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An Investıgatıon of Pre-Servıce Elementary School Teachers' Knowledge Concernıng Quadrilaterals

Berna CANTÜRK GÜNHAN[*a]

[a]Dokuz Eylül University, Faculty of Education, İzmir/Türkiye

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Abstract

The purpose of this study was to examine pre-service teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) about quadrilaterals. The research was a case study. Within the scope of the research, five open-ended questions concerning quadrilaterals were asked to pre-service teachers, who are at five different geometrical thinking levels. According to the research, it was determined that of the pre-service teachers, the SMK of those whose geometrical thinking levels were low was poor and they confused the relationships among quadrilaterals. In the light of the research, it was suggested that emphasis be placed on making the pre-service teachers acquire the SMK and PCK while they were being trained and the atmosphere where they can share these knowledge be created.

Introduction

Geometry is an important part of the elementary and secondary school mathematics curriculum. "The study of geometry contributes to helping students develop the skills of visualization, critical thinking, intuition, perspective, problem solving, conjecturing, deductive reasoning, logical argument and proof" (Jones, 2002, p 122). Perception of geometrical shapes contributes to problem solving (Martin & Strutchens, 2000). Teachers must know how to teach the concepts as regards geometry in order for the students to acquire the desired skills. However, a number of studies indicate that teachers' subject matter knowledge (SMK) and their pedagogical content knowledge (PCK) are not enough (Barrantes & Blanco, 2006; Hershkowitz & Vinner, 1984; Van der Sandt & Nieuwoudt, 2003). Therefore, students make mistakes in matters of geometry. To overcome these mistakes, it is necessary to assess the teachers' knowledge regarding these concepts.

The competence, which a teacher, who is one of the most important elements in the educational system, possesses, greatly affects the students' learning process (Ball & McDiarmid, 1988). The knowledge the teachers possess constitutes this fitment. Also, as Hill, Blunk, Charambous, Lewis, Phelps, Sleep and Ball (2008) pointed out "there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction".

Shulman (1987) categorized the type of information which teachers should have as follows: subject matter knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational purposes. On the other hand, Grossman, who is Shulman's PhD student, sums up the type of information teachers should have under the four headings: subject matter knowledge, pedagogical knowledge, pedagogical content knowledge, and curriculum knowledge (Swenson, 1998).

* **Author:** bernagunhan@gmail.com

Of these types of knowledge, subject matter is an essential component of teacher knowledge (Ball & McDiarmid, 1990). SMK is knowledge of the content of the discipline, consisting both of substantive knowledge (the key facts, concepts, principles and explanatory frameworks in a discipline) and syntactic knowledge (the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community) (Shulman, 1986). Mathematics educators performed studies similar to those of Shulman's studies (Hill, Rowan & Ball, 2005; Ma, 1999). In one of the studies regarding teachers' knowledge in mathematics teaching, Ma (1999) indicated that mathematics teachers should understand mathematics in depth while teaching it to children and should be able to reflect it in their teaching. Ball (1990), on the other hand, bases the mathematical knowledge the teacher should possess on the following three criteria: A teacher's knowledge of mathematical concepts and operations must be correct; a teacher should know the underlying rules and understandings and s/he is able to explain their reasons; a teacher should understand and assess the relationships among the mathematical ideas. A teacher need not be only proficient in regards to their mathematical knowledge; they must also be skilled in conveying it. Pedagogical content knowledge, another type of knowledge, is involved in this process, on which Shulman first focused momentously in 1986. Shulman, in his study in 1986, defined PCK as follows:

> .....the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others. PCK also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (Shulman, 1986, p. 9).

In his following studies, Shulman continued to define PCK as follows:

> PCK represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented instruction. It is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue (Shulman, 1987, p. 8).

As will be understood from Shulman's two definitions, a teacher presents the SMK that s/he has by finding several ways to represent them by adapting the learning materials with consideration to the students' individual differences, that is to say by transforming them into the form the students will be able to understand. In this context, the most important point distinguishing a math teacher from a mathematician is the ability to demonstrate the interaction of the knowledge of mathematics with the knowledge of pedagogy, namely to educationally-effectively present the knowledge of mathematics according to the varieties of basic skills and talents the students have (Shulman, 1987).

Some researchers suggested that two of PCK's constituents are knowledge of students' understanding and potential misunderstandings of a subject area and knowledge of instructional strategies and representations for teaching a particular topic (An, Kulm & Wu, 2004; Grossman, 1990; Kulm, R. Capraro, M. Capraro, Burghardt & Ford, 2001; Marks, 1990; Schoenfeld, 1998; Winsor, 2003). In addition, the fact that teachers should know popular understandings and misunderstandings such as what pre-learning the students possess concerning the topic, where they experience difficulties and which aspects they find interesting was emphasized by the researchers (Cochran, DeRuiter & King, 1993; Fennema & Franke, 1992; National Council of Teachers of Mathematics [NCTM], 2000; Tirosh, Fischbein, Graeber & Wilson, 1998). This study also focuses on these components: knowledge of students' understanding and potential misunderstandings as well as knowledge of instructional strategies.

Students' understanding of geometry and acquiring geometrical thinking depend on a teacher's knowing geometry in depth and teaching it in an effective way. The studies conducted demonstrate that the teachers whose SMK and PCK are high help students think and learn (Ball 1988a, Carpenter,

Fennema, Peterson, Chiang, & Loef; 1989; Ma, 1999; Rovegno, 1992; Wilson & Winwberg, 1989). Lenhart (2010) also found a relationship between middle school math teacher pedagogical content knowledge and student "Standards of Learning" scores in geometry and measurement. In the learning process of students, it is of great importance for the SMK and the PCK of the existing teachers and pre-service teachers to be high. However, Ball, Lubienski and Mewborn (2001) pointed out that elementary and secondary, pre-service or experienced teachers have widespread weaknesses in understanding the basic concepts and relationships in mathematics. Some other researchers indicated that pre-service and in-service middle school teachers' content knowledge in geometry is low (Fuys, Geddes & Tischler, 1988; Mayberry, 1983). Also, Hershkowitz and Vinner (1984) found that the pre-service and in-service teachers lacked basic geometrical knowledge, skills and analytical thinking ability. On the other hand, Barrantes and Blanco (2006) stated that most of the prospective teachers come to the university with the same lack of knowledge and experiences about geometry. They observed that the standard teaching techniques led the pre-service teachers to not employing different materials and sources. Van der Sandt and Nieuwoudt (2003), in the study they conducted, examined the content knowledge of both 7th grade geometry teachers and pre-service teachers. It was observed that both groups showed an insuffient content knowledge in the teaching processes and they failed to reach the desired geometric thinking levels. Chinnappan, Nason and Lawson (1996) examined pre-service secondary mathematics teachers' pedagogical and content knowledge of trigonometry and geometry. They reported that the student teacher showed a general lack of integration of mathematical knowledge with pedagogical principles and the teacher's pedagogical content knowledge seemed to be poorly developed. Gal (1998), in one of his studies indicated that high school teachers were not aware of students' levels of thinking about special segments with triangles, and for this reason, the teachers were wrong in some instructional decisions.

Studies also show that teachers and their students tended to exhibit similar patterns of misconceptions (Swafford, Jones & Thornton,1997). Some studies reported that there was a confusion in geometric concepts, which is sometimes caused by teachers and sometimes by students (Baturo & Nason, 1996; Clements, 1999; Currie & Pegg, 1998; Fujita, 2008; Fuller, 1997; Heaton, 1992; Lehrer, Jenkins & Osana, 1998; Monaghan, 2000; Okazaki & Fujita, 2007; Wu & Ma, 2005). For instance, Menon (1998) found that 54 pre-service elementary teachers have a procedural understanding of area and perimeter rather than a conceptual and relational understanding (cited in Kow & Yeo, 2008). Cunningham and Roberts (2010), found that despite being given the definitions of the altitude of a triangle and the diagonal of a polygon, pre-service elementary school teachers have limited understanding of these concepts. On the other hand, Aslan-Tutak (2011) observed that secondary education mathematics pre-service teachers have misconceptions in the relationships between similarity and equality and on the subjects of reflection and symmetry.

Quadrilaterals were chosen in this study because it is one of the most basic concepts in geometry, and a field in which students experience misconceptions and difficulties (Clements, 1999; Currie & Pegg, 1998; Monaghan, 2000; Wu & Ma, 2005). Monaghan (2000) noted that many students had deficiencies in understanding the properties of quadrilaterals. Quadrilaterals are taught both in elementary and secondary education. In fact, the geometrical concepts are defined in four basic categories in pre-school curriculum: circular, square, triangle and rectangle (Clements, 1998, Ministry of National Education [MNE], 2006). In addition, quadrilateral types confront students in daily life. For this reason, it is of great significance for the teachers to have enough mathematical knowledge while teaching these concepts. We encounter some studies in literature where the teachers and pre-service teachers confuse the relationships among quadrilaterals (Fujita, 2008; Okazaki & Fujita, 2007). In a study with pre-service primary teachers in their first and third year of university in Scotland, Fujita and Jones (2006a) reported that they did not seem to have a good understanding of the hierarchical relationship between quadrilaterals. For example, the pre-service teachers regarded "a rectangle" as a special case of "a square" and they could draw a correct image of a square but they defined it incorrectly. Furthermore, even after two years of education, their understanding did not seem to improve. In another study, Yee Han (2003) found that in-service teachers did not have a good understanding of quadrilaterals. Fujita

and Jones (2006b) in their research to determine the perceptions of trainee primary teachers concerning parallelogram, demonstrated that only a minority of 105 candidates studying at the second year in the university have the knowledge of parallelogram. The cited research studies mainly focus on pre-service and in-service teachers' subject matter knowledge and less has been done with pedagogical content knowledge about quadrilaterals.

Successful geometry teaching depends on the geometry knowledge the teacher has and his/her ability to teach it in an effective way. In addition, the fact that understanding geometry in depth by teachers will help students cope with the difficulties they encounter. According to Ball (1988b), teachers' PCK and their thoughts about mathematics are found in the years when they are students at the university and the structuring of this process continues in the course of their career. The most important step to be able to contribute to this process of structuring is experienced in the institutions that train the teachers. Besides, it is necessary that mathematics teachers, when they are students at the university, should primarily be aware of their concept knowledge and be capable of associating the concepts they know correctly (National Research Council [NRC], 2000). In this context, an important mission falls to the institutions training teachers. Primarily, it is necessary that to the extent a pre-service teacher, having already graduated or about to graduate from the teacher-training institutions, possesses these types of knowledge be known. The purpose of this study is to investigate both the SMK that the pre-service Elementary School Teachers possess regarding quadrilaterals included in the curriculum of the primary education, and the PCK that they have concerning mistakes, which the 5th grade students had about the quadrilaterals, the potential reasons for these mistakes, and strategies to overcome these mistakes. This research study aimed at answering the following research questions:

1. What is the pre-service elementary school teachers' subject matter knowledge about quadrilaterals on fifth grade curriculum?

2. What is the pre-service elementary school teachers' pedagogical content knowledge about quadrilaterals on fifth grade curriculum?

Method

Research Design

Case study, which one of the qualitative research methods, was preferred in the study. Because the present study is intended for presenting SMK that pre-service elementary school teachers have concerning quadrilaterals and PCK that they possess the elementary students' mistakes, the reasons of these mistakes, and strategies to overcome these mistakes. The SMK and PCK of pre-service elementary school teachers were analyzed within the context of classroom teacher training curriculum.

Participants

Data were collected from the five junior student teachers (1 female and 4 male) enrolled in the Elementary School Teacher Training Program at a public university. The sampling of maximum diversity was used while the pre-service teachers forming the sampling of the study were selected. The purpose in the method of this sampling is not to generalize via ensuring the diversity, but to discover what kind of commonalities and similarities exist between the situations exhibited in diversity (Yıldırım & Şimşek, 2004). While five pre-service teachers were selected, "The Van Hiele Geometry Test (VHGT)" was applied to thirty-two third year students at the end of the Teaching Mathematics II Course, which they were taught in the spring semester. These questions were used to determine the students' understanding of geometric concepts as identified by P.M. Van Hiele and his wife D. Van Hiele-Geldof (1959; cited in Lee, 1999). Based on this test, it was possible to determine the geometric thinking level of the students. Geometrical thinking levels are made up of five levels and summarized by the abilities each level measures as shown in Table 1 (Lee, 1999). This test was translated into Turkish by Duatepe

and she applied it to pre-service elementary school teachers (Duatepe, 2000). In her study, Cronbach Alpha reliability measures were found as .82; .51; .70; .72 and .59 for the first, second, third, fourth and fifth level, respectively.

Table 1. *The Abilities That Each Level of Van Hiele Geometry Test Measures*

Level	Items	Measured the Abilities
1	1 – 5	Identify, Recognize geometric shapes based on an individual's visual abilities, (Visualization)
2	6 – 10	Measure properties of geometric shapes by observing and drawing a picture, (Analysis)
3	11 – 15	Verify figures, hierarchically, by analyzing the properties of figures, (Abstraction)
4	16 – 20	Understand proof meaning in the context of definitions, axioms, and theorems, (Deduction)
5	21 – 25	Identify the consistency of set axioms and compare axiomatic systems. (Rigor)

In order for the students' levels to be determined, it is necessary that they should at least answer three of the five questions at each level. In the study, in consequence of the Van Hiele Geometry Test applied to the pre-service teachers, there is 1 teacher at the fifth level, 1 teacher at the fourth level, 10 at the third level, 12 at the second level and 8 at the first level. One person was selected for the research from each of the five levels. Instead of expressing them with their names, the pre-service teachers taking part in the study were expressed/specified via a kind of coding in the form of PT1 (pre-service teacher at the 1st level-male), PT2 (pre-service teacher at the 2nd level-male), PT3 (pre-service teacher at the 3rd level-male), PT4 (pre-service teacher at the 4th level-male) and PT5 (pre-service teacher at the 5th level-female).

Elementary School Teacher Education Program

Elementary school teacher training is a four-year program. In the first year, pre-service teachers take the courses of Basic Mathematics I and II, which are intended for mathematics content knowledge (MCK) and until the end of the third year, they take courses such as "educational psychology", "teaching principles and methods", "teaching technologies and developing materials" and "class management", which are intended for Pedagogical Knowledge. For PCK, on the other hand, they take Mathematics Teaching I and Mathematics Teaching II only in the third year. In the content of Mathematics Teaching-I, which is taken in the fall semester, the focus is on topics such as teaching and learning strategies to be utilized in mathematics teaching, the scope, aim and characteristics of elementary education mathematics curriculum, and the significant skills in mathematics teaching (association, representations, communication, reasoning, problem solving). On the other hand, in the content of Mathematics Teaching-II, which is taken in the spring semester, the pre-service teachers receive training as to how they will practice the learning methods aimed at the acquisitions included in the learning fields of "Geometry, Numbers, Data, Measurement", included in the Ministry of National Education (MNE) curriculum, as well as how they can carry out evaluations in the subject of mathematics. In particular, they are expected to be familiar with the program in Mathematics Teaching-I and Mathematics Teaching-II, and to be aware of the relationships between the mathematics concepts, and knowledge of students' mistakes and of what must be taken into account while teaching the students these concepts. During two semesters of the activities developed by pre-service teachers had been evaluated with them on discussion about whether these activities could be achieved concepts or not, whether or not they were referred to any misconceptions and the use of techniques which method would be more appropriate in order to achieve concepts. The researcher observed the practices of pre-service teachers in the subject of Mathematics Teaching, and the diversity in the

performances the pre-service teachers exhibited in their practices drew the researcher's attention. The diversity was observed form that pre-service teachers reflect the mathematical concepts as right or wrong. The reason for diversity was questioned examining their SMK and PCK about quadrilaterals when the knowledge the participants acquired in the subject of Mathematics Teaching, which they took for two semesters, was new. However, the pre-service teachers created lots of activities intended for the so-called acquisitions in curriculum, but none of those experiences were directly related to quadrilaterals.

Instrument

The pre-service teachers were asked five open-ended problems concerning quadrilaterals taught under the name of the field of geometry learning that is included in the elementary curriculum. A number of suggestions on open-ended questions for SMK and PCK of pre-service teachers on the fact that interviews and multiple choice items can be used, on observation and analyses that can be made, and on in-class interactions can be studied in depth in the investigations carried out in the literature (Ball & Bass 2000; Hill et al. 2005; Ma, 1999; Manizade & Mason, 2011; Zhou, Peverly & Xin, 2006). In this study, open-ended questions were also asked to the pre-service teachers. The problems were formed to assess pre-service teachers' subject matter knowledge and pedagogical content knowledge. In order for the questions formed by the researcher to be appraised whether these questions were appropriate with the acquisitions taught regarding quadrilaterals in the curriculum, the views of two mathematics educators were received apart from the researcher. The questions were finalized in accordance with the views of the mathematics educators. In the questions, the texts are different but the questions asked in their choices are the same and are as follows:

a. Find the answer to the problem.

b. List two common mistakes students may make while performing (i) and/or (ii)

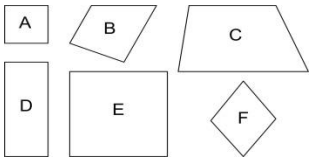
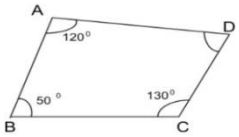
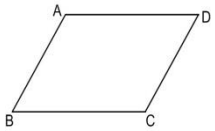
c. Describe possible sources for each of these mistakes.

d. How will you fix these mistakes? [*]

In the questions, the "a" choice is related to measuring the pre-service teachers' subject matter knowledge of quadrilaterals. Subject matter knowledge was categorized as pre-service teachers' knowledge on basic concepts and their relationships with basic drawing. The "b", "c" and "d" choices are related to measuring the pre-service teachers' pedagogical content knowledge of quadrilaterals. Pedagogical content knowledge was categorized as knowledge of students' mistakes, the reasons for these mistakes and instructional strategies aiming at overcoming them. The text part of the questions, the categories of SMK and PCK have been given in Table 2.

[*] These choices of questions were adapted from Işıksal's (2006) study.

Table 2. *Problems, Categories of SMK and PCK*

Problems	Categories of SMK and PCK
1. Of the shapes below, find the regular quadrilaterals.  	SMK- basic principles-1a (The pre-service teachers are expected to know the relationship of quadrilaterals and to distinguish the regular quadrilaterals). (Correct answer is A and E) PCK- knowledge on students' mistakes- 1b, the reasons for the mistakes-1c and instructional strategies-1d
2. "Both pairs of mutual edge are made up of parallel line pieces and the shape is a regular polygon." Which of the quadrilaterals do the properties given above belong to?	SMK- basic principles-2a (The pre-service teachers are expected to find the regular polygon by using the relationship of quadrilaterals) (Correct answer is square) PCK- knowledge on students' mistakes- 2b, the reasons for the mistakes-2c and instructional strategies-2d
3. How much degree is the angle of D in the below quadrilateral?  	SMK- basic principles-3a (The pre-service teachers are expected to find the angle by using the features of quadrilaterals) (Correct answer is 60 degree) PCK- knowledge on students' mistakes- 3b, the reasons for the mistakes-3c and instructional strategies-3d
4. Draw the ABCD parallelogram's heights belonging to D corner.  	SMK- basic drawing-4a (The pre-service teachers are expected to draw the heights by using the features of quadrilaterals) PCK- knowledge on students' mistakes- 4b, the reasons for the mistakes-4c and instructional strategies-4d
5. How many symmetry lines the ABCD rectangle has? Draw.  	SMK- basic drawing-5a (The pre-service teachers are expected to draw the symmetry lines by using the features of quadrilaterals) PCK- knowledge on students' mistakes- 5b, the reasons for the mistakes-5c and instructional strategies-5d

Data Analysis

In the present research, the data were collected making interviews via five open-ended questions. Each interview lasted for about 45 minutes. The analysis of these interviews, on the other hand, was carried out with content analysis. Content analysis is a technique used for the characterization and comparison of the data from the interviews (Altunışık, Çoşkun, Bayraktaroğlu & Yıldırım, 2004). The data were evaluated within the framework of the two research questions of the study. First, the answers given to the "a" choice of the questions asked for the evaluation of the SMK of classroom pre-service teachers were categorized as pre-service teachers' knowledge on basic concepts and their relationships with basic drawing. Answer to these questions were summarized and evaluated, with respect to

whether or not pre-service teachers brought with them knowledge. Secondly, the answers given to "b", "c" and "d" choices of the questions asked for the evaluation of the classroom pre-service teachers' PCK were categorized as knowledge of students' conceptions, the reasons for the mistakes and instructional strategies aiming at overcoming them (see Table 2). Later, it was ensured that the answers of pre-service teachers were separated to significant divisions via coding operation. The views of the participants at the different geometrical thinking levels were compared and codings were created based on these comparisons. Another mathematics educator also carried out a coding operation. Coding shema was presented in Table 3 in detail. The consistency between the two codings was calculated based on P = $\frac{Na}{Na + Nd}$ formula determined by Miles and Huberman (1994) ("P is the percent of consistency", "Na is the consistency amount" and "Nd is the inconsistency amount"). It is stated that in the codings carried out using this formula, there is at least a 70% reliability rate (Miles & Huberman, 1994). After the coding operation was completed, the reliability among the researchers was calculated. The percentage agreement between the two raters was roughly 93%. This percentage calculated demonstrates that the codings made were reliable.

Results

Findings of the research have been discussed within the framework of two research questions formed to deeply understand and examine the pre-service teachers' subject matter and pedagogical content knowledge concerning the concept of quadrilaterals included in the curriculum.

Pre-service elementary school teachers' subject matter knowledge

While the SMK of pre-service teachers was being evaluated, regarding quadrilaterals and their relationships, their knowledge of necessary drawings and how they reflect this knowledge was also examined.

While the information derived from the first three questions regarding quadrilaterals encompassed knowledge of basic concepts and their relationships, those obtained from the fourth and fifth questions involved knowledge of basic drawings in quadrilaterals. If we are to examine the answers the pre-service teachers gave to these questions respectively, the pre-service teachers are asked to distinguish the regular quadrilaterals in the given shape in "*Problem 1"*. It was observed that of the pre-service teachers, PT5, PT4 and PT3 were correctly able to distinguish the regular quadrilaterals, that is, the square shapes. PT2 and PT1, on the other hand, indicated rhombus among the regular quadrilaterals, whereas, square is a special case of rhombus and rectangle. This finding, as in the study of Okazaki and Fujita (2007), indicates that two pre-service teachers did not seem to have the hierarchical relationships between quadrilaterals. In *"Problem 2"*, the pre-service teachers are asked to determine the quadrilaterals of the given characteristics. As is the case in *"Problem 1"*, while PT5, PT4 and PT3 said that the quadrilaterals bearing the given characteristics are squares, PT2 and PT1 failed to answer, instead stating that it might be a rectangle or a parallelogram. This finding shows that the concepts of regular quadrilaterals and the characteristics of rectangles in PT2 and PT1 pre-service teachers were not completely developed. On the other hand, in *"Problem 3"*, the pre-service teachers are asked to find the fourth angle of the quadrilateral using the measurements of three angles which were given. It was observed that all of the pre-service teachers found the required value. The pre-service teachers are asked to draw the heights belonging to the D corner of the parallelogram given in "*Problem 4*". While PT5, PT4, PT3 and PT2 pre-service teachers drew the heights correctly, PT1 pre-service teacher drew the height belonging only to the BC edge of the D corner. When he was asked why he did not draw the height belonging to the other AB edge, he said that did not occur to him. The fifth problem asked the pre-service teachers to draw the symmetry line of a rectangle. While the pre-service teachers, PT5, PT4 and PT3 made the correct drawing, the rectangle has two symmetry lines; the other pre-service teachers

could not make this. PT2 and PT1 suggested and showed that a rectangle has four symmetry lines. The mistake these pre-service teachers made is that they confused the rectangle's symmetry lines by counting diagonals with the square's symmetry lines. That is to say, the facts that pre-service teachers confused the characteristics of quadrilaterals were reflected in their drawings.

In brief, we can say that in this sample the pre-service teachers' quadrilateral knowledge, whose Van Hiele geometric thinking levels were low, was weaker than that of the other pre-service teachers. In addition, it was observed that these pre-service teachers (PT2 and PT1) confused the characteristics of quadrilaterals. Similar results were found in the study Fujita (2008), Okazaki and Fujita (2007) made.

Pre-service elementary school teachers' pedagogical content knowledge

Pedagogical content knowledge was categorized as knowledge of students' conceptions, reasons for their mistakes, and the instructional strategies aiming at overcoming them. In Table 3, the mistakes, reasons for the mistakes, and the instructional strategies aiming at overcoming them are given.

As seen in the Table 3, pre-service teachers classified the mistakes the students might make as conceptual, drawing, or operational mistakes, as well as mistakes arising from the question's not being understood. The answers the pre-service teachers gave concerning the reasons of these mistakes and the instructional strategies aiming at overcoming them are examined in detail below.

Table 3. *The Mistakes, Their Reasons and the Instructional Strategies Aiming at Overcoming Them*

Type of Mistake	**Reasons of the Mistake**	**Instructional Strategies Aiming at Overcoming the Mistake**
Conceptual Mistake	The concept's not being completely formed	Use of concrete material
	Lack of knowledge	Examples from daily life
	Misconception	The activities to make them reach the concept
	The constructivism-based education's not being given	The teaching methods based on constructivism
	Not paying attention to crucial points of the concepts	Paying attention to crucial points of the concepts
	The hierarchical order of quadrilaterals and their characteristics' not being known	Discovery learning Consolidation via game Showing and recounting
Drawing Mistake	The concept's not being completely formed	Paying attention to misconceptions
	Carelessness	Solving problems including different drawings
Operational Mistake	The concept's not being completely formed	Increasing the problem solving
	Carelessness	Developing four operations skills
The Mistake arising from the question's not being understood	Carelessness	Carefully reading of the question

In *"Problem 1",* PT5 said that some students would only give rhombus as an example for regular quadrilateral and would not give square as a regular polygon, and stated that as a reason for this, the concept of regular quadrilaterals was not completely formed in the students. She suggested that in order for the students to grasp the concept of a regular quadrilateral and its characteristics, the teachers should help students' learning by employing concrete materials and providing examples from

daily life. PT4 stated that students might also indicate a rhombus and a rectangle as regular quadrilaterals. He said that students might lack knowledge and have misconceptions as the reason for these mistakes. He stated that teachers could employ concrete materials and activities to help students understand the concept during the lesson in order for the mistakes to be corrected. PT3, on the other hand, said that the students could not indicate the regular quadrilaterals and this was due to the concepts' not being completely formed because the students were not provided with the education based on constructivism. In order for this situation to be corrected, he stated that the learning methods based on constructivism such as cooperative learning and drama were employed during lecturing. PT2 states that students can show the wrong shapes because the concept of a regular quadrilateral was not formed in the students. In order for this mistake to be corrected, he said that teachers should teach by showing different regular quadrilaterals during the lesson. PT1 failed to state, in a logical way and in mathematical language, the mistakes the students might make, and reasons for the neither mistakes nor what must be done to overcome them.

In *"Problem 2"*, PT5 stated the mistakes the students may make in the second problem as follows;

> *"To the question of what kind of quadrilateral is square when some characteristics of the square are provided, students can answer as rectangle, trapezoid or parallelogram. Students cannot match what are given with the concepts.* The reason for these mistakes is that *"students do not know well the characteristics of quadrilaterals and the relationships between them".* In for the mistakes to be overcome*, "teachers must provide the environments for students to discover the characteristics of quadrilaterals and ensure that students should strengthen their characteristics through mathematical games".*

PT4, on the other hand, stated that students might indicate a rectangle or a parallelogram as the quadrilateral bearing these characteristics and this might be due to their misconceptions. It was pointed out that students should be ensured to learn by discovery in order for these mistakes to be corrected. PT3 suggested the mistakes the students might make as follows:

> *"Because there is the expression of "*pairs of mutual edge*" in the characteristics of the given quadrilateral in the question, students may think that the asked quadrilateral can be "rectangle" or "parallelogram" because of the words "*parallel*"."*

PT3 stated that the reason for students' mistakes was that the concepts were not completely learned and the teacher might not pay attention to the points at which conceptual errors might occur. It was suggested by PT3 that for these mistakes not to happen, the teachers should be careful about the situations where misconceptions might occur during the lesson. PT2, on the other hand, stated that students might not completely be able to comprehend the question. In this case, the pre-service teacher said that the teacher must ensure that the question be read carefully. When considering the comments of the pre-service teacher, he can be regarded as incompetent in his answer which he gave, because he also experience problems in the concept of a regular quadrilateral. PT1 stated that students would say that the quadrilateral bearing the characteristics in the question might be a square (which is the correct answer) instead of a rectangle. He could not state the reason for the mistake. He pointed out that the solution to the question should be taught in order for the mistake to be corrected. Because the concept of regular polygon was wrongly formed in PT1, it was observed that his answers were in this direction. This finding matches up with the Fujita and Jones's (2006) discovery where pre-service teachers wrongly did the association among the quadrilaterals.

In "*Problem 3*", PT5, PT4 and PT3, similarly, specified the mistakes the students might make. Three pre-service teachers said that students would consider the quadrilateral given in the question as if it was a parallelogram and would resort to sum up the angles of the successive corners and equalize them. They stated that the source of the mistake was misconceptions and that in order for this mistake to be overcome, use of activities should be employed during the lesson (PT5, PT4 and PT3), solving the sample questions should be increased (PT5), and making use of mathematical games through the use of reinforcement (PT4 and PT3). PT2 and PT1 remarked that students would make operational mistakes in

this question and this might be due to carelessness. In order for the mistakes to be overcome, they suggested that addition and subtraction operations should be done (PT2) and many more problems should be solved (PT1). It was observed that the pre-service teachers whose Van Hiele geometric level points were lower did not ground the students' mistakes on conceptual inadequacy or misconceptions, but on operational mistakes. This result is compatible with the findings of Menon (1998), who reported that pre-service elementary teachers have a procedural understanding of area and perimeter rather than a conceptual and relational understanding.

In "*Problem 4*", five pre-service teachers also remarked that students might make drawing mistakes. PT5 and PT3 showed the students' mistakes by drawing. PT4, PT2 and PT1 said that students would try to draw the heights of the parallelogram inside the shape. As the reason for these mistakes, all five pre-service teachers remarked that the concept of height was not completely formed and usual drawings about the quadrilaterals might have been made in the lessons. In order for these mistakes to be overcome, it was stated that the sample questions containing the drawings of shapes in different position should be solved (PT5, PT4, PT2, and PT1) and the misconceptions should be taken into account during the lessons (PT3).

In "*Problem 5*", PT5, PT4 and PT3 who correctly drew the symmetry lines of the rectangle, remarked that one mistake the students might make is to see "*the shape as if it had been square and might count the symmetry lines passing from the corners*". This mistake arises from the students' confusion of the characteristics of quadrilaterals. To overcome this confusion, they said that the characteristics of quadrilaterals should be comprehended well and students should not be made to memorize, but *"they should be made to understand the characteristics of quadrilaterals over visual materials in the lessons"*. PT2 and PT1, who indicated incorrectly the symmetry line of the rectangle, stated that the students could draw the symmetry line from everywhere of the rectangle. They also pointed out that the mistake was due to students' not knowing how to take the symmetry of a shape and in order for the mistake to be overcome, how to take the symmetry of a shape should be shown and taught in detail.

The summarized form of the answers, which the pre-service teachers gave to the questions, is provided in Table 4. As can be seen from Table 4, in this sample, as the pre-service teachers' Van Hiele geometric thinking levels improve, their PCK increases. In the studies in literature, while teachers should make use of the teaching technologies (Baki, 2008) and of multiple representations (Bütün, 2005) in order to ensure that the students should be able to comprehend a certain topic or concept, it was observed that the pre-service teachers in the sample of this study, even though they took the subject of "teaching technologies and developing materials", did not reflect any of this in their PCK. Based on the answers the pre-service teachers provided within the scope of this study, the explanations reflecting the PCK of the pre-service teachers whose SMK was weaker can also be said to be poor.

Table 4. *The Summation of Pre-Service Teachers' Answers*

		PT5	PT4	PT3	PT2	PT1
P1	*Mistake*	*Conceptual*	*Conceptual*	*Conceptual*	*Conceptual*	-
	Instruction Strategies	Concrete material, Samples from the daily life	Concrete material, Activity	Teaching methods based on constructivism	Showing and explanation	-
P2	*Mistake*	*Conceptual*	*Conceptual*	*Conceptual*	*Failing to understand the question*	-
	Instruction Strategies	Learning by discovery, Reinforcing via game	Learning by discovery	Being careful about the crucial points of the concepts	Carefully reading of the question	-
P3	*Mistake*	*Conceptual*	*Conceptual*	*Conceptual*	*Operational*	*Operational*
	Instruction Strategies	Activity, Problem solving	Activity, Reinforcer, Game	Activity, Reinforcer, Game	Addition and subtraction operations	Increasing the problem solving
P4	*Mistake*	*Drawing*	*Drawing*	*Drawing*	*Drawing*	*Drawing*
	Instruction Strategies	Solving the problems including different drawings	Solving the problems including different drawings	Being careful about misconceptions	Solving the problems including different drawings	Solving the problems including different drawings
P5	*Mistakes*	*Conceptual*	*Conceptual*	*Conceptual*	*Conceptual*	*Conceptual*
	Instruction Strategies	Concrete material	Concrete material	Concrete material	Showing and explanation	Showing and explanation

Discussion, Conclusion & Implementation

In this study, the SMK and the PCK of the pre-service teachers at different geometrical thinking levels regarding quadrilaterals of elementary education were examined. Concerning the SMK of quadrilaterals, it was determined that the SMK of the pre-service teachers in interpreting and making sense of the quadrilaterals was not sufficiently deep. In this study, the pre-service teachers confused the characteristics of quadrilaterals were reflected in their drawings and they confused the hierarchical relationships between quadrilaterals. The results support the findings of previous studies. Such as, Fujita and Jones (2006), Fujita (2008), Fuys et al. (1988), Mayberry (1983), Okazaki and Fujita (2007) and Yee Han (2003) who state that pre-service and in-service middle school teachers' geometry content knowledge is not adequate. Therefore, it is necessary that while training elementary school teachers, their shortcomings in the SMK are determined, precautions are taken to overcome these shortcomings and practices towards improving pre-service teachers' geometric thinking skills also are carried out. Besides, it should not be ignored that SMK has an important effect in the planning and teaching process (Rowland, Martin, Barber & Heal, 2001). As Ball (1998b) pointed out, teachers should comprehend the mathematical concepts and their relationships in selecting and forming useful mathematical activities for the students and in both interpreting and evaluating their opinions resiliently. In this context, it can be said that it is significant for the accuracy of the SMK the teachers have about the concepts to establish their shortcomings in order for the geometrical concepts to be learned and used by the students correctly.

According to the PCK results, the pre-service elementary school teachers grouped the mistakes the students may make under the following headings; conceptual mistake, drawing mistakes, operational mistake and the mistakes arising from the question's not being understood. It was determined that pre-service teachers mostly mentioned the conceptual mistakes as mistake. The pre-service teachers suggested that concrete materials be used in the lessons in order for the conceptual mistakes not to occur. This finding is consistent with the literature, because students learn more meaningfully in the environments in which concrete materials are employed (MNE, 2005). The pre-service teachers, on the other hand, suggested that in order for the misconceptions or wrong learning not to occur, the environments of learning by discovery should be created by employing the activities leading to concept acquisition, the learning of methods based on constructivism should be used, care should be given regarding the crucial points of the geometric concepts and there should be reinforcement. Similarly, Van De Walle (2004) indicated that employing various activities in the lessons has positive effects on learning geometry. In addition, it was determined that one pre-service teacher (PT5-the one having the highest level of geometrical thinking) emphasized the importance of associating the geometrical concepts with daily life like Van Hiele (1986). Of the mistakes that the students might make, pre-service teachers suggested that the usual drawings about the quadrilaterals might have been done in the lessons to correct the drawing mistakes; this situation can cause the student to form the concept in a limited structure. Therefore, it should be noted that making the students draw is effective in their learning characteristics of the shapes (Burger & Shaughnessy, 1986) and the pre-service teachers also proposed that the samples containing the shapes in different positions be included in the lessons. In order for the operational mistakes that the pre-service teachers who are at the first two levels of geometrical thinking remarked on to be corrected, they stated that the number of problem solving should be increased and in the case of mistakes due to the fact that the question is not completely understood, the question should be carefully read. In addition to these results, the fact that none of the pre-service teachers mentioned teaching technologies in their proposals they made in order for the possible mistakes to be corrected demonstrates that they failed to completely reflect the knowledge they acquired in such courses as "teaching technologies and material development" towards improving their pedagogical knowledge. Therefore, different teaching methods and use of materials should be provided in the instructional practices the pre-service teachers can make in mathematics teaching courses, and the environments where pre-service teachers can share their SMK and PCK should be created after the practices. Thus, the pre-service teachers can improve their SMK and PCK before starting their careers. Also, based on the use of various geometrical software programs in the lessons, the students are provided with the environments where they can create the geometrical concepts on their own (Laborde, Kynigos, Hollebrands & Strasser, 2006), students can create geometrical drawings and can carry out interactive examinations on the dynamic, geometrical shapes the teacher prepared (MNE, 2005).

In addition to all of these, it was discovered that pedagogical content knowledge of the pre-service teachers at the 1st and 2nd geometrical thinking levels in the sample of this study was not at the sufficient level and they interpreted the relations between the quadrilaterals incorrectly. That is to say, pre-services teachers have been reflected their geometrical thinking levels on their PCKs. This is showed that SMK and PCK are interconnected. The researchers conducted indicate that the majority of pre-service elementary and secondary mathematics teachers' geometric thinking stages were below level-III (Abstraction) and level-IV (Deduction) (Duatepe, 2000; Durmuş, Toluk & Olkun, 2002; Hershkowitz & Vinner, 1984; Knight, 2006; Mayberry, 1983;). In particular, understanding hierarchical relations between quadrilaterals requires the third level of van Hiele Geometric Thinking. Therefore, the importance for the improvement of Van Hiele Geometrical Thinking Levels should be included while training the pre-service teachers. In light of the fact that one of the characteristics of Van Hiele levels is "geometrical experience, the most important factor affecting the advancement via levels" (Van de Walle, 2004), the enhancement of pre-service teachers' geometric experiences should be taken into account. Experience will also help pre-service teachers improve their content training/teaching knowledge (Cochran et al., 1993; Foss & Kleinsasser, 1996).

For this case study, the results suggest that the pre-service elementary school teachers must improve their geometric concepts, they must be provided with the knowledge of noticing the misconceptions the students might have about the topics of geometry and they must be taught the knowledge of how they must cope with the so-called misconceptions. As a matter of fact, teacher training for a contemporary geometry education should be disseminated sufficiently and effectively (De Villers, 1996). Moreover, what is needed in teacher training is forming a balance between content and pedagogical knowledge (Mapolelo, 1999). In this context, if the balance between SMK and PCK of the teachers and pre-service teachers is well created and their knowledge is improved, their integration with teaching curriculum, their self-confidence, their awareness in their shortcomings in content knowledge and their openness to students' interests, ideas participations and questions will become that much easier and comfortable (Babbington & Lomas, 2004), and the efficiency of educational practices will also increase (Cochran et al., 1993; Fuys et al. 1988; Hershkowitz & Vinner, 1984).

In the prospective studies, the examination of pre-service teachers' SMK and PCK of geometry can be instructive in the reorganization to be carried out in the teacher-training curriculum. In these curriculums, some courses intended for improving pre-service teachers' SMK and PCK could be provided. Because it is important for pre-service teachers to experience the instructional approaches, which they are expected to employ, primarily themselves in terms of their learning, the environments that will enable them to practice in the lessons should be created.

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