_O v_1], \quad (7)$$

The traditional regression estimator under systematic random sampling scheme and its MSE as given by

$$\hat{\bar{a}}_{reg} = \bar{y}_{ss} + b(\bar{X} - \bar{x}_{ss}) \quad (8)$$

$$MSE(\hat{\bar{a}}_{reg}) = \bar{Y}^2 v_O(1-\rho^2). \quad (9)$$

Following Swain [11], one may propose the following ratio and product estimators as given by

$$\hat{\hat{a}}_{hr1} = \bar{y}_{ss}\left(\frac{\bar{X}}{\bar{x}_{ss}}\right)^2 \quad (10)$$

$$\hat{\hat{a}}_{hr2} = \bar{y}_{ss}\left(\frac{\bar{x}_{ss}}{\bar{X}}\right)^2 \quad (11)$$

The MSEs of $\hat{\hat{a}}_{hr1}$ *and* $\hat{\hat{a}}_{hr2}$ are given by

$$MSE(\hat{\hat{a}}_{hr1}) = \bar{Y}^2\left[v_o + \frac{1}{4}v_1 - v_{01}\right], \quad (12)$$

$$MSE(\hat{\hat{a}}_{hr2}) = \bar{Y}^2\left[v_o + \frac{1}{4} + v_{01}\right]. \quad (13)$$

Singh and Solanki [8] family of estimators under systematic random sampling as

$$\hat{\hat{a}}_{ss} = \omega_{ss1}\bar{y}_{ss}\left[\frac{\bar{X}'}{(\alpha\bar{x}'_{ss}+(1-\alpha)\bar{X}')}\right]^g + \omega_{ss2}\bar{y}_{ss}exp\left[\frac{\delta(\bar{X}'-\bar{x}'_{ss})}{\bar{X}'+\bar{x}'_{ss}}\right] \quad (14)$$

where $\bar{X}' = \alpha\bar{X} + b$ and $\bar{x}' = \alpha\bar{x}_{ss} + b$. Some members of $\hat{\hat{a}}_{ss}$ are given in Table1 according to suitable variable of $\alpha, g, \delta$, a and b. The MSE of $\hat{\hat{a}}_{ss}$ is

$$MSE(\hat{\hat{a}}_{ss}) = \bar{Y}^2 + w_{ss1}^2\zeta_A + w_{ss2}^2\zeta_C + 2\omega_{ss1}\omega_{ss2}\zeta_D - 2\omega_{ss1}\zeta_B - 2\omega_{ss2}\zeta_E \quad (15)$$

where

$$\zeta_A = \bar{Y}^2[1 + v_o - 4\alpha V g v_{01} + \alpha^2 V^2 g(2g+1)v_1],$$

$$\zeta_B = \bar{Y}^2\left[1 - \alpha V g v_{01} + \frac{g(g+1)}{2}\alpha^2 V^2 v_1\right]$$

$$\zeta_C = \bar{Y}^2\left[1 + v_o - 2\delta V v_{01} + \left\{\frac{\delta(\delta+2)}{4}V^2 + \frac{\delta^2 V^2}{4}\right\}v_1\right]$$

$$\zeta_D = \bar{Y}^2\left[1 + v_o - V(2\alpha g + \delta)v_{01} + \frac{A'' V^2}{8}v_1\right]$$

$$\zeta_E = \bar{Y}^2\left[1 - \frac{\delta V}{2}v_{01} + \frac{\delta(\delta+2)}{8}V^2 v_1\right]$$

$V = \frac{a\bar{X}}{a\bar{X}+b}$ and $A'' = [(2\alpha g + \delta)^2 + 2(2\alpha^2 g + \delta)]$.

which is minimum for

$$\omega_{ss1}^{opt} = \left[\frac{\zeta_B\zeta_C - \zeta_D\zeta_E}{\zeta_A\zeta_C - \zeta_D^2}\right], \omega_{ss2}^{opt} = \left[\frac{\zeta_A\zeta_E - \zeta_B\zeta_D}{\zeta_A\zeta_C - \zeta_D^2}\right].$$

The minimum MSE of $\hat{\hat{a}}_{ss}$ is

$$MSE_{min}(\hat{\hat{a}}_{ss}) = \left[\bar{Y}^2 - \frac{\zeta_C\zeta_B^2+\zeta_A\zeta_E^2-2\zeta_B\zeta_D\zeta_E}{\zeta_A\zeta_C-\zeta_D^2}\right] \quad (16)$$

**Table 1**: Family members of Singh and Solanki [8]

| **Estimators (α, g, δ )=(1,1,-1)** | **a** | **b** |
|---|---|---|
| $\hat{\hat{a}}_{ss1} = w_{ss1}\bar{y}_{ss}\left(\frac{\bar{X}}{\bar{x}_{ss}}\right) + w_{ss2}\bar{y}_{ss}\left(\frac{\bar{x}_{ss}-\bar{X}}{\bar{X}+\bar{x}_{ss}}\right)$ | 1 | 0 |
| $\hat{\hat{a}}_{ss2} = w_{ss1}\bar{y}_{ss}\left(\frac{\bar{X}+C_x}{\bar{x}_{ss}+C_x}\right) + w_{ss2}\bar{y}_{ss}\left(\frac{\bar{x}_{ss}-\bar{X}}{(\bar{X}+\bar{x}_{ss})+2C_x}\right)$ | 1 | $C_x$ |
| $\hat{\hat{a}}_{ss3} = w_{ss1}\bar{y}_{ss}\left(\frac{S_x\bar{X}+1}{S_x\bar{x}_{ss}+1}\right) + w_{ss2}\bar{y}_{ss}\left(\frac{S_x(\bar{x}_{ss}-\bar{X})}{S_x(\bar{X}+\bar{x}_{ss})+2}\right)$ | $S_x$ | 1 |
| $\hat{\hat{a}}_{ss4} = w_{ss1}\bar{y}_{ss}\left(\frac{\bar{X}+\rho}{\bar{x}_{ss}+\rho}\right) + w_{ss2}\bar{y}_{ss}\left(\frac{(\bar{x}_{ss}-\bar{X})}{(\bar{X}+\bar{x}_{ss})+2\rho}\right)$ | 1 | ρ |
| $\hat{\hat{a}}_{ss5} = w_{ss1}\bar{y}_{ss}\left(\frac{S_x\bar{X}+C_x}{S_x\bar{x}_{ss}+C_x}\right) + w_{ss2}\bar{y}_{ss}\left(\frac{\rho(\bar{x}_{ss}-\bar{X})}{S_x(\bar{X}+\bar{x}_{ss})+2C_x}\right)$ | $S_x$ | $C_x$ |
| $\hat{\hat{a}}_{ss6} = w_{ss1}\bar{y}_{ss}\left(\frac{\rho\bar{X}+C_x}{\rho\bar{x}_{ss}+C_x}\right) + w_{ss2}\bar{y}_{ss}\left(\frac{\rho(\bar{x}_{ss}-\bar{X})}{\rho(\bar{X}+\bar{x}_{ss})+2C_x}\right)$ | ρ | $C_x$ |
| $\hat{\hat{a}}_{ss7} = w_{ss1}\bar{y}_{ss}\left(\frac{S_x\bar{X}+\rho}{\rho\bar{x}_{ss}+C_x}\right) + w_{ss2}\bar{y}_{ss}\left(\frac{\rho(\bar{x}_{ss}-\bar{X})}{\rho(\bar{X}+\bar{x}_{ss})+2C_x}\right)$ | $S_x$ | ρ |

Singh et al. [9] introduced the following estimator under systematic random sampling as

$$\hat{\hat{a}}_s = [w_{s1}\bar{y}_{ss} + w_{s2}(\bar{X} - \bar{x}_{ss})]\left(\frac{\bar{X}}{\bar{x}_{ss}}\right) \quad (17)$$

The MSE $\hat{\hat{a}}_s$ is

$$\mathrm{MSE}(\hat{\hat{a}}_s) = \bar{Y}^2 + w_{s1}^2\zeta_{As} + w_{s2}^2\zeta_{Bs} + 2w_{s1}w_{s2}\zeta_{Cs} - 2w_{s1}\zeta_{Ds} - 2w_{s2}\zeta_{Es}, \quad (18)$$

where

$\zeta_{As} = \bar{Y}^2[1 + V_O + 3V_1 - 4V_{01}], \qquad \zeta_{Bs} = \bar{X}^2 V_1, \quad \zeta_{Cs} = \bar{X}\bar{Y}[2V_1 - V_O],$

$\zeta_{Ds} = \bar{Y}^2[1 + V_1 - V_{01}], \qquad \zeta_{Es} = \bar{X}\bar{Y}V_1.$

which is minimum for

$$\omega_{s1}^{opt} = \left[\frac{\zeta_{Bs}\zeta_{Ds} - \zeta_{Cs}\zeta_{Es}}{\zeta_{As}\zeta_{Bs} - \zeta_{Cs}^2}\right], \omega_{s2}^{opt} = \left[\frac{\zeta_{As}\zeta_{Es} - \zeta_{Cs}\zeta_{Ds}}{\zeta_{As}\zeta_{Bs} - \zeta_{Cs}^2}\right]$$

The minimum MSE of $\hat{\hat{a}}_s$ is

$$MSE_{min}(\hat{\hat{a}}_s) = \left[\bar{Y}^2 - \frac{\zeta_{Bs}\zeta_{Ds}^2 + \zeta_{As}\zeta_{Es}^2 - 2\zeta_{Cs}\zeta_{Ds}\zeta_{Es}}{\zeta_{As}\zeta_{Bs} - \zeta_{Cs}^2}\right] \qquad (19)$$

Now we are adapting Koyuncu [3] estimator to systematic random sampling. Koyuncu [3] class of estimators under systematic random sampling will be

$$\hat{\hat{a}}_{nk} = \left[w_{nk1}\bar{y}_{ss} + w_{nk2}\left(\frac{\bar{x}_{ss}}{\bar{X}}\right)^{\gamma}\right] exp\left[\frac{\eta(\bar{X} - \bar{x}_{ss})}{\eta(\bar{X} + \bar{x}_{ss}) + 2\xi}\right] \qquad (20)$$

We generated some member of $\hat{\hat{a}}_{nk}$ using suitable $\gamma$, $\eta$ and $\xi$, are given in Table 2.

**Table 2**: Some family members of Koyuncu [3]

| Estimators | $\gamma$ | $\eta$ | $\xi$ |
|---|---|---|---|
| $\hat{a}_{nk1} = \left[\omega_{nk1}\bar{y}_{ss} + \omega_{nk2}\left(\frac{\bar{x}_{ss}}{\bar{X}}\right)\right]\exp\left[\frac{C_x(\bar{X} - \bar{x}_{ss})}{C_x(\bar{X} + \bar{x}_{ss}) + 2\beta_2(x)}\right]$ | 1 | $C_x$ | $\beta_2(x)$ |
| $\hat{a}_{nk2} = \left[\omega_{nk1}\bar{y}_{ss} + \omega_{nk2}\left(\frac{\bar{x}_{ss}}{\bar{X}}\right)\right]\exp\left[\frac{\beta_2(x)(\bar{X} - \bar{x}_{ss})}{\beta_2(x)(\bar{X} + \bar{x}_{ss}) + 2C_x}\right]$ | 1 | $\beta_2(x)$ | $C_x$ |
| $\hat{a}_{nk3} = \left[\omega_{nk1}\bar{y}_{ss} + \omega_{nk2}\left(\frac{\bar{x}_{ss}}{\bar{X}}\right)\right]\exp\left[\frac{(\bar{X} - \bar{x}_{ss})}{(\bar{X} + \bar{x}_{ss}) + 2C_x}\right]$ | 1 | 1 | $C_x$ |
| $\hat{a}_{nk4} = \left[\omega_{nk1}\bar{y}_{ss} + \omega_{nk2}\left(\frac{\bar{x}_{ss}}{\bar{X}}\right)\right]\exp\left[\frac{(\bar{X} - \bar{x}_{ss})}{(\bar{X} + \bar{x}_{ss}) + 2\beta_2(x)}\right]$ | 1 | 1 | $\beta_2(x)$ |

The MSE $\hat{\hat{a}}_{nk}$ is

$$MSE(\hat{\hat{a}}_{nk}) = \bar{Y}^2 + w_{nk1}^2\zeta_{Ank} + w_{nk2}^2\zeta_{Bnk} + 2w_{nk1}w_{nk2}\zeta_{Cnk} - 2w_{nk1}\zeta_{Dnk} - 2w_{nk2}\zeta_{Enk} \qquad (21)$$

where

$\zeta_{Ank} = \bar{Y}^2\left[1 + v_O + \frac{1}{2}\theta^2 v_1 - \theta v_{o1}\right], \qquad \zeta_{Bnk} = 1 + (b^2 + 2c')v_1,$

$$\zeta_{Cnk} = \bar{Y}\left[1 + (c' - \frac{b'\theta}{2} + \frac{3}{8}\theta^2)v_1 + \left(b' - \frac{\theta}{2}\right)v_{o1}\right],$$

$$\zeta_{Dnk} = \bar{Y}^2\left[1 + \frac{3}{8}\theta^2 v_1 - \frac{\theta}{2}v_{o1}\right], \qquad \zeta_{Enk} = \bar{Y}[1 + c'V_1],$$

$$\alpha' = \frac{\gamma(\gamma-1)}{2}, b' = \gamma - \frac{\theta}{2}, c' = \frac{3}{8}\theta^2 - \frac{\theta\gamma}{2} + \alpha'.$$

$\hat{\bar{a}}_{nk}$ is minimum for

$$\omega_{nk1}^{opt} = \left[\frac{\zeta_{Bnk}\zeta_{Dnk} - \zeta_{Cnk}\zeta_{Enk}}{\zeta_{Ank}\zeta_{Bnk} - \zeta_{Cnk}^2}\right], \omega_{nk2}^{opt} = \left[\frac{\zeta_{Ank}\zeta_{Enk} - \zeta_{Cnk}\zeta_{Dnk}}{\zeta_{Ank}\zeta_{Bnk} - \zeta_{Cnk}^2}\right].$$

The minimum MSE of $\hat{\bar{a}}_{nk}$ is

$$MSE_{min}(\hat{\bar{a}}_{nk}) = \left[\bar{Y}^2 - \frac{\zeta_{Bnk}\zeta_{Dnk}^2 + \zeta_{Ank}\zeta_{Enk}^2 - 2\zeta_{Cnk}\zeta_{Dnk}\zeta_{Enk}}{\zeta_{Ank}\zeta_{Bnk} - \zeta_{Cnk}^2}\right] \tag{22}$$

## 3. Suggested estimators in absence non-response

### *3.1 First suggested family of estimators in absence non-response*

Mohanty and Sahoo [4] have used transformed supplementary variables as given in (1) and (2). Koyuncu [3] have suggested an exponential class of estimators. Taking motivation from Mohanty and Sahoo [4] and Koyuncu [3], we suggest the following class of estimators under systematic random sampling

$$\hat{\bar{a}}_{N1} = \left[\eta_u + w_{1p}\left\{\frac{\bar{u}_{ss}}{\bar{U}}\right\}^{\gamma} + w_{2p}\bar{y}_{ss}\right] exp\left\{\frac{\alpha(\bar{u}_{ss} - \bar{U})}{2C_k\bar{U} + \bar{u}_{ss} - \bar{U}}\right\} \tag{23}$$

where $\eta_u = \frac{\bar{y}_{ss}}{2}\left\{\left(\frac{\bar{U}}{\bar{u}_{ss}}\right)^{\frac{1}{2}} + \left(\frac{\bar{u}_{ss}}{\bar{U}}\right)^{\frac{1}{2}}\right\}$. Some members of $\hat{\bar{a}}_{N1}$ are given in Table 3.

Let we express $\hat{\bar{a}}_{N1}$ in terms of $\varepsilon_0$ $and$ $\varepsilon_1$ as follows

$$\hat{\bar{a}}_{N1} = \left[\bar{Y}\left(1 + \frac{1}{8}\varepsilon_U^2 + \varepsilon_O\right) + w_{1P}(1 + \gamma\varepsilon_u) + \frac{\gamma(\gamma-1)}{2}\varepsilon_U^2 + w_{2p}\bar{y}(1 + \varepsilon_o)\right](1 - a\varepsilon_o + b\varepsilon_u^2)$$

The bias of $\hat{\bar{a}}_{N1}$ is

$$Bias(\hat{\bar{a}}_{N1}) = \bar{Y}\left\{\left(b + \frac{1}{8}\right)v_u - av_{ou}\right\} + w_{1P}\{1 + cv_u\} + w_{2p}\bar{y}\{1 + bv_u - av_{ou}\} \tag{24}$$

where

$$a=\frac{\alpha}{2c_k},\qquad b=\frac{\alpha}{4c_k^2}+\frac{\alpha^2}{8c_k^2},\qquad c=b-a\gamma+\frac{\gamma(\gamma-1)}{2}$$

The MSE of $\hat{\bar{a}}_{N1}$ is

$$MSE(\hat{\bar{a}}_{N1})=\left[L_1+w_{1P}^2\Lambda_{A1}+w_{2P}^2\Lambda_{B1}+2w_{1p}w_{2p}\Lambda_{C1}-2w_{1p}\Lambda_{D1}-w_{2p}\Lambda_{E1}\right] \quad (25)$$

where

$$L_1=\bar{Y}^2\{v_0+a^2v_u-2av_{ou}\}$$

$$\Lambda_{A1}=\{1+(\gamma-a)^2\,v_u+2cv_{ou}\}$$

$$\Lambda_{B1}=\bar{Y}^2\{1+v_0+(a^2+2b)v_u-4av_{ou}\}$$

$$\Lambda_{C1}=\bar{Y}\{1+(c-a(\gamma-a)+b)v_u+(\gamma-2a)v_{ou}\}$$

$$\Lambda_{D1}=\bar{Y}\left\{a(\gamma-a)v_u-\left(b+\frac{1}{8}\right)v_u+(2a-\gamma)v_{ou}\right\}$$

$$\Lambda_{E1}=2\bar{Y}^2\left\{-v_o-\left(a^2+b+\frac{1}{8}\right)v_u+3av_{ou}\right\}$$

**Table 3.** Some members of first suggested family

| Estimators | Values of constants | | |
|---|---|---|---|
| | $\gamma$ | $\alpha$ | $c_k$ |
| $\hat{\bar{a}}_{N1(1)}=\left[\eta_u+\omega_{1p}\left\{\frac{\bar{u}_{ss}}{\bar{U}}\right\}+\omega_{2p}\bar{y}\right]\exp\left\{\frac{(\bar{u}_{ss}-\bar{U})}{(2C_x\bar{U}+\bar{u}_{ss}-\bar{U})}\right\}$ | 1 | 1 | $C_x$ |
| $\hat{\bar{a}}_{N1(2)}=\left[\eta_u+\omega_{1p}\left\{\frac{\bar{u}_{ss}}{\bar{U}}\right\}+\omega_{2p}\bar{y}\right]\exp\left\{\frac{(\bar{u}_{ss}-\bar{U})}{(2\rho\bar{U}+\bar{u}_{ss}-\bar{U})}\right\}$ | 1 | 1 | $\rho$ |
| $\hat{\bar{a}}_{N1(3)}=\left[\eta_u+\omega_{1p}\{\frac{\bar{u}_{ss}}{\bar{U}}\}^{\frac{1}{2}}+\omega_{2p}\bar{y}\right]\exp\left\{\frac{(\bar{u}_{ss}-\bar{U})}{(2N\bar{U}+\bar{u}_{ss}-\bar{U})}\right\}$ | $\frac{1}{2}$ | 1 | N |
| $\hat{\bar{a}}_{N1(4)}=\left[\eta_u+\omega_{1p}\{\frac{\bar{u}_{ss}}{\bar{U}}\}^{\frac{1}{2}}+\omega_{2p}\bar{y}\right]\exp\left\{\frac{(\bar{u}_{ss}-\bar{U})}{(2\beta_2(x)\bar{U}+\bar{u}_{ss}-\bar{U})}\right\}$ | $\frac{1}{2}$ | 1 | $\beta_2(x)$ |
| $\hat{\bar{a}}_{N1(5)}=\left[\eta_u+\omega_{1p}\{\frac{\bar{u}_{ss}}{\bar{U}}\}^{\frac{1}{3}}+\omega_{2p}\bar{y}\right]\exp\left\{\frac{(\bar{u}_{ss}-\bar{U})}{(2N\bar{U}+\bar{u}_{ss}-\bar{U})}\right\}$ | $\frac{1}{3}$ | 1 | N |

| | | | |
|---|---|---|---|
| $\hat{\hat{a}}_{N1(6)} = \left[\eta_u + \omega_{1p}\{\frac{\bar{u}_{ss}}{\bar{U}}\}^{\frac{1}{3}} + \omega_{2p}\bar{y}\right]\exp\left\{\frac{(\bar{u}_{ss}-\bar{U})}{(2\beta_2(x)\bar{U}+\bar{u}_{ss}-\bar{U})}\right\}$ | $\frac{1}{3}$ | 1 | $\beta_2(x)$ |
| $\hat{\hat{a}}_{N1(7)} = \left[\eta_u + \omega_{1p}\{\frac{\bar{u}_{ss}}{\bar{U}}\}^{\frac{1}{2}} + \omega_{2p}\bar{y}\right]$ | $\frac{1}{2}$ | 0 | - |
| $\hat{\hat{a}}_{N1(8)} = \left[\eta_u + \omega_{1p}\{\frac{\bar{u}_{ss}}{\bar{U}}\}^{\frac{1}{3}} + \omega_{2p}\bar{y}\right]$ | $\frac{1}{3}$ | 0 | - |
| $\hat{\hat{a}}_{N1(9)} = \left[\eta_u + \omega_{1p}\left\{\frac{\bar{u}_{ss}}{\bar{U}}\right\} + \omega_{2p}\bar{y}\right]\exp\left\{\frac{(\bar{u}_{ss}-\bar{U})}{(2\bar{U}+\bar{u}_{ss}-\bar{U})}\right\}$ | 1 | 1 | 1 |

By minimizing MSE $\hat{\hat{a}}_{N1}$ we get the optimum value of $\omega_1, \omega_2$ i.e.

$$\omega_1{}^{opt} = \left[\frac{\Lambda_{B1}\Lambda_{D1}-\frac{\Lambda_{C1}\Lambda_{E1}}{2}}{\Lambda_{A1}\Lambda_{B1}-\Lambda_{C1}{}^2}\right] \text{ and } \omega_2{}^{opt} = \left[\frac{-\Lambda_{C1}\Lambda_{D1}-\frac{\Lambda_{A1}\Lambda_{E1}}{2}}{\Lambda_{A1}\Lambda_{B1}-\Lambda_{C1}{}^2}\right].$$

Hence, minimum mean square error $\hat{\hat{a}}_{N1}$ i.e.

$$MSE_{min}(\hat{\hat{a}}_{N1}) = [L_1 - \frac{\Lambda_{B1}\Lambda_{D1}^2+\frac{\Lambda_{A1}\Lambda_{E1}^2}{4}-\Lambda_{C1}\Lambda_{D1}\Lambda_{E1}}{\Lambda_{A1}\Lambda_{B1}-\Lambda_{C1}^2}]. \quad (26)$$

*3.2. Second suggested family of estimators in absence non-response*

Taking motivation from Mohanty and Sahoo [4] and Koyuncu [3], we suggest another class of estimators as

$$\hat{\hat{a}}_{N2} = \left[\eta_z + \omega_{1p}\left\{\frac{\bar{z}_{ss}}{\bar{Z}}\right\}^{\gamma} + \omega_{2p}\bar{y}_{ss}\right]\exp\left\{\frac{\alpha(\bar{z}_{ss}-\bar{Z})}{(2c_k\bar{Z}+\bar{z}_{ss}-\bar{Z})}\right\}. \quad (27)$$

where $\eta_z = \frac{\bar{y}_{ss}}{2}\left\{\left(\frac{\bar{Z}}{\bar{z}_{ss}}\right)^{\frac{1}{2}} + \left(\frac{\bar{z}_{ss}}{\bar{z}}\right)^{\frac{1}{2}}\right\}$.

Let we express $\hat{\hat{a}}_{N2}$ in terms of $\varepsilon_0$, $\varepsilon_z$ as follows

$$\hat{\hat{a}}_{N2} = \left[\bar{Y}\left(1+\frac{1}{8}\varepsilon_z{}^2+\varepsilon_0\right) + \omega_{1p}\left(1+\gamma\varepsilon_z+\frac{\gamma(\gamma-1)}{2}\varepsilon_z{}^2\right) + \omega_{2p}\bar{Y}(1+\varepsilon_0)\right](1-\alpha\varepsilon_z + b\varepsilon_z{}^2).$$

The bias of $\hat{\hat{a}}_{N2}$ is

$$Bias(\hat{\hat{a}}_{N2}) = \bar{Y}\left\{\left(b+\frac{1}{8}\right)v_z - \alpha v_{0z}\right\} + \omega_{1p}(1+cv_z) + \omega_{2p}\bar{Y}(1+bv_z-\alpha v_{0z}). \quad (28)$$

The MSE of $\hat{\hat{a}}_{N2}$ is

$$MSE\ (\hat{\hat{a}}_{N2}) = [L_2 + \omega_{1p}{}^2\Lambda_{A2} + \omega_{2p}{}^2\Lambda_{B2} + 2\omega_{1p}\omega_{2p}\Lambda_{C2} - 2\omega_{1P}\Lambda_{D2} - \omega_{2p}\Lambda_{E2}]. \quad (29)$$

where
$L_2 = \bar{Y}^2\{v_z + \alpha^2 v_0 - 2\alpha v_{0z}\}$,
$\Lambda_{A2} = \{1 + (\gamma - \alpha)^2 v_z + 2cv_z\}$,
$\Lambda_{B2} = \bar{Y}^2\{1 + v_0 + (\alpha^2 + 2b)v_z - 4\alpha v_z\}$,
$\Lambda_{C2} = \bar{Y}\{1 + (c - \alpha(\gamma - \alpha) + b)v_z + (\gamma - 2\alpha)v_{0z}\}$,
$\Lambda_{D2} = \bar{Y}\left\{\alpha(\gamma - \alpha)v_z - \left(b - \frac{1}{8}\right)v_z + (2\alpha - \gamma)v_{0z}\right\}$,
$\Lambda_{E2} = \bar{Y}^2\left\{-v_0 - \left(\alpha^2 + b + \frac{1}{8}\right)v_z + 3\alpha v_z\right\}$.

Note that we can generate new estimators from $\hat{\hat{a}}_{N2}$ using suitable $\gamma$, $\alpha$ and $c_k$ values. For numerical example we have generated $\hat{\hat{a}}_{N2(1)}$, $\hat{\hat{a}}_{N2(2)} \cdots \hat{\hat{a}}_{N2(9)}$ estimators using the same constants of $\gamma$, $\alpha$ and $c_k$ in Table 3 respectively.

By minimizing MSE of $\hat{\hat{a}}_{N2}$ , we get the optimum values of $\omega_1, \omega_2$ i.e.

$$\omega_1{}^{opt} = \left[\frac{\Lambda_{B2}\Lambda_{D2} - \frac{\Lambda_{C2}\Lambda_{E2}}{2}}{\Lambda_{A2}\Lambda_{B2} - \Lambda_{C2}{}^2}\right] \text{ and } \omega_2{}^{opt} = \left[\frac{-\Lambda_{C2}\Lambda_{D2} - \frac{\Lambda_{A2}\Lambda_{E2}}{2}}{\Lambda_{A2}\Lambda_{B2} - \Lambda_{C2}{}^2}\right].$$

Hence, minimum mean square error $\hat{\hat{a}}_{N2}$ i.e.

$$MSE_{min}(\hat{\hat{a}}_{N2}) = [L_2 - \frac{\Lambda_{B2}\Lambda_{D2}{}^2 + \frac{\Lambda_{A2}\Lambda_{E2}{}^2}{4} - \Lambda_{C2}\Lambda_{D2}\Lambda_{E2}}{\Lambda_{A2}\Lambda_{B2} - \Lambda_{C2}{}^2}]. \quad (30)$$

## 4. Non-response

### 4.1. *Existing and adapted estimators in presence non-response*

Strikes and holidays etc. may be the common reasons of non-response, however; issue of non-response is wider in mail surveys as compare to personal interviews. Hansen and Hurwitz [2] sub-sampling scheme is a traditional way to handle or reduce the non-response problem. Suppose the population is divided into response group $N_1$, non-response group $N_2$ and $n_1$, $n_2$ be their respected systematic random samples. As $n_1$ is a sample of non-respondents so a sub-sample having size $n_g = \frac{n_2}{l}$ is selected where $l$ is the rate of inverse sampling i.e. $(l > 1)$.

Suppose non-response is present in Y (variate of interest) but not in X (auxiliary variate), Hansen and Hurwitz [2] unbiased estimator for the estimation of mean of Y under systematic random sampling as

$$\hat{\hat{a}}'_{ss} = \frac{n_1\bar{y}_{n1} + n_2\bar{y}_{g2}}{n}, \quad (31)$$

where $\bar{y}_{n1} = \frac{\sum_{j=1}^{n_1} y_{ij}}{n_1}$ and $\bar{y}_{g2} = \frac{\sum_{j=1}^{n_g} y_{ij}}{n_g}$

The variance of $\hat{\hat{a}}'_{ss}$ is

$$V(\hat{\hat{a}}'_{ss}) = \bar{Y}^2 v_0 + \omega S^2_{Y(2)}, \tag{32}$$

where $\omega = \frac{N_2(l-1)}{nN}$.

Traditional ratio and product estimators under non-response with their MSEs are as follows

$$\hat{\hat{a}}'_r = \bar{y}'_{ss}\left[\frac{\bar{X}}{\bar{x}_{ss}}\right], \tag{33}$$

$$\hat{\hat{a}}'_p = \bar{y}'_{ss}\left[\frac{\bar{x}_{ss}}{\bar{X}}\right], \tag{34}$$

$$MSE\ (\hat{\hat{a}}'_r) = \bar{Y}^2\{v'_0 + v_1 - 2v_{01}\}, \tag{35}$$

$$MSE\ \left(\hat{\hat{a}}'_p\right) = \bar{Y}^2\{v'_0 + v_1 + 2v_{01}\} \tag{36}$$

where $v'_0 = v_0 + \omega\frac{S^2_{Y(2)}}{\bar{Y}^2}$.

The traditional regression estimator with their MSE under non-response as given by

$$\hat{\hat{a}}'_{reg} = \bar{y}'_{ss} + b(\bar{X} - \bar{x}_{ss}), \tag{37}$$

$$MSE\ \left(\hat{\hat{a}}'_{reg}\right) = \bar{Y}^2 v_o(1 - p^2) + \omega S^2_{Y(2)}. \tag{38}$$

Following Swain [11], one can propose the ratio and product estimators using square root transformation under non-response as follows

$$\hat{\hat{a}}'_{hr1} = \bar{y}'_{ss}(\frac{\bar{X}}{\bar{x}_{ss}})^{\frac{1}{2}}, \tag{39}$$

$$\hat{\hat{a}}'_{hr2} = \bar{y}'_{ss}(\frac{\bar{x}_{ss}}{\bar{X}})^{\frac{1}{2}}. \tag{40}$$

The MSEs of $\hat{\hat{a}}'_{hr1}$ and $\hat{\hat{a}}'_{hr2}$ are given by

$$MSE\ (\hat{\hat{a}}'_{hr1}) = \bar{Y}^2\left\{v'_0 + \frac{1}{4}v_1 - v_{01}\right\}, \tag{41}$$

$$MSE\ (\hat{\hat{a}}'_{hr2}) = \bar{Y}^2\left\{v'_0 + \frac{1}{4}v_1 + v_{01}\right\}. \tag{42}$$

Mohanty and Sahoo [4] estimators in case of non-response under systematic random sampling are as follows

$$\hat{\bar{a}}'_{msr1} = \bar{y}'_{ss}\frac{\bar{U}}{\bar{u}_{ss}},\ \hat{\bar{a}}'_{msp1} = \bar{y}'_{ss}\left[\frac{\bar{u}_{ss}}{\bar{U}}\right],\ \hat{\bar{a}}'_{msr2} = \bar{y}'_{ss}\frac{\bar{Z}}{\bar{z}_{ss}},\ \hat{\bar{a}}'_{msp2} = \bar{y}'_{ss}\left[\frac{\bar{z}_{ss}}{\bar{Z}}\right] \quad (43)$$

The MSEs of these estimators are given by

$$MSE(\hat{\bar{a}}'_{msr1}) = \bar{Y}^2\{v'_0 + v_U - 2v_{0u}\}, \quad (44)$$

$$MSE\left(\hat{\bar{a}}'_{msp1}\right) = \bar{Y}^2\{v'_0 + v_U + 2v_{0u}\}, \quad (45)$$

$$MSE(\hat{\bar{a}}'_{msr2}) = \bar{Y}^2\{v'_0 + v_z - 2v_{0z}\}, \quad (46)$$

$$MSE\left(\hat{\bar{a}}'_{msp2}\right) = \bar{Y}^2\{v'_0 + v_z + 2v_{0z}\}. \quad (47)$$

Singh and Solanki [8] estimators under non-response will be

$$\hat{\bar{a}}'_{ss} = \omega_{ss1}\bar{y}'_{ss}\left[\frac{\bar{X}'}{(\alpha\bar{x}'_{ss}+(1-\alpha)\bar{X}')}\right]^g + \omega_{ss2}\bar{y}'_{ss}\exp\left[\frac{\delta(\bar{X}'-\bar{x}'_{ss})}{\bar{X}'+\bar{x}'_{ss}}\right], \quad (48)$$

The minimum MSE of $\hat{\bar{a}}'_{ss}$ is

$$MSE_{min}(\hat{\bar{a}}'_{ss}) = [\bar{Y}^2 - \frac{\zeta'_C\zeta_B^2+\zeta'_A\zeta_E^2-2\zeta_B\zeta'_D\zeta_E}{\zeta'_A\zeta'_C-\zeta_D'^2}] \quad (49)$$

where

$$\zeta'_A = \bar{Y}^2\{1 + v'_0 - 4\alpha V g v_{01} - \alpha^2 V^2 g(2g+1)v_1\},$$

$$\zeta'_C = \bar{Y}^2\left\{1 + v'_0 - 2\delta V v_{01} + \{\frac{\delta(\delta+2)}{4}V^2 + \frac{\delta^2V^2}{4}\right\}v_1$$

$$\zeta'_D = \bar{Y}^2\left\{1 + v'_0 - V(2\alpha g + \delta)v_{01} + \frac{A''V^2}{8}v_1\right\}$$

Singh et al. [9] introduced the following estimator under systematic random sampling in presence of non response

$$\hat{\bar{a}}'_s = [\omega_{s1}\bar{y}'_{ss} + \omega_{s2}(\bar{X} - \bar{x}_{ss})](\frac{\bar{X}}{\bar{x}_{ss}}) \quad (50)$$

The minimum MSE for $\hat{\bar{a}}'_s$ is

$$MSE_{min}(\hat{\bar{a}}'_s) = \left[\bar{Y}^2 - \frac{\zeta_{Bs}\zeta_{Ds}^2+\zeta'_{As}\zeta_{Es}^2-2\zeta_{Cs}\zeta_{Ds}\zeta_{Es}}{\zeta'_{As}\zeta_{Bs}-\zeta_{Cs}^2}\right], \quad (51)$$

Where $\zeta'_{As} = \bar{Y}^2[1 + v'_0 + 3v_1 - 4v_{01}]$.

Following Koyuncu [3] we are adapting following class of estimators in case of non-response under systematic random sampling as

$$\hat{\bar{a}}'_{nk} = \left[\omega_1 \bar{y}'_{ss} + \omega_2 \left(\frac{\bar{x}_{ss}}{\bar{X}}\right)^{\gamma}\right] \exp\left[\frac{\alpha(\bar{X} - \bar{x}_{ss})}{\eta(\bar{X} + \bar{x}_{ss}) + 2\xi}\right]. \tag{52}$$

The minimum MSE of $\hat{\bar{a}}'_{nk}$ is

$$MSE_{min}(\hat{\bar{a}}'_{nk}) = \left[\bar{Y}^2 - \frac{\zeta_{Bnk}\zeta_{Dnk}^2 + \zeta'_{Ank}\zeta_{Enk}^2 - 2\zeta_{Cnk}\zeta_{Dnk}\zeta_{Enk}}{\zeta'_{Ank}\zeta_{Bnk} - \zeta_{Cnk}^2}\right], \tag{53}$$

where $\zeta'_{Ank} = \bar{Y}^2[1 + v'_0 + \frac{1}{2}\theta^2 v_1 - \theta v_{01}]$

*4.2. First suggested family of estimators in presence non-response*

In this section we propose $\hat{\bar{a}}'_{N1}$, $\hat{\bar{a}}'_{N2}$ and $\hat{\bar{a}}'_{N3}$ estimators under non-response. $\hat{\bar{a}}'_{N1}$ under non-response is given by

$$\hat{\bar{a}}'_{N1} = \left[\eta'_u + \omega_{1p}\left(\frac{\bar{u}_{ss}}{\bar{U}}\right)^{\gamma} + \omega_{2p}\bar{y}'_{ss}\right] \exp\left[\frac{\alpha(\bar{u}_{ss} - \bar{U})}{2c_k(\bar{U} + \bar{u}_{ss} - \bar{U})}\right]. \tag{54}$$

where $\eta'_u = \frac{\bar{y}_{ss}{}'}{2}\{\left(\frac{\bar{U}}{\bar{u}_{ss}}\right)^{\frac{1}{2}} + \left(\frac{\bar{u}_{ss}}{\bar{U}}\right)^{\frac{1}{2}}\}$

The minimum mean square error of $\hat{\bar{a}}'_{N1}$ is

$$MSE_{min}(\hat{\bar{a}}'_{N1}) = \left[L'_1 - \frac{\Lambda'_{B1}\Lambda_{D1}^2 + \frac{\Lambda_{A1}\Lambda'^2_{E1}}{4} - \Lambda_{C1}\Lambda_{D1}\Lambda'_{E1}}{\Lambda_{A1}\Lambda'_{B1} - \Lambda_{C1}^2}\right], \tag{55}$$

where

$L'_1 = \bar{Y}^2\{v_0 + a^2 v_u - 2\alpha v_{0u}\}$,

$\Lambda'_{B1} = \bar{Y}^2\{1 + v_0 + (a^2 + 2b)v_u - 4\alpha v_{0u}\}$,

$\Lambda'_{E1} = 2\bar{Y}^2\left\{-v_0 - \left(a^2 + b + \frac{1}{8}\right)v_u + 3\alpha v_{0u}\right\}$,

$\hat{\bar{a}}'_{N1(1)}, \hat{\bar{a}}'_{N1(2)}, \ldots, \hat{\bar{a}}'_{N1(9)}$ can be generated from $\hat{\bar{a}}'_{N1}$ using the same constants in Table3.

*4.3. Second suggested family of estimators in presence non-response*

$\hat{\bar{a}}'_{N2}$ under non-response is given by

$$\hat{\bar{a}}'_{N2} = \left[\eta'_z + \omega_{1p}\left(\frac{\bar{z}_{ss}}{\bar{Z}}\right)^{\gamma} + \omega_{2p}\bar{y}'_{ss}\right] \exp\left[\frac{\alpha(\bar{z}_{ss} - \bar{Z})}{2c_k(\bar{Z} + \bar{z}_{ss} - \bar{Z})}\right].$$

where $\eta'_z = \frac{\bar{y}_{ss}{}'}{2}\{\left(\frac{\bar{Z}}{\bar{z}_{ss}}\right)^{\frac{1}{2}} + \left(\frac{\bar{z}_{ss}}{\bar{Z}}\right)^{\frac{1}{2}}\}$

$\hat{\bar{a}}'_{N2(1)}, \hat{\bar{a}}'_{N2(2)}, \ldots, \hat{\bar{a}}'_{N2(9)}$ can be generated from $\hat{\bar{a}}'_{N2}$ using the same constants in Table3.

The minimum mean square error of $\hat{\bar{a}}'_{N2}$ is

$$MSE_{min}(\hat{\hat{a}}'_{N1}) = \left[L'_2 - \frac{\Lambda'_{B2}\Lambda^2_{D2} + \frac{\Lambda_{A2}\Lambda'^2_{E2}}{4} - \Lambda_{C2}\Lambda_{D2}\Lambda'_{E2}}{\Lambda_{A2}\Lambda'_{B2} - \Lambda^2_{C2}}\right],$$

where

$L'_2 = \bar{Y}^2\{v_0 + a^2 v_z - 2\alpha v_{0z}\}$,

$\Lambda'_{B2} = \bar{Y}^2\{1 + v_0 + (a^2 + 2b)v_z - 4\alpha v_{0z}\}$,

$\Lambda'_{E2} = 2\bar{Y}^2\left\{-v_0 - \left(a^2 + b + \frac{1}{8}\right)v_z + 3\alpha v_{0z}\right\}$,

## 5. Efficiency Comparison

Here we perform efficiency comparison for the proposed estimators for (*j* = 1, 2) by looking at the MSE of the reviewed estimators as given below

### *5.1 In absence of non-response*

Observation (1):

$MSE_{min}(\hat{\hat{a}}_{Nj}) < MSE(\hat{\hat{a}}_r)$

If

$$\frac{\Lambda_{Bj}\Lambda^2_{Dj} + \frac{\Lambda_{Aj}\Lambda^2_{Ej}}{4} - \Lambda_{Cj}\Lambda_{Dj}\Lambda_{Ej}}{\Lambda_{Aj}\Lambda_{Bj} - \Lambda^2_{Cj}} - \bar{Y}^2\left[\frac{L_j}{\bar{Y}^2} - (v_O + v_1 - 2v_O v_1)\right] > 0$$

Observation (2):

$MSE_{min}(\hat{\hat{a}}_{Nj}) < MSE(\hat{\hat{a}}_{reg})$

If

$$\frac{\Lambda_{Bj}\Lambda^2_{Dj} + \frac{\Lambda_{Aj}\Lambda^2_{Ej}}{4} - \Lambda_{Cj}\Lambda_{Dj}\Lambda_{Ej}}{\Lambda_{Aj}\Lambda_{Bj} - \Lambda^2_{Cj}} - \bar{Y}^2\left[\frac{L_j}{\bar{Y}^2} - v_O(1 - \rho^2)\right] > 0$$

Observation (3):

$MSE_{min}(\hat{\hat{a}}_{Nj}) < MSE(\hat{\hat{a}}_{hr1})$

if

$$\frac{\Lambda_{Bj}\Lambda^2_{Dj} + \frac{\Lambda_{Aj}\Lambda^2_{Ej}}{4} - \Lambda_{Cj}\Lambda_{Dj}\Lambda_{Ej}}{\Lambda_{Aj}\Lambda_{Bj} - \Lambda^2_{Cj}} - \bar{Y}^2\left[\frac{L_j}{\bar{Y}^2} - (v_O + \frac{1}{4}v_1 - v_{O1})\right] > 0$$

Observation (4):

$MSE_{min}(\hat{\hat{a}}_{Nj}) < MSE(\hat{\hat{a}}_{ss})$

if

$$\frac{\Lambda_{Bj}\Lambda^2_{Dj} + \frac{\Lambda_{Aj}\Lambda^2_{Ej}}{4} - \Lambda_{Cj}\Lambda_{Dj}\Lambda_{Ej}}{\Lambda_{Aj}\Lambda_{Bj} - \Lambda^2_{Cj}} - \frac{\zeta_C\zeta^2_B + \zeta_A\zeta^2_E - 2\zeta_B\zeta_D\zeta_E}{\zeta_A\zeta_C - \zeta^2_D} - (L_j - \bar{Y}^2) > 0$$

Observation (5):

$MSE_{min}(\hat{\hat{a}}_{Nj}) < MSE(\hat{\hat{a}}_s)$

if

$$\frac{\Lambda_{Bj}\Lambda^2_{Dj} + \frac{\Lambda_{Aj}\Lambda^2_{Ej}}{4} - \Lambda_{Cj}\Lambda_{Dj}\Lambda_{Ej}}{\Lambda_{Aj}\Lambda_{Bj} - \Lambda^2_{Cj}} - \frac{\zeta_{Bs}\zeta^2_{Ds} + \zeta_{As}\zeta^2_{Es} - 2\zeta_{Cs}\zeta_{Ds}\zeta_{Es}}{\zeta_{As}\zeta_{Bs} - \zeta^2_{Cs}} - (L_j - \bar{Y}^2) > 0$$

From above observations we can argue that the new estimators perform better than all of the reviewed estimators. Also we can develop such type of efficiency conditions for the non-response case.

## 6. Numerical illustration

For assessing the performance of proposed and reviewed estimators, we use the data set available in Murthy [5] concerning volume of the timber considered as ($Y$) and length of the timber as ($X$). Details of the population descriptives are, $N = 176,\ n = 16,$

$\bar{Y} = 282.61,\ S_y = 155.73,\ \bar{X} = 6.99,\ S_x = 2.95,\ \rho = 0.87, \rho_y = -0.0019, \rho_x = -0.0014$

**Table 4:** PRE of estimators in absence of non-response

| Est. | PRE | Est. | PRE | Est. | PRE | Est. | PRE |
|---|---|---|---|---|---|---|---|
| $\bar{y}_{ss}$ | 100 | $\hat{\bar{a}}_{ss6}$ | 414.55 | $\hat{\bar{a}}_{N1(1)}$ | 11211.40 | $\hat{\bar{a}}_{N2(1)}$ | 142760.30 |
| $\hat{\bar{a}}_r$ | 397.90 | $\hat{\bar{a}}_{ss7}$ | 414.66 | $\hat{\bar{a}}_{N1(2)}$ | 1408.10 | $\hat{\bar{a}}_{N2(2)}$ | 24059.29 |
| $\hat{\bar{a}}_{reg}$ | 414.33 | $\hat{\bar{a}}_s$ | 416.18 | $\hat{\bar{a}}_{N1(3)}$ | 1938.01 | $\hat{\bar{a}}_{N2(3)}$ | 22338.42 |
| $\hat{\bar{a}}_{hr1}$ | 209.65 | $\hat{\bar{a}}_{msr1}$ | 357.29 | $\hat{\bar{a}}_{N1(4)}$ | 4183.70 | $\hat{\bar{a}}_{N2(4)}$ | 48238.17 |
| $\hat{\bar{a}}_{ss1}$ | 414.90 | $\hat{\bar{a}}_{msr2}$ | 157.99 | $\hat{\bar{a}}_{N1(5)}$ | 5490.35 | $\hat{\bar{a}}_{N2(5)}$ | 54513.73 |
| $\hat{\bar{a}}_{ss2}$ | 414.58 | $\hat{\bar{a}}_{nk1}$ | 318.68 | $\hat{\bar{a}}_{N1(6)}$ | 20354.27 | $\hat{\bar{a}}_{N2(6)}$ | 202103.60 |
| $\hat{\bar{a}}_{ss3}$ | 414.63 | $\hat{\bar{a}}_{nk2}$ | 654.36 | $\hat{\bar{a}}_{N1(7)}$ | 1916.07 | $\hat{\bar{a}}_{N2(7)}$ | 22085.33 |
| $\hat{\bar{a}}_{ss4}$ | 414.40 | $\hat{\bar{a}}_{nk3}$ | 603.89 | $\hat{\bar{a}}_{N1(8)}$ | 5397.21 | $\hat{\bar{a}}_{N2(8)}$ | 53588.54 |
| $\hat{\bar{a}}_{ss5}$ | 414.77 | $\hat{\bar{a}}_{nk4}$ | 406.67 | $\hat{\bar{a}}_{N1(9)}$ | 1021.35 | $\hat{\bar{a}}_{N2(9)}$ | 17453.57 |

**Table 5**: PRE of estimators under non-response for ($l$=2)

| Est. | PRE | Est. | PRE | Est. | PRE | Est. | PRE |
|---|---|---|---|---|---|---|---|
| $\bar{y}'_{ss}$ | 100 | $\hat{\bar{a}}'_{ss6}$ | 183.68 | $\hat{\bar{a}}'_{N1(1)}$ | 11236.01 | $\hat{\bar{a}}'_{N2(1)}$ | 140980.50 |
| $\hat{\bar{a}}'_r$ | 149.79 | $\hat{\bar{a}}'_{ss7}$ | 183.91 | $\hat{\bar{a}}'_{N1(2)}$ | 1412.76 | $\hat{\bar{a}}'_{N2(2)}$ | 23786.80 |
| $\hat{\bar{a}}'_{reg}$ | 401.91 | $\hat{\bar{a}}'_s$ | 181.75 | $\hat{\bar{a}}'_{N1(3)}$ | 1920.70 | $\hat{\bar{a}}'_{N2(3)}$ | 22031.55 |
| $\hat{\bar{a}}'_{hr1}$ | 171.36 | $\hat{\bar{a}}'_{msr1}$ | 350.57 | $\hat{\bar{a}}'_{N1(4)}$ | 4146.28 | $\hat{\bar{a}}'_{N2(4)}$ | 47575.43 |
| $\hat{\bar{a}}'_{ss1}$ | 184.32 | $\hat{\bar{a}}'_{msr2}$ | 157.31 | $\hat{\bar{a}}'_{N1(5)}$ | 5425.86 | $\hat{\bar{a}}'_{N2(5)}$ | 53728.51 |
| $\hat{\bar{a}}'_{ss2}$ | 183.36 | $\hat{\bar{a}}'_{nk1}$ | 419.88 | $\hat{\bar{a}}'_{N1(6)}$ | 20114.98 | $\hat{\bar{a}}'_{N2(6)}$ | 199192.01 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\hat{\bar{a}}'_{ss3}$ | 182.36 | $\hat{\bar{a}}'_{nk2}$ | 456.49 | $\hat{\bar{a}}'_{N1(7)}$ | 1898.96 | $\hat{\bar{a}}'_{N2(7)}$ | 21781.94 |
| $\hat{\bar{a}}'_{ss4}$ | 182.37 | $\hat{\bar{a}}'_{nk3}$ | 447.84 | $\hat{\bar{a}}'_{N1(8)}$ | 5333.80 | $\hat{\bar{a}}'_{N2(8)}$ | 52816.64 |
| $\hat{\bar{a}}'_{ss5}$ | 184.11 | $\hat{\bar{a}}'_{nk4}$ | 426.78 | $\hat{\bar{a}}'_{N1(9)}$ | 1024.74 | $\hat{\bar{a}}'_{N2(9)}$ | 17256.09 |

**Table 6**: PRE of estimators under non-response for ($l$=2.5)

| Est. | PRE | Est. | PRE | Est. | PRE | Est. | PRE |
|---|---|---|---|---|---|---|---|
| $\bar{y}'_{ss}$ | 100 | $\hat{\bar{a}}'_{ss6}$ | 183.15 | $\hat{\bar{a}}'_{N1(1)}$ | 11249.37 | $\hat{\bar{a}}'_{N2(1)}$ | 140140.90 |
| $\hat{a}'_{r}$ | 149.52 | $\hat{\bar{a}}'_{ss7}$ | 183.38 | $\hat{\bar{a}}'_{N1(2)}$ | 1415.19 | $\hat{\bar{a}}'_{N2(2)}$ | 23658.17 |
| $\hat{\bar{a}}'_{reg}$ | 401.96 | $\hat{\bar{a}}'_{s}$ | 181.22 | $\hat{\bar{a}}'_{N1(3)}$ | 1912.58 | $\hat{\bar{a}}'_{N2(3)}$ | 21886.09 |
| $\hat{\bar{a}}'_{hr1}$ | 170.91 | $\hat{\bar{a}}'_{msr1}$ | 347.34 | $\hat{\bar{a}}'_{N1(4)}$ | 4128.72 | $\hat{\bar{a}}'_{N2(4)}$ | 47261.28 |
| $\hat{\bar{a}}'_{ss1}$ | 183.78 | $\hat{\bar{a}}'_{msr2}$ | 156.98 | $\hat{\bar{a}}'_{N1(5)}$ | 5395.38 | $\hat{\bar{a}}'_{N2(5)}$ | 53355.95 |
| $\hat{\bar{a}}'_{ss2}$ | 183.22 | $\hat{\bar{a}}'_{nk1}$ | 418.98 | $\hat{\bar{a}}'_{N1(6)}$ | 20001.92 | $\hat{\bar{a}}'_{N2(6)}$ | 197810.60 |
| $\hat{\bar{a}}'_{ss3}$ | 182.32 | $\hat{\bar{a}}'_{nk2}$ | 457.81 | $\hat{\bar{a}}'_{N1(7)}$ | 1890.94 | $\hat{\bar{a}}'_{N2(7)}$ | 21638.12 |
| $\hat{\bar{a}}'_{ss4}$ | 182.74 | $\hat{\bar{a}}'_{nk3}$ | 449.02 | $\hat{\bar{a}}'_{N1(8)}$ | 5303.85 | $\hat{\bar{a}}'_{N2(8)}$ | 52450.41 |
| $\hat{\bar{a}}'_{ss5}$ | 183.58 | $\hat{\bar{a}}'_{nk4}$ | 426.92 | $\hat{\bar{a}}'_{N1(9)}$ | 1026.51 | $\hat{\bar{a}}'_{N2(9)}$ | 17162.87 |

**Table 7:** PRE of estimators under non-response for ($l$=3)

| Est. | PRE | Est. | PRE | Est. | PRE | Est. | PRE |
|---|---|---|---|---|---|---|---|
| $\bar{y}'_{ss}$ | 100 | $\hat{\bar{a}}'_{ss6}$ | 182.62 | $\hat{\bar{a}}'_{N1(1)}$ | 11263.39 | $\hat{\bar{a}}'_{N2(1)}$ | 139332.80 |
| $\hat{a}'_{r}$ | 149.24 | $\hat{\bar{a}}'_{ss7}$ | 182.65 | $\hat{\bar{a}}'_{N1(2)}$ | 1417.68 | $\hat{\bar{a}}'_{N2(2)}$ | 23534.32 |
| $\hat{\bar{a}}'_{reg}$ | 402.01 | $\hat{\bar{a}}'_{s}$ | 180.70 | $\hat{\bar{a}}'_{N1(3)}$ | 1904.80 | $\hat{\bar{a}}'_{N2(3)}$ | 21745.62 |
| $\hat{\bar{a}}'_{hr1}$ | 170.46 | $\hat{\bar{a}}'_{msr1}$ | 344.19 | $\hat{\bar{a}}'_{N1(4)}$ | 4111.90 | $\hat{\bar{a}}'_{N2(4)}$ | 46957.92 |
| $\hat{\bar{a}}'_{ss1}$ | 183.26 | $\hat{\bar{a}}'_{msr2}$ | 156.65 | $\hat{\bar{a}}'_{N1(5)}$ | 5366.03 | $\hat{\bar{a}}'_{N2(5)}$ | 52995.99 |
| $\hat{\bar{a}}'_{ss2}$ | 182.61 | $\hat{\bar{a}}'_{nk1}$ | 418.11 | $\hat{\bar{a}}'_{N1(6)}$ | 19892.99 | $\hat{\bar{a}}'_{N2(6)}$ | 196475.90 |
| $\hat{\bar{a}}'_{ss3}$ | 182.80 | $\hat{\bar{a}}'_{nk2}$ | 459.14 | $\hat{\bar{a}}'_{N1(7)}$ | 1883.24 | $\hat{\bar{a}}'_{N2(7)}$ | 21499.25 |
| $\hat{\bar{a}}'_{ss4}$ | 182.22 | $\hat{\bar{a}}'_{nk3}$ | 450.19 | $\hat{\bar{a}}'_{N1(8)}$ | 5274.99 | $\hat{\bar{a}}'_{N2(8)}$ | 52096.56 |
| $\hat{\bar{a}}'_{ss5}$ | 183.05 | $\hat{\bar{a}}'_{nk4}$ | 427.06 | $\hat{\bar{a}}'_{N1(9)}$ | 1028.32 | $\hat{\bar{a}}'_{N2(9)}$ | 17073.12 |

10 % weight is assumed for non-response. So numerical results are provided only for 10 % percent weight of missing values and consider last 18 values as non-respondents. Further, we check the proposed and existing estimators on different choices of $l$ i.e ($l$ = 2, 2.5, 3). Some important results from the population of non-respondents are as follows: $S^2_{Y(2)} = 41.46$; $\bar{Y}_2 = 182.61$ and $N_2$ = 18. The PREs of all the ratio type proposed and existing estimators available in Table 4-7.

For absence of non-response, $PRE(.) = \dfrac{MSE(\bar{y}_{ss})}{MSE(.)} \times 100$

For presence of non-response, $PRE(.) = \dfrac{MSE(\bar{y}'_{ss})}{MSE(.)} \times 100$

For non-response problem we take 10% values as non-response with different choices of $l$ i.e ($l$ = 2, 2.5, 3). By assuming all these choices we see that the PREs of the proposed and reviewed estimators are not really affected. No doubt, numerical results are not same on all these percentages of weights but the behavior of the numerical results is same in all situations.

## 7. Conclusion

In this study, we have proposed two new classes of estimators for estimating population mean in systematic random sampling scheme alongside the non-response problem. Some members of two new classes are developed. We determine the properties of the proposed families of estimators. Theoretical efficiency conditions are also performed under which proposed families of estimators perform better as compared to reviewed ones. On the premise of results given in Tables 4-7, we conclude that the proposed classes of estimators are preferable over its competitive estimators under systematic random sampling scheme. Hence, it is advise-able to use the proposed classes of estimators.

## References


[1] W.G. Cochran, 1977, *Sampling Technique*, 3rd ed., John Wiley and Sons, New York.

[2] M.H. Hansen, W.N. Hurwitz, 1946, The problem of non-response in sample surveys, *Journal of the American Statistical Association*, 41, 517-529.

[3] N. Koyuncu, 2012, Efficient estimators of population mean using auxiliary attributes, *Applied Mathematics and Computation*, 218, 10900-10905.

[4] S. Mohanty, J. Sahoo, 1995, A note on improving the ratio method of estimation through linear transformation using certain known population parameters, *Sankhya: The Indian Journal of Statistics, Series B*, 57, 93-102.

[5] M.N. Murthy, 1967, *Sampling: Theory and Methods, Statistical Publishing Society*, Calcutta.

[6] S. Riaz, G. Diana, J. Shabbir, 2017, Modified classes of estimators in circular systematic sampling, Hacettepe J. Math. Stat, Doi: 10.15672/HJMS.20158412762.

[7] N. D. Shukla, 1971, Systematic sampling and product method of estimation, *In Proceeding of all India Seminar on Demography and Statistics*, BHU, Varanasi, India.

[8] H.P. Singh, R.S. Solanki, 2012, An efficient class of estimators for the population mean using auxiliary information in systematic sampling, *Journal of Statistical Theory and Practice*, 6, 274-285.

[9] R. Singh, S. Malik, M.K. Chaudhary, H.K. Verma, A.A. Adewara, 2012, A general family of ratio-type estimators in systematic sampling, *Journal of Reliability and Statistical Studies*, 5, 73-82.

[10] A.K.P.C. Swain, 1964, The use of systematic sampling in ratio estimate, *Jour. Ind. Stat. Assoc*, 2, 160-164.

[11] A.K.P.C. Swain, 2014, On an improved ratio type estimator of finite population mean in sample surveys ratio, *Revista Investigacion Operacional*, 35,1, 49-57.

[12] H.K. Verma, 2014, Some improved estimators in systematic sampling under non-response, *National Academy Science Letters*, 37, 91-95.