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Measurement of Environmental Sustainability Using Slack-Based Measure and Data Envelopment Analysis*

Research Article / Araştırma Makalesi

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ABSTRACT

Research on environmental protection and sustainability focus increasingly on undesirable outputs (such as waste and pollutants) of production and consumption processes. Thus, technological improvements leading to fewer undesirable outputs in production processes have become main concerns. In research on sustainability, the relationship between resource efficiency and performance optimization can be examined using different quantitative methods. In this regard, the Slack Based Measure (SBM) model and the traditional input-oriented (BCC, Banker-Charnes-Cooper) Data Envelopment Analysis model provide important capabilities for assessing the efficient use of resources from sustainability perspective. This study examines whether there is a difference in efficiency between SBM (SBM-UO) Data Envelopment Analysis model and the standard input-oriented BCC Data Envelopment Analysis (BCC DEA-UO) model, within the context of sustainability efficiency, using an industrial case example. Since slack values are considered, the results of efficiency measurements made with SBM (SBM-UO) show lower or equal values compared to BCC Data Envelopment Analysis with undesirable outputs (BCC DEA-UO).

Keywords: *Data Envelopment Analysis, SBM, Sustainability, Slack Based Measure*.

1. INTRODUCTION

Sustainable development is defined as fulfilling present needs without endangering the ability of future generations to meet their own needs. The concept of sustainability is described by the World Commission on Environment and Development as "*development that meet the needs of the present without compromising the ability of future generations to meet their own needs*" (WCED, 1987). The systems approach to sustainability aims to study the interactions and feedback across environmental, economic and social systems and coherency of the unit. Developing tools to establish and analyze the balance between systems is an operational requirement.

During industrial activities, resources are used, and waste is created. There is a need to create a template that would minimize resource usage and waste production (Barbier, 1987). In industrial systems, the production of a certain amount of output is done using a specific amount of capital such as equipment and factories. At the same time, various inputs such as labor and raw materials are also included in the systems (Meadows et al., 1972). Furthermore, industries and production processes have impacts on natural resources throughout the entire process, including raw material extraction, energy consumption, production of final products, waste generation, consumption of these products by consumers, and the disposal of these products (WCED, 1987). In this sense, industrial production processes have impact on economic, social and environmental scale.

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Environmental changes can affect individuals' wellbeing through various channels. The impacts caused by fluctuating product prices in the markets, changes in the quality and quantity of non-market goods (such as air quality), changes in production factors reflected in wages, and the individual risks that people personally encounter are key elements in this sense (Freeman, Herriges and Kling, 2014). Even in the best scenario, the use of scarce resources is expected to reach 15% by 2030 and 75% by 2050 and a potential raw material shortage of 8 billion tons and 29 billion tons is estimated to occur in 2030 and 2050 respectively. This growing demand for natural resources can increase the cost of inputs, decrease profits, lead companies to increase product prices and cause lower levels of economic growth (Lacy and Rutqvist, 2015). Moreover, the reduction in natural resources may cause an increase in diseases, negative changes in daily activities, and a decrease in life expectancy (Freeman, Herriges and Kling, 2014).

In sustainability research, the relationship between resource efficiency and performance optimization can be examined in various ways depending on the selected quantitative method. In this regard, Slack Based Measure (SBM) model and the input-oriented traditional BCC (Banker-Charnes-Cooper) Data Envelopment Analysis (DEA) model provide important tools for assessing the efficient use of resources from a sustainability perspective (Zhou et al., 2018). While both classical SBM and DEA models provide valuable contributions to decision-making processes, they also present different approaches in terms of performance measurement and improvement recommendations (Tone, 2001). This study aims specifically at answering the following questions: "*How does the relationship between resource efficiency and performance optimization in sustainability practices differ while using the SBM (SBM-UO) model with undesirable outputs compared to the standard input-oriented BCC DEA (BCC DEA-UO) model with undesirable outputs, and how the contribution of these models to decision-making processes can be evaluated?*"

The paper is organized as follows. Section 2 provides the theoretical background on Data Envelopment Analysis (DEA), Slack-Based Measure (SBM) model and reviews related literature on efficiency measurement within the context of sustainability, with a particular focus on undesirable outputs. Section 3 describes the methodology on SBM-UO and BCC DEA-UO models in detail. The dataset including inputs, outputs, and undesirable outputs is highlighted. Section 4 presents empirical results, comparing the efficiency scores obtained from both models. Section 5 discusses the implications of these results for sustainable resource utilization. Finally, Section 6 concludes the study by summarizing key findings and suggesting directions for future research.

2. THEORETICAL BACKGROUND/ CONCEPTUAL FRAMEWORK

2.1. Eco-Efficiency and Input-Output Indicators in the Literature

The concept of *eco-efficiency* is essential when we evaluate how effectively economic activities use the natural resources and services. Eco-efficiency is measured by comparing the produced values (such as income, quality goods and services, employment, and GDP) with their environmental impacts. This corresponds to the comparison of the economic benefits provided by a product or service with the environmental impacts it generates (Zhang et al., 2008). This relative measure combining and balancing both environmental and economic benefits, primarily aims at minimizing resource consumption (such as energy, water, and raw materials) while reducing negative environmental outputs (such as waste and emissions), and maintaining or enhancing the value of the produced goods (Maxime et al., 2006).

Environment is affected in many ways by industrial and human activities. Over the past 20-30 years, regulations have been implemented to limit negative effects. Additionally, some institutions and companies have adopted more proactive and conscious approaches toward environmental issues. Thus, measuring environmental performance has become very significant in this perspective and, tools and materials for performance evaluations have become central. Quantitative evaluation of environmental performance enables the adoption of more sustainable and eco-friendly practices. In this context, performance evaluation tools facilitate more consciously decision-makings providing quantitative assessments. Analytical performance indicators allow making comparisons between different factories of a company, various companies, or units operating under similar environmental conditions, thereby environmentally efficient decision making can be secured. Generally, the concept of environmental efficiency assessment highlights three categories of parameters: *inputs, desirable outputs, and undesirable outputs*. Inputs represent the resources, raw materials specific to the analysed activity, while outputs include both desired outcomes and undesirable effects. DEA is one of the quantitative decision-making tools providing an environmental performance assessment. This method assesses units' efficiency performance on a scale ranging from 0 to 1, distinguishing between well performing and poorly performing ones (Tyteca, 1996). Table 1 presents a detailed overview of the inputs and outputs used in DEA studies in the literature. The fields of these studies vary at both micro and macro levels, with the same variation occurring for the inputs and outputs.

2.2. Data Envelopment Analysis and Other Methods

2.2.1 Data Envelopment Analysis (DEA)

DEA is a data driven approach used to assess the performance of similar *Decision-Making Units* (DMUs) by considering multiple inputs and outputs. It is widely used across different fields and industries in various countries. Examples include performance assessments of cities, regions, countries, companies and many other entities like, schools, hospitals, shops and others. DEA is a valuable method to provide organizations and institutions meaningful insights (Cooper, Seiford & Zhu, 2011). There are over 20000 publications in which the efficiency of various DMUs are measured using DEA, a non-parametric method that can generate efficiency scores based on inputs and outputs (Emrouznejad, Yang, Khoveyni, & Michali, 2022).

Table 1: Applications, inputs, and outputs in the literature

Field of Study	Inputs	Outputs	Researchers
26 OECD Countries 1995-1997	- Employee	- GDP - SOx - CO_2 - CO - NOx	Zhou, Poh & Ang (2007)
Beijing 2005-2009	- Employee - Capital - Energy	- GDP - Wastewater - Solid Waste - Gas emission	Li, Yang & Liu (2013)
USA (Energy plants)	- Capacity - Fuel Cons.	- Energy Production - SO_2 - CO - CO_2	Sueyoshi & Goto (2012)
20 Countries 1990-2011	- Employee - Investment	- Financial Return - CO_2 - N_2O - SO_2 - CH_4	Halkos, Tzeremes & Kourtzidis (2016)
30 Province in China 2005-2010	- Employee - Capital - Energy	- Power Capacity - CO_2 – Nox - SO_2	Zhou et al. (2013)
27 Global Airline Companies	- Employee - Capacity - Fuel Cons.	- Income (income and related metrics) - CO_2	Chang, Park, Jeong & Lee (2014)
29 Province in China 1998-2009	- Employee - Capital - Energy	- GDP - Wastewater - Solid Waste - Gas emission	Song, Song, An & Yu (2013)

CCR (Charnes, Cooper and Rhodes) DEA model, introduced by Charnes, Cooper and Rhodes (1978), evaluates the relative efficiency among different homogeneous DMUs. The efficiency is derived by referencing a frontier function obtained from the most efficient DMU. This is a constant returns to scale (CRS) model (Charnes, Cooper & Rhodes, 1978). BCC (Banker, Charnes, and Cooper) DEA model was developed and introduced by Banker, Charnes, and Cooper (1984). This model includes the variable returns to scale (VRS) form. Although there have been developments in both CCR and BCC DEA models over the years, these two models continue to dominate research and applications (Zarrin & Brunner, 2022).

2.2.1.1 CCR Model

In CCR model, there are n DMUs, denoted as DMU_j $(j = 1,2,\dots,n)$ and each DMU have common inputs and outputs. The inputs consist of m elements, represented as X_{ij} $(i = 1,2,\dots,m)$ while the outputs include s elements and denoted as Y_{rj} $(r = 1,2,\dots,s)$. The performance and efficiency of the observed DMUs are evaluated based on these inputs and outputs. CCR model is represented by Equations 1.a to 1.d below. In the model and notation DMU_0 is used to denote the unit being measured, and the model is formulated accordingly (Charnes, Cooper & Rhodes, 1978).

$$Max\, h_0 = \sum_{r=1}^{s} u_r y_{r0} \tag{1.a}$$

Constraints:

$$\sum_{i=1}^{m} v_i x_{i0} = 1 \tag{1.b}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, (j = 1,2,\dots,n) \tag{1.c}$$

$$u_r \geq \varepsilon\ (r = 1,2,\dots,s) \text{ and } v_i \geq \varepsilon\ (i = 1,2,\dots,m) \tag{1.d}$$

where x_{ij} are inputs (with input indexes $i = 1,2,\dots,m$), y_{rj} are outputs (with output indexes $r = 1,2,\dots,s$), u_r is the weight of output r, v_i is the weight for input i, h_0 is the relative efficiency score for DMU_0 and ε is a very small non-negative number.

2.2.1.2 BCC Model

After the introduction of CCR Model, one of the most critical developments in DEA is BCC Model which was introduced by Banker, Charnes and Cooper (1984). BCC is a model of variable returns to scale (VRS). In this model, the main difference is the inclusion of the convexity constraint as shown in Equation 2, which enables the VRS. In VRS model, one unit increase in inputs does not always lead to a proportional increase in outputs. Like CCR model, this model can be either input oriented or output oriented. The output-oriented model aims to maximize output while keeping inputs under control, whereas the input-oriented model aims to minimize inputs while maintaining a fixed level of output (Martic, Novakovic & Baggi, 2009).

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{2}$$

The input-oriented BCC model including the variable u_0, is shown Equations 3.a to 3.d (Banker, Charnes and Cooper, 1984).

$$Max\, h_0 = \sum_{r=1}^{s} u_r y_{r0} - u_0 \tag{3.a}$$

Constraints:

$$\sum_{i=1}^{m} v_i x_{i0} = 1 \tag{3.b}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, (j = 1,2,\dots,n) \tag{3.c}$$

$$u_r \geq \varepsilon\ (r = 1,2,\dots,s) \text{ and } v_i \geq \varepsilon\ (i = 1,2,\dots,m) \tag{3.d}$$

Additionally, it is possible to formulate the version of BCC model as shown in Equations 4.a to 4.e by including the convexity constraint given in Equation 2, which differs from CCR model (Banker, Charnes and Cooper, 1984). s_r^+ and s_i^- represent the output slack value (shortfall) and input slack value (excess), respectively.

$$Min\, \alpha - \varepsilon\left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\right) \tag{4.a}$$

Constraints:

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0},\ (r = 1,2,\dots,s) \text{ and } (j = 1,2,\dots,n) \quad (4.b)$$
$$\sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = \alpha . x_{i0},\ (\text{i}= 1,2,\dots,m) \text{ and } (j = 1,2,\dots,n) \quad (4.c)$$
$$\sum_{j=1}^{n} \lambda_j = 1 \quad (4.d)$$
$$s_r^+, s_i^-, \lambda_j \geq 0 \quad (4.e)$$

2.2.1.3 SBM Model

SBM model is introduced by Kaoru Tone in 2001. Its main difference from previous models is that it directly measures the efficiency using a fractional function based on input excess and output shortfalls. Let X represents matrix of inputs; Y represents matrix of outputs. λ is any non-negative vector. The expression of a specific Decision-Making Unit is denoted as $DMU_0 = (x_0, y_0)$. In this context, the slack values are defined as follows: $s^+ \geq 0$ with $s^+ \in R^s$ represents shortfall/output slacks, while $s^- \geq 0$ with $s^- \in R^m$ represents the input surplus/input slacks. SBM efficiency is derived using the optimization problem given by Equation 5.a to 5.d (Tone, 2001).

$$Min\ \rho = \frac{1 - \left(\frac{1}{m}\right)\sum_{i=1}^{m} \frac{s_i^-}{x_{i0}}}{1 + \left(\frac{1}{s}\right)\sum_{r=1}^{s} \frac{s_r^+}{y_{r0}}} \quad (5.a)$$

Constraints:

$$x_0 = X\lambda + s^- \quad (5.b)$$
$$y_0 = Y\lambda - s^+ \quad (5.c)$$
$$s^+, s^-, \lambda \geq 0 \quad (5.d)$$

In addition, transformations such as $\boldsymbol{S}^+ = t.s^+; \boldsymbol{S}^- = t.s^-; \boldsymbol{\Lambda} = t\lambda$ have been done to obtain the following Equations 6.a to 6.e (Tone, 2001).

$$Min\ \tau = t - \frac{1}{m}\sum_{i=1}^{m} \frac{S_i^-}{x_{i0}} \quad (6.a)$$

Constraints:

$$t + \frac{1}{s}\sum_{r=1}^{s} \frac{S_r^+}{y_{r0}} = 1 \quad (6.b)$$
$$tx_0 = X\Lambda + S^- \quad (6.c)$$
$$tx_0 = X\Lambda - S^+ \quad (6.d)$$
$$\boldsymbol{S}^+, \boldsymbol{S}^-, \boldsymbol{\Lambda} \geq 0 \text{ and } t > 0 \quad (6.e)$$

In order to define DMU (x_0, y_0) as efficient in terms of SBM, ρ^* must take the value of 1. This condition is equivalent to the case where $s^{+*}$and s^{-*} are 0. The standard CCR model is radial, and the presence of output or input slack values does not directly indicate the inefficiency for any DMU. However, since SBM model directly takes slacks into account, the presence of any slack indicates the inefficiency according to SBM model (Tone, 2001).

2.2.2. Methods Used for Undesirable Outputs

Addressing undesirable outputs has been a challenging topic for researchers. There are multiple methods to handle undesirable outputs, researchers should select the method they consider most suitable for their needs. These methods can be classified into four different categories:

1. Ignoring undesirable outputs in the production function,
2. Treating undesirable outputs as normal inputs,
3. Treating undesirable outputs as normal outputs,
4. Evaluating undesirable outputs by applying the necessary transformations.

One of the methods used in this field is SBM with undesirable outputs (SBM-UO). In addition to that there is the use of the standard BCC DEA (BCC DEA-UO) where undesirable outputs are treated as standard inputs (Halkos & Petrou, 2019). In sustainability research handling undesirable outputs, SBM model and the input-oriented traditional BCC DEA model are two critical tools while evaluating the efficient use of resources from a sustainability perspective (Zhou et al., 2018).

2.2.2.1 SBM Model with Undesirable Outputs

SBM model with undesirable outputs (SBM-UO) is introduced by Kaoru Tone in 2004. It is a model that measures efficiency using a fractional solution based on input excesses, undesirable output excesses, and desirable output shortfalls. Let X represent the input matrix, Y^g the desirable output matrix, and Y^b the undesirable output matrix. λ is any non-negative vector. The expression of a specific Decision-Making Unit is denoted as $DMU_0 = \left(x_0, y_0^g, y_0^b\right)$. In this context, the slack variables are defined as follows: $s_r^g \geq 0$ with $s_r^g \in R^{s_1}$ are the slack values of desirable outputs, $s_z^b \geq 0$ with $s_z^b \geq\in R^{s_2}$ are the slack values of undesirable outputs, and $s_i^- \geq 0$ with $s_i^- \in R^m$ are the slack values of inputs. To measure the efficiency of $\left(x_0, y_0^g, y_0^b\right)$, the fractional objective function in Equation 7.a is defined (Tone, 2004).

$$Min\,\rho = \frac{1 - \left(\frac{1}{m}\right)\sum_{i=1}^{m}\frac{s_i^-}{x_{i0}}}{1 + \frac{1}{s_1 + s_2}\left(\sum_{r=1}^{s_1}\frac{s_r^g}{y_{r0}^g} + \sum_{z=1}^{s_2}\frac{s_z^b}{y_{z0}^b}\right)} \tag{7.a}$$

Constraints:

$$x_0 = X\lambda + s^- \tag{7.b}$$
$$y_0^g = Y^g\lambda - s^g \tag{7.c}$$
$$y_0^b = Y^b\lambda + s^b \tag{7.d}$$
$$s^g, s^b, s^-, \lambda \geq 0 \tag{7.e}$$

After that, the fractional model is transformed into a linear form, as in the standard SBM, using the Charnes-Cooper transformation. By multiplying both the numerator and denominator by the scalar variable t, the fractional structure is transformed into a linear form. Additionally, $\boldsymbol{S^+} = t.s^+, \boldsymbol{S^g} = t.s^g, \boldsymbol{S^b} = t.s^b, \boldsymbol{\Lambda} = t\lambda$ are applied to obtain the following Equation 8.a (Tone, 2004).

$$Min\,\tau = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{i0}} \tag{8.a}$$

Constraints:

$$t + \frac{1}{s_1 + s_2}\left(\sum_{r=1}^{s_1}\frac{S_r^g}{y_{r0}^g} + \sum_{r=1}^{s_2}\frac{S_z^b}{y_{z0}^b}\right) = 1 \tag{8.b}$$
$$tx_0 = X\Lambda + S^- \tag{8.c}$$
$$ty_0^g = Y^g\Lambda - S^g \tag{8.d}$$
$$ty_0^b = Y^b\Lambda + S^b \tag{8.e}$$
$$\boldsymbol{S^-}, \boldsymbol{S^g}, \boldsymbol{S^b}, \boldsymbol{\Lambda} \geq 0 \text{ and } t > 0 \tag{8.f}$$

The optimal solution in this linear programming model is defined as $\tau^*, S^{-*}, S^{b*}, S^{g*}, \Lambda^*, t^*$. Accordingly, it can be expressed, as in Equation 9, that ρ^*, s^{+*}, $s^{g*}, s^{b*}, \lambda^*$ are also optimal solutions (Tone, 2004).

$$\rho^* = \tau^*; s^{-*} = \frac{S^{-*}}{t^*}; s^{g*} = \frac{S^{g*}}{t^*}; s^{b*} = \frac{S^{b*}}{t^*} =; \lambda^* = \frac{\Lambda^*}{t^*} \tag{9}$$

To define DMU as efficient in terms of SBM-UO, ρ^* must be equal to 1. This situation is equivalent to slack variables s^{+*}, s^{g*} and s^{b*} being equal to 0. So, in this optimal solution, there is no input excesses (slack), no undesirable output excesses (slack), and no shortfalls (slack) of desirable outputs. For inefficient DMUs to become efficient, they must eliminate input slacks, prevent generating undesirable output slacks, and increase the levels of desirable outputs by eliminating shortfalls, as stated in Equations 10.a to 10.c (Tone, 2004).

$$x_0 \Leftarrow x_0 - s^{-*} \quad (10.a)$$

$$y_0^g \Leftarrow y_0^g + s^{g*} \quad (10.b)$$

$$y_0^b \Leftarrow y_0^b - s^{b*} \quad (10.c)$$

Below SBM-UO is expressed using j as the DMU index ($j = 1,2,\dots,n$), i as the input index ($i = (1,2,3,..m)$, r as the desirable output index ($r = 1,2,\dots,s_1$), z as the undesirable output index ($z = 1,2,\dots,s_2$), S_i^- as the input slack value, S_r^g as the desirable outputs slack value, S_z^b as the undesirable outputs slack value, x_{ij} as the input value, y_{rj}^g as the desirable outputs value, y_{zj}^b as the undesirable outputs value and Λ_j as the weight of the j[th] unit.

$$Min\ \tau = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{i0}} \quad (11.a)$$

Constraints:

$$t + \frac{1}{s_1 + s_2}\left(\sum_{r=1}^{s_1}\frac{S_r^g}{y_{r0}^g} + \sum_{r=1}^{s_2}\frac{S_z^b}{y_{z0}^b}\right) = 1 \quad (11.b)$$

Input Constraints: $\sum_{j=1}^{n}\Lambda_j x_{ij} + S_i^- = tx_{i0}$ with $i = 1,\dots,m$ (11.c)

Desirable Output Constraints: $\sum_{j=1}^{n}\Lambda_j y_{rj}^g - S_r^g = ty_{r0}^g$ with $r = 1,\dots,s_1$ (11.d)

Undesirable Output Constraints: $\sum_{j=1}^{n}\Lambda_j y_{zj}^b + S_z^b = ty_{z0}^b$ with $z = 1,\dots,s_2$ (11.e)

$$\sum_{j=1}^{n}\Lambda_j = t \text{ with } j = 1,\dots,n \quad (11.f)$$

$$\Lambda_j, S_z^b, t, S_r^g, S_i^- \geq 0 \quad (11.g)$$

2.2.2.2 BCC Data Envelopment Analysis with Undesirable Outputs

We have considered the standard BCC DEA model with undesirable outputs (BCC DEA-UO) as an alternative to SBM-UO. In this model, undesirable outputs are treated as standard inputs. DMUs must reduce both inputs and undesirable outputs at the same time to be efficient and enhance the eco-efficiency. The input/undesirable output-oriented BCC (BCC DEA-UO) model can be represented as below Equations 12.a to 12.g. If θ value is equal to 1, it indicates an efficient unit; otherwise, it is inefficient unit (Zhang et al., 2008).

$$Min\ \theta - \varepsilon E^T\left(S_z^b + S_r^g + S_i^-\right) \quad (12.a)$$

Input Constraints: $\sum_{j=1}^{n}\Lambda_j x_{ij} + S_i^- = \theta x_{i0}$ with $i = 1,\dots,m$ (12.b)

Desirable Output Constraints: $\sum_{j=1}^{n}\Lambda_j y_{rj}^g - S_r^g = y_{r0}^g$ with $r = 1,\dots,s_1$ (12.c)

Undesirable Output Constraints: $\sum_{j=1}^{n}\Lambda_j y_{zj}^b + S_z^b = \theta y_{z0}^b$ with $z = 1,\dots,s_2$ (12.d)

$$\sum_{j=1}^{n}\Lambda_j = t \text{ with } j = 1,\dots,n \quad (12.e)$$

$$\Lambda_j, S_z^b, S_r^g, S_i^- \geq 0 \quad (12.f)$$

$$\varepsilon > 0 \quad (12.g)$$

3. METHODOLOGY

In this study, DEA is applied for different DMUs that are having undesirable outputs. This application aims to assess the efficiency of these DMUs by evaluating their manufacturing activities from a sustainability perspective and comparing the differences between SBM-UO and the standard BCC DEA (BCC DEA-UO). 26 different DMUs operating in different regions, and their sustainability performance have been evaluated using SBM-UO and standard BCC DEA (BCC DEA-UO). The data is retrieved from the OECD database.

In our analyses, outputs have been categorized into two different groups: desirable and undesirable outputs. Different inputs have been included. The purpose is providing a more detailed perspective while measuring the efficiency of related activities and assessing their environmental sustainability. The relevant inputs and outputs are shown in the Table 2.

Table 2: Inputs and Outputs List

Inputs	Desirable Outputs	Undesirable Outputs
Energy Consumption(S_1^-)	Gross Value Added (S_1^g)	Carbon dioxide (S_1^b)
Number of Employees (S_2^-)		Methane(S_2^b)
		Nitrogen dioxide(S_3^b)
		Solid waste (S_4^b)

In both applications, normalization has been first applied for data adjustments since inputs and outputs contain numerical values of different magnitudes. Since the same units are used for each DMU, the data is independent of the unit. The raw data is shared in Table 3.

Table 3: Inputs and Outputs Data

DMU	S_1^-	S_2^-	S_1^g	S_1^b	S_2^b	S_3^b	S_4^b
DMU1	904.9	763	77.6	191.9	3.37	16.90	5.36
DMU2	260.4	333.6	10.8	63.6	0.72	6.48	2.71
DMU3	471.1	391.1	30.9	82.6	7.28	8.29	2.71
DMU4	392.6	263.8	26.5	81.7	1.00	10.15	1.95
DMU5	540.2	199.3	15.4	73.0	8.51	12.13	1.35
DMU6	540.0	79.7	10.3	43.5	1.68	3.24	1.34
DMU7	229.3	50.8	6.7	32.9	0.33	3.93	1.26
DMU8	126.4	34.0	4.1	10.6	0.10	2.61	0.94
DMU9	90.0	154.3	4.3	22.8	6.63	3.79	0.70
DMU10	155.5	56.3	7.6	13.7	0.19	2.62	0.49
DMU11	87.9	66.1	7.7	26.0	0.33	2.36	0.55
DMU12	9.2	12.0	0.4	1.6	0.02	0.24	0.41
DMU13	95.1	141.0	5.6	17.5	0.42	2.84	0.49
DMU14	156.8	36.1	1.8	16.2	0.21	3.04	0.40
DMU15	25.7	26.2	15.9	7.2	0.03	0.95	0.40
DMU16	45.5	60.1	1.0	9.0	0.85	1.37	0.43
DMU17	58.9	52.6	2.0	13.8	1.27	2.00	0.33
DMU18	67.3	88.1	2.9	11.3	0.28	1.45	0.31
DMU19	83.7	76.1	3.0	13.9	0.42	4.00	0.29
DMU20	73.7	22.2	1.0	5.1	0.06	0.73	0.20
DMU21	6.2	21.9	1.1	2.7	0.02	0.47	0.15
DMU22	43.2	29.6	4.8	5.6	0.12	0.83	0.11
DMU23	2.4	3.3	0.4	1.4	0.01	0.14	0.06
DMU24	19.7	29.3	0.8	4.7	0.02	0.78	0.05
DMU25	1.0	3.6	0.1	1.5	0.13	0.18	0.03
DMU26	3.6	12.0	0.4	1.2	0.08	0.64	0.04

First, we have applied SBM-UO, and the efficiency of each DMU has been measured using Equations 11.a to 11.g. For each DMU, the values of $(S_1^-), (S_2^-), (S_1^g), (S_1^b), (S_2^b), (S_3^b), (S_4^b), \Lambda_j, t, \tau$ have been determined. Then, the same

DMUs have been evaluated for their sustainable efficiency by using the standard BCC DEA with undesirable outputs (BCC DEA-UO), as stated in Equations 12.a to 12.g. For each DMU, the values of $(\boldsymbol{S_1^-}), (\boldsymbol{S_2^-}), (\boldsymbol{S_1^g}), (\boldsymbol{S_1^b}), (\boldsymbol{S_2^b}), (\boldsymbol{S_3^b}), (\boldsymbol{S_4^b}), \boldsymbol{\lambda_j}, \boldsymbol{\theta}$ have been determined.

4. RESULTS

The efficiency values obtained from the analyses by using SBM-UO and BCC DEA-UO are presented in Table 4. DMU1, DMU15, DMU22, DMU23, DMU24, DMU25, and DMU26 have been identified as efficient units in both methods. Based on the initial analysis results, they do not contain any slack values and operate with full efficiency. Consequently, these efficient DMUs serve as reference points and benchmarks for other inefficient DMUs within the models. DMUs with an efficiency value less than 1 can develop their improvement plans for their sustainable performance enhancement by taking into account inputs and outputs of these efficient DMUs.

Table 4: Results of DMUs with SBM-UO and BCC DEA-UO models

DMU	SBM-UO	BCC DEA-UO	DMU	SBM-UO	BCC DEA-UO
DMU1	1.00	1.00	DMU14	0.05	0.20
DMU2	0.04	0.11	DMU15	1.00	1.00
DMU3	0.37	0.63	DMU16	0.04	0.18
DMU4	0.37	0.64	DMU17	0.05	0.20
DMU5	0.05	0.28	DMU18	0.05	0.30
DMU6	0.08	0.23	DMU19	0.05	0.27
DMU7	0.09	0.25	DMU20	0.07	0.35
DMU8	0.10	0.27	DMU21	0.27	0.62
DMU9	0.04	0.17	DMU22	1.00	1.00
DMU10	0.10	0.40	DMU23	1.00	1.00
DMU11	0.11	0.35	DMU24	1.00	1.00
DMU12	0.20	0.86	DMU25	1.00	1.00
DMU13	0.06	0.28	DMU26	1.00	1.00

Furthermore, as shown in Figures 1 and 2, there are obvious differences between the inefficient units in SBM-UO and BCC DEA-UO models. In Figure 1, the efficiency values of the DMUs in BCC DEA-UO model show a more uniform and homogenous distribution between 0 and 1. On the other hand, SBM-UO model assesses their efficiency differently from BCC DEA-UO model and contains more values closer to zero. This shows that SBM-UO model tends to measure the efficiency of DMUs more precisely, especially as slack values increase. As Tone (2001) stated, compared to BCC DEA model, SBM model directly evaluates the efficiency by focusing on input excesses (slacks) and output shortfalls (slacks). SBM model is unit-independent and involves monotonicity based on slack values.

In this study, we observe that SBM-UO model used for environmental efficiency measurements shows the same behaviour as SBM model. SBM-UO directly takes into account input excesses (slacks), undesirable output excesses (slacks), and desirable output shortfalls (slacks) while calculating efficiency performances. As seen in Figure 1 and Figure 2, the presence of slack values in inputs or undesirable outputs has led to a greater reduction in efficiency in SBM-UO compared to BCC DEA-UO.

Tone (2001) found that when comparing SBM and CCR DEA models, the efficiency values of inefficient units were lower or equal in SBM model compared to the CCR DEA model. Similarly, in this study, we make the same observation form our results that the efficiency values of all DMUs are equal or lower in SBM-UO model compared to the BCC DEA-UO model.

Figure 1: The efficiency distribution of DMUs under SBM-UO and BCC DEA-UO

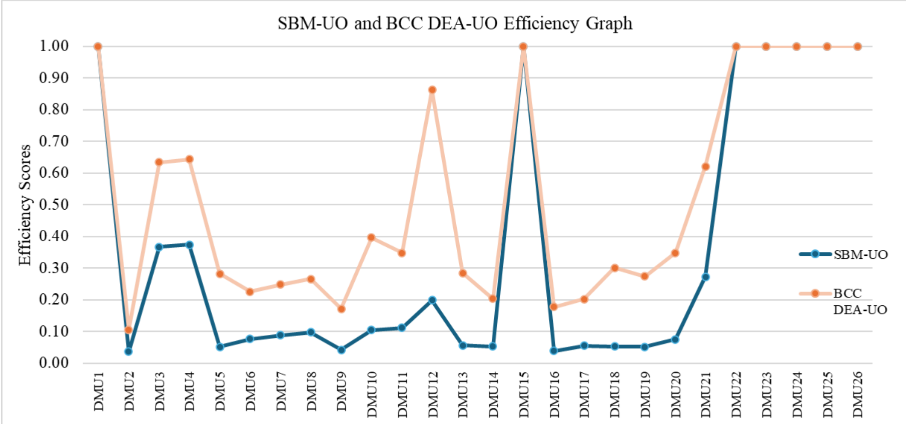
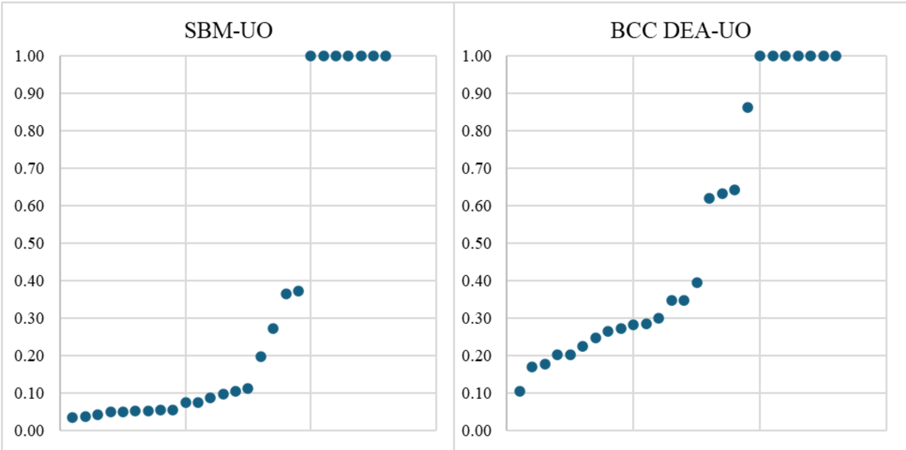


Figure 2: The comparison of DMUs (Decision-Making Units) between SBM-UO and BCC DEA-UO



5. CONCLUSION

SBM-UO and BCC DEA-UO are two widely used efficiency measurement methods. However, they differ in terms of the efficiency concepts and the insights they provide. Standard DEA evaluates the efficiency of DMUs relatively by comparing them with other DMUs through a proportional assessment of multiple inputs and outputs. In contrast, SBM focuses on measuring efficiency based on slacks (Tone, 2001). Similarly, in efficiency evaluation, SBM-UO directly considers both the excesses (slacks) in inputs and excesses (slacks) in undesirable outputs, as well as the shortfalls (slacks) in desirable outputs, as a result it provides an analysis from the perspective of slacks. Although both methods can detect the inefficient DMUs, SBM-UO provides a superior analysis in terms of the depth of the inefficiency.

In the context of environmental protection and sustainability, undesirable outputs resulting from production activities (e.g., pollutants and waste) are increasingly becoming the focus of policies and research. Technological progress that minimizes undesirable outputs in every production sector has become a centre of attention as a driver in sustainability. Thus, it is important to choose wisely, methods evaluating the impact of undesirable inputs and look for the appropriate methods to reduce the ones that are most important.

This study highlights the importance of selecting appropriate efficiency measurement models in sustainability research, particularly in industries that produce undesirable outputs such as pollutants and waste. The general observation in our study is that both the selection of inputs and outputs and the choice of models are important in studies conducted within the scope of sustainability. These mentioned differences between the two models can be considered as critical factors that researchers should carefully consider since SBM-UO and BCC DEA-UO may significantly impact the analysis and interpretation of efficiency.

Based on our results, SBM-UO has an ability to handle slacks in both desirable and undesirable outputs and provides a more comprehensive view of the inefficiencies in systems. This is especially valuable in industries with a considerable environmental footprint. On the other hand, BCC DEA-UO model relies on a more traditional input output approach. It may be more suitable for cases where the main purpose is maximizing resource utilization. Therefore, objectives of the research, the characteristics of the industry being studied and the role of environmental considerations in the overall efficiency evaluation should be considered while deciding which model to use.

Future research could also further explore the application of SBM-UO in different sectors. Especially those focused on sustainability and green technologies to increase desirable outputs while minimizing undesirable outputs. SBM-UO is a useful tool because of its adaptability in taking environmental factors into account. It helps in assessing how well industries are preserving or increasing productivity while reducing their ecological impact. In order to create more dynamic and predictive models, future research could also investigate the possibility of combining SBM-UO with other innovative techniques. This could lead to the creation of real time decision making. It can contribute the companies to continuously monitor and optimize their environmental and resource efficiency.

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REFERENCES

Banker, R.D., Charnes, A., & Cooper, W. W. (1984). Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis. Management Science, 1078-1092.

Barbier, E.B. (1987). The Concept of Sustainable Economic Development. Environmental Conservation, 101-110. doi:10.1017/S0376892900011449

Chang, Y.T., Park, H.S., Jeong, J.B. & Lee, J.W. (2014). Evaluating economic and environmental efficiency of global airlines: A SBM-DEA approach. *Transportation Research Part D: Transport and Environment*, *27*, 46-50.

Charnes, A., Cooper, W. & Rhodes, E. (1978). Measuring the efficiency of decission making units. European Journal of Operational Research, 429-444.

Cooper, W.W., Seiford, L.M. & Tone, K. (2008). Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software. Kluwer Academic Publishers.

Cooper, W.W., Seiford, L.M. & Zhu, J. (2011). Data Envelopment Analysis: History, Models, and Interpretations. In W. Cooper, L. Seiford, & J. Zhu, Handbook on Data Envelopment Analysis, 1-6.

Emrouznejad, A., Yang, G.L., Khoveyni, M. & Michali, M. (2022). Data envelopment analysis: Recent developments and challenges. *The Palgrave Handbook of Operations Research*, 307-350.

Freeman, A.M., Herriges, J.A., & Kling, C.L. (2014). The Measurement of Environmental and Resource Values. London: RFF Press.

Halkos, G., & Petrou, K.N. (2019). Treating undesirable outputs in DEA: A critical review. Economic Analysis and Policy, 62, 97-104.

Halkos, G., Tzeremes, N. & Kourtzidis, S. (2016). Measuring Sustainability Efficiency Using a Two-Stage Data Envelopment. Journal of Industrial Ecology, 20, 1159-1175.

Lacy, P., & Rutqvist, J. (2015). Waste to wealth: The circular economy advantage. Palgrave Macmillan.

Li, X.G., Yang, J., & Liu, X.J. (2013). Analysis of Beijing's environmental efficiency and related factors using a DEA model that considers undesirable outputs. *Mathematical and Computer Modelling*, *58*(5-6), 956-960.

Martić, Milan, Novaković, Marina S., Baggia, Alenka (2009). Data envelopment analysis -basic models and their utilization. Organizacija (Kranj), 42(2), 37-43.

Maxime, D., Marcotte, M., & Arcand, Y. (2006). Development of eco-efficiency indicators for the Canadian food and beverage industry. *Journal of Cleaner Production*, *14*(6-7), 636-648.

Meadows, D.H., Meadows, D.L., Randers, J. & Behres III, W. (1972). The limits to growth: A report for the Club of Rome's project on the predicament of mankind. New York: Universe Books.

OECD. (n.d.). OECD Data Explorer. Retrieved 01 2025, from https://data-explorer.oecd.org/?lc=en

Song, M., Song, Y., An, Q. & Yu, H. (2013). Review of environmental efficiency and its influencing factors in China: 1998-2009. Renewable and Sustainable Energy Reviews, 20, 8-14.

Sueyoshi, T. & Goto, M. (2012). DEA radial measurement for environmental assessment and planning: Desirable procedures to evaluate fossil fuel power plants. Energy Policy, 41, 422-432.

Tone, K. (2001). A Slacks-Based Measure of Efficiency in Data Envelopment Analysis. European Journal of Operational Research, 130, 498-509.

Tone, K. (2004). Dealing with undesirable outputs in DEA: a Slacks-Based Measure (SBM). Tokyo: GRIPS Report Series.

Tyteca, D. (1996). On the Measurement of the Environmental Performance of Firms-A Literature Review and a Productive Efficiency Perspective. Journal of Environmental Management, 281-308.

WCED (1987) World Commission on Environment and Development Our Common Future. Oxford University Press. Retrieved 12 14, 2024

Zarrin, M. & Brunner, J. O. (2022). Analyzing the accuracy of variable returns to scale data envelopment. European Journal of Operational Research, 1286-1301.

Zhang, B., Bi, J., Fan, Z., Yuan, Z. & Ge, J. (2008). Eco-efficiency analysis of industrial system in China: A data envelopment analysis approach. Ecological Economics, Volume 68(1-2), 306-316.

Zhou, H., Yang, Y., Chen, Y. & Zhu, J. (2018). Data envelopment analysis application in sustainability: The origins, development and future directions. European Journal of Operational Research, 1-16.

Zhou, P., Poh, K. & Ang, B. (2007). A non-radial DEA approach to measuring environmental performance. European Journal of Operational Research, 178(1), 1-9.

Zhou, Y., Xing, X., Fang, K., Liang, D. & Xu, C. (2013). Environmental efficiency analysis of power industry in China based on an entropy SBM model. Energy Policy, 57, 68-75.