Loss Calculation Technique With Randomize Load Curves

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ABSTRACT

Calculating feeder losses accurately is an important part of evaluating designs for electric power distribution systems. Historically, these losses have been calculated one of three ways: (1) using a peak load calculation and the load factor method, (2) using customer class statistics normalized for a month, season, or year, or (3) using customer class statistics together with feeder measurements to reflect the variation in load every hour of the year. The first two methods require far less data but provide far less accuracy than the third method. In this paper, the authors present a method of calculating losses that achieves better accuracy than the first two methods without the large data requirements of the third method.

Keywords: Load Factor Method, Measurements, Lagrange Interpolation, Advanced Metering Infrastructure (AMI), Loss Calculation.

Rastgele Yük Eğrisi Metodu ile Elektrik Kayıplarının Hesaplanması

ÖΖ

Fiderlerin elektrik kayıplarının dogru biçimde hesaplanması elektrik dağıtım sistemlerini değerlendirirken çok önemli bir paya sahiptir. Geçmişte, bu kayıpların hesaplanması üç değişik metod ile yapılırdı: (1) en yüksek yük hesaplaması ve yük faktörü metodunun kullanımiı ile (2) normalize edilmiş müşteri sınıf istatistiklerinin aylık, sezonluk ve yıllık kullanımı ile ve (3) müşteri sınıf istatisliklerinin fider ölçümlerinide kullanarak yükün yıl içinde her saat değişimini dikkate alarak kullanımıdır. Bunlardan ilk ikisi daha az bilgi gereksinimine ihtiyaç duymasına rağmen üçüncü metoda göre daha az doğruluk payları vardır. Bu çalışmada, elektrik kayıplarının hesaplanması için yeni bir metod kullanılacak ve bu ilk iki metoddan daha doğru sonuclar elde edilecek ve ayni zamanda üçüncü metodun gereksinim duydugu geniş bilgi ihtiyacına da gerek kalmayacaktır.

Anahtar Kelimeler : Yük Ölçüm Metodu, Ölçümler, Lagrange Enterpolasyonu, İleri seviye ölçüm altyapısı (AMI), Kayıp Hesaplanması.

1. INTRODUCTION

When planning engineers are laying out new feeders or reconfiguring or updating existing feeders, they want to either choose the least expensive design that meets certain criteria or else choose the design with the highest cost-benefit ratio. Part of calculating the costs of each design includes the cost of the electrical energy losses that will be incurred on the feeder over time. The accuracy with which these losses can be calculated may impact the decision of which design, if any, is implemented.

The accuracy of the calculation of feeder losses is largely dependent upon the accuracy of the load model [1-3]. In the past, many engineers worked only with a peak load model [4-6]. For a new feeder, there would be rough assumptions about how much load would be located in various areas, but when planning upgrades to existing

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feeders, there would often be a peak feeder reading from a circle chart that would assist in modeling the peak load. A power flow calculation [7] would then produce the total losses for the feeder, and the load factor would be used to estimate the losses for the whole year.

As the cost of metering equipment fell, utilities became able to generate load statistics for different types of customers to help them create "typical" load curves for more accurate calculation of losses [8-9]. By gathering hourly metered measurements for just a few meters of a few different customer types, the utility could generate an approximate load curve for each type of customer for each season or month for each type of day (weekday/weekend). These load curves would be scaled at each customer meter based on the kWh of consumption billed for each month of the year. Power flow calculations would then be done for each hour for each type of day for each season or month, and the losses would be calculated at each hour. By modeling the timevarying nature of loads, a more accurate estimate of losses could be achieved [10-11].

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When flow measurements are available at the start of the feeder, these "typical" load curves could be adjusted to match the feeder flow measurements, resulting in even greater accuracy. With one measurement for each hour of the year, 8760 power flow calculations could be made to calculate the losses for the year. With the increased data and increased calculations, a large increase in accuracy was possible.

In rare cases, a utility may have hourly metered loads at every point of service, often called AMI (advanced metering infrastructure) [12-16]. As of the writing of this paper, however, very few utilities have extensive installations of such meters combined with the electrical model to make effective use of this data.

Without AMI, using feeder flow measurements to adjust customer class load statistics offers the most accurate calculation of losses. Feeder flow measurements, however, are not available on all feeders, nor are they helpful when adding significant numbers of new loads, as these new loads must be modeled in some timevarying way to be used together with the feeder measurements. In this paper, the authors present a method of modeling loads that makes effective use of a peak load measurement or estimate as well as customer class load statistics, without requiring feeder measurements for every hour of the year.

In section 2, the older load modeling and loss calculation methodologies will be presented in more detail, followed by a presentation of the new load modeling method in section 3. These methods are then compared in section 4 for a few feeders for which both monthly kWh billing data and hourly feeder measurements were available. Finally, in section 5, the work is summarized and some conclusions are drawn.

2. PREVIOUS METHODOLOGIES

Loss calculation methodologies have advanced in direct correspondence with the availability of more detailed measurements on the power system. Starting with the load factor method requiring no more than a measurement or estimate of the total feeder flow and progressing to the more precise time-varying load flow analysis using AMI measurements, electrical power system engineers have seen the complexity of the mathematical problem rise together with increased accuracy.

2.1. Load Factor Method

The earliest and simplest method uses a single measurement (or estimate) of the peak feeder demand and a rough model of how that load is distributed, as well as a load factor. The peak feeder demand may either be estimated or read from a circle chart. The load distribution varies from an assumed even distribution of load over the feeder to a model representing each transformer kVA where the load is assumed proportional to the transformer rating. Regardless of how the load is modeled, a power flow analysis is performed using the estimated peak demand on the feeder to calculate the energy losses at peak.

The load factor (LF) represents the ratio of average annual energy consumption to peak demand, as shown in Equation 1.

$$LF = \frac{P_{avg}}{P_{peak}} \tag{1}$$

Using the peak load flow results and the load factor, the average losses may be calculated using Equation 2, and these average losses are multiplied by the number of hours in a year (assuming 8760 hours in a year) to provide the total annual losses, as shown in Equation 3.

$$Loss_{avg} = Loss_{peak} * LF^2 \tag{2}$$

$$Loss_{total} = Loss_{avg} * 8760 \tag{3}$$

This approach to calculating annual losses on a feeder leads to an underestimation of the losses and a further underestimation of the cost of losses, as proven in reference [17].

2.2. Customer Class Load Statistics Method

Of course, in order to bill their customers correctly, utilities have been measuring more than merely the peak demand at the feeder even since the earliest power systems infrastructure was being built. In order to make use of these measurements in a load flow analysis, however, one must be able to convert the total monthly kilowatt-hour consumption into a kilowatt demand at a given hour of the year.

By metering a few loads, a generic load curve could be estimated for various customer types for various seasons or months and for various types of days. The monthly kilowatt-hours read from customer meters (or anticipated consumption based on energy sales to similar customers) could then be divided among the hours of the month based on the typical load curve and the types of days. For example, in Figure 1, we have a sample plot for two different customer types (residential and commercial) and two different types of day (weekday and weekend). The vertical axis of Figure 1 shows the kW demand at each hour based on a total monthly consumption of 1000 kWh. For a residential customer with only 500 kWh, the load curve would take the same shape but each value would be halved.



Fig. 1. Comparison of kW demand for two different customer types and Days

The number of distinct daily curves depends on the variation in customer load. For some utilities, four seasons and two types of day (weekday/weekend) may prove sufficiently accurate. For other utilities, there will be different curves for each day of the week for each month of the year, resulting in 2016 hourly curve points (12 months x 7 day types x 24 hours/day). Modeling holidays independently would add additional curve points to be analyzed.

A more precise formulation is given in Equations 7-10 of reference [11].

2.1. Customer Class Load Statistics With Measurement Matching

An obvious shortfall of the customer class load statistics method is that the load is assumed to be the same for many days in a given month. Without representing the variation in load, the total losses will be underestimated by the same proof as given in [11]. If feeder measurements are available, the loads produced by the customer load statistics and monthly kWh billing data can be scaled at each hour to match the feeder flow measurement. This then provides the variation in load needed to greatly improve the accuracy of the total feeder losses calculation.

2.2. Fully Metered Loads (AMI)

When matching feeder measurements, the total feeder flow is accurate but the actual distribution of the load is only as accurate as the customer load statistics. If the actual loads were known at each point of service, then the feeder measurement would not be needed (or would only be needed to help scale unmetered loads or to identify significant errors in the metered loads). This type of information is provided by advanced metering infrastructure (AMI). While the extent of AMI deployment has been increasing over the last several years, such detailed measurements are not available in much of the service territory around the world.

Table 1 below shows the variations in amount and types of data required as well as the variation in amount of calculations required. The rows are listed in order of increasing accuracy which corresponds with increasing data requirements and increasing computational cost. In this case, the customer load statistics method uses two daily curves per month (weekday and weekend), resulting in 576 time points (12 months x 2 day types per month x 24 hours per day type).

Table 1 provides inspiration for another load modeling technique introduced below. Since greater accuracy is possible with greater measurements and increased analysis times, another row could be introduced between the customer load statistics and the customer load statistics scaled for feeder measurements: a row that uses the peak feeder flow as well as the monthly kWh consumption which also models time variation for all 8760 hours of the year, not just for 576 hours per year. It is this new method to which we now turn.

3. RANDOMIZED CUSTOMER CLASS LOAD STATISTICS METHODOLOGY

When neither AMI measurements nor feeder flow measurements are available, or when feeder flow measurements are insufficient due to large load growth on the feeder, the planning engineer is left with only two sets of data to use in modeling the load: (1) an expected peak load for the whole feeder and (2) customer class load statistics and either actual or estimated monthly kWh consumption data. The load factor method makes use of the former but not the latter, with rather poor accuracy. The customer class load statistics method makes use of the latter but not the former with somewhat improved accuracy. Even greater accuracy is available to the engineer if both the feeder peak load can be used as well as the anticipated kWh consumption.Our proposal is to take the "typical" daily load curves calculated from the customer class load statistics and scale them randomly throughout the month such that the following two conditions are met: (1) the total consumption matches the provided kWh consumption forecasts and (2) that the peak load on the feeder matches the forecasted feeder peak. The feeder peak helps to put an upper bound on the randomized scaling. A third piece of data, the ratio of the average load to the minimum load, is used to set the lower bound on the randomized scaling. While this ratio is not as likely to be known for individual feeders, the utility

Method	Method Measurement Data Requirements	
Load Factor	Peak Feeder Flow	1
Customer Load	Monthly kWh Consumption per Service	576
Statistics	Point	
Customer Load	Hourly Feeder Flow Measurements +	8760
Statistics	Monthly kWh Consumption per Service	
Scaled for Feeder	Point	
Measurements		
Power Flow with AMI	Hourly Load Measurements at Every	8760
Data	Service Point	

Table 1. Comparison of Data Required For Each Methods

will often have a rule-of-thumb number that may be used (for utilities involved in both the generation and distribution of electric energy, this ratio is part of planning their base load generation and thus likely has good data behind it). Even if the minimum-to-average load ratio is not chosen accurately, it will not prevent the model from achieving both objectives listed above.



Fig. 2. Scaled Residential Load Curves

In Figure 1, the residential customer type has a minimum load point on the weekend and a maximum load point on the weekday. The three constraints listed above can be explained in terms of the scaled residential load curves shown in Figure 2. The upper bound of the scaling is determined by the average-to-peak ratio (B / A in Figure 2) and the lower bound of the scaling is determined by the minimum-to-peak ratio (C / A in Figure 2). The average must be maintained so that the total consumption on the feeder matches the total kWh billed to the customers. The maximum scaling factor (SF) for the ith type of day can be calculated in terms of the average load curve (scaling factor of 1.0) using Equation 4.

$$SF_{Max_i} = \frac{DailyAvg_i}{DailyPeak_i} * \frac{MonthlyPeak}{MonthlyAvg}$$
(4)

The minimum scaling factor can likewise be calculated using Equation 5.

$$SF_{Min_i} = \frac{DailyAvg_i}{DailyMin_i} * \frac{MonthlyMin}{MonthlyAvg}$$
(5)

The curves in Figure 2 assume an average-to-peak ratio of 0.65 and a minimum-to-peak ratio of 0.4. Using these values, the scaling factors for each curve are calculated as shown in Table 2.

and (1.0, SFmax). These three points may be interpolated using Lagrange interpolation to provide the randomization functions, as shown in Figure 3. For each customer type, then, a distinct Lagrange interpolation function must be calculated. Note that, although the maximum scaling factor is larger for the weekend, the peak will still (on average) be on a weekday, since the weekday curve starts out higher.



Fig. 3. Randomization Function produced by Lagrange Interpolation

Since the load curve shape is to be maintained, all loads of a given type shall have the same scaling factor for the entire day; i.e., the entire daily curve gets scaled 28 to 31 times a month, depending on the number of days in the month. Additionally, in order for the feeder peak and minimum to correspond to the monthly peak-to-average and minimum-to-peak ratios, and, in respect of the fact that loads are often temperature-dependent, the same random number must be supplied to each Lagrange interpolation function for each day of the month. Thus, all loads on the system will see their highest weekday on the same day of the month, and all loads on the system will see their highest weekend on the same day of the month. Since some loads peak on the weekdays and some loads peak on the weekend, the different load types will peak on one of two different days, so the scaling factors must be calculated based on the curve data for different types weighted based on the percent of load on the feeder allocated to a particular type of customer.

4. SIMULATION RESULTS

Table 3 below shows loss results for load factor calculation for 8 different feeders. As we discussed

Type of Day	DailyAvg DailyPeak	DailyAvg DailyMin	SF _{Max}	SF _{Min}
Weekday	0.793	1.245	1.220	0.766
Weekend	0.817	1.294	1.257	0.796

Table 2. Scaling Factors For Each Curve

The randomization function that satisfies these three constraints can be depicted as a cumulative distribution function which must pass through (0, SFmin), (0.5, 1.0)

previously, loss calculation with load factor method is underestimated, so loss values are very lower than its original measurement values. Table 3 also provides extra

-	Table 3. Load Factor Loss Calculation							
	Load Factor calculation							
				Load				
Feeder#	MaxKWFlow	KWFlowSum	KWFlowAvg	Factor	LossesKWMax	LossesKW		
Feeder1	4496.15	1623715.19	2255.16	0.50	81.30	14725.74		
Feeder2	4312.54	2023898.59	2810.97	0.65	58.72	17961.45		
Feeder3	3751.95	1322135.87	1836.30	0.49	86.72	14956.37		
Feeder4	3381.83	1506666.53	2092.59	0.62	18.86	5199.40		
Feeder5	6815.34	2795443.59	3882.56	0.57	139.46	32586.05		
Feeder6	531.45	245696.83	341.25	0.64	2.95	875.96		
Feeder7	3754.92	1619877.46	2249.83	0.60	37.49	9690.08		
Feeder8	5623.38	2600546.54	3611.87	0.64	77.71	23082.48		

information to calculate losses by using load factor method.

method and customer class statistic methods are under or overestimated with these methods.

Another way of calculation the losses is to get customer class statistics which basically dependent on accuracy of customer class curves. Table 4 shows detail calculation of losses by using customer class statistics method.

When we don't have actual measurements or AMI data, system losses needs to be calculated by different method than load factor and customer class statistic method to be able to plan for future power systems more accurately.

Table 4. Customer Class Curve Loss Calculation	
Customer Class Curve Loss	

	Customer Cla Calculation	ss Curve Loss		
Feeder #	LossesKW	KWFlow	KVARFlow	MaxKWFlow
Feeder1	51209.86	2915378.20	627254.78	5847.75
Feeder2	22764.78	1905225.87	1154151.19	3810.41
Feeder3	17377.86	1491892.73	465625.82	2986.67
Feeder4	7937.17	1657220.31	995658.61	3311.64
Feeder5	54290.76	2650325.64	1603754.01	5293.32
Feeder6	1899.71	312417.75	189837.58	624.63
Feeder7	14785.36	1664536.00	924510.92	3324.75
Feeder8	31064.30	2192025.61	1289755.67	4385.54

Also Table 5 shows the actual measurement for losses for each feeder. Notice that losses calculated by load factor

Table 6 shows the randomized load curve method results among with power flow results. It is interesting to see that

	Measurements of losses						
Feeder #	LossesKW	KWFlow	KVARFlow	MaxKWFlow			
Feeder1	34005.36	1623715.19	-383872.48		4496.15		
Feeder2	24419.53	2023898.59	1206260.58		4312.54		
Feeder3	27513.36	1322135.87	-1603911.28		3751.95		
Feeder4	6807.19	1506666.53	894542.87		3381.83		
Feeder5	56435.99	2795443.59	1666101.43		6815.34		
Feeder6	1428.90	245696.83	145831.94		531.45		
Feeder7	14684.76	1619877.46	962173.18		3754.92		
Feeder8	32849.89	2600546.54	464311.18		5623.38		

losses by calculated randomized load are between load factor method and customer class load curve method.

measurements of little use, the utility needs another means of calculating the load and losses to evaluate their

Feeder #LossesKWKWFlowKVARFlowMaxKWFlowFeeder150706.682876338.63597331.386265.69Feeder222528.531879711.771138401.854083.04Feeder317158.101471901.64439409.833199.89Feeder47828.451635012.71982224.683547.60Feeder553911.622615039.341581825.665671.22Feeder61882.49308235.83187260.53669.28Feeder714639.231642283.51910893.223561.53Feeder830748.302162656.371271488.034699.70	Randomized Load Curve						
Feeder150706.682876338.63597331.386265.69Feeder222528.531879711.771138401.854083.04Feeder317158.101471901.64439409.833199.89Feeder47828.451635012.71982224.683547.60Feeder553911.622615039.341581825.665671.22Feeder61882.49308235.83187260.53669.28Feeder714639.231642283.51910893.223561.53Feeder830748.302162656.371271488.034699.70	Feeder #	LossesKW	KWFlow	KVARFlow	MaxKWFlow		
Feeder222528.531879711.771138401.854083.04Feeder317158.101471901.64439409.833199.89Feeder47828.451635012.71982224.683547.60Feeder553911.622615039.341581825.665671.22Feeder61882.49308235.83187260.53669.28Feeder714639.231642283.51910893.223561.53Feeder830748.392162656.371271488.034699.70	Feeder1	50706.68	2876338.63	597331.38	6265.69		
Feeder317158.101471901.64439409.833199.89Feeder47828.451635012.71982224.683547.60Feeder553911.622615039.341581825.665671.22Feeder61882.49308235.83187260.53669.28Feeder714639.231642283.51910893.223561.53Feeder830748.392162656.371271488.034699.70	Feeder2	22528.53	1879711.77	1138401.85	4083.04		
Feeder47828.451635012.71982224.683547.60Feeder553911.622615039.341581825.665671.22Feeder61882.49308235.83187260.53669.28Feeder714639.231642283.51910893.223561.53Feeder830748.392162656.371271488.034699.70	Feeder3	17158.10	1471901.64	439409.83	3199.89		
Feeder553911.622615039.341581825.665671.22Feeder61882.49308235.83187260.53669.28Feeder714639.231642283.51910893.223561.53Feeder830748 302162656 371271488 034699 70	Feeder4	7828.45	1635012.71	982224.68	3547.60		
Feeder61882.49308235.83187260.53669.28Feeder714639.231642283.51910893.223561.53Feeder830748 302162656 371271488 034699 70	Feeder5	53911.62	2615039.34	1581825.66	5671.22		
Feeder7 14639.23 1642283.51 910893.22 3561.53 Feeder8 30748 30 2162656 37 1271488 03 4699 70	Feeder6	1882.49	308235.83	187260.53	669.28		
Fooder 8 30748 30 2162656 37 1271488 03 4699 70	Feeder7	14639.23	1642283.51	910893.22	3561.53		
Feeders 30746.39 2102050.57 1271468.05 4093.70	Feeder8	30748.39	2162656.37	1271488.03	4699.70		

Table 6. Loss Results with Randomized Load Curve

Among these loss calculation technique, proposed technique provides the most accurate data when there is no measurement or AMI data. Table 7 below shows the accuracy comparison of three methods based on actual measurement values.

design decisions. In this paper, a novel method of calculating customer loads at every service location is presented. This method does not have the large measurement requirements of the more accurate methodologies but makes effective use of peak load

Table 7. Comparison of Accuracy Among Methods

Feeder #	Load factor LossesKW	Randomized Load Curve LossesKW	Average Load Curve LossesKW	Measurement LossesKW	Load factor Accuracy	Randomi zed Load Curve Accuracy	Customer Load Curve Accuracy
Feeder1	14725.74	50706.68	51209.86	34005.36	0.43	0.67	1.51
Feeder2	17961.45	22528.53	22764.78	24419.53	0.74	1.08	0.93
Feeder3	14956.37	17158.10	17377.86	27513.36	0.54	1.60	0.63
Feeder4	5199.40	7828.45	7937.17	6807.19	0.76	0.87	1.17
Feeder5	32586.05	53911.62	54290.76	56435.99	0.58	1.05	0.96
Feeder6	875.96	1882.49	1899.71	1428.90	0.61	0.76	1.33
Feeder7	9690.08	14639.23	14785.36	14684.76	0.66	1.00	1.01
Feeder8	23082.48	30748.39	31064.30	32849.89	0.70	1.07	0.95

As it is seen from Table 7, average accuracy with load factor method is 0.62 while accuracies are 1.01 and 1.06 with randomized load curve and customer class load curve respectively.

5. CONCLUSION

When a utility needs to evaluate the impact of a design decision on the operating costs of a feeder, they must often incorporate the impact of the design on the cost of losses on the feeder. Historically, these losses have been calculated one of three ways: (1) using a peak load calculation and the load factor method, (2) using customer class statistics normalized for a month, season, or year, or (3) using customer class statistics together with feeder measurements to reflect the variation in load every hour of the year. Some utilities are beginning to have AMI data available which allows them to significantly reduce the uncertainty of the load. When AMI data is not available and either feeder measurements are not available or else the topology and loading of the feeder have changed so as to make the feeder measurement/estimation and customer class statistics to greatly improve the accuracy of the feeder loss calculation. In the eight feeders simulated, this novel method presented 37% improvement in loss calculation relative to using the load factor method and 4% improvement in loss calculation relative to using normalized customer load statistics.

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