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Terminal and Fast Terminal Sliding Mode Controls of a SISO Coupled Tank System: A Comparative Performance

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ABSTRACT

In this study, both a terminal sliding mode controller and a fast terminal sliding mode controller are devised with the aim of enhancing the stability of the liquid level within a single-input, single-output coupled tank system. The foundational elements of the proposed controllers are established through an overview of the mathematical model governing the coupled tank system, alongside the conventional sliding mode control method. In the conventional sliding mode control design, the target can be reached in infinite time. To overcome this problem and produce more efficient results, a terminal sliding mode controller has been designed. Furthermore, an advanced design for a fast terminal sliding mode controller is introduced, with the objective of achieving expedited convergence towards the target. This design extends upon the findings derived from the terminal sliding mode controller. Subsequently, a comparative analysis of the outcomes obtained from the three methodologies is conducted. Simulation outcomes corroborate the superior efficacy of the proposed approach over conventional sliding mode control methods in accurately regulating the desired liquid level.

SISO Bağlantılı Tank Sisteminin Terminal ve Hızlı Terminal Kayan Mod Kontrolleri: Karşılaştırmalı Bir Performans Çalışması

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Sorumlu Yazar

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ÖZ

Bu çalışmada, tek girişli, tek çıkışlı bir bağlı tank sistemine yönelik olarak hem bir terminal kayan kipli kontrolcü hem de hızlı terminal kayan kipli kontrolcü tasarlanmıştır. Önerilen kontrolcülerin temel unsurları, bağlı tank sistemini yöneten matematiksel modelin genel bir değerlendirmesi ile geleneksel kayan kipli kontrol yöntemi üzerinden belirlenmiştir. Geleneksel kayan kipli kontrol tasarımında, hedefe ulaşım sonsuz bir zaman alabilir. Bu sorunu aşmak ve daha verimli sonuçlar elde etmek için bir terminal kayan kipli kontrolcü tasarlanmıştır. Ayrıca, hedefe hızlı bir şekilde yaklaşmayı hedefleyen gelişmiş bir hızlı terminal kayan kipli kontrolcü tasarımı tanıtılmıştır. Bu tasarım, terminal kayan kipli kontrolcüden elde edilen bulgulara dayanmaktadır. Ardından, üç metodun elde edilen sonuçlarının karşılaştırmalı bir analizi yapılmıştır. Simülasyon sonuçları, önerilen yaklaşımın geleneksel kayan kipli kontrol yöntemlerine göre istenen sıvı seviyesini daha doğru bir şekilde düzenlemede üstün etkinliğini doğrulamaktadır.

1. INTRODUCTION

The coupled tank system, which is widely employed in laboratory environments and various industries, serves as a fundamental mechanism for liquid distribution [1]. It is possible to use the coupled tank systems in different ways with the model structure and tank features that can be modified based on specific requirements. Using pop-up valves positioned at the transitions between the tanks enables manipulation of the liquid flow direction and structural adjustments. Several studies have added value to the literature by exploring both the single input single output (SISO) and multi input multi output (MIMO) configurations of the coupled tank system [2-4].

Following the development of a controller employing an appropriate control law tailored for the system, the coupled tank system becomes readily deployable in experimental configurations. The primary objective of the designed controller is to regulate the liquid level within the tank, ensuring it stays within the desired range. This necessitates the generation of a precise control signal [5]. Controlling such a complex system is challenging. Numerous significant studies in existing literature demonstrate the control of liquid flow through various noteworthy controller designs [6-8]. The compatibility and proper design of controllers with the system are essential as they affect system performance. Controller performance can be evaluated by observing the values for rise time, steady-state error, settling time, and overshoot.

There are many noteworthy studies on liquid level control in the literature. Experiments in these studies have been carried out with different control methods such as the PID controller, backstepping controller, fuzzy controller, and sliding mode controller, resulting in successful outcomes [6,9-12].

Sliding mode control (SMC) stands out as a nonlinear control approach renowned for its ease of implementation and robustness [13]. The SMC is a variable structure control (VSC) based approach [14]. Following the popularization of SMC through the work of Vadim Utkin in the 1960s, its applications expanded, and studies were conducted combining the SMC law with different techniques [15]. SMC has garnered significant attention and utilization across various domains due to its robustness and ease of implementation. Its applicability extends to a wide array of complex systems, encompassing discrete-time, nonlinear, large-scale, and higher-order systems.

An example of the main benefits of SMC is that it is resistant to internal disturbances and parameter changes. However, unmodeled dynamics may destabilize the controller and damage the system, leading to a phenomenon known as chattering. Various approaches have been introduced to reduce the adverse effects of chattering [16].

Switching and equivalent control laws are used in the SMC design. In the process of crafting the switching control law, particular emphasis should be placed on the careful selection of the sliding surface coefficient and switching gain coefficients. These parameters play a crucial role in shaping the behavior and performance of the control system. A judicious choice of these coefficients can contribute significantly to the stability and effectiveness of the control law, ensuring robust performance in the face of uncertainties and disturbances. Insufficient attention to the meticulous selection of coefficients and the meticulous design of the control law can lead to a substantial deterioration in the controller's performance. This can also lead to the occurrence of chattering. The overarching aim of the control law is to guide the trajectory of the nonlinear system towards a user-defined surface. The control law attempts to maintain the trajectory at the target after the system trajectory drives onto the sliding surface. System states can change during the control process, and this event can be considered a type of VSC. The term SMC originates from the presence of a sliding surface. The purpose of using the equivalent control law in the SMC design is to limit the control gain of the switching control law.

With its robust developed features, SMC has often been used together with different practical control methods, such as fuzzy logic, adaptive approach, and integral and terminal approach, to represent a strategic endeavor aimed at augmenting the efficacy of control systems [5,17]. The conventional SMC method uses linear sliding surfaces with a non-finite convergence time. The terminal sliding mode control (TSMC) approach overcomes this problem by using a fractional exponent term in the definition of the sliding surface [18]. In the controller crafted using the TSMC approach, despite the absence of a nonlinear characteristic in the sliding surface, assured convergence to zero within a finite timeframe is achievable. When juxtaposed with conventional SMC, the TSMC method exhibits enhanced system performance and superior stability.

However, it's noteworthy that the chattering phenomenon persists to some extent and remains unresolved, indicating that complete elimination is not attained [19,20].

The TSMC method is prone to a singularity phenomenon that can adversely affect the system's performance. The singularity phenomenon can result in infinite control quantities or an inability to guarantee convergence [21]. If disturbances persist in the TSMC that prevent reaching equilibrium, it becomes challenging to maintain a consistent approach toward zero. To overcome these problems, the fast terminal sliding mode control (FTSMC) approach, which is derived from TSMC, has been proposed [22]. Similar to TSMC, the FTSMC approach ensures convergence to zero within a finite timeframe. However, noteworthy distinctions arise in their convergence behaviors: the convergence graph in FTSMC showcases a faster trajectory compared to controllers devised using the TSMC method. The system states of closed-loop systems have been observed to exhibit better performance in the FTSMC. While the FTSMC significantly reduces the chattering phenomenon, the singularity problem still exists and remains unsolved. Ongoing studies are exploring solutions to address this issue through the application of the FTSMC method [22,23].

This study focuses on developing both a terminal sliding mode controller and a fast terminal sliding mode controller tailored for regulating liquid levels within a coupled tank system. The primary objective here is to devise a resilient control signal capable of mitigating the chattering phenomenon commonly encountered in traditional SMCs and achieving the target within a finite timeframe. The stability of the proposed controllers was scrutinized employing the Lyapunov theorem, a cornerstone of stability analysis in control systems. Furthermore, a comparative assessment was conducted between the proposed terminal sliding mode controller and the fast terminal sliding mode controller vis-a-vis the conventional SMC. This evaluation aimed to gauge the efficacy of the simulation results obtained from each controller configuration.

2. COUPLED TANK SYSTEM

The coupled tank system refers to a setup consisting of identical tanks in which liquid circulation occurs. The system used in this study consists of two identical tanks with a reservoir positioned beneath them, as illustrated in Figure 1. The reservoir functions as a liquid-filled tank from which the pump draws liquid, while any surplus water in tank 2 is discharged into this reservoir to regulate the liquid level. The pump immersed in the reservoir fills the first tank with liquid. The speed of the liquid flowing through the pump may change. Through the open valve pipe between identical tanks, the liquid flows toward the second tank, and from the open outlet hole of the second tank, the liquid flows into the reservoir. Liquid flows freely between tanks and into the reservoir through open valved holes.

The control aim in this scenario is to attain and uphold the liquid level within the second tank to the desired level. Throughout the process, the flow rate of the liquid flowing through the pump is regulated to attain the desired liquid level.

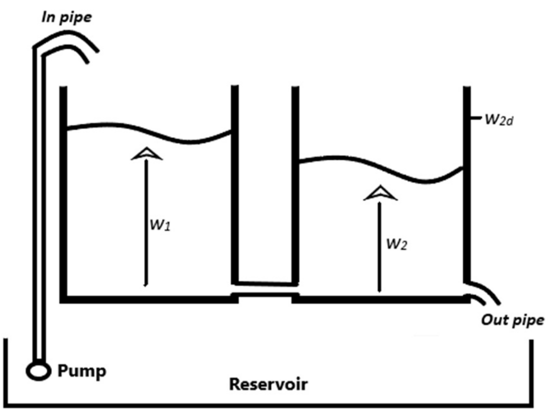

Figure 1. Coupled tank system

The rate of liquid flowing between the tanks f_{i1} and the rate at which liquid exits while flowing into the reservoir f_{i2} can be defined as follows (Equations 1 and 2) [6]

$$f_{i1} = p_{12}\sqrt{2g(w_1 - w_2)} \tag{1}$$

$$f_{i2} = p_2\sqrt{2gw_2} \tag{2}$$

where, w_1 and s w_2 represent the liquid levels in tanks 1 and 2, respectively; g denotes the gravitational acceleration, p_{12} signifies the area of the pipe between the coupled tanks, and p_2 stands for the area of the outlet in the 2nd tank. Utilizing (Equation 1 and 2), the equations governing the liquid flow balances can be expressed as (Equations 3 and 4) [6]:

$$P\frac{dw_1}{dt} = f_i - f_{i1} \tag{3}$$

$$P\frac{dw_2}{dt} = f_{i1} - f_{i2} \tag{4}$$

where f_i is the rate at which liquid flows into the system through the pump and P stands for the cross-sectional area encompassing both tanks.

Upon differentiating the (Equations 3 and 4) for the liquid levels are left on one side of the equation, the mathematical equation for coupled tanks can be written as (Equations 5 and 6) [6]

$$\frac{dw_1}{dt} = -b_1\sqrt{|w_1 - w_2|}\,\mathrm{sgn}(w_1 - w_2) + \frac{f_i}{P} \tag{5}$$

$$\frac{dw_2}{dt} = b_1\sqrt{|w_1 - w_2|}\,\mathrm{sgn}(w_1 - w_2) - b_2\sqrt{w_2} \tag{6}$$

where

$$b_1 = \frac{p_{12}\sqrt{2g}}{P} \qquad b_2 = \frac{p_2\sqrt{2g}}{P} \tag{7}$$

and the signum function for the coupled tank system can be described as (Equation 8)

$$\mathrm{sgn}(w_1 - w_2) = \begin{cases} 1 & w_1 - w_2 > 0 \\ 0 & w_1 - w_2 = 0 \\ -1 & w_1 - w_2 < 0. \end{cases} \tag{8}$$

The fact that the derivatives of w_1 and w_2 are zero indicates that the liquid levels are in balance, expressed as follows (Equation 9):

$$\frac{dw_1}{dt} = \dot{w}_1 = 0 \qquad \frac{dw_2}{dt} = \dot{w}_2 = 0. \tag{9}$$

With the information in (Equation 9), can be rearranged as follows (Equation 10):

$$f_i = b_1\sqrt{|w_1 - w_2|}\,\mathrm{sgn}(w_1 - w_2)P. \tag{10}$$

Because the liquid flow from the pump is directed to the 1st tank, the inequality $w_1 > w_2$ is acknowledged. Therefore, the system should satisfy $sgn(w_1 - w_2) \geq 0$ and This necessitates a rearrangement of the mathematical model of the system as follows (Equations 11 and 12):

$$\dot{w}_1 = -b_1\sqrt{w_1 - w_2} + \frac{f_i}{P} \tag{11}$$

$$\dot{w}_2 = b_1\sqrt{w_1 - w_2} - b_2\sqrt{w_2}. \tag{12}$$

System output and control input can be defined with the following transformations for use in subsequent equations [6]

$$\begin{aligned} y_1 &= w_2 \\ y_2 &= w_1 - w_2 \\ u &= f_{i.} \end{aligned} \tag{13}$$

The derivatives of y_1 and y_2 in (Equation 13) can be written as follows [6]:

$$\begin{aligned} \dot{y}_1 &= -b_1\sqrt{y_1} + b_2\sqrt{y_2} \\ \dot{y}_2 &= b_1\sqrt{y_1} - 2b_2\sqrt{y_2} + \frac{f_i}{P} \\ y &= y_1 = w_2. \end{aligned} \tag{14}$$

Consider $z = [z_1, z_2]^T$. Define a transformation $z = T(y)$, according to the following rule:

$$\begin{aligned} z_1 &= y_1 \\ z_2 &= -b_1\sqrt{y_1} + b_2\sqrt{y_2} \end{aligned} \tag{15}$$

The inverse transformation $y = T^{-1}(z)$

$$\begin{aligned} y_1 &= z_1 \\ y_2 &= \frac{\left(b_1\sqrt{z_1} + z_2\right)}{b_2} \end{aligned} \tag{16}$$

Upon inspection, the dynamic representation in (Equation 14) can be recast as:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{b_1 b_2}{2}\left(\frac{\sqrt{y_1}}{\sqrt{y_2}} - \frac{\sqrt{y_2}}{\sqrt{y_1}}\right) + \frac{b_1 b_1}{2} - b_2 b_2 + \frac{b_2 u}{2P\sqrt{y_2}} \end{aligned} \tag{17}$$

Consequently, the system dynamics can be compactly represented as:

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = f + gu \quad (17)$$
$$y = z_1$$

where

$$f = \frac{b_1 b_2}{2}\left(\frac{\sqrt{y_1}}{\sqrt{y_2}} - \frac{\sqrt{y_2}}{\sqrt{y_1}}\right) + \frac{b_1 b_1}{2} - b_2 b_2 \quad (18)$$

and

$$g = \frac{b_2}{2P\sqrt{y_2}}. \quad (19)$$

3. METHODS

3.1. Sliding Mode Control

The assertion regarding SMC as a robust nonlinear control method applicable to both linear and nonlinear systems resonates with established literature in control theory. SMC has become a popular method frequently used in modern control systems because of its easy application and fast response. The statement regarding the existence of two fundamental principles in sliding mode controller design aptly encapsulates a cornerstone of control theory discourse. Initially, the user needs to designate a suitable surface, termed the sliding surface, for the system. The second principle involves guiding the system trajectory onto the sliding surface and maintaining the system's dynamics and trajectory there [24].

Within the SMC framework, two distinct control units operate: switching control and equivalent control. Switching control is responsible for guiding the system trajectory onto the sliding surface. To ensure the highest level of performance, it is necessary to design the switching control correctly. The sliding surface pertaining to the switching control law can be rigorously delineated as [25,26]:

$$S_c = \beta_c e_c + \dot{e}_c \quad (20)$$

where β_c is a positive real number, e_c represents the tracking error, $\dot{e}_c$ while stands for its derivative.

Considering the tracking error as $e_c = w_2 - w_{2d}$ for the coupled tank system and its time derivative as $\dot{e}_c = \dot{w}_2 = \dot{z}_1$, the reformulation of the sliding surface in (Equation 21) yields (Equation 22):

$$S_c = \beta_c (w_2 - w_{2d}) + \dot{z}_1. \quad (21)$$

Using (Equations 17 and 18), the time derivative of the sliding surface can be obtained as (Equation 23)

$$\begin{aligned}\dot{S}_c &= \beta_c \dot{z}_1 + \ddot{z}_1 \\ &= \beta z_2 + f + gu\end{aligned} \quad (22)$$

The SMC law, which includes switching and equivalent controls, can be defined as follows (Equation 24) [26]:

$$\begin{aligned}u_c &= u_{csw} + u_{ceq} \\ &= \frac{1}{g}(-f - \beta z_2 - k\,\mathrm{sgn}(S_c))\end{aligned} \quad (24)$$

where u_{csw} is switching control, u_{ceq} is equivalent control and k is a positive real number.

A Lyapunov function candidate, aimed at assessing the stability of the control system, can be identified as follows [26,27]:

$$V_c = \frac{S_c^2}{2}. \tag{25}$$

The time derivative of V_c is defined as (Equation 26)

$$\begin{aligned}\dot{V}_c &= S_c \dot{S}_c \\ &= S_c(f + gu_c + \beta_c z_2) \\ &= S_c(-k\,\mathrm{sgn}(S_c)) \\ &= -k\left|S_c\right| \\ &\le 0.\end{aligned} \tag{26}$$

In this case, the solution to the equation yields a negative result. In accordance with the Lyapunov theorem, it is necessary for $\dot{V}_c$ to satisfy less than zero demonstrates that the stability of the system has been proven and also confirms that the system trajectory converges to the sliding surface and remains there.

3.2. Terminal Sliding Mode Control

A nonlinearity can be introduced into the system dynamics, potentially leading to improved performance compared to conventional SMC approaches [23,26,28]. The incorporation of the terminal attractor approach into the SMC has been demonstrated in previous studies to enable convergence in finite time, leading to the proposal of TSMC. The TSMC can be applied to control high-order systems and can be designed as follows for a coupled tank system characterized by a single input, single output configuration (Equation 27) [20]:

$$S_t = \beta_t e_t^{m/n} + \dot{e}_t \tag{27}$$

where β_t is a positive real number, represents the tracking error, while $\dot{e}_t$ stands for its derivative, m and n positive odd numbers and n must be greater than m.

Once the sliding surface is enforced $S_t = 0$, the closed-loop dynamics simplify to a nonlinear differential equation:

$$\dot{e}_t = -\beta_t e_t^{m/n} \tag{28}$$

Due to the presence of a fractional exponent, this equation exhibits finite-time convergence. Solving it via separation of variables yields the convergence time:

$$T = \frac{e_t(0)^{1-m/n}}{\beta_t(1-m/n)} \tag{29}$$

The condition $0 < m/n < 1$ ensures $T < \infty$, establishing strict finite-time stability[32].

Considering the tracking error as $e_t = w_2 - w_{2d}$ for the coupled tank system and its time derivative as $\dot{e}_t = \dot{w}_2 = \dot{z}_1$, the reformulation of the sliding surface in (Equation 27) yield (Equation 30):

$$S_t = \beta_t (w_2 - w_{2d})^{m/n} + \dot{z}_1. \tag{30}$$

By employing (Equations 17 and 18), one can compute the time derivative of the sliding surface as (Equation 31):

$$\begin{aligned}\dot{S}_t &= \ddot{z}_1 + \frac{m}{n}\beta_t e_t^{\frac{m-n}{n}} \dot{z}_1 \\ &= f + gu_t + \beta_t \frac{m}{n}(w_2 - w_{2d})^{\frac{m-n}{n}} z_2\end{aligned} \tag{31}$$

The TSMC law can be selected as (Equation 32) [26]

$$\begin{aligned}u_t &= u_{tsw} + u_{teq} \\ &= \frac{1}{g}(-f - \beta_t \frac{m}{n}(w_2 - w_{2d})^{\frac{m-n}{n}} z_2 - k\,\mathrm{sgn}(S_c))\end{aligned} \tag{32}$$

where u_{tsw} is switching control, u_{teq} is equivalent control and k is a positive real number.

The Lyapunov function employed in the TSMC design can be specified as follows (Equation 33) [26,27]:

$$V_t = \frac{S_t^2}{2}. \tag{33}$$

V_t can be derived as (Equation 34)

$$\begin{aligned}\dot{V}_t &= S_t \dot{S}_t \\ &= S_t(f + gu_t + \frac{m}{n}\beta_t(w_2 - w_{2d})^{\frac{m-n}{n}} z_2) \\ &= S_t(-k\,\mathrm{sgn}(S_t)) \\ &= -k|S_t|.\end{aligned} \tag{34}$$

According to the theory of finite-time stability, the Lyapunov function derivative must fulfill the condition $\dot{V}_t \le -mV_t^n$, where $m > 0$ and $0 < n < 1$, in order to guarantee convergence within a finite time horizon. By introducing the variable transformation $S_t = \sqrt{2V_t}$, the derivative of the Lyapunov function takes the form:

$$\dot{V}_t = -k\sqrt{2}V_t^{1/2} \tag{35}$$

which is clearly consistent with the finite-time stability condition by identifying $m = k\sqrt{2}$ and $n = 1/2$. This confirms that, under the proposed the TSMC framework, the system trajectories reach the sliding surface $S_t = 0$ in finite time, thereby ensuring finite-time convergence properties.

3.3. Fast Terminal Sliding Mode Control

The statement suggests that the FTSMC possesses the capability to drive the system trajectory to zero within a finite timeframe. This assertion aligns with the fundamental objective of FTSMC, which is to expedite convergence towards the desired target, thus enhancing the overall performance of the control system. The ability to achieve such convergence within a finite time underscores the efficacy of the FTSMC approach, particularly in scenarios where rapid response and precise control are paramount.

The sliding surface, which is very important for shaping the performance characteristics, can be expressed as follows for FTSMC [29]:

$$S_f = \dot{e}_f + ae_f + \beta_f e_f^{\frac{m}{n}} \tag{37}$$

where a and β_f are positive real numbers, e_f represents the tracking error, while $\dot{e}_f$ stands for its derivative and m and n positive odd numbers and n must be greater than m.

Once the sliding manifold is reached ($S_f = 0$), the closed-loop error dynamics simplify to a combination of a nonlinear terminal attractor and a linear damping term. Specifically, the expression

$$\dot{e}_f = -\beta_f e_f^{\frac{m}{n}} - a e_f \tag{38}$$

ensures finite-time convergence by exploiting the fast settling behavior of the nonlinear term near the origin, while the linear term enhances transient performance. The convergence time is analytically given by [30, 31]:

$$T = \frac{1}{a(1-\frac{m}{n})} \ln\left(\frac{a e_f(0)^{1-\frac{m}{n}} + \beta_f}{\beta_f} \right) \tag{39}$$

which guarantees that the tracking error e_f reaches zero within a finite time interval[32].

In the context of the coupled tank system, (Equation 40) provides a representation of the sliding surface, which can be further reformulated as follows:

$$S_f = \beta_f (w_2 - w_{2d})^{\frac{m}{n}} + a(w_2 - w_{2d}) + \dot{z}_1. \tag{40}$$

derivating S_f in (Equation 41) yield

$$\dot{S}_f = f + g u_f + \beta_t \frac{m}{n} (w_2 - w_{2d})^{\frac{m-n}{n}} z_2 + a z_2. \tag{41}$$

The FTSMC law can be defined as [29]

$$u_f = \frac{1}{g}(-f - a z_2 - \beta \frac{m}{n} (w_2 - w_{2d})_f^{\frac{m-n}{n}} z_2 - k \operatorname{sgn}(S_f)) \tag{42}$$

The Lyapunov candidate for FTSMC stability can be selected as (Equation 43) [26-28]

$$V_f = \frac{S_f^2}{2}. \tag{43}$$

The derivative of V_f can be written as (Equation 44)

$$\begin{aligned} \dot{V}_f &= S_f \dot{S}_f \\ &= S_f \left(f + g u_f + \frac{m}{n} \beta_f (w_2 - w_{2d})^{\frac{m-n}{n}} z_2 - a z_2 \right) \\ &= S_f \left(-k \operatorname{sgn}(S_f) \right) \\ &= -k \left| S_f \right|. \end{aligned} \tag{44}$$

To establish finite-time stability, the Lyapunov function derivative must fulfill the condition $\dot{V}_f = -m V_f^n$, where $m > 0$ and $0 < n < 1$ [33]. By substituting the relation $S_f = \sqrt{2V_f}$ into the derivative expression, we obtain:

$$\dot{V}_f = -k\sqrt{2}S_f^{1/2} \tag{45}$$

This satisfies the required inequality with $m = k\sqrt{2}$ and $n = 1/2$, confirming that the proposed FTSMC strategy ensures convergence of system trajectories to the equilibrium point within a finite time interval.

4. RESULT AND DISCUSSION

The diagram shown in Figure 2 shows the signal flow between the controller (SMC/TSMC/FTSMC), the connected tank system and the signal processing blocks. Specifically, the diagram includes the reference signal input, the measured output from the plant, and the derivative of the output fed back to the controller.

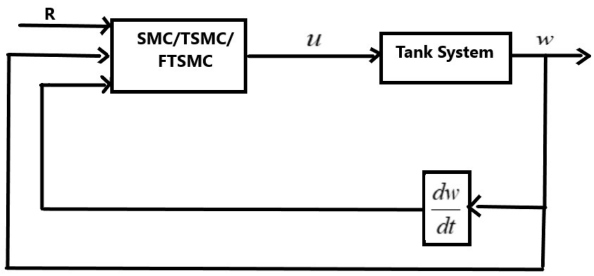

Figure 2. Block diagram of the controls

The SMC, TSMC, and FTSMC laws defined in (Equations 24, 32, and 42), respectively, are compared with each other. The saturation function can be used in these equations instead of the sign function to dampen the chattering effect resulting from the SMC law. The saturation function can be defined as follows (Equation 46):

$$sat(S) = \begin{cases} S & |S| < 1 \\ sgn(S) & |S| >= 1. \end{cases} \tag{46}$$

The primary goal of the controllers is to ensure that the liquid level in Tank 2 reaches and maintains 5 cm during the first 80 seconds, and subsequently tracks a reference of 10 cm for the remainder of the operation. The experiment duration was set as 300 seconds. The liquid level in the 1st tank is initially fixed at a precise measurement of 0.1 cm, and the 2nd tank is completely empty. In (Table 1), the parameters defining the coupled tank system outlined in (Equations 5 and 6) are presented.

Table 1. Coupled tank system parameters

Symbol	Value	Meaning
w_1	$0.1\,cm$	Initial liquid level for tank 1
w_2	$0\,cm$	Initial liquid level for tank 2
w_{2d}	$5\ cm$	Desired liquid level for tank 2
g	$981\ cm/s^2$	Acceleration due to gravity
P	$208.2\ cm^2$	Cross-sectional area
p_{12}	$0.58\ cm^2$	tank 2 Outlet Area
p_2	$0.24\ cm^2$	Area of the interconnecting pipe between tanks 1 and 2

While applying the SMC law, the β_s parameter of the controller was set to 0.015 and the k parameter was set to 45. For the TSMC and FTSMC laws, the constant values used in the equation of the controller were chosen as β_t = 0.015, β_f = 0.015, $a = 0{,}015$ and $k = 45$ for comparison under the same conditions. The selection of constant parameters is determined through iterative experimentation and refinement. In the simulation, the SMC, TSMC, and FTSMC methods were tested on coupled tanks. While Tank 1 is not the primary control target, its level is illustrated in (Figure 3) to highlight the influence of the interconnected dynamics and to support the interpretation of the overall system behavior. The outputs obtained after the tests of the SMC, TSMC, and FTSMC methods on the coupled tanks are combined in (Figure 4) for better observation.

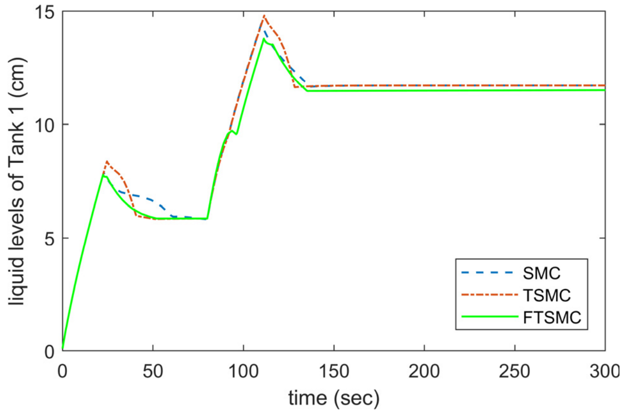

Figure 3. Evolutions of Tank 1 levels

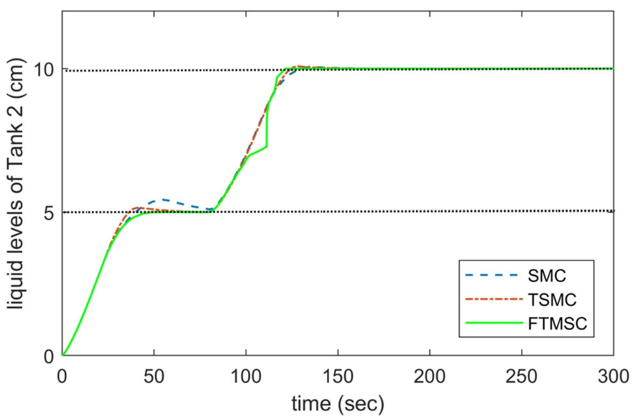

Figure 4. Evolutions of outputs

In order to examine the continuity and adaptation capability of the controllers, the reference level was increased from 5 cm to 10 cm at 80 seconds. This change enables the evaluation of controller performance under a varying setpoint condition and better reflects real-world operational scenarios.

A diagram showing the control signals of the three control laws on the coupled tanks is combined in (Figure 5) for better observation.

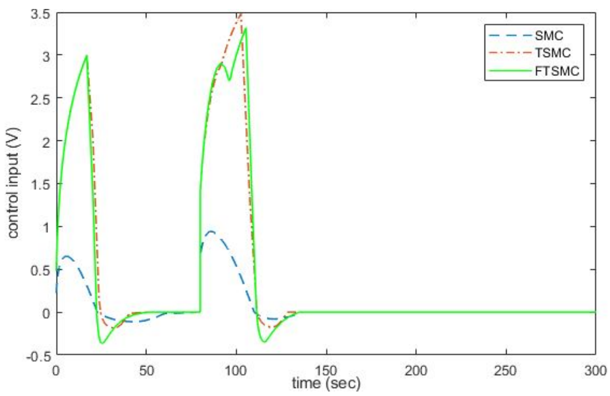

Figure 5. Evolution of the control signals

As illustrated in (Figures 3 and 4), while the conventional SMC was able to drive the system output to the desired reference level, the TSMC and FTSMC controllers demonstrated superior performance in terms of convergence speed and reduced steady-state error. The enhanced responsiveness and smoother control behavior of TSMC and FTSMC can be clearly observed in the transient and steady-state responses shown in these figures. Furthermore, (Table 2) presents a detailed comparison of the numerical performance indices obtained from the experimental tests, including Settling time, Overshoot, and Steady-state error values. These metrics quantitatively confirm that both TSMC and FTSMC outperform conventional SMC, with FTSMC yielding the best overall results.

Table 2. Performans results

	Settling time	Overshoot	Steady-state error
SMC	106.7073	7.8319	0.0056
TSMC	55.8702	2.7081	0.0005
FTSMC	44.3647	0.0231	0.0011

The performance indices presented in (Table 3) clearly demonstrate that the proposed FTSMC outperforms both SMC and TSMC in terms of tracking accuracy. With the lowest IAE (287.75) and ISE (968.57) values among all controllers, FTSMC effectively minimizes both the total and squared tracking errors. This indicates a faster and more precise convergence to the desired reference levels. While TSMC also shows improved performance over SMC, the superiority of FTSMC suggests that the incorporation of fractional powers enhances the robustness and error correction capabilities of the controller.

Table 3. Performans indices

	IAE	ISE
SMC	325.33	1268.33
TSMC	306.80	1157.28
FTSMC	287.75	968.57

5. CONCLUSION

In this study, controllers were designed using SMC-derived TSMC and FTSMC laws to manage the liquid level of the coupled tank system in the SISO structure. The differences and advantages of these approaches were observed by comparing the examined TSMC and FTSMC laws with the conventional SMC law. Simulation tests were conducted to evaluate the efficacy of the designed controllers for the coupled tank system within the simulated environment. From the evolution of outputs, TSMC and FTSMC, it is observed that the conventional SMC tends to approach zero in finite time, but if we interpret the performance results table, it is clear that the overshoot, settling time and steady-state error values are better. According to the results, the proposed methods are compatible with the system and provide more effective outputs than the conventional SMC. The next goal after the liquid level control of the SISO coupled tank system, which will be a light for future studies, is to examine the proposed methods on the cross-coupled four-tank system with simulation and experimental results.

As a potential extension of this study, the robustness of the proposed FTSMC approach can be further evaluated by introducing external disturbances to the system. For instance, a disturbance scenario involving a sudden opening of an external valve connected to Tank 1 could be implemented to simulate unexpected variations in flow dynamics. Such an intervention would allow assessment of the controller’s ability to reject disturbances and maintain output tracking accuracy. Future work will focus on incorporating this type of disturbance into both simulation and experimental setups to investigate the disturbance rejection performance and overall resilience of the controller under more realistic operating conditions.

6. REFERENCES

1. Pan, H., Wong, H., Kapila, V. & Queiroz, M.S. (2005). Experimental validation of a nonlinear backstopping liquid level controller for a state coupled two tank systemç *Control Engineering Practice, 13*(1), 27-40.

2. Le, H.T., Lee, C.Y. & Lee, S.R. (2010). Dynamic adaptive backstepping and saturated proportional integral sliding mode control for uncertain nonlinear systems. *International Journal of Precision Engineering and Manufacturing, 11*(4), 527-537.
3. Shamily, S., Praveena & Bhuvaneswari, N.S. (2015). Intelligent control and adaptive control for interacting system. *IEEE Technological Innovation in ICT for Agriculture and Rural Development (TIAR)*.
4. Kadri, M.B. (2017). Model-free fuzzy adaptive control for MIMO systems. *Arabian Journal for Science and Engineering, 42*, 2799-2808.
5. Geylani, O. & Ata, B. (2022). Adaptive sliding mode control of a coupled tank system. *2022 International Congress on Human-Computer Interaction, Optimization and Robotic Applications (HORA)*, IEEE.
6. Almutairi, N.B. & Zribi, M. (2006). Sliding mode control of coupled tanks. *Mechatronics, 16*, 427-441.
7. Khan, M.K. & Spurgeon, S.K. (2006). Robust MIMO water level control in interconnected twin tanks using second-order sliding mode control. *Control Engineering Practice, 14*, 375-386.
8. Abbas, H., Asghar, S. & Qamar, S. (2012). Sliding mode control for coupled-tank liquid level control system. *10th International Conference on Frontiers of Information Technology, 4*, 4.
9. Malmborg, J. & Eker, J. (1997). Hybrid control of a double tank system. *Proceedings of the 1997 IEEE International Conference on Control Applications.*
10. Dharamniwas, Ahmad, A., Redhu, V. & Gupta, U. (2012). Liquid level control by using fuzzy logic controller. *International Journal of Advances in Engineering & Technology, 4*(1), 537-549.
11. Meng, X., Yu, H., Wu, H. & Xu, T. (2020). Disturbance observer-based integral backstepping control for a two-tank liquid level system subject to external disturbances. *Mathematical Problems Engineering*, 1-22.
12. Yasar, O. (2024). Climbing with robots: a second order controller design for accurate wheel motion positioning. *Cukurova University Journal of the Faculty of Engineering, 39*(1), 175-187.
13. Qureshi, M.S., Das, S., Swarnkar, P. & Gupta, S. (2018). Design and implementation of sliding mode control for uncertain systems. *Materials Today: Proceedings, 5*(2), Part 1, 4299- 4308.
14. Utkin, V.I., Young, K.D. & Özgüner, Ü.A. (1996). Control engineer's guide to sliding mode control. *IEEE Transactions*.
15. Sabanovic, A. (2011). Variable structure systems with sliding modes in motion control. *IEEE Transactions on Industrial Informatics, 7*(2), 212- 223.
16. Suryawanshi, P.V., Shendge, P.D. & Phadke, S.B. (2016). A boundary layer sliding mode control design for chatter reduction using uncertainty and disturbance estimator. *International Journal of Dynamics and Control,* 456-465.
17. Wang, S., Hou, L., Dong, L. & Xiao, H. (2011). Adaptive fuzzy sliding mode control of uncertain nonlinear SISO systems. *2011 International Conference on Advances in Engineering, Procedia Engineering, 24*, 33-37.
18. Yorgancıoğlu, F. & Redf, S. (2019). Fast nonsingular terminal decoupled sliding-mode control using time-varying sliding surfaces. *Turkish Journal of Electrical Engineering and Computer Sciences, 27*(3), 1922-1937.
19. Zhang, Q., Yu, H. & Wang, X. (2013). Integral terminal sliding mode control for a class of nonaffine nonlinear systems with uncertainty. *Mathematical Problems Engineering*, 1-11.
20. Burange, P.R., Parvat, B.J. & Kadu, C.B. (2015). Design of TSMC for coupled tank process. *International Conference on Energy Systems and Applications*.
21. Chen, Z., Yuan, X., Wu, X., Yuan, Y. & Lei, X. (2019). Global fast terminal sliding mode controller for hydraulic turbine regulating system with actuator dead zone. *Journal of the Franklin Institute-engineering and Applied Mathematics, 356*(15), 8366-8387.
22. Ghogare, M.G., Patil, S.L., Patil, C.Y. & Chaudhari, L.R. (2023). Experimental application of global fast terminal sliding mode controller to TITO system. *Ecti Transactions On Electrical Engineering, Electronics, And Communications, 21*(2), 249727.
23. Ghogare, M.G., Patil, S.L. & Patil, C.Y. (2022). Experimental validation of optimized fast terminal sliding mode control for level system. *ISA Transactions, 26*, 486-497.
24. Kayisli, K., Tuncer, S. & Poyraz, M. (2008). Active power factor correction using a sliding mode controller. *Pamukkale University Journal of Engineering Science, 14*(3), 253-260.
25. Sancak, C., Coskun, M.Y. & Itik, M. (2016). Vision based sliding mode control of micro displacement of a conjugated electroactive polymer actuator. *Pamukkale University Journal of Engineering Sciences, 22*(8), 629-635.

26. Shtessel, Y., Edwards, C., Fridman, L. & Levant, A. (2014). Sliding mode control and observation. *Control Engineering.* New York: Springer New York.

27. Ata, B. (2020). Bir top ve bir çubuk sisteminin ayrıklaştırılmış geri adımlamalı kayan kipli kontrolü. *Cukurova University Journal of the Faculty of Engineering, 35*(3), 659-668.

28. Zhongqiang, W.U. (2016). Terminal sliding mode control of MIMO linear time delay systems. *2nd International Conference on Advances in Mechanical Engineering and Industrial Informatics (AMEII 2016*).

29. Parvat, B.J. & Patre, B.M. (2017). Fast terminal sliding mode controller for square multivariate processes with experimental application. *International Journal of Dynamics and Control, 5*, 1139-1146.

30. Amirkhani, S., Mobayen, S., Iliaee, N., Boubaker, O. & Hosseinnia, H. (2019). Fast terminal sliding mode tracking control of nonlinear uncertain mass–spring system with experimental verifications. *International Journal of Advanced Robotic Systems,* 1-10.

31. Fang, Q., Mao, P., Shen, L. & Wang, J. (2022). A global fast terminal sliding mode control for trajectory tracking of unmanned aerial manipulation. *Measurement and Control, 56*(3-4), 763-776.

32. Yu, X. & Zhihong, M. (2002). Fast terminal sliding-mode control design for nonlinear dynamical systems. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 49*(2), 261-264.

33. Song, Y. & Su, J. (2022). A unified lyapunov characterization for finite time control and prescribed time control. *International Journal of Robust and Nonlinear Control, 33*, 2509-2949.