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On Lightlike Hypersurfaces of An Indefinite f-Kenmotsu Space Form

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Abstract
In the present study, we consider a f-Kenmotsu space form $\overline{M}(c)$ and we investigate its lightlike hypersurfaces. We prove the non-existence of these type hypersurfaces of an f-Kenmotsu space form when f is a constant function and it takes different values from $-c$ or $3c$ and so we give a chracterization of lightlike hypersurfaces on a f-Kenmotsu space form. Finally, we obtain some related properties.

Keywords: *Indefinite f-Kenmotsu space form; Lightlike hypersurface; Second fundamental form.*

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1. Introduction

It is well-known that one has there different types of submanifolds in semi-Riemannian manifolds as spacelike, timelike and lightlike. These conditions are determined by the characteristic structure of the induced metric on the tangent space. When a semi-Riemannian manifold is given, we directly obtain a natural existence of lightlike subspaces, due to the degeneracy of the metric, there are fundamental differences between the study of lightlike submanifolds and classical theory of Riemannian and semi-Riemannian submanifolds ([4] and [8]). Moreover, this topis is quite new and it has a developable aspect. Many authors focus on this topic and they investigate lightlike hypersurfaces in different ambient spaces in ([1]-[3], [5], [6], [10]).

Motivated by the previous works, we investigate the characteristic properties of lightlike hypersurfaces of an indefinite f Kenmotsu space form and we get some conditions on the non-existence of lightlike hypersurfaces of an f-Kenmotsu space form when f is a constant function and it is not equal to $-c$ or $3c$. Thus, we give a characterization of these hypersurfaces on $\overline{M}(c)$ and we obtain some related results. On the other hand, we compute Gauss and Codazzi equations of these type hypersurfaces.

2. Preliminaries

Let $\overline{M}$ be a $(2n+1)$-dimensional differentiable manifold. We can say that it has a $(\varphi,\ \xi,\ \eta)$-structure if we have a $(1,\ 1)$ tensor field φ, a vector field ξ and a 1-form η which satisfy

$$\eta(\xi) = 1 \quad \text{and} \quad \varphi^2 = -I + \eta \otimes \xi \tag{2.1}$$

where I denotes the identity transformation. If the $\overline{M}$ endowed with a $(\varphi,\ \xi,\ \eta)$-structure admits a compatible semi-Riemannian metric $\overline{g}$ such that

$$\overline{g}(\varphi x,\ \varphi y) = \overline{g}(x,\ y) - \varepsilon\eta(x)\eta(y) \quad \text{and} \quad \eta(x) = \varepsilon\overline{g}(x,\ \xi) \tag{2.2}$$

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for all vector fields $x,\ y\in\chi\left(\overline{M}\right)$, then we call it an indefinite almost contact metric manifold. Here,

$$\overline{g}\left(\xi,\ \xi\right)=\varepsilon\quad\text{and}\quad\varepsilon=\begin{cases}1, & \text{if } \xi \text{ is spacelike}\\ -1, & \text{if } \xi \text{ is timelike.}\end{cases}$$

Furthermore an indefinite almost contact metric manifold $\overline{M}$ is called an indefinite f-Kenmotsu manifold if the following properties hold

$$\left(\overline{\nabla}_x\varphi\right)y=f\left\{-\overline{g}\left(\varphi x,\ y\right)\xi+\eta\left(y\right)\varphi x\right\}\quad\text{and}\quad\overline{\nabla}_x\xi=f\varphi^2x \tag{2.3}$$

where f denotes a smooth function defined on $\overline{M}$. An indefinite f-Kenmotsu manifold is a natural extension of an f-Kenmotsu manifold defined by Olszak in [7].

Now, for an indefinite f-Kenmotsu manifold $\overline{M}$, if its Riemannian curvature tensor $\overline{R}$ satisfies

$$\begin{aligned}\overline{R}\left(x,\ y\right)z\ =\ & \frac{c-3f}{4}\left\{\overline{g}\left(y,\ z\right)x-\overline{g}\left(x,\ z\right)y\right\}\\ & +\frac{c+f}{4}\left\{\overline{g}\left(\varphi y,\ z\right)\varphi x-\overline{g}\left(\varphi x,\ z\right)\varphi y\right.\\ & -2\overline{g}\left(\varphi x,\ y\right)\varphi z+\overline{g}\left(x,\ z\right)\eta\left(y\right)\xi\\ & \left.-\overline{g}\left(y,\ z\right)\eta\left(x\right)\xi+\eta\left(x\right)\eta\left(z\right)y-\eta\left(y\right)\eta\left(z\right)x\right\}\end{aligned} \tag{2.4}$$

for all vector fields $x,\ y$ and z on $\overline{M}$, then it is called an indefinite f-Kenmotsu space form and we denote it $\overline{M}\left(c\right)$. Moreover, if f is a constant function which is equal to α then it is also called α-Kenmotsu space form. Also, it is said that Kenmotsu space form is a 1-Kenmotsu space form.

Now, we recall some fundamental properties which we use in the next section from [9]. Let $\left(M,\ g,\ S\left(TM\right)\right)$ be a lightlike hypersurface of semi-Riemannian manifold $\left(\overline{M},\ \overline{g}\right)$ and $\overline{\nabla}$ be the Levi-Civita connection on $\overline{M}$ with respect to $\overline{g}$, where $S\left(TM\right)$ denotes the screen distribution. Then we have

$$\overline{\nabla}_xy=\nabla_xy+h\left(x,y\right) \tag{2.5}$$

and

$$\overline{\nabla}_xV=-A_Vx+\nabla_x^{\perp}V \tag{2.6}$$

for all vector fields $x,\ y\in\Gamma\left(TM\right)$ and $V\in\Gamma\left(ltr\left(TM\right)\right)$, where $ltr\left(TM\right)$ is the lightlike transversal vector bundle of M. Moreover, it is said that $\nabla_xy,\ A_Vx\in\Gamma\left(TM\right)$ and $h\left(x,y\right),\ \nabla_x^{\perp}V\in\Gamma\left(ltr\left(TM\right)\right)$ and also it can be easily seen that ∇ is a torsion free linear connection on M, h is a $\Gamma\left(ltr\left(TM\right)\right)$-valued symmetric $F\left(M\right)$-bilinear form on $\Gamma\left(TM\right)$, A_V is a $F\left(M\right)$-linear operator on $\Gamma\left(TM\right)$ and $\nabla^{\perp}$ is a linear connection on the vector bundle $ltr\left(TM\right)$.

Let us suppose that $\left\{\widetilde{E},\ \widetilde{N}\right\}$ is a pair of sections on $U\subset M$. Thus one can define a symmetric $F\left(U\right)$-bilinear form $\widetilde{B}$ and a 1-form ρ on U as follows:

$$\widetilde{B}\left(x,\ y\right)=\overline{g}\left(h\left(x,\ y\right),\ \widetilde{E}\right) \tag{2.7}$$

and

$$\rho\left(x\right)=\overline{g}\left(\nabla_x^{\perp}\widetilde{N},\ \widetilde{E}\right) \tag{2.8}$$

for each $x,\ y\in\Gamma\left(TM|_U\right)$. Hence, by using (2.5), (2.6) we locally get

$$\overline{\nabla}_xy=\nabla_xy+\widetilde{B}\left(x,y\right)\widetilde{N} \tag{2.9}$$

and

$$\overline{\nabla}_xV=-A_{\widetilde{N}}x+\rho\left(x\right)\widetilde{N} \tag{2.10}$$

respectively, where $\widetilde{B}$ is called a local second fundamental form, $A_{\widetilde{N}}$ denotes a shape operator and ∇ is the induced linear torsion free connection. Furthermore, (2.9) and (2.10) are called Gauss and Weingarten formulas of the lightlike hypersurface of $\overline{M}$, respectively.

Now, let $\widetilde{P}$ be the projection of TM on $S\left(TM\right)$. Then the local Gauss and Weingarten formulas can be given by

$$\nabla_x\widetilde{P}y=\nabla_x^{\circ}\widetilde{P}y+\widetilde{C}\left(x,\ \widetilde{P}y\right)\widetilde{E} \tag{2.11}$$

and

$$\nabla_x \widetilde{E} = -A^{\circ}_{\widetilde{E}} x - \rho(x) \widetilde{E} \tag{2.12}$$

where $\nabla^{\circ}_x \widetilde{P}y,\ A^{\circ}_{\widetilde{E}} x \in S(TM)$ and $\widetilde{C}$ denotes a 1-form on U. Then we obtain

$$g\left(A_{\widetilde{N}} x,\ \widetilde{P}y\right) = \widetilde{C}\left(x,\ \widetilde{P}y\right), \quad \overline{g}\left(A_{\widetilde{N}} x,\ \widetilde{N}\right) = 0 \tag{2.13}$$

and

$$g\left(A^{\circ}_{\widetilde{E}} x,\ \widetilde{P}y\right) = \widetilde{B}\left(x,\ \widetilde{P}y\right), \quad \overline{g}\left(A^{\circ}_{\widetilde{E}} x,\ \widetilde{N}\right) = 0 \tag{2.14}$$

for all vector fields $x,\ y \in \Gamma(TM)$.

Let $\overline{R}$ and R be curvature tensors with respect to the connections $\overline{\nabla}$ and ∇, respectively. So we get a relation between $\overline{R}$ and R as

$$\begin{aligned}\overline{R}(x,\ y)z &= R(x,\ y)z + A_{h(x,\ z)}y - A_{h(y,\ z)}x \\ &\quad + (\nabla_x h)(y,\ z) - (\nabla_y h)(x,\ z).\end{aligned} \tag{2.15}$$

Also, we state that the induced connection on M satisfies

$$(\nabla_x g)(y,\ z) = \widetilde{B}(x,\ y)\widetilde{\omega}(z) + \widetilde{B}(x,\ z)\widetilde{\omega}(y) \tag{2.16}$$

for any $x,\ y,\ z \in \Gamma(TM)$, where $\widetilde{\omega}$ is a differential 1-form locally defined on M as follow:

$$\widetilde{\omega}(x) = \overline{g}\left(x,\ \widetilde{N}\right) \tag{2.17}$$

for each $x \in \Gamma(TM)$.

3. Lightlike Hypersurfaces of Indefinite f-Kenmotsu Space Forms

Lemma 3.1. *Let M be a lightlike hypersurface of $\overline{M}(c)$. Then*

(i) we compute the Gauss formulae of M like that

$$\begin{aligned}R(x,\ y)z &= \frac{c-3f}{4}\{\overline{g}(y,\ z)x - \overline{g}(x,\ z)y\} \\ &\quad + \frac{c+f}{4}\{\overline{g}(\varphi y,\ z)\widetilde{\sigma}x - \overline{g}(\varphi x,\ z)\widetilde{\sigma}y \\ &\quad - 2\overline{g}(\varphi x,\ y)\widetilde{\sigma}z + \overline{g}(x,\ z)\eta(y)\xi \\ &\quad - \overline{g}(y,\ z)\eta(x)\xi + \eta(x)\eta(z)y \\ &\quad - \eta(y)\eta(z)x\} - \widetilde{B}(x,\ z)A_{\widetilde{N}}y + \widetilde{B}(y,\ z)A_{\widetilde{N}}x.\end{aligned} \tag{3.1}$$

(ii) the Codazzi formulae of M is given as

$$\begin{aligned}(\nabla_y h)(x,\ z) - (\nabla_x h)(y,\ z) &= \frac{c+f}{4}\{\overline{g}(\varphi y,\ z)\widetilde{\upsilon}(x) \\ &\quad - \overline{g}(\varphi x,\ z)\widetilde{\upsilon}(y) - 2\overline{g}(\varphi x,\ y)\widetilde{\upsilon}(z)\}\widetilde{N}\end{aligned} \tag{3.2}$$

for any $x,\ y,\ z \in \Gamma(TM)$.

Proof. Let M be a lightlike hypersurface of an indefinite f-Kenmotsu space form $\overline{M}(c)$. For any $x \in \Gamma(TM)$, we directly have

$$\varphi x = \widetilde{\sigma}x + \widetilde{\upsilon}(x)\widetilde{N} \tag{3.3}$$

where $\widetilde{\upsilon}(x) = g(x,\ V),\ V = -\varphi\widetilde{E}$ and $\widetilde{\sigma}$ is a tensor field of type $(1,\ 1)$ defined on M. From (2.4) and (2.15), we deduce

$$\begin{aligned}\overline{R}(x,\ y)z &= \frac{c-3f}{4}\{\overline{g}(y,\ z)x - \overline{g}(x,\ z)y\} \\ &\quad + \frac{c+f}{4}\{\overline{g}(\varphi y,\ z)\varphi x - \overline{g}(\varphi x,\ z)\varphi y \\ &\quad - 2\overline{g}(\varphi x,\ y)\varphi z + \overline{g}(x,\ z)\eta(y)\xi \\ &\quad - \overline{g}(y,\ z)\eta(x)\xi + \eta(x)\eta(z)y - \eta(y)\eta(z)x\} \\ &\quad - A_{h(x,\ z)}y + A_{h(y,\ z)}x - (\nabla_x h)(y,\ z) + (\nabla y h)(x,\ z).\end{aligned} \tag{3.4}$$

By virtue of (3.3) and from (3.4), then we get (3.1) and (3.2) by considering the tangential and transversal vector bundle parts. □

Lemma 3.2. *For a lightlike hypersurface M of an indefinite f-Kenmotsu space form $\overline{M}(c)$. We have the following*

$$\overline{g}\left(R\left(x,\ \widetilde{E}\right)z,\ \widetilde{N}\right) = -\frac{c-3f}{4}\overline{g}(x,\ z) - \frac{c+f}{4}\{\widetilde{v}(z)\widetilde{\omega}(\varphi x) + 2\widetilde{v}(x)\widetilde{\omega}(\varphi z) - \eta(x)\eta(z)\}.$$

Proof. It can be easily seen from Lemma 3.1. □

Lemma 3.3. *For a lightlike hypersurface M of an indefinite f-Kenmotsu space form $\overline{M}(c)$. Then we have*

$$\widetilde{B}(y,\ U) = \widetilde{C}(y,\ V)$$

for any $y \in \Gamma(TM)$, where $U = -\varphi\widetilde{N}$.

Proof. By virtue of definition $\widetilde{B}$, we have

$$\begin{aligned}\widetilde{B}\left(y,\ \varphi\widetilde{N}\right) &= \overline{g}\left(h\left(y,\ \varphi\widetilde{N}\right),\ \widetilde{E}\right) = \overline{g}\left(\overline{\nabla}_y\varphi\widetilde{N},\ \widetilde{E}\right)\\ &= -\overline{g}\left(\overline{\nabla}_y\widetilde{N},\ \varphi\widetilde{E}\right) + \overline{g}\left(\left(\overline{\nabla}_y\varphi\right)\widetilde{N},\ \widetilde{E}\right).\end{aligned}$$

From (2.3) and (2.13), it follows that

$$\widetilde{B}\left(y,\ \varphi\widetilde{N}\right) = -\overline{g}\left(\overline{\nabla}_y\widetilde{N},\ \varphi\widetilde{E}\right) = g\left(A_N y,\ \varphi\widetilde{E}\right) = \widetilde{C}\left(y,\ \varphi\widetilde{E}\right)$$

which completes the proof. □

Theorem 3.1. *We can not find a lightlike hypersurface of an indefinite f-Kenmotsu space form $\overline{M}(c)$ with parallel second fundamental form where $f \neq -c$ for all values on $\overline{M}$.*

Proof. We assume that M is a lightlike hypersurface of $\overline{M}(c)$ which satisfies our hypothesis conditions. By setting $y = \widetilde{E}$ and $z = \varphi\widetilde{N}$ in (3.2), it yields that

$$-\frac{3c+3f}{4}\left\{\widetilde{v}(x) - 2\overline{g}\left(x,\ \varphi\widetilde{E}\right)\right\} = 0$$

and taking $x = \varphi\widetilde{N}$ in the last equation, we deduce that

$$f = -c$$

which is a contradiction. Thus we get desired result. □

Theorem 3.2. *We can not find a lightlike hypersurfaces of an indefinite f-Kenmotsu space form $\overline{M}(c)$ with parallel screen distribution where $f \neq 3c$ for all values on $\overline{M}$.*

Proof. We assume that M is a lightlike hypersurface of $\overline{M}(c)$ which satisfies our hypothesis conditions. By using (2.4), then we derive

$$\overline{g}\left(\overline{R}\left(\widetilde{E},\ \varphi\widetilde{N}\right)\varphi\widetilde{E},\ \widetilde{N}\right) = \frac{3c-f}{4}. \tag{3.5}$$

Furthermore, we have

$$\begin{aligned}\overline{g}\left(\overline{R}(x,\ y)\widetilde{P}z,\ \widetilde{N}\right) &= \overline{g}\left(R(x,\ y)\widetilde{P}z,\ \widetilde{N}\right)\\ &= \left(\nabla_x\widetilde{C}\right)\left(y,\ \widetilde{P}z\right) - \left(\nabla_y\widetilde{C}\right)\left(x,\ \widetilde{P}z\right)\\ &\quad +\rho(y)\widetilde{C}\left(x,\ \widetilde{P}z\right) - \rho(x)\widetilde{C}\left(y,\ \widetilde{P}z\right).\end{aligned} \tag{3.6}$$

from [3]. By virtue of (3.6), we get

$$\overline{g}\left(\overline{R}\left(\widetilde{E},\ \varphi\widetilde{N}\right)\varphi\widetilde{E},\ \widetilde{N}\right)=0. \tag{3.7}$$

Now, by considering together (3.5) and (3.7) then it follows that

$$f=3c$$

which is a contradiction and thus we complete the proof. □

Lemma 3.4. *Let us assume that M is a lightlike hypersurface of an indefinite f-Kenmotsu manifold $\overline{M}$. If V is a principle vector field, then we have*

$$\widetilde{B}\left(V,\ U\right)=\widetilde{C}\left(V,\ V\right)=0.$$

Proof. By using (2.3) and (2.9), it follows that

$$\overline{\nabla}_x U=-\overline{\nabla}_x\varphi\widetilde{N}=-\varphi\overline{\nabla}_x\widetilde{N}-\left(\overline{\nabla}_x\varphi\right)\widetilde{N}$$

which means

$$\nabla_x U+\widetilde{B}\left(x,\ U\right)\widetilde{N}=\varphi A_{\widetilde{N}}x-\rho\left(x\right)\varphi\widetilde{N}+\overline{g}\left(x,\ U\right)\xi. \tag{3.8}$$

By virtue of (3.3) and from (3.8), then we derive that

$$\nabla_x U+\widetilde{B}\left(x,\ U\right)\widetilde{N}=\widetilde{\sigma}A_{\widetilde{N}}x+\widetilde{v}\left(A_{\widetilde{N}}x\right)\widetilde{N}-\rho\left(x\right)\varphi\widetilde{N}+\overline{g}\left(x,\ U\right)\xi.$$

Now by comparing the transversal vector bundle parts of both sides of the last equation, it yields that

$$\widetilde{B}\left(x,\ U\right)=\widetilde{v}\left(A_{\widetilde{N}}x\right)=-g\left(A_{\widetilde{N}}x,\ \varphi\widetilde{E}\right)=\widetilde{C}\left(x,\ V\right)$$

which gives us the assertion. □

Lemma 3.5. *We assume that M is a lightlike hypersurface of an indefinite f-Kenmotsu space form $\overline{M}\left(c\right)$. We compute the Codazzi formulae like that*

$$\begin{aligned}\left(\nabla_x A_{\widetilde{N}}\right)y-\left(\nabla_y A_{\widetilde{N}}\right)x &= \frac{c-3f}{4}\left\{\widetilde{\omega}\left(y\right)x-\widetilde{\omega}\left(x\right)y\right\}\\ &+\frac{c+f}{4}\left\{\overline{g}\left(y,\ U\right)\varphi x-\overline{g}\left(x,\ U\right)\varphi y\right.\\ &+2\overline{g}\left(\varphi x,\ y\right)U+\widetilde{\omega}\left(x\right)\eta\left(y\right)\xi\\ &\left.-\widetilde{\omega}\left(y\right)\eta\left(x\right)\xi\right\}+\rho\left(y\right)A_{\widetilde{N}}x-\rho\left(y\right)A_{\widetilde{N}}x.\end{aligned}$$

Proof. It can be easily seen by straightforward computations, thus we omit it. □

Let us consider an orthonormal basis $\left\{e_1,\ \ldots,\ e_{n-2},\ \ldots,\ e_{2n-4},\ \xi,\ \widetilde{E},\ \varphi\widetilde{E},\ \varphi\widetilde{N}\right\}$ of $\Gamma\left(TM\right)$ such that

$$\varphi e_i=e_{n-2+i},\quad \varphi e_{n-2+i}=-e_i\quad \text{and}\quad \varphi\xi=0$$

for each $i=1,\ \ldots,m-2$.

Lemma 3.6. *Let M be a lightlike hypersurface of an indefinite f-Kenmotsu manifold $\overline{M}$. Then*

$$\begin{aligned}A_{\widetilde{N}}U &= \sum_{i=1}^{2n-4}\frac{\widetilde{C}\left(U,\ e_i\right)}{\varepsilon_i}e_i+\widetilde{C}\left(U,\ \xi\right)\xi\\ &+\widetilde{C}\left(U,\ U\right)V+\widetilde{C}\left(U,\ V\right)U\end{aligned} \tag{3.9}$$

and

$$A_{\widetilde{N}}\widetilde{E}=\sum_{i=1}^{2n-4}\frac{\widetilde{C}\left(\widetilde{E},\ e_i\right)}{\varepsilon_i}e_i+\widetilde{C}\left(\widetilde{E},\ \xi\right)\xi+\widetilde{C}\left(\widetilde{E},\ U\right)V \tag{3.10}$$

where $\left\{\varepsilon_i\right\}$ denotes the signature of the basis $\left\{e_i\right\}$.

Proof. By virtue of assumption, we can write

$$A_{\widetilde{N}}U=\sum_{i=1}^{2n-4}\lambda_i e_i+\gamma\xi+\alpha_1\widetilde{E}+\alpha_2\varphi\widetilde{E}+\alpha_3\varphi\widetilde{N}.$$

By taking into account of (2.13), then we deduce that $\lambda_i=\dfrac{\widetilde{C}\left(U,\ e_i\right)}{\varepsilon_i}$, $\gamma=\widetilde{C}\left(U,\ \xi\right),\ \alpha_1=0,\ \alpha_2=-\widetilde{C}\left(U,\ U\right)$ and $\alpha_3=-\widetilde{C}\left(U,\ V\right)$. Thus it yields (3.9). In a similar way, we obtain (3.10). □

Theorem 3.3. *There are no lightlike hypersurfaces of an indefinite f-Kenmotsu manifold $\overline{M}$ with $f\neq 3c$ satisfying*

$$g\left(\left(\nabla_{\widetilde{E}}A_{\widetilde{N}}\right)U,\ V\right)=g\left(\left(\nabla_U A_{\widetilde{N}}\right)\widetilde{E},\ V\right)$$

and

$$\widetilde{B}\left(U,\ U\right)=0.$$

Proof. Putting $y=U$ and $x=\widetilde{E}$ in Lemma 3.5, it follows that

$$\left(\nabla_{\widetilde{E}}A_{\widetilde{N}}\right)U-\left(\nabla_U A_{\widetilde{N}}\right)\widetilde{E}=-\frac{3c-f}{4}U+\rho\left(U\right)A_{\widetilde{N}}\widetilde{E}-\rho\left(\widetilde{E}\right)A_{\widetilde{N}}U.$$

By using (3.9) and (3.10), then we arrive at

$$\begin{aligned}\left(\nabla_{\widetilde{E}}A_{\widetilde{N}}\right)U-\left(\nabla_U A_{\widetilde{N}}\right)\widetilde{E} &= -\frac{3c-f}{4}U+\rho\left(U\right)\left\{\frac{\widetilde{C}\left(\widetilde{E},\ e_i\right)}{\varepsilon_i}e_i\right.\\ &\quad\left.+\widetilde{C}\left(\widetilde{E},\ \xi\right)\xi+\widetilde{C}\left(\widetilde{E},\ U\right)V\right\}\\ &\quad-\rho\left(\widetilde{E}\right)\left\{\frac{\widetilde{C}\left(U,\ e_i\right)}{\varepsilon_i}e_i+\widetilde{C}\left(U,\ \xi\right)\xi\right.\\ &\quad\left.+\widetilde{C}\left(U,\ U\right)V+\widetilde{C}\left(U,\ V\right)U\right\}.\end{aligned}$$

By taking into account of Lemma 3.4, then we derive

$$g\left(\left(\nabla_{\widetilde{E}}A_{\widetilde{N}}\right)U-\left(\nabla_U A_{\widetilde{N}}\right)\widetilde{E},\ V\right)=-\frac{3c-f}{4}U-\rho\left(\widetilde{E}\right)\widetilde{B}\left(U,\ U\right)$$

which implies the desired result. □

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