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# Calculation of earthwork volume with Artificial Neural Networks (ANN)

*Yapay Sinir Ağları (YSA) ile hafriyat hacim hesabı*

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**Abstract**

Volume calculations are critical for cost analysis, planning and environmental sustainability in disciplines such as civil engineering, environmental engineering, geodesy and mining. In this study, volume values based on digital elevation models (DEMs) generated by various methods were compared to examine the accuracy of earthwork volume calculations. In the DEM generation process, polynomials and multiquadric interpolation methods, which are traditional methods, and feedforward neural network (FFNN) and radial basis neural network (RBFNN), which are artificial neural network methods, were used. Within the scope of the study, earthwork volume calculations were performed using the DEMs produced by these methods and a detailed evaluation of each method was carried out in terms of accuracy, performance and computation time. The results show that artificial neural network based methods provide good accuracy and consistency compared to conventional methods, especially in complex topographies. These findings provide an important contribution to more accurate cost estimation and better assessment of environmental impacts in earthworks projects.

**Keywords:** Artificial neural networks, Interpolation, Volume

***Öz***

*Hacim hesaplamaları, inşaat mühendisliği, çevre mühendisliği, jeodezi, madencilik gibi disiplinlerde maliyet analizi, planlama ve çevresel sürdürülebilirlik için kritik bir öneme sahiptir. Bu çalışmada, hafriyat hacim hesaplamalarının doğruluğunu incelemek amacıyla, çeşitli yöntemlerle üretilen sayısal yükseklik modellerine (SYM) dayalı hacim değerleri karşılaştırılmıştır. SYM oluşturma sürecinde, geleneksel yöntemler olan polinomlar ve multikuadrik enterpolasyon yöntemleri ile yapay sinir ağları yöntemlerinden ileri beslemeli yapay sinir ağı (İBYSA) ve radyal tabanlı yapay sinir ağı (RTYSA) kullanılmıştır. Çalışma kapsamında, bu yöntemlerle üretilen SYM'ler kullanılarak toprak hacim hesaplamaları gerçekleştirilmiş ve her bir yöntemin doğruluk, performans ve hesaplama süresi açısından detaylı bir değerlendirmesi yapılmıştır. Elde edilen sonuçlar, yapay sinir ağı tabanlı yöntemlerin, özellikle karmaşık topoğrafyalarda, geleneksel yöntemlere kıyasla iyi bir doğruluk ve tutarlılık sağladığını ortaya koymuştur. Bu bulgular, hafriyat projelerinde maliyet tahminlerinin daha doğru yapılabilmesi ve çevresel etkilerin daha iyi değerlendirilebilmesi açısından önemli bir katkı sağlamaktadır.*

**Anahtar kelimeler**: *Yapay sinir ağları, Enterpolasyon, Hacim*

## 1. Introduction

Volume calculations are of great importance for project planning, cost analysis and environmental sustainability applications in civil engineering, environmental engineering, geodesy, mining and related disciplines. Especially in earthworks projects, the accuracy of volume calculations plays a decisive role in critical processes such as determining project costs, optimising resource utilisation and minimising environmental impact. The accuracy and reliability of the methods used in volume calculations are therefore of great importance.

Digital Elevation Models (DEMs) are used as the basic tool for such calculations and provide a three-dimensional representation of the Earth's topography, allowing volume calculations to be performed. DEMs can be produced using a variety of interpolation and modelling methods, and the choice of these methods has a direct impact on the accuracy of the results obtained.

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A review of the literature on volume calculation reveals that Easa (1988) and Chambers (1989) proposed a Simpson-based method, Chen & Lin (1991) introduced a cubic spline method, and Easa (1998) developed a method based on the cubic Hermite polynomial. Additionally, Mukherji (2012) employed finite element analysis, Yanalak (2005) utilized interpolation methods, and Khalil (2014) applied multiple regression analysis techniques. Yakar and Yılmaz (2008) utilised digital close range photogrammetry to calculate the volume of a natural hill, while Yilmaz (2017) evaluated the performance of interpolation methods in calculating the volume of earthworks in the field. Furthermore, the use of unmanned aerial vehicles (UAVs) in earthworks calculations has been extensively investigated in the studies conducted by Akgul et al. (2018), Ulvi (2018), Şahin and Yılmaz (2021), Lee and Lee (2022), and Hasegawa (2023).

In this study, the accuracy of earthwork volume calculations was examined by performing calculations based on DEMs produced by classical and flexible calculation methods. Polynomials and multiquadratic interpolation methods were used as classical methods, while artificial neural network-based approaches such as FFNN and RBFNN were evaluated within the scope of flexible computing methods. Each of these methods was applied to a theoretical test area with a complex topographic structure, and volume calculations were performed using the DEMs obtained. The primary objective of the present study is to undertake a comparative analysis of these methodologies with regard to accuracy, performance and computation time, and to provide a framework for determining the most appropriate methodology to be employed in earthwork projects. Moreover, this study offers an original contribution by systematically comparing traditional interpolation techniques with artificial neural network-based methods for DEM-based volume estimation, a perspective that has been limited or lacking in the existing literature. The findings indicated that neural network-based methods exhibited superior accuracy and consistency in comparison to conventional methods, particularly in the context of complex topographies. These results provide an important contribution both in terms of improving cost estimation in engineering applications and evaluating environmental impacts more effectively.

## 2. Methods

### 2.1. Polynomial interpolation

In this method, the land surface can be expressed by *n*th order orthogonal or non-orthogonal polynomials formed by reference points with known (x,y,z) coordinates. Since non-orthogonal polynomial is used in this study, the relevant expression is as follows:

$$z(x,y) = \sum_{i=0}^{n} \sum_{j=0}^{n} a_{ij}\, x^i y^j \qquad (1)$$

$a_{ij}$, denotes the unknown coefficients of the polynomial and n denotes the degree of the surface. In the solution of polynomial expressions, if the number of reference points is greater than the unknowns, the unknown coefficients of the polynomial are calculated according to the least squares method (İnal & Yiğit, 2004). Thus, the height value of the interpolation point is easily calculated with the polynomial whose coefficients are determined.

### 2.2. Multiquadric interpolation

In this method, which aims to express the land surface with a single function, a trend surface is first passed through the study area, after which residual height values ($\Delta z_i$) are calculated (İnal & Yiğit, 2004, Yanalak and Baykal, 2001). The most general equation of the multiquadric surface,

$$\Delta z = \sum_{i=1}^{n} c_i [Q(x_i, y_i, x, y)] \qquad (2)$$

$\Delta z$ is the sum of n number of $Q$ surfaces which are functions of $y$ and $x$. The series of circular hyperboloids in two leaves are utilised as $Q$ surfaces in this study. After the coefficients $c_i$ are calculated by creating a system of linear equations based on the values ($x_i$, $y_i$, $\Delta z_i$) of $n$ number of reference points, the $z_0$ value of any interpolation point whose position is known is calculated from the following equation:

$$z_0 = z(x_0,\, y_0) + \Delta z_0 = \sum_{i=1}^{n} c_i [(x_i - x_0)^2 + (y_i - y_0)^2 + \delta^2]^{1/2} \qquad (3)$$

The geometric parameter $\delta$, which is a fixed number determined by the user, is indicative of the surface's smoothness or sharpness (Güler, 1985; Hardy, 1971, 1990).

### 2.3. Artificial neural networks (ANN)

ANNs are inspired by the human nervous system and have the ability to make predictions from data through the learning process. In this way, ANNs are used in tasks such as data analysis, prediction and classification with high accuracy rates and are at the centre of artificial intelligence studies due to these advantages. In this study, FFNN and RBFNN methods were used.

#### 2.3.1. Feedforward neural network (FFNN)

FFNNs, also known as multilayer perceptrons (MLPs), are one of the most basic and widely used types of artificial neural networks. In an FFNN, data travels in a single direction from the input layer to hidden layer and finally to the output layer. As illustrated in Figure 1 (a), the network structure comprises interconnected layers, namely the input layer, one or more hidden layers, and the output layer. Each layer consists of multiple neurons or nodes that apply a weighted sum of the inputs followed by a non-linear activation function. This structure enhances the ability of FFNNs to model complex, non-linear relationships between inputs and outputs.

The aim of ANNs is to define the association in the sample input-output data set and to adjust the weight and bias values between neurons with the selected training algorithms. Thus, the network can generate output values for new test data. In the training of the network, an input data set is first presented to the network and these data are transmitted through the network in the forward direction (from the input layer to the outputlayer). In each layer, neurons multiply the inputs by weights, add bias and pass them through an activation function. This process produces the output of the network. Then, the error between this output and the expected output (target value) is calculated. In the backpropagation stage, the error is propagated in a backwards direction, and the weights are updated to minimise the error. The training process is repeated until a specified number of iterations is reached, or until the error is sufficiently small (Rumelhart et al., 1985; Haykin, 1999).

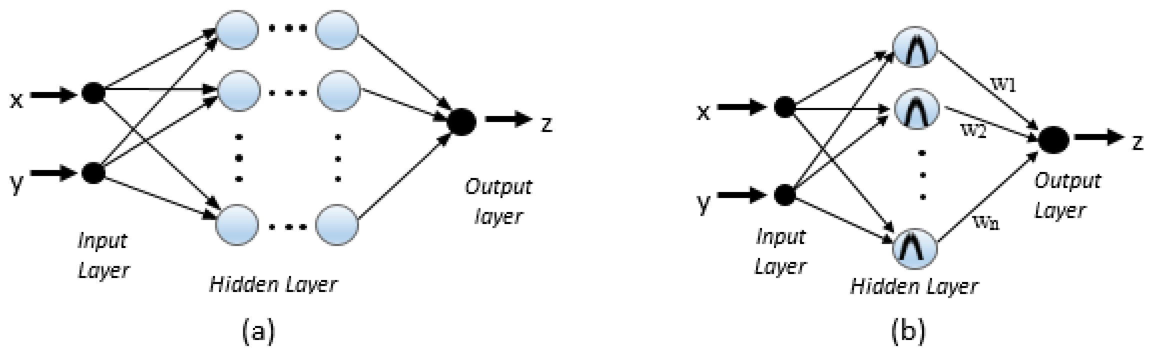



**Figure 1**. a) FFNN and b) RBFNN structures

#### 2.3.2. Radial basis function neural network (RBFNN)

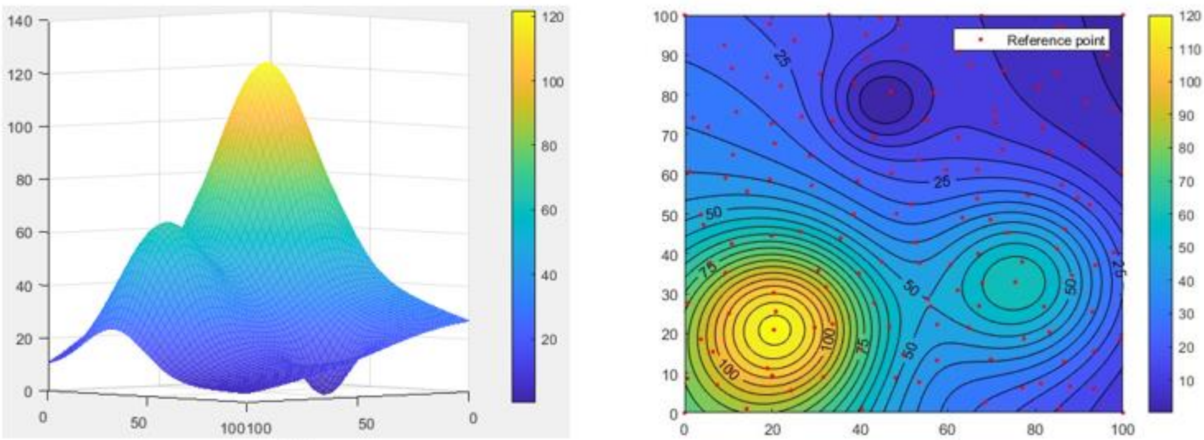
RBFNN is a type of feed-forward artificial neural network that has been employed extensively in a variety of applications. A distinctive feature of RBFNN is the use of radial basis activation functions in its hidden layers, as illustrated in Figure 1 (b). Furthermore, the number of radially symmetric hidden layers in RBFNN cannot be more than one.

In this network structure, while the transformation from the intermediate layer to the output layer is linear, the transformation from the input layer to the intermediate layer is non-linear due to the radial-based activation functions used. This feature enables RBF networks to effectively model non-linear problems. The processing elements in the input layer transmit the input data directly to the hidden layer without any processing. The outputs of the processing elements in the hidden layer compute a function of the distance between the ANN inputs and the centre of the radial function (which is usually a Gaussian function). Finally, the output layer produces the final result by summing the weighted outputs from the intermediate layer (Moody & Darken, 1989; Park & Sandberg, 1991).

## 3. Application and results

In this study, a function as in Eq. (4) was utilised as the test area. This function is a revised version of a functional test surface employed by Franke (1979As shown in Figure 2(a), the test surface, which covers an area of 100m*100m with elevations ranging from 1 to 121 metres, exhibits a complex topography characterised by two hills and a pit. In practice, firstly, 100 scattered points in a homogeneous distribution representing the state of the model were determined as reference points. The distribution of these points on the surface is illustrated in Figure 2(b). Based on these reference points, DEMs were created with polynomials, multiquadric interpolation, FFNN and RBFNN methods at 1, 5 and 10m grid intervals. Then, the reference surface was considered as zero and volume calculations were made according to the trapezoidal method. All computations and figure plotting were performed in the MATLAB R2023b environment.

$$z(x,y) = \begin{pmatrix} 0.75exp\left[\frac{\left(\frac{9x}{100}-2\right)^2+\left(\frac{9y}{100}-2\right)^2}{4}\right] + 0.75exp\left[\frac{\left(\frac{9x}{100}+1\right)^2}{49} - \frac{\left(\frac{9y}{100}+1\right)^2}{10}\right] \\ +0.5exp\left[-\frac{\left(\frac{9x}{100}-7\right)^2+\left(\frac{9y}{100}-3\right)^2}{4}\right] - 0.2exp\left[-(\frac{9x}{100}-4)^2-(\frac{9y}{100}-7)^2\right] \end{pmatrix} * 100 \qquad (4)$$




(a) (b)

**Figure 2.** (a) The perspective view of the test surface and (b) distribution of reference points

The functionally expressed test surface offers a significant advantage in the calculation of the actual volume value of complex and irregular geometries. Thanks to this approach, the actual volume value of the test surface ($V_{actual}$) was determined as 406970 m³ by taking the integral of the test surface. This result allows the calculation of the actual error and relative error values as in Eq. (5) for the volume values obtained from DEMs generated by different methods for different grid spacings.

$$E_{relative}(\%) = \left(\frac{V_{estimated}-V_{actual}}{V_{actual}}\right) * 100 \qquad (5)$$

In the polynomial interpolation method for the test area, non-orthogonal polynomials of 5th, 6th and 7th degree were utilised to generate DEMs for three different grid intervals, and volume values were calculated. Upon analysis of the results presented in Table 1, it is evident that the volumetric results obtained using the 7th degree polynomial are more accurate than the others. Consequently, while the relative errors of the volume values calculated with the 7th degree polynomial at 1, 5 and 10 m grid spacing in Table 1 are 0.10%, 0.24% and 0.56%, respectively, the volume differences in Figure 3 are 413 m³, 990 m³ and 2285 m³.

In the study, the relative errors of the volume values calculated with the DEMs generated at 1, 5 and 10 m grid intervals using the multiquadric interpolation method are calculated as 0.04%, 0.06% and 0.35%, respectively (see Table 1). Furthermore, Figure 3 demonstrates that the volume differences are 158 m³, 242 m³ and 1416 m³ at three different grid intervals, respectively. When the volume differences calculated by two different interpolation methods are compared, it is understood that the multiquadric interpolation method produces more reliable and accurate results than polynomials.

**Table 1.** Statistical results obtained from volume calculations

| Method | Grid interval | | | | | |
|---|---|---|---|---|---|---|
| | 1m | | 5m | | 10m | |
| | Volume ($m^3$) | Error % | Volume ($m^3$) | Error % | Volume ($m^3$) | Error % |
| 5th degree polynomial interp. | 409422 | 0.60 | 407902 | 0.23 | 403113 | 0.95 |
| 6th degree polynomial interp. | 408456 | 0.37 | 408824 | 0.46 | 409572 | 0.64 |
| 7th degree polynomial interp. | 407383 | 0.10 | 407960 | 0.24 | 409255 | 0.56 |
| Multiquadric interp. | 407128 | 0.04 | 406728 | 0.06 | 405554 | 0.35 |
| RBFNN | 406923 | 0.01 | 406878 | 0.02 | 404979 | 0.50 |
| FFNN | 406943 | 0.01 | 406817 | 0.04 | 406334 | 0.16 |

The choice of network architecture and input parameters is very important in modelling the test surface with FFNN. In this application, (*x,y*) data is used as input information and (*z*) height value is used as output information. The optimal network architecture for FFNN was identified through a systematic trial-and-error approach. The following parameters were determined as a result of the experiments: Levenberg-Marquardt learning algorithm, learning rate 0.02 and momentum coefficient 0.9, two intermediate layers. The architecture of FFNN is 2:15:10:1. According to this expression, there are 15 neurons in the first interlayer and 10 neurons in the second interlayer. It was observed that hyperbolic tangent sigmoid, logarithmic sigmoid and linear functions yielded superior results, respectively. Consequently, the differences in volume values of 1, 5 and 10 m grid intervals obtained with the FFNN model, which demonstrated optimal performance in the study area, were 27 $m^3$, 153 $m^3$ and 1099 $m^3$, respectively, and the relative errors were calculated as 0.01%, 0.04% and 0.16%.

In the process of DEM generation using the RBFNN method, the most widely used gaussian function in the literature is used as a radial basis function. There are (*x,y*) in the input layer and height value (*z*) in the output layer. In the repeated trials, the optimum network architecture was determined as 2:40:1 for DEM with 1m grid and 2:51:1 for DEM with 5 and 10m grid. In addition, since the variation of the propagation parameter ($\delta$) of the Gaussian function has a significant effect on the results, the value of the propagation parameter was determined as 0.225 at the end of repeated trials. As a result, the differences of the volume values of 1, 5 and 10 m grid spacing obtained with the RBFNN model, which showed the best performance in the study area, were 47$m^3$, 93$m^3$ and 1991$m^3$, respectively, and the relative errors were calculated as 0.01%, 0.02% and 0.56%.

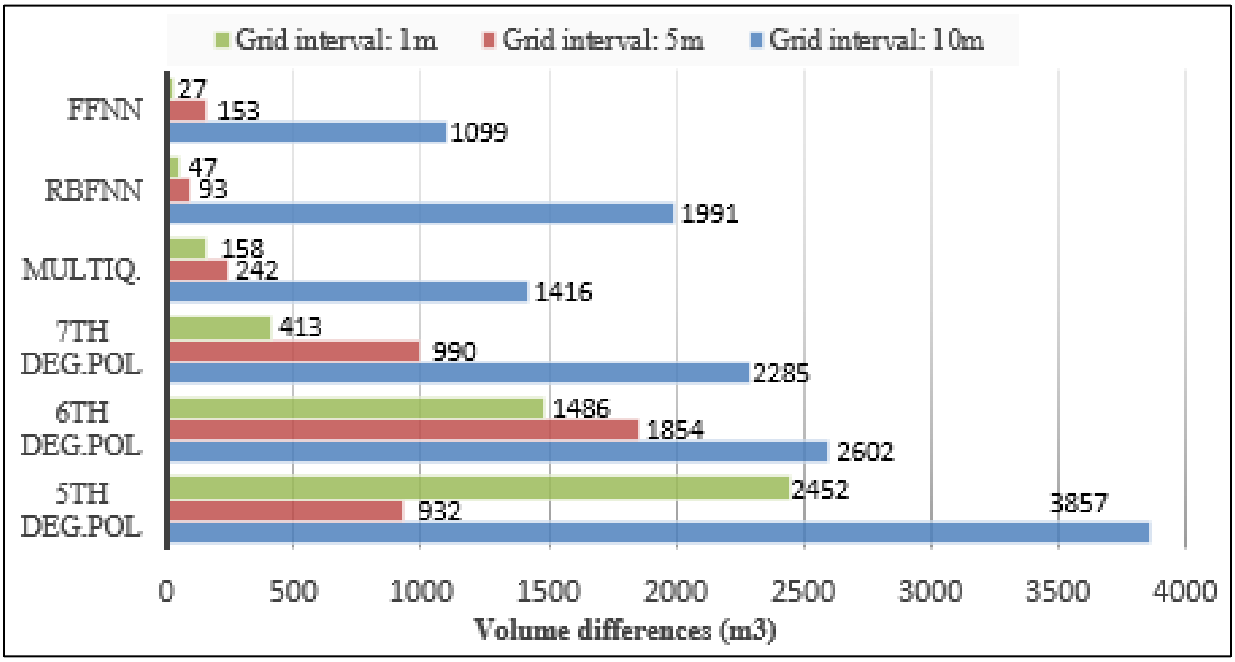



**Figure 3.** Differences obtained from volume calculations

## 4. Conclusion

This study emphasises the importance of accurate and precise calculations in many areas where volume calculations are widely used, and provides a concrete example of how artificial neural network-based approaches can create an impact in such applications. From this perspective, for a theoretical test surface, polynomials, multiquadric interpolation, FFNN, and RBFNN methods were employed to generate DEM, and volume values were determined according to 1, 5, and 10 m grid spacing. A general analysis of the study's results indicates the following key points:

-At small grid spacing, such as 1m, volume values are typically calculated with high accuracy.
-FFNN and RBFNN, which are artificial neural networks, demonstrate volume values with good accuracy in comparison to classical methods.
- Since the determination of the network architecture and the selection of input parameters in FFNN and RBFNN methods are generally based on trial and error, this process requires longer computation times compared to classical approaches.

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**Author contribution**

Leyla Cakir's contributions encompassed the selection of the research topic, the literature review, the methodology, the analysis, the interpretation of the results, and the writing. Nazan Yılmaz contributed to the interpretation of the results.

**Declaration of ethical code**

The authors of this article declare that the materials and methods used in this study do not require ethical committee approval and/or legal-specific permission.

**Conflicts of interest**

The authors declare that they have no conflict of interest.

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