Çukurova Üniversitesi İİBF Dergisi

Cilt:17.Sayı:2.Aralık 2013
ss.15-28

Predicting the Direction of Gold Price Returns: Integrating Composite Artificial Neural Network Models by Markov Chain Process

Altın Fiyatına ait Getiri Yönünün Tahmini: Bileşik Yapay Sinir Ağları Yöntemi ile Markov Zincirleri Sürecinin Bütünleştirilmesi

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ABSTRACT

In this study, we first modeled daily gold returns as the discrete state Markov chain process, and second we trained an Artificial Neural Network (ANN) model in order to estimate direction of gold return. The trained model provides valuable information about the direction of next day return.

Keywords: Gold price returns, Artificial Neural Network Model, Markov process

ÖZET

Bu çalışmada, öncelikle günlük ortalama altın fiyatına ait getiriler Markov zincirleri olarak modellenmiş ve daha sonra bir sonraki gün için getirinin yönünü tahmin eden Yapay Sinir Ağı Modelleri tahmin edilmiştir. Tahmin edilen modeller bir sonraki güne ait getirinin yönü konusunda önemli bilgiler vermektedir.

Anahtar Kelimeler: Altın Fiyatına Ait Getiriler, Yapay Sinir Ağı Modeli, Markov Süreci

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1.Introduction

Forecasting gold price return is very important for individual and corporate investors and also for governments. Recently many of the considerable studies used neural networks models for forecasting gold prices. Mirmirani and Li (2004) suggested that neural networks and genetic algorithm systems provide better predictions when compared with traditional econometric models. Parisi et al. (2008), analyzed recursive and rolling neural network models to forecast one-step-ahead sign variations in gold price. Different combinations of techniques and sample sizes are studied for feed forward and ward neural networks. Their results showed the rolling ward networks exceed the recursive ward networks and feed forward networks in forecasting gold price sign variation. Their results support the use of neural networks with a dynamic framework to forecast the gold price sign variations, recalculating the weights of the network on a period-by-period basis, through a rolling process. They were validated using the block bootstrap methodology with an average sign prediction of 60.68% with a standard deviation of 2.82% for the rolling ward net. Ismail et al. (2009), estimated two multiple linear regression model. The first model considered all possible independent variables which considered having influence on the gold prices such as Commodity Research Bureau future index, USD/Euro Foreign Exchange Rate, Inflation rate, Money Supply (M1), New York Stock Exchange, Standard and Poor 500, Treasury Bill and US Dollar index. They stated that the first model was appeared to be useful for predicting the price of gold with 85.2% of sample variations in monthly gold prices. The second model considered the following four independent variables the Commodity Research Bureau future index (lagged one), USD/Euro Foreign Exchange Rate (lagged one), Inflation rate (lagged two) and M1 (lagged two) to be significant. They also stated that, in terms of prediction, the second model achieved high level of predictive accuracy. The amount of variance explained was about 70% and the regression coefficients also provide a means of assessing the relative importance of individual variables in the overall prediction of gold price. Xu, L-P. & Luo, M-Zo (2011) stated that after 2008 international financial crisis, the price of international gold shows a V-mode fluctuation, reaching the historical peak of $1, 400 per ounce. They stated that the gold price has been rising, which will exert great influences on investment decisions of their government, enterprises and individuals. They used gold monthly price in London from January 1973 to November 2010 to establish ARIMA model and make analysis and prediction of the gold price trend in the first half of 2011. They conclude that international gold price will continue to rise in the short term, and provides policy-making suggestion to adjustment of their nation's foreign capital reserve structure and accumulation of gold reserve. Xu (2011) used the gray prediction method and attempted to establish a form from the limited size of the Chinese gold futures price data sample. Xu stated that the model fits well, and it is able to describe the sample data pointed on the continuous time, with good prediction results, and the model could serve as a guiding reference on the short-term changes of China gold futures price. Askari and Askari (2011) have been investigated the accuracies of two different grey models include original GM (1, 1) and modified GM (1, 1) using Fourier series. They have been compared the performance of these models with ARIMA as a conventional forecasting model. Numerical results were showed that the modified GM (1, 1) provides better performance in model fitting and model forecasting. Zhou et al. (2012), proposed an improved EMD meta-learning rate-based model for gold price forecasting. First, they adopt the EMD method to divide the time series data into different subsets. Second, a back-propagation neural network model (BPNN) is used to function as the prediction model in their system. they update the online learning rate of BPNN instantly as well as the weight matrix. Finally, a rating method was used to identify the most suitable BPNN model for further prediction. Their experiment results show that system has a good forecasting performance. In the study of Yazdani-Chamzini et al. (2012), adaptive neuro-fuzzy inference system (ANFIS) and artificial neural network (ANN) model have been used for modeling the gold price, and compared with the traditional statistical model of ARIMA (autoregressive integrated moving average). The three performance measures, the coefficient of determination (R 2), root mean squared error (RMSE), mean absolute error (MAE), are utilized to evaluate the performances of different models developed. The results of the study showed that the ANFIS model outperforms other models (i.e. ANN and ARIMA model), in terms of different performance criteria during the training and validation phases. Their sensitivity analysis showed that the gold price changes are highly dependent upon the values of silver price and oil price**.** Miswan et al. (2013) model and forecast the prices of Malaysian gold called *kijang emas* using Box-Jenkins methodology. To find the best model, They applied parameter estimates using ordinary least squares and maximum likelihood estimates were computed. Based on the Akaike information criteria and mean absolute percentage error, they stated that the model estimated with OLS was found to perform better.

In this study, I applied the same methodology that I used in my previous study (Kılıç, 2013) which was integrated ANN models by Markov chain process to forecast the direction of Turkish Lira/US Dollar exchange rate returns. By using the same methodology, the daily gold price returns are modeled as a stochastic process with sixthteen discrete state spaces of Markov chain that the conditional probability of any next future return state depends only on the present return of the state and is independent any other states of the past returns. Thus, I transformed returns into sixthteen equal discrete categorical intervals of states, from high loss (negative return) to positive high return. Objective of modeling the returns as Markov chain process is to calculate probability of positive return for the next step (day) given the present state. These probabilities are used as inputs to the ANN models for training process for prediction of next day's return direction (positive/non positive).

We trained three ANN models; Model (0), Model (1) and Model (0,1). Classification results are given for the training and testing sample. In the study the whole sample data were randomly divided two equal groups as training and testing sample; training sample was used to train the models, testing sample was used to evaluate the models in terms of the classification achievements.

The rest of this article is organized as follows. Section 2 includes the sample selection, methodology and empirical results; section 3 concludes the article and discusses some future research perspectives.

2.The Sample, Methodology and Results

2.1.The Sample

The sample data covers 4226 daily average price of gold (Ons/Dolar) between the period of 08.08.1995- 31.05.2013. The data were obtained from the electronic data delivery system of the Central Bank of Turkey.

Daily returns of gold (R_t) are computed as a percentage change of the daily closing price of gold (P_t); $R_t = (P_t - P_{t-1})/P_{t-1}$. Here, *t* represents the days (*t*=1…4226). The average (expected) return is calculated (μ_R) as approximately 0.0004 with a standard deviation of (σ_R) 0.0104 for the period considered. The standard deviation is extremely high in comparison to expected return implying high volatility, and enormous risk for the investors.

2.2.The Methodology and Results

2.2.1.Modeling the gold returns as the discrete categorical states of Markov chain process

In the study, the daily returns are assumed to be a stochastic process with sixthteen discrete state spaces $\{S_1,..,S_{17}\}$ with Markov chain that conditional probability of any next future return state (S_j^t) depends only on the present return of the state (S_i^{t-1}) and is independent any other states of the past returns $P\left(S_j^t \middle| S_i^{t-1}\right)$.

As stated previously, objective of modeling the gold returns as discrete categorical states is to calculate probability of positive/non positive return for the next step (day) given the present state. In section 2.2.2, these probabilities are used as inputs to the ANN models for training process for prediction of next day's return direction. Thus, we transformed returns into sixthteen equal discrete categorical intervals of states, from high loss (negative return) to positive high return.

In Table 5 of Appendix A the total number of return transitions, occurring from the present day to the next day, from states S_i to S_j , were calculated for the whole period considered. We can easily compute the one step (one day) conditional transition probability matrix $P\left(S_j^t \middle| S_i^{t-1}\right)$ from Table 5 of Appendix A, from state *i* to *j* by dividing the row elements by row total. This matrix is given in Table 6 of Appendix A. Here, when the return in state S_1 in the present day, conditional probability of it will be going to S_{17} in the next day is $P(S_2^t \middle| S_3^{t-1}) =$ 0.0122. Similarly, conditional probability of passing from state S_9 to S_{13} is $P(S_2 \middle| S_4^{t-1}) = 0.0366$.

From Table 6 of Appendix A probability of non-positive return in the next day (step) given the present state *i* can be calculated by equation (1).

$$P(R_t \leq 0) \middle| S_i^{t-1} = \sum_{j=1}^{9} P\left[(S_{ij}^t) \middle| S_i^{t-1} \right] \qquad (1)$$

And probability of positive return is given the present state *i* can be calculated by equation (2).

$$P(R_t > 0) \middle| S_i^{t-1} = 1 - P\left[(R_t \leq 0) \middle| S_i^{t-1}) \right] \qquad (2)$$

Table 1 gives each of the states, return range, percentage of occurrence the states, and probability of positive return in the next day (step) given the present state for the period considered.

Table 1: States, return ranges, percentage of occurrence of states, and probability of positive return in the next step

Present states (S_i^{t-1})	**Return Range**	**% of occurrence of states**	**Probability of positive return for next step given the preset state** $P(R_t > 0) \mid S_i^{t-1}$
S_1	$-0,0104 \le R_t$	0,1011	0,4778
S_2	$-0,0104 < R_t \le -0,0091$	0,0194	0,4634
S_3	$-0,0091 < R_t \le -0,0078$	0,0267	0,4956
S_4	$-0,0078 < R_t \le -0,0065$	0,0388	0,4634
S_5	$-0,0065 < R_t \le -0,0052$	0,0353	0,5570
S_6	$-0,0052 < R_t \le -0,0039$	0,0452	0,4660
S_7	$-0,0039 < R_t \le -0,0026$	0,0774	0,4771
S_8	$-0,0026 < R_t \le -0,0013$	0,0575	0,4979
S_9	$-0,0013 < R_t \le 0,0000$	0,0902	0,5276
S_{10}	$0,0000 < R_t \le 0,0013$	0,0750	0,5426
S_{11}	$0,0013 < R_t \le 0,0026$	0,0663	0,5321
S_{12}	$0,0026 < R_t \le 0,0039$	0,0679	0,5157
S_{13}	$0,0039 < R_t \le 0,0052$	0,0445	0,5213
S_{14}	$0,0052 < R_t \le 0,0065$	0,0445	0,5053
S_{15}	$0,0065 < R_t \le 0,0078$	0,0417	0,5568
S_{16}	$0,0078 < R_t \le 0,0091$	0,0270	0,5088
S_{17}	$R_t > 0,0091$	0,1415	0,5117

2.2.2.Training of the ANN models

By using the information (probabilities of positive or non positive return in the next step given the present state i) provided by Markov chain process in the previous section, we trained three ANN models; Model (0), Model(1) and Model(0,1).

Table 2: Inputs and outputs of the trained models

	Inputs	Output
Model (0)	$R_{t-1} \le 0$	$y'_t = 0$
	$P(R_t \le 0) \mid S_i^{t-1}$	$y'_t = 1$
Model (1)	$R_{t-1} > 0$	$y'_t = 0$
	$P(R_t > 0) \mid S_i^{t-1}$	$y'_t = 1$
Model (0,1)	R_{t-1}	$y'_t = 0$
	$P(R_t > 0) \mid S_i^{t-1}$	$y'_t = 1$

Inputs of the trained models are given in Table 2. Inputs of the Model (0) are non-positive return of the present state and probability of non-positive return in the next state given the present state. Inputs of the Model (1) are positive return of the present state and probability of positive return in the next state given the present state. Inputs of the Model (0,1) are present state return (either positive or non-positive) and probability of positive return in the next day given the present state i .

Outputs of the three models are the same; prediction of either non-positive or positive returns for the next state and defined by following function:

$$y_i^{'} = \begin{cases} 0, & if\ R_t \le 0 \\ 1, & if\ R\ \ > 1 \end{cases} \tag{3}$$

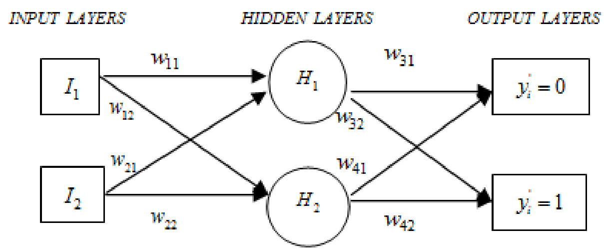

Figure 1: Network diagrams of the estimated ANN models

All of the three models trained have the same architecture that is given Figure 1. The ANN models consist of two input and output nodes in the input and output layer, and one hidden layer with two nodes between input and output layer.

By using gradient decent multilayer perceptron procedure (Kılıç, 2013), the three models were trained. Adjusted connection weights for Model (0), Model (1) and Model(0,1) are given in the Table 3. For example, for Model (0) connection weights between input node I_1 and hidden layer node H_1 and H_2 (w_{11}, w_{12}) are 0.287 and 1.130 respectively. Weights between hidden layer node H_1 and output node $y_i^{'} = 0$ and $y_i^{'} = 1$ are -0.0370 and -0.6580 respectively. Weights between hidden layer node H_2 and output layer nodes $y_i^{'} = 0$ and $y_i^{'} = 0$ are 0.007 and 0.330 respectively.

Table 3: Adjusted weights for the three ANN models

Model (0)		H_1	H_2	$y_i' = 0$	$y_i' = 1$
Input Layer	I_1	,287	,130		
	I_2		-,452		
Hidden Layer 1	H_1			-,238	,155
	H_2			,007	,330
Model (1)					
Input Layer	I_1	,419	,442		
	I_2	,435	-,027		
Hidden Layer 1	H_1			,237	,331
	H_2			,378	,157
Model (0.1)					
Input Layer	I_1	,188	-,146		
	I_2	,366	-,231		
Hidden Layer 1	H_1			,195	,406
	H_2			,400	,304

Table 4 gives observed and predicted classification results of daily direction of gold returns by the estimated three ANNs model. Classification results are given for the training and testing sample. In the study the whole sample data were randomly divided two equal groups as training and testing sample; training sample was used to train (estimate) the models, testing sample was used to evaluate the models in terms of the classification achievements.

In last column of Table 4 for the testing sample we can see that if the present return is non-positive, the Model (0) predicts next day non positive direction ($R_t \leq 0$) 67.5% correctly for testing sample. If the present return is positive the Model (1) predicts next day positive direction ($R_t > 0$) 98.5% correctly.

However, if the present return is positive, the Model (1) does not accurately predicts negative direction (1.8%). If the present return is either positive or non-positive the Model (0,1) correctly predicts next day non-positive return 31.6%. In order to eliminate these unreliable predictions and to make more accurate prediction, the three models can be integrated together for the prediction process.

Table 4: Classification achievement of estimated ANN models

		Predicted		
Model (0)	Observed	*D≤0*	*D>1*	Correct classification (%)
Training	*D≤0*	296	136	68,5
	D>1	317	176	35,7
	Overall	66,3%	33,7%	51,0
Testing	*D≤0*	355	171	67,5
	D>1	336	192	36,4
	Overall	65,6%	34,4%	51,9
Model (1)				
Training	*D≤0*	10	679	1,5
	D>1	11	822	98,7
	Overall	1,4%	98,6%	54,7
Testing				
	D≤0	6	326	1,8
	D>1	6	387	98,5
	Overall	1,7%	98,3%	54,2
Model(0.1)				
Training	*D≤0*	287	721	28,5
	D>1	285	818	74,2
	Overall	27,1%	72,9%	52,3
Testing	*D≤0*	307	664	31,6
	D>1	312	832	72,7
	Overall	29,3%	70,7%	53,9%

Flowchart of integrated use of these models in prediction process is given in Figure 2. Here, if the present return is positive the Model (1) should be used. If the Model (1) predicts positive return, prediction is 98.5% correct, stop the prediction. If the Model (1) predicts non-positive return do not use model (1); use model (0,1). If Model (0,1) predicts positive direction it is prediction 72.7% correct, stop prediction. If Model (0,1) predicts non-positive direction it is 31.6% correct stop prediction.

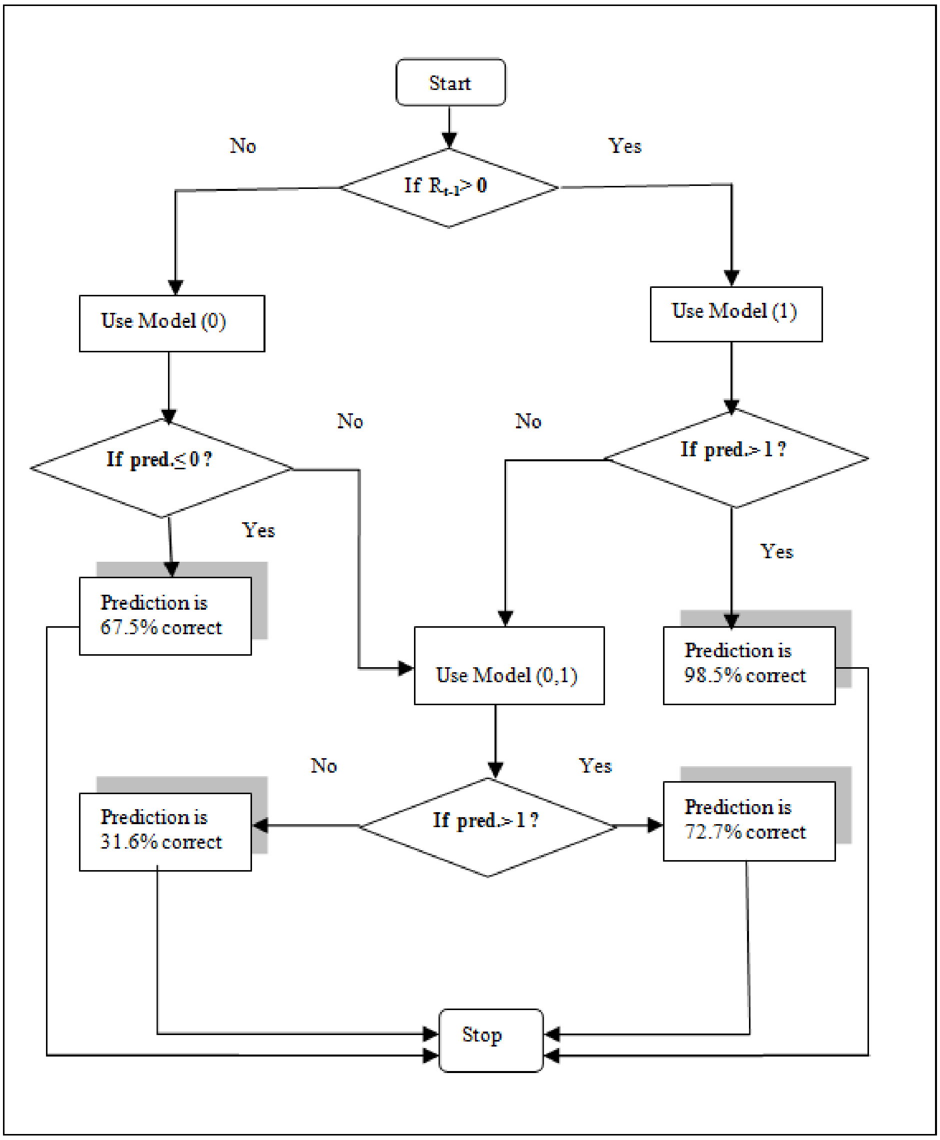

Figure 2: Flowchart of using the three combined models for prediction

If the present return is non-positive the Model (0) should be used. If the Model (0) predicts non-positive return, prediction is 67.5% correct stop the prediction. If the

Model (0) predicts positive return, do not use Model (0); use Model (01). If the prediction is positive return it is 72.7% correct, stop the prediction. Otherwise prediction is 31.6% correct.

Overall average integrated use of the three models provides 67.50% correct prediction for the direction of returns. This means that if an investor determines his/her daily buying-selling strategy according to the prediction of the integrated models, his/her daily investment strategy in gold could be profitable 67.50% of the time in the long run.

3. Conclusion

This study combined Markov chain process with the ANN models. Composite use of ANN models provides valuable information about daily direction of the gold return given the present state.

In our previous study (Kılıç, 2013) which used the same methodology for forecasting the direction of TL/Dollar exchange rate return provided 65.8% correct prediction for the direction of returns. This study provided little more prediction accuracy (67.5%) for gold price returns.

Finally, this study uses daily average returns and first order Markov chain process. Further similar analysis can also be performed by considering second order Markov chain process for gold prices by considering returns of smaller time intervals, such as an intraday hourly change which may provide more information for the direction of next day return.

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Appendix A

Table 5: Number of occurrence of the transitions from states S_i to S_j

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	*Row total*
S_1	77	7	16	13	17	21	16	21	35	21	21	21	21	18	16	13	73	427
S_2	14	1	1	3	4	8	4	2	7	2	2	6	2	7	6	1	12	82
S_3	16	4	3	3	6	5	10	4	6	5	9	6	10	5	4	0	17	113
S_4	25	4	3	9	6	6	17	4	14	14	4	16	6	4	9	5	18	164
S_5	15	1	6	7	7	7	6	9	8	17	14	11	2	4	7	6	22	149
S_6	25	7	3	10	7	9	11	18	12	10	15	12	13	9	6	5	19	191
S_7	30	8	8	9	14	13	31	19	39	24	24	29	8	15	13	4	39	327
S_8	26	4	2	5	6	15	17	18	29	28	19	12	9	10	10	4	29	243
S_9	20	7	9	17	9	12	33	23	50	31	34	32	19	16	19	13	37	381
S_{10}	19	8	7	12	6	12	26	29	26	29	18	22	20	13	14	10	46	317
S_{11}	22	4	7	9	6	12	31	19	21	27	25	18	10	13	14	6	36	280
S_{12}	19	5	6	7	9	8	37	16	32	24	18	23	15	16	4	6	42	287
S_{13}	13	3	5	8	4	11	21	6	19	15	10	9	8	11	9	10	26	188
S_{14}	21	2	4	7	12	11	12	14	10	13	13	10	6	7	9	8	29	188
S_{15}	9	4	11	8	4	6	15	5	16	13	8	18	10	7	11	2	29	176
S_{16}	7	1	8	2	8	6	4	7	13	7	9	9	4	1	3	7	18	114
S_{17}	69	12	14	35	24	29	36	29	44	37	37	33	25	32	22	14	106	598

Table 6 : One step conditional transition probability matrix from S_i to S_j

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	Row total
S_1	0,1803	0,0164	0,0375	0,0304	0,0398	0,0492	0,0375	0,0492	0,0820	0,0492	0,0492	0,0492	0,0492	0,0422	0,0375	0,0304	0,1710	1
S_2	0,1707	0,0122	0,0122	0,0366	0,0488	0,0976	0,0488	0,0244	0,0854	0,0244	0,0244	0,0732	0,0244	0,0854	0,0732	0,0122	0,1463	1
S_3	0,1416	0,0354	0,0265	0,0265	0,0531	0,0442	0,0885	0,0354	0,0531	0,0442	0,0796	0,0531	0,0885	0,0442	0,0354	0,0000	0,1504	1
S_4	0,1524	0,0244	0,0183	0,0549	0,0366	0,0366	0,1037	0,0244	0,0854	0,0854	0,0244	0,0976	0,0366	0,0244	0,0549	0,0305	0,1098	1
S_5	0,1007	0,0067	0,0403	0,0470	0,0470	0,0470	0,0403	0,0604	0,0537	0,1141	0,0940	0,0738	0,0134	0,0268	0,0470	0,0403	0,1477	1
S_6	0,1309	0,0366	0,0157	0,0524	0,0366	0,0471	0,0576	0,0942	0,0628	0,0524	0,0785	0,0628	0,0681	0,0471	0,0314	0,0262	0,0995	1
S_7	0,0917	0,0245	0,0245	0,0275	0,0428	0,0398	0,0948	0,0581	0,1193	0,0734	0,0734	0,0887	0,0245	0,0459	0,0398	0,0122	0,1193	1
S_8	0,1070	0,0165	0,0082	0,0206	0,0247	0,0617	0,0700	0,0741	0,1193	0,1152	0,0782	0,0494	0,0370	0,0412	0,0412	0,0165	0,1193	1
S_9	0,0525	0,0184	0,0236	0,0446	0,0236	0,0315	0,0866	0,0604	0,1312	0,0814	0,0892	0,0840	0,0499	0,0420	0,0499	0,0341	0,0971	1
S_{10}	0,0599	0,0252	0,0221	0,0379	0,0189	0,0379	0,0820	0,0915	0,0820	0,0915	0,0568	0,0694	0,0631	0,0410	0,0442	0,0315	0,1451	1
S_{11}	0,0786	0,0143	0,0250	0,0321	0,0214	0,0429	0,1107	0,0679	0,0750	0,0964	0,0893	0,0643	0,0357	0,0464	0,0500	0,0214	0,1286	1
S_{12}	0,0662	0,0174	0,0209	0,0244	0,0314	0,0279	0,1289	0,0557	0,1115	0,0836	0,0627	0,0801	0,0523	0,0557	0,0139	0,0209	0,1463	1
S_{13}	0,0691	0,0160	0,0266	0,0426	0,0213	0,0585	0,1117	0,0319	0,1011	0,0798	0,0532	0,0479	0,0426	0,0585	0,0479	0,0532	0,1383	1
S_{14}	0,1117	0,0106	0,0213	0,0372	0,0638	0,0585	0,0638	0,0745	0,0532	0,0691	0,0691	0,0532	0,0319	0,0372	0,0479	0,0426	0,1543	1
S_{15}	0,0511	0,0227	0,0625	0,0455	0,0227	0,0341	0,0852	0,0284	0,0909	0,0739	0,0455	0,1023	0,0568	0,0398	0,0625	0,0114	0,1648	1
S_{16}	0,0614	0,0088	0,0702	0,0175	0,0702	0,0526	0,0351	0,0614	0,1140	0,0614	0,0789	0,0789	0,0351	0,0088	0,0263	0,0614	0,1579	1
S_{17}	0,1154	0,0201	0,0234	0,0585	0,0401	0,0485	0,0602	0,0485	0,0736	0,0619	0,0619	0,0552	0,0418	0,0535	0,0368	0,0234	0,1773	1