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Atangana-Baleanu Kesirli Coupled Burgers Denklemi Üzerine Güçlü Bir Çalışma

Hakkı GÜNGÖR[1*]

[1]Ufuk Üniversitesi, Meslek Yüksekokulu, Bilgisayar Teknolojileri Bölümü, 06800, Ankara

[1]https://orcid.org/0000-0002-9546-665X
*Sorumlu yazar: hakki.gungor@ufuk.edu.tr

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ÖZ

Bu çalışma, Atangana-Baleanu anlamında kesirli türevleri içeren, bağlı Burgers denklemi için oldukça doğru sayısal çözümler elde etmek amacıyla yeni Atangana-Baleanu yönteminin uygulanmasını araştırmaktadır. Bunu başarmak için, Atangana-Baleanu q-Elzaki homotopi analiz dönüşümü yöntemi (ABq-EHATM), sistemi etkili bir şekilde ele almak için bir hesaplama tekniği olarak kullanılmıştır. Bu yöntem, kesirli hesaplamanın avantajlarını q-Elzaki ve homotopi analiz çerçevelerinin esnekliğiyle benzersiz bir şekilde birleştirerek, doğrusal olmayanlıkların ve bellek etkilerinin ele alınması konusunda yeni bir bakış açısı sunmaktadır. Bu hibrit yaklaşımla elde edilen sayısal çözümlerin ayrıntılı bir incelemesi sunulmakta ve önerilen yöntemin hem hassasiyeti hem de hesaplama verimliliği vurgulanmaktadır. Ayrıca, kesirli mertebeler aralığı için Maple yazılımı kullanılarak sayısal simülasyonlar gerçekleştirilmekte ve bu da kesirli parametrelerdeki değişimlerin genel sistem dinamiklerini nasıl etkilediğinin kapsamlı bir şekilde incelenmesine olanak sağlamaktadır. Sonuçlar ABq-EHATM'nin yalnızca etkili ve esnek bir yaklaşım değil, aynı zamanda karmaşık doğrusal olmayan kesirli diferansiyel denklemlerle başa çıkmak için sağlam ve güvenilir bir alternatif olduğunu açıkça göstermektedir. Genel olarak, bu çalışma ABq-EHATM'nin pratik ve yenilikçi bir uygulamasını vurgulayarak potansiyel faydalarını göstermekte ve kesirli diferansiyel denklem modellemesi alanında güçlü bir araç olarak önemini vurgulamaktadır.

A Robust Study on Atangana-Baleanu Fractional Coupled Burgers Equation

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ABSTRACT

This study explores the application of the novel Atangana–Baleanu method to obtain highly accurate numerical solutions for coupled Burgers equation, which incorporates fractional derivatives in the sense of Atangana–Baleanu. To achieve this, the Atangana–Baleanu q-Elzaki homotopy analysis transform method (ABq-EHATM) is utilized as a computational technique to address the system effectively. This method uniquely combines the advantages of fractional calculus with the flexibility of q-Elzaki and homotopy analysis frameworks, offering a new perspective on handling nonlinearities and memory effects. A detailed investigation of the numerical solutions obtained through this hybrid approach is presented, emphasizing both the precision and computational efficiency of the proposed method. Furthermore, numerical simulations are carried out using Maple software for a range of fractional orders, allowing for a thorough examination of how variations in the fractional parameters influence the overall system dynamics. The results clearly demonstrate that the ABq-EHATM is not only an effective and flexible approach, but also a robust and reliable alternative for tackling complex nonlinear fractional differential equations. Overall, this study highlights a practical and innovative application of the ABq-EHATM, illustrating its potential benefits and underscoring its significance as a powerful tool in the field of fractional differential equation modeling.

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1. Introduction

Fractional differential equations have recently acquired popularity due to their successful applications in a variety of scientific and engineering domains. For example, a fractional derivative can be used to characterize an earthquake's nonlinear oscillations. In dynamic systems such as earthquakes, which are nonlinear and where past behaviors can influence future responses, fractional derivatives offer a powerful tool to capture this memory dependence. In contrast, partial derivatives generally represent local and instantaneous changes, and thus may fall short in accurately describing systems where historical effects are dominant. In this context, the ability of fractional derivatives to more accurately and realistically model nonlinear oscillations makes them a preferable choice for the mathematical characterization of such physical phenomena. Research on this subject is progressing rapidly, and new applications are being discovered every day. Genetics, medicine, biology, chemistry, geology, economics, statistics, pharmacy, psychology and other fields fall under the umbrella of engineering and physics. It has numerous uses in many fields of science (Miller et al., 1993; Machado et al., 2011; Baleanu et al., 2012; Atangana, 2018; Shah et al., 2022).

Fractional calculus is also applied in domains such as control theory, heat conduction, electricity, mechanics, chaos, and fractals. For example, fractional differentiation is used to generalize Maxwell's equations (Engheta, 1998). Some derivatives, such as the Grünwald, Riemann-Liouville and Caputo definitions, are based on the singular kernel and are discussed in the literature. Recent examples of nonsingular kernels used to define fractional derivatives include the Atangana-Baleanu and Caputo-Fabrizio fractional derivatives (Podlubny, 1999; Kilbas et al., 2006; Caputo and Fabrizio, 2015; Atangana and Baleanu, 2016; Hristov, 2019). This derivative addresses some of the limitations associated with traditional fractional derivatives, such as the singular kernels found in the Riemann-Liouville and Caputo formulations. By employing an exponential kernel, it avoids the singularities present in conventional definitions. The AB derivative is particularly suitable for modeling physical systems that display smoother memory behaviors. In this study, the Caputo derivative is preferred due to its compatibility with classical initial conditions and its suitability for modeling physical problems more effectively. There are a few integral transform approaches available in the literature for solving fractional differential equations. The Elzaki transform (ET) is an important transform in this context. Numerous academics have examined some major techniques for tackling real-world problems, as well as numerical simulations produced by the novel integral transform (Elzaki, 2011).

Burgers equations are applied to products of thermodynamics, gas dynamics, elasticity, diffusion theory, shock wave theory, and turbulence issues.

v is a real constant

$$u_t + uu_x - vu_{xx} = 0 \quad (1)$$

The equation (1), known as the Burgers equation, was first investigated by Bateman (Bateman, 1915). Burger has done substantial research on this equation, particularly as a model of turbulence. As a result, the equation became known as the Burgers equation (Partal, 2011).

The Burgers equation has the following initial and boundary conditions:

$$\begin{cases} u(x,0) = f(x), & a \le x \le b \\ u(a,t) = \alpha, \quad u(b,t) = \beta, & 0 \le t \le T \end{cases} \quad (2)$$

The Burgers equation has similar properties to the Navier-Stokes equation due to the nonlinear convective term ux and the viscosity term vuxx. Therefore, before moving on to the numerical solutions of the Navier-Stokes equation, it is a suitable start to study the simpler model, the Burgers equation. Therefore, the Burgers equation is used as a model to test the stability and accuracy of the numerical solution methods of the Navier-Stokes equation (Jain and Holla, 1978; Uçar et al., 2023).

To date, several scientists have employed various numerical solution approaches to determine the numerical solutions to the Burgers equation. It has been found that numerical solutions to the equation become challenging at very low viscosity values. Jain and Holla investigated the numerical solution of the one and two-dimensional Burgers equations using the finite difference approach and cubic spline functions (Prakasha et al., 2019; Yağmurlu and Gagir, 2022).

The time-fractional coupled Burgers equation is (Esipov, 1995; Liu and Hou, 2011),

$$\begin{cases} \frac{\partial^{\alpha_1} u}{\partial t^{\alpha_1}} = \frac{\partial^2 u}{\partial x^2} + 2u\frac{\partial^{\alpha_2} u}{\partial x^{\alpha_2}} - \frac{\partial(uv)}{\partial x}, \\ \frac{\partial^{\beta_1} v}{\partial t^{\beta_1}} = \frac{\partial^2 v}{\partial x^2} + 2v\frac{\partial^{\beta_2} u}{\partial x^{\beta_2}} - \frac{\partial(uv)}{\partial x}, \qquad 0 < \alpha_i, \beta_i < 1, \qquad t > 0. \end{cases} \quad (3)$$

The Burgers turbulence model is a fundamental framework in fluid dynamics, widely used to study shock waves and nonlinear flow behavior. Many researchers have explored its theoretical aspects to better understand physical flow dynamics and assess alternative approximation methods. Eq. (3) offers an advantage over other numerical formulations due to its simplicity in capturing viscous diffusion and nonlinear advection.

In this study the Atangana- Baleanu fractional coupled Burgers equation is examined as (Ahmet et al.,2019);

$$ {}^{AB}D_t^{\alpha} u(x,y,t) + u(x,y,t)\frac{\partial u(x,y,t)}{\partial x} + v(x,y,t)\frac{\partial u(x,y,t)}{\partial y} = \frac{1}{Re}\left[\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2}\right], \quad (4)$$

$$
{}^{AB}D_t^{\beta} v(x,y,t) + u(x,y,t)\frac{\partial v(x,y,t)}{\partial x} + v(x,y,t)\frac{\partial v(x,y,t)}{\partial y} = \frac{1}{Re}\left[\frac{\partial^2 v(x,y,t)}{\partial x^2} + \frac{\partial^2 v(x,y,t)}{\partial y^2}\right],
$$

where $0 < \zeta < 1,\ 0 < \mu < 1,\ 0 < \alpha < 1,\ 0 < \beta < 1, \tau > 0.$ Re is Reynolds number. Re represents the Reynolds number. As is well-known, when the Reynolds number is large, a shock wave with a cusp forms, and achieving numerical stability near this shock wave is often challenging. (Yağmurlu and Gagir, 2021).

The coupled Burgers equation is used in a variety of applications and is essential for understanding and modeling nonlinear systems. Solving equations can be quite analytically complex, especially in the coupled case. For this reason, numerical methods are widely used in solving problems (Güngör, 2024). The conventional Fourier integral serves as the foundation for the Elzaki Transform. The Elzaki transform is mathematically simple and has fundamental features. Tarig Elzaki created the Elzaki transform to make it easier to solve ordinary and partial differential equations in the time domain. Typically, Fourier, Laplace, and Sumudu transforms are useful mathematical tools for solving differential equations; however, the Elzaki transform and some of its essential features are also used (Baleanu, et al., 2022).

The primary objective of this study is to introduce a novel methodological approach, referred to as the Atangana- Baleanu q-Elzaki Homotopy Analysis Transform Method (ABq-EHATM), and to employ it for obtaining innovative numerical solutions to the Atangana- Baleanu fractional coupled Burgers equation. The structure of this paper is systematically organized as follows.

In the second section, the fundamental definitions of fractional derivatives are elaborated, and the Elzaki transform is introduced as a significant tool for their computation. Section three provides a comprehensive exposition of the Atangana- Baleanu q-Elzaki Homotopy Analysis Transform Method (ABq-EHATM), detailing its mathematical formulation and applicability. In section four, numerical solutions to the Atangana- Baleanu fractional coupled Burgers equation are presented, demonstrating the effectiveness and practical implementation of the proposed approach. Finally, the fifth section summarizes the key findings of the study and offers conclusive remarks based on the obtained results.

2. Main definitions and Theorems

In this section, fundamental definitions and key theorems will be presented and discussed.

Definition 2.1. Let $\vartheta(t)$ be a continuously differentiable function. The Caputo derivative of order $\mu > 0$ is mathematically expressed as (Baleanu, et al., 2012);

$$ {}^{C}D_t^{\mu}[\vartheta(t)] = \frac{1}{\Gamma(n-\mu)} \int_0^t \frac{\vartheta^{(n)}(s)}{(t-s)^{1+\mu-n}} ds, \tag{5} $$

$n-1 < \mu \le n,\ n \in \mathbb{N},\ t > 0,\ \vartheta(t) \in C^{m-1}[0,t],\ s(t) = t^{n-\mu-1}$ is the singular kernel. ${}^{C}D_t^{\mu};\ t \in \mathbb{Z}, \mu \in \mathbb{R}^+$ gives *n* th integer and fractional order derivatives of $\vartheta(t)$(Kürkçü et al., 2019)

Definition 2.2. The fractional derivative of order μ of the function $\vartheta(t)$ is mathematically defined as follows (Anaç, 2022);

$$ D_t^{\mu}[\vartheta(t)] = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dt^n} \int_{t_0}^t (s-t)^{n-\mu-1} [\vartheta_0(s) - \vartheta(s)]\, ds. \tag{6} $$

Definition 2.3. The Atangana-Baleanu (AB) fractional derivative of order $0 < \mu < 1$ for a function $\vartheta(t)$ is defined in terms of the Mittag-Leffler function as follows (Atangana and Baleanu, 2016);

$$ {}^{ABC}D_t^{\mu}[\vartheta(t)] = \frac{N(\mu)}{1-\mu} \int_0^t \vartheta'(\tau)\, E_{\mu}\left[-\frac{\mu(t-\tau)^{\mu}}{1-\mu}\right] d\tau, \qquad 0 < \mu \le 1, \tag{7} $$

where $N(\mu)$ is the normalization function, which satisfies $N(\mu) = 1$ when $\mu = 0, \mu = 1$ and E_{μ} denotes the Mittag-Leffler function, defined as;

$$ E_{\mu} = \sum_{k=0}^{\infty} \frac{\tau^k}{\Gamma(\mu k + 1)} $$

This fractional derivative is widely used in modeling complex systems and non-local phenomena due to its non-singular kernel and memory effects.

Definition 2.4

The Caputo-Fabrizio fractional derivative of order $0 < \mu$ for a differentiable function $\vartheta(t)$ is expressed as (Caputo, 1969; Caputo and Fabrizio, 2015);

$$ {}^{CF}D_{0,t}^{\mu}[\vartheta(t)] = \frac{M(\mu)}{1-\mu} \int_0^t \vartheta'(\tau)\, exp\left[-\frac{\mu(t-\tau)}{1-\mu}\right] d\tau, \qquad t > 0, \tag{8} $$

where $M(\mu)$ is the normalization function such that $M(0) = M(1) = 1$, and $g \in H^1(a,b), b > a.$ Here, $M(\mu)$ represents a normalization function that ensures the operator satisfies fundamental properties of fractional calculus. The presence of the exponential kernel differentiates this derivative from classical fractional derivatives, as it provides a non-singular memory effect, making it suitable for real-world applications involving systems with smooth fading memory behavior (Alkan and Anaç, 2024).

Definition 2.5 The Elzaki transform applied to the Atangana-Baleanu (AB) fractional derivative of order ${}^{AB}D_t^{\mu}[\vartheta(t)]$ under the Atangana-Baleanu-Caputo (ABC) operator is formulated as follows: (Haroon et al., 2022)

$$ {}^{ABC}E_{\alpha}\left[{}^{ABC}D_t^{\alpha}\vartheta(t)\right] = \frac{N(\alpha)}{\alpha\omega^{\alpha}+1-\alpha}\left[\frac{{}^{ABC}E_{\alpha}\,\vartheta(t)}{\omega} - \omega\vartheta(0)\right]. \tag{9}$$

2.1. The basic principle of the q-homotopy analysis transform method

To illustrate the basic concept of the suggested approach (Veeresha, P et al, 2019; Alkan and Anaç 2024) let's look at the fractional-order nonlinear non-homogeneous partial differential equation:

$$D_t^{\mu}u(x,t) + Ku(x,t) + Su(x,t) = f(x,t), t \in (0,\infty), n-1 < \mu \leq n, \tag{10}$$

Where $D_t^{\mu}u(x,t)$ presents Caputo fractional derivative of the function $u(x,t)$, K and S, respectively, correspond to linear and nonlinear operators, and $f(x,t)$, represent the source term.

Applying the Laplace transform (LT**)** to Equation (10), Equation (11) is obtained as follows:

$$s^{\mu}\mathcal{L}[u(x,t)\,] - \sum_{k=0}^{n-1} s^{\mu-k-1}u^{(k)}(x,0) + \mathcal{L}[Ku(x,t)\,] + \mathcal{L}[Su(x,t)\,] = \mathcal{L}[f(x,t)\,]. \tag{11}$$

Rewriting the Eq. (11), Eq. (12) is obtained by

$$\mathcal{L}[u(x,t)\,] - \frac{1}{s^{\mu}}\sum_{k=0}^{n-1} s^{\mu-k-1}u^{(k)}(x,0) + \frac{1}{s^{\mu}}\{\mathcal{L}[Ku(x,t)] + \mathcal{L}[Su(x,t)\,] - \mathcal{L}[f(x,t)\,]\} = 0. \tag{12}$$

Via the homotopy analysis method, the nonlinear operator of $\psi(x,t;q)$ is described by

$$N[\psi(x,t;q)] = \mathcal{L}[\psi(x,t;q)\,] - \frac{1}{s^{\mu}}\sum_{k=0}^{n-1} s^{\mu-k-1}\psi^{(k)}(x,t;q)(0^{+})$$
$$+\frac{1}{s^{\mu}}\{\mathcal{L}[K\psi(x,t;q)] + \mathcal{L}[S\psi(x,t;q)\,] - \mathcal{L}[f(x,t)\,]\},$$

where $q \in \left[0,\frac{1}{n}\right]$, and $\psi(x,t;q)$ is real function of x, t, and q.

A homotopy is generated by

$$(1-nq)\mathcal{L}[\psi(x,t;q) - u_0(x,t)] = hqS[\psi(x,t;q)], \tag{13}$$

where $h \neq 0$ is an auxiliary parameter, and $q \in \left[0,\frac{1}{n}\right], n \geq 1$ is the embedding parameter, $u_0(x,t)$ is an initial guess of $u(x,t)$ and $\psi(x,t;q)$ is an unknown function. By considering the cases $q = 0$ and $q = \frac{1}{n}$, the corresponding solutions of Equation (13) can be determined.

$$\psi(x,t;0) = u_0(x,t), \qquad \psi\left(x,t;\frac{1}{n}\right) = u(x,t),$$

Thus, as q varies from 0 to $\frac{1}{n}$, the solution $\psi(x,t;q)$ gradually approaches $u_0(x,t)$ starting from $u(x,t)$. Expanding around q using Taylor's theorem, the resulting expression is obtained as follows:

$$\psi(x,t;q) = u_0(x,t) + \sum_{i=1}^{\infty} u_m(x,t)q^m, \tag{14}$$

where,

$$u_m(x,t) = \frac{1}{m!}\frac{\partial^m \psi(x,t;q)}{\partial q^m}|_{q=0}.$$

Eq. (14) converges at $q = \frac{1}{n}$ for the convenient $u_0(x,t)$, n and h. Therefore, the numerical solution of the nonlinear equation is obtained as

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\left(\frac{1}{n}\right)^m.$$

By differentiating the $0-th$ order deformation Equation (14) $m-$times with respect to q and subsequently dividing by $m!$, we obtain the following expression for $q = 0$:

$$\mathcal{L}[u_m(x,t) - k_m u_{m-1}(x,t)] = h\mathcal{R}_m(\vec{u}_{m-1}),$$

where the vector notation is defined as

$$\vec{u}_m = \{u_0(x,t), u_1(x,t), \dots, u_m(x,t)\}. \tag{15}$$

Applying the inverse ABCET (IABCET) to Equation (15), we derive the following result:

$$u_m(x,t) = k_m u_{m-1}(x,t) + h\mathcal{L}^{-1}[\mathcal{R}_m(\vec{u}_{m-1})],$$

where

$$\mathcal{R}_m(\vec{u}_{m-1}) = \mathcal{L}[u_{m-1}(x,t)] - \left(1 - \frac{k_m}{n}\right) \left(\sum_{k=0}^{n-1} s^{\mu-k-1}u^{(k)}(x,0) + \frac{1}{s^\mu}\mathcal{L}[f(x,t)\,]\right)\frac{1}{s^\mu}\mathcal{L}[\mathcal{R}u_{m-1} + H^*{}_{m-1}],$$

The coefficient k_m is given by

$$k_m = \begin{cases} 0, & m \le 1, \\ n, & m > 1. \end{cases} \tag{16}$$

Here, $H^*{}_m$ represents homotopy polynomials, which are formulated as

$$H^*{}_m = \frac{1}{m!}\frac{\partial^m \psi(x,t;q)}{\partial q^m}|_{q=0}\,, \psi(x,t;q) = \psi_0 + q\psi_1 + q^2\psi_2 + \cdots. \tag{17}$$

Utilizing Eqs. (16)-(17), one gets

$$u_m(x,t) = (k_m + h)u_{m-1}(x,t) - \left(1 - \frac{k_m}{n}\right)\mathcal{L}^{-1}\left(\sum_{k=0}^{n-1} s^{\mu-k-1}u^{(k)}(x,0) + \frac{1}{s^\mu}\mathcal{L}[f(x,t)\,]\right)$$

$$+h\mathcal{L}^{-1}\left\{\frac{1}{s^\mu}\mathcal{L}[\mathcal{R}u_{m-1} + H^*{}_{m-1}]\right\}$$

Via ABq-EHATM, then it is obtained as

$$u(x,t)=\sum_{m=0}^{\infty}u_m(x,t).$$

3. Analysis of the ABq-EHATM

Now, we proceed with the analysis of the Atangana-Baleanu time fractional partial differential equations (ABTFPDE), which are expressed as follows:

$$^{ABC}D_t^{\alpha}u(x,t)+Kw(x,t)+Sw(x,t)=\zeta(x,t), t\in(0,\infty), n-1<\alpha\leq n. \tag{18}$$

In this context, K and S correspond to linear and nonlinear operators, respectively, while $\zeta(x,t)$ represents a nonhomogeneous function. Additionally, $^{ABC}D_t^{\alpha}$ denotes the Atangana-Baleanu-Caputo fractional derivative (ABCFD) of order α, which plays a crucial role in the mathematical formulation of fractional differential equations.

By applying the Atangana-Baleanu Elzaki Transform (ABET**)** to Equation (18) and incorporating the initial condition (IC), Equation (19) is obtained as follows:

$$\frac{N(\alpha)}{\alpha\omega^{\alpha}+1-\alpha}\left[\frac{^{ABC}E_{\alpha}\ u(x,t)}{\omega}-\omega u(x,0)\right]+{}^{ABC}E_{\alpha}\ [Ku(x,t)+Su(x,t)-\zeta(x,t)]=0. \tag{19}$$

Rewriting the Eq. (19), Eq. (20) is obtained by

$$^{ABC}E_{\alpha}\ u(x,t)-\omega^2u(x,0)+\frac{\alpha\omega^{\alpha+1}+\omega-\omega\alpha}{N(\alpha)} \times {}^{ABC}E_{\alpha}\ [Ku(x,t)+Su(x,t)-\zeta(x,t)]=0. \tag{20}$$

Via the homotopy analysis method, the nonlinear operator of $\psi(x,t;q)$ is described by

$$N[\psi(x,t;q)]={}^{ABC}E[\psi(x,t;q)]-\omega^2\psi(x,t;q)(0^+)+\frac{\alpha\omega^{\alpha+1}+w-w\alpha}{N(\alpha)} \times {}^{ABC}E_{\alpha}\ [K\psi(x,t;q)+S\psi(x,t;q)-\zeta(x,t)],$$

where $q\ \epsilon\left[0,\frac{1}{n}\right]$.

A homotopy is generated by

$$(1-nq)^{ABC}E_{\alpha}\ [\psi(x,t;q)-u_0(x,t)]=hqH^*(x,t)^{ABC}E_{\alpha}\ [\psi(x,t;q)], \tag{21}$$

where $h\neq0$ serves as an auxiliary parameter, and $^{ABC}E_{\alpha}$ denotes Atangana-Baleanu-Caputo Elzaki Transform (ABCET). By considering the cases $q=0$ and $q=\frac{1}{n}$, the corresponding solutions of Equation (21) can be determined.

$$\psi(x,t;0)=u_0(x,t), \psi\left(x,t;\frac{1}{n}\right)=u(x,t).$$

Thus, as q varies from 0 to $\frac{1}{n}$, the solution $\psi(x,t;q)$ gradually approaches $u_0(x,t)$ starting from $u(x,t)$. Expanding around q using Taylor's theorem, the resulting expression is obtained as follows:

$$\psi(x,t;q) = u_0(x,t) + \sum_{i=1}^{\infty} u_m(x,t)q^m, \tag{22}$$

where,

$$u_m(x,t) = \frac{1}{m!}\frac{\partial^m \psi(x,t;q)}{\partial q^m}|_{q=0}.$$

Eq. (22) converges at $q = \frac{1}{n}$ for the convenient $u_0(x,t)$, n and h. Therefore, the numerical solution of the nonlinear equation is obtained as

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\left(\frac{1}{n}\right)^m. \tag{23}$$

By differentiating the $0-th$ order deformation Equation (22) $m-$times with respect to q and subsequently dividing by $m!$, we obtain the following expression for $q = 0$:

$$^{ABC}E_\alpha\,[u_m(x,t) - k_m u_{m-1}(x,t)] = hH^*(x,t)\mathcal{R}_m(\vec{u}_{m-1}),$$

where the vector notation is defined as

$$\vec{u}_m = \{u_0(x,t), u_1(x,t), \dots, u_m(x,t)\}. \tag{24}$$

Applying the inverse ABCET (IABCET) to Equation (24), we derive the following result:

$$u_m(x,t) = k_m u_{m-1}(x,t) + h\left(^{ABC}E_\alpha\right)^{-1}[H^*(x,t)\mathcal{R}_m(\vec{u}_{m-1})],$$

where

$$\mathcal{R}_m(\vec{u}_{m-1}) = {}^{ABC}E_\alpha\,[u_{m-1}(x,t)] - \left(1 - \frac{k_m}{n}\right)\omega^2 u_0(x,t)$$
$$+\frac{\alpha\omega^{\alpha+1} + w - w\alpha}{N(\alpha)}\,{}^{ABC}E_\alpha\,[Ku_{m-1}(x,t) + S_{m-1}(x,t) - \zeta(x,t)].$$

The coefficient k_m is given by

$$k_m = \begin{cases} 0, & m \le 1, \\ n, & m > 1, \end{cases} \tag{25}$$

Here, $H^*{}_m$ represents homotopy polynomials, which are formulated as

$$H^*{}_m = \frac{1}{m!}\frac{\partial^m \psi(x,t;q)}{\partial q^m}|_{q=0},$$

and

$$\psi(x,t;q) = \psi_0 + q\psi_1 + q^2\psi_2 + \cdots. \tag{26}$$

Utilizing Eqs. (25)-(26), one gets

$$w_m(x,t) = (k_m + h)w_{m-1}(x,t) - \left(1 - \frac{k_m}{n}\right)\omega^2 u_0(x,t)$$

$$+h\left({}^{ABC}E_{\alpha}\right)^{-1}\left[\left(\frac{\alpha\omega^{\alpha+1}+w-w\alpha}{N(\alpha)}\,{}^{ABC}E_{\alpha}\,\left[Ku_{m-1}(x,t)+H^{*}{}_{m-1}(x,t)-\zeta(x,t)\right]\right)\right].$$

Via ABq-EHATM, then it is obtained as

$$u(x,t)=\sum_{c=0}^{\infty}u_c(x,t).$$

4. Applications

In the part, the application of the ABq-EHATM to Atangana-Baleanu time-fractional Coupled Burgers equation (ABTFCBE) is presented.

Example 4.1.

Let us analyze the nonlinear ABCTFCBE (Liu and Hou, 2011).

$$\begin{aligned}{}^{AB}D_t^{\alpha}u(x,y,t)+u(x,y,t)\frac{\partial u(x,y,t)}{\partial x}+v(x,y,t)\frac{\partial u(x,y,t)}{\partial y}\\=\frac{1}{Re}\left[\frac{\partial^2 u(x,y,t)}{\partial x^2}+\frac{\partial^2 u(x,y,t)}{\partial y^2}\right],\\{}^{AB}D_t^{\beta}v(x,y,t)+u(x,y,t)\frac{\partial v(x,y,t)}{\partial x}+v(x,y,t)\frac{\partial v(x,y,t)}{\partial y}\\=\frac{1}{Re}\left[\frac{\partial^2 v(x,y,t)}{\partial x^2}+\frac{\partial^2 v(x,y,t)}{\partial y^2}\right],\end{aligned} \tag{27}$$

$$0<x<1,\qquad 0<y<1,\qquad 0<\alpha<1,\qquad 0<\beta<1,\qquad t>0.$$

with ICs

$$\begin{aligned}u(x,0)=\frac{3}{4}-\frac{1}{4+4e^{\frac{R(-4x+4y)}{32}}}.\\v(x,0)=\frac{3}{4}+\frac{1}{4+4e^{\frac{R(-4x+4y)}{32}}}.\end{aligned} \tag{28}$$

where, R is Reynolds number.

Applying the ABCET to Eqs. (28) and using ICs, then it is found by

$$\frac{N(\alpha)}{\alpha w^{\alpha}+1-\alpha}\left(\frac{{}^{ABC}\mathbb{E}_{\alpha}[u(x,t)]}{w}-wu(x,0)\right)+{}^{AB}\mathbb{E}_{\alpha}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{1}{Re}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]\right]=0. \tag{29}$$

$$\frac{N(\alpha)}{\alpha w^{\alpha}+1-\alpha}\left(\frac{{}^{ABC}\mathbb{E}_{\alpha}[v(x,t)]}{w}-wv(x,0)\right)+{}^{AB}\mathbb{E}_{\alpha}\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}-\frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]\right]=0. \tag{30}$$

Rewriting the Eq. (29-30), then it is obtained as

$$ {}^{AB}\mathbb{E}_\alpha[u(x,t)] - w^2 u(x,0) + \frac{\alpha w^\alpha + 1 - \alpha}{N(\alpha)} \times {}^{AB}\mathbb{E}_\alpha\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{1}{Re}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]\right] = 0. \tag{31} $$

$$ {}^{AB}\mathbb{E}_\alpha[v(x,t)] - w^2 v(x,0) + \frac{\alpha w^\alpha + 1 - \alpha}{N(\alpha)} \times {}^{AB}\mathbb{E}_\alpha\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - \frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]\right] = 0. \tag{32} $$

Via the homotopy analysis method, the nonlinear operator of $\varphi(x,t;q), \psi(x,t;q)$ is described by

$$ \begin{aligned} N^1[\varphi(x,y,\tau;q), \psi(x,y,\tau;q)] &= {}^{AB}\mathbb{E}_\alpha[\varphi(x,y,\tau;q)] - w^2\varphi(x,t;q)(0^+) \\ &+ \frac{\alpha w^\alpha + 1 - \alpha}{N(\alpha)}\left\{{}^{AB}\mathbb{E}_\alpha\left[\varphi(x,y,\tau;q)\frac{\partial\varphi(x,y,\tau;q)}{\partial x} + \psi(x,y,\tau;q)\frac{\partial\varphi(x,y,\tau;q)}{\partial y}\right.\right. \\ &\left.\left. - \frac{1}{Re}\left[\frac{\partial^2\varphi(x,y,\tau;q)}{\partial x^2} + + \frac{\partial^2\psi(x,y,\tau;q)}{\partial y^2}\right]\right]\right\}, \end{aligned} \tag{33} $$

$$ \begin{aligned} N^2[\varphi(x,y,\tau;q), \psi(x,y,\tau;q)] &= {}^{AB}\mathbb{E}_\alpha[\psi(x,y,\tau;q)] - w^2\psi(x,t;q)(0^+) \\ &+ \frac{\alpha w^\alpha + 1 - \alpha}{N(\alpha)}\left\{{}^{AB}\mathbb{E}_\alpha\left[\varphi(x,y,\tau;q)\frac{\partial\psi(x,y,\tau;q)}{\partial x} + \psi(\zeta,\mu,\tau;q)\frac{\partial\psi(x,y,\tau;q)}{\partial y}\right.\right. \\ &\left.\left. - \frac{1}{Re}\left[\frac{\partial^2\psi(x,y,\tau;q)}{\partial y^2} + \frac{\partial^2\psi(x,y,\tau;q)}{\partial y^2}\right]\right]\right\}, \end{aligned} \tag{34} $$

where $q \in \left[0, \frac{1}{n}\right]$.

By implementing the proposed algorithm, the deformation equation of order $m - th$ is formulated as follows:

$$ {}^{AB}\mathbb{E}_\alpha[u_m(x,t) - k_m u_{m-1}(x,t)] = h\mathcal{R}_{1,m}(\vec{u}_{m-1}), \tag{35} $$

$$ {}^{AB}\mathbb{E}_\alpha[v_m(x,t) - k_m v_{m-1}(x,t)] = h\mathcal{R}_{2,m}(\vec{v}_{m-1}), \tag{36} $$

where,

$$ \begin{aligned} \mathcal{R}_{1,m}(\vec{u}_{m-1}) = {}^{AB}\mathbb{E}_\alpha[u_{m-1}(x,\mathrm{y},t)] - \left(1 - \frac{k_m}{n}\right)w^2 u_0(x,\mathrm{y},t) + \frac{\alpha w^\alpha + 1 - \alpha}{N(\alpha)} \\ {}^{AB}\mathbb{E}_\alpha\left[\sum_{j=0}^{i} u_j(x,\mathrm{y},t)\frac{\partial u_{i-j}(x,\mathrm{y},t)}{\partial x} + \sum_{j=0}^{i} v_j(x,\mathrm{y},t)\frac{\partial u_{i-j}(x,\mathrm{y},t)}{\partial y}\right. \\ \left. - \frac{1}{Re}\left[\frac{\partial^2 u_{m-1}(x,\mathrm{y},t)}{\partial x^2} + \frac{\partial^2 u_{m-1}(x,\mathrm{y},t)}{\partial y^2}\right]\right]. \end{aligned} \tag{37} $$

$$\mathcal{R}_{2,m}(\vec{v}_{m-1}) = {}^{AB}\mathbb{E}_\alpha[v_{m-1}(x,\mathrm{y},t)] - \left(1-\frac{k_m}{n}\right) w^2 v_0(x,y,t) + \frac{\alpha w^\alpha + 1 - \alpha}{N(\alpha)} \tag{38}$$

$$ {}^{AB}\mathbb{E}_\alpha\left[\sum_{j=0}^{i} v_j(x,\mathrm{y},t)\frac{\partial v_{i-j}(x,\mathrm{y},t)}{\partial x} + \sum_{j=0}^{i} v_j(x,\mathrm{y},t)\frac{\partial v_{i-j}(x,\mathrm{y},t)}{\partial y}\right.$$

$$\left. - \frac{1}{Re}\left[\frac{\partial^2 v_{m-1}(x,\mathrm{y},t)}{\partial x^2} + \frac{\partial^2 v_{m-1}(x,\mathrm{y},t)}{\partial y^2}\right]\right].$$

Implementing the inverse ABCET (IABCET) to Eq. (35-36), then it is acquired by

$$u_m(x,\mathrm{y},t) = k_m u_{m-1}(x,y,t) + h\left({}^{AB}\mathbb{E}_\alpha\right)^{-1}\left[\mathcal{R}_{1,m}(\vec{u}_{m-1})\right]. \tag{39}$$

$$v_m(x,y,t) = k_m v_{m-1}(x,y,t) + h\left({}^{AB}\mathbb{E}_\alpha\right)^{-1}\left[\mathcal{R}_{2,m}(\vec{v}_{m-1})\right]. \tag{40}$$

With ICs, it is obtained by

$$u_0(x,y,t) = \frac{3}{4} - \frac{1}{4+4e^{\frac{R(-4x+4y)}{32}}}. \tag{41}$$

$$v_0(x,y,t) = \frac{3}{4}\frac{3}{4} + \frac{1}{4+4e^{\frac{R(-4x+4y)}{32}}}.$$

Putting the values of $m = 1, m = 2$ in the Eqs. (39-40), Eqs. (42)-(45) are found as

$$u_1(x,y,t) = \frac{hR\left(1-\alpha+\frac{\alpha t^\alpha}{\Gamma(\alpha+1)}\right).e^{-\frac{R(x-y)}{8}}}{128.N(\alpha)\left(1+e^{-\frac{R(x-y)}{8}}\right)^2}. \tag{42}$$

$$v_1(x,y,t) = -\frac{hR\left(1-\beta+\frac{\beta t^\beta}{\Gamma(\beta+1)}\right).e^{-\frac{R(x-y)}{8}}}{128.N(\beta)\left(1+e^{-\frac{R(x-y)}{8}}\right)^2}. \tag{43}$$

$$u_2(x,y,t) = \frac{(n+h)hR\left(1-\alpha+\frac{\alpha t^\alpha}{\Gamma(\alpha+1)}\right)e^{-\frac{R(x-y)}{8}}}{128N(\alpha)\left(1+e^{-\frac{R(x-y)}{8}}\right)^2}$$

$$+ \frac{1}{4096N(\alpha)^2\left(1+e^{-\frac{R(x-y)}{8}}\right)^4 N(\beta)\Gamma(\beta)\Gamma(\alpha)}$$

$$\times\left(h^2 e^{-\frac{R(x-y)}{8}}R^2\left((N(\beta)\Gamma(\beta)\left(\frac{\alpha\Gamma(\alpha+1)t^{2\alpha}}{\Gamma(2\alpha+1)}+\Gamma(1-\alpha)t^{\alpha}\right)\right.\right. \tag{44}$$

$$-\left(N(\alpha)\left(\frac{\alpha\Gamma(\beta+1)t^{2\alpha}}{\Gamma(\alpha+\beta+1)}+(1-\alpha)t^{\beta}\right)\right.$$

$$+\Gamma(\beta)\left(N(\beta)\left(1-\alpha+\frac{\alpha t^{\alpha}}{\Gamma(\alpha+1)}\right)(-1+\alpha)-(-1+\beta)N(\alpha)\right)\Big)\Gamma(\alpha)\Big)e^{-\frac{R(x-y)}{8}}$$

$$+N(\beta)\Gamma(\beta)\left(e^{-\frac{R(x-y)}{4}}-1\right)\left(-\frac{\alpha\Gamma(\alpha+1)t^{2\alpha}}{\Gamma(2\alpha+1)}\right.$$

$$\left.\left.\left.-(1-\alpha)t^{\alpha}+(-1+\alpha)\left(1-\alpha+\frac{\alpha t^{\alpha}}{\Gamma(\alpha+1)}\right)\Gamma(\alpha)\right)\right)\right).$$

$$v_2(x,y,t)=-\frac{(n+h)hR\left(1-\beta+\frac{\beta t^{\beta}}{\Gamma(\beta+1)}\right)e^{-\frac{R(x-y)}{8}}}{128N(\beta)\left(1+e^{-\frac{R(x-y)}{8}}\right)^2}$$

$$-\frac{1}{4096N(\beta)^2\left(1+e^{-\frac{R(x-y)}{8}}\right)^4N(\alpha)\Gamma(\beta)\Gamma(\alpha)}$$

$$\times\left(h^3e^{-\frac{R(x-y)}{8}}R^2\left(\left(\left(\frac{\beta\Gamma(\beta+1)t^{2\beta}}{\Gamma(2\beta+1)}+(1-\beta)t^{\beta}\right)N(\alpha)\Gamma(\alpha)+\Gamma(\beta)\left(-N(\beta)\left(\frac{\beta\Gamma(\alpha+1)t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}+(1-\beta)t^{\alpha}\right)\right.\right.\right.\right. \tag{45}$$

$$\times\left(\left((1-\beta)N(\alpha)+N(\beta)(-1+\alpha)\right)\left(1-\beta+\frac{\beta t^{\beta}}{\Gamma(\beta+1)}\right)\Gamma(\alpha)\right)e^{-\frac{R(x-y)}{8}}+\left(\frac{\beta\Gamma(\beta+1)t^{2\beta}}{\Gamma(2\beta+1)}-(1-\beta)\,t^{\beta}\right.$$

$$\left.\left.+\left(1-\beta+\frac{\beta t^{\beta}}{\Gamma(\beta+1)}\right)(1-\beta)\Gamma(\beta)\right)+\left(e^{-\frac{R(x-y)}{4}}-1\right)N(\alpha)\Gamma(\alpha)\right)\Bigg).$$

Therefore, the ABq-EHATM solution of Eq. (28) is found as

$$u(x,y,t)=u_0(x,y,t)+\sum_{m=1}^{\infty}u_m(x,y,t)\left(\frac{1}{n}\right)^m. \tag{46}$$

$$v(x,y,t)=v_0(x,y,t)+\sum_{m=1}^{\infty}v_m(x,y,t)\left(\frac{1}{n}\right)^m. \tag{47}$$

2D graphs for $u(x,y,t), v(x,y,t)$ solution with ABq-EHATM by distinct α,β values are shown in Figure 1.

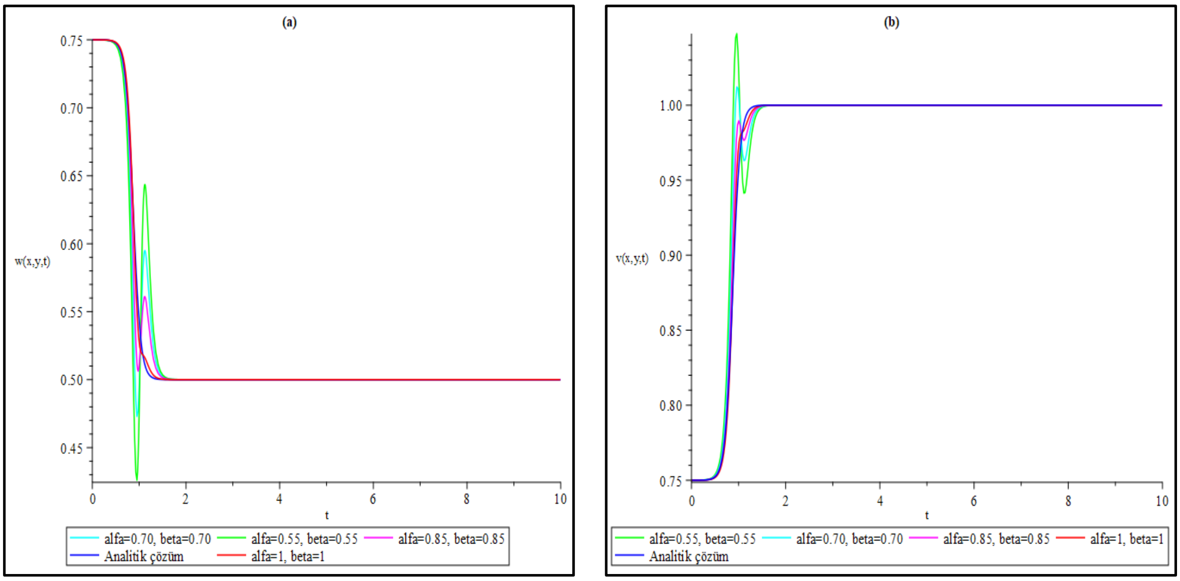

Figure 1. 2D plots of $u(x,y,t), v(x,y,t)$ solutions with ABq-EHATM at $t = 0.5, R = 100, \mathrm{h} = -1, \mathrm{n} = 1$ with distinct α, β.

Figure 2 shows 3D graphs for the $u(x,y,t), v(x,y,t)$ solution of ABq-E

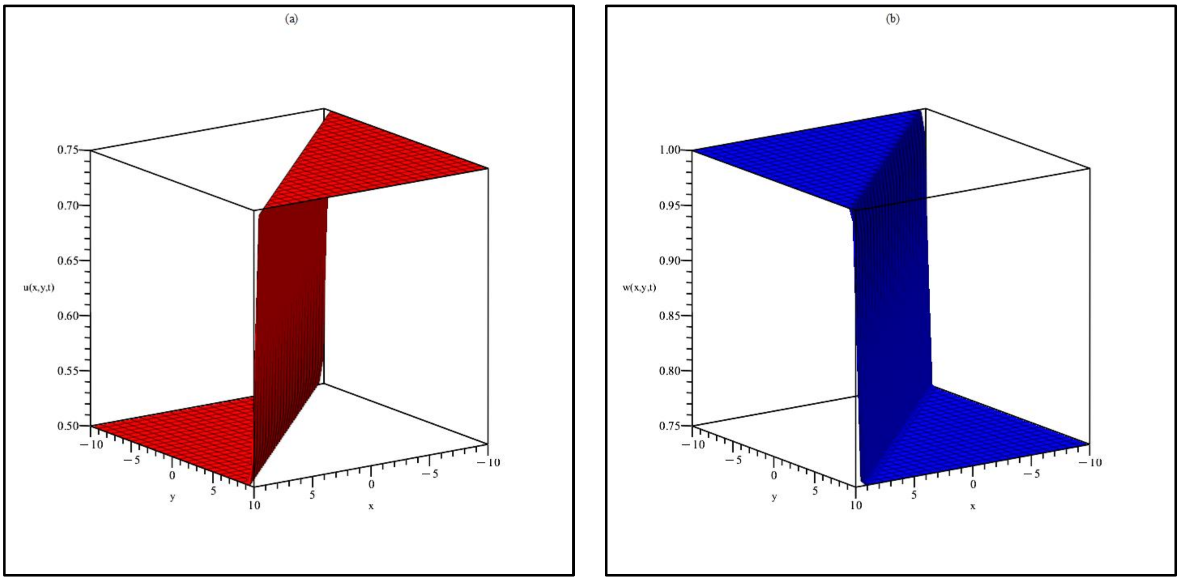

Figure 2. Three-dimensional visual representations of the solutions $u(x,y,t)$, and $v(x,y,t)$ obtained using the ABq-EHATM method for $t = 0.5, R = 100, \mathrm{n} = 1$, and $\mathrm{h} = -1$, considering various values of the fractional parameters α and β.

Table 1 presents the numerical results of the $u(x,y,t)$ obtained using the ABq-EHATM method for different fractional values of α.

Table 1. The numerical values of $u(x, y, t)$ solutions with ABq-EHATM at various fractional values α for $t = 0.5, Re = 100, \mathrm{h} = -1, \mathrm{n} = 1$ ve $\beta = 1$.

x	y	$\alpha = 0.55$	$\alpha = 0.7$	$\alpha = 0.85$	$\alpha = 1$
0.1	0.1	0.5051186770	0.5088260690	0.5166198748	0.5273437500
	0.2	0.5003980018	0.5389457141	0.5712559823	0.5974078528
	0.3	0.6266025790	0.6509116808	0.6704624271	0.6855030474
	0.4	0.7077119587	0.7165529014	0.7235970002	0.7289501769
	0.5	0.7372185060	0.7399346407	0.7420932709	0.7437282216
0.2	0.1	0.5707221498	0.5432957620	0.5260105116	0.5173540324
	0.2	0.5051186770	0.5088260690	0.5166198748	0.5273437500
	0.3	0.5003980018	0.5389457141	0.5712559823	0.5974078528
	0.4	0.6266025790	0.6509116808	0.6704624271	0.6855030474
	0.5	0.7077119587	0.7165529014	0.7235970002	0.7289501769
0.3	0.1	0.5439768322	0.5274194867	0.5162491980	0.5097284239
	0.2	0.5707221498	0.5432957620	0.5260105116	0.5173540324
	0.3	0.5051186770	0.5088260690	0.5166198748	0.5273437500
	0.4	0.5003980018	0.5389457141	0.5712559823	0.5974078528
	0.5	0.6266025790	0.6509116808	0.6704624271	0.6855030474
0.4	0.1	0.5158212171	0.5099438254	0.5059235302	0.5035112213
	0.2	0.5439768322	0.5274194867	0.5162491980	0.5097284239
	0.3	0.5707221498	0.5432957620	0.5260105116	0.5173540324
	0.4	0.5051186770	0.5088260690	0.5166198748	0.5273437500
	0.5	0.5003980018	0.5389457141	0.5712559823	0.5974078528
0.5	0.1	0.5048422224	0.5030509491	0.5018211703	0.5010779842
	0.2	0.5158212171	0.5099438254	0.5059235302	0.5035112213
	0.3	0.5439768322	0.5274194867	0.5162491980	0.5097284239
	0.4	0.5707221498	0.5432957620	0.5260105116	0.5173540324
	0.5	0.5051186770	0.5088260690	0.5166198748	0.5273437500

Table 2 displays the numerical values of $v(x, y, t)$ solutions with ABq-EHATM at various fractional values α.

Table 2. The numerical values of $v(x,y,t)$ solutions with ABq-EHATM at various fractional values α for $t = 0.5, Re = 100, \mathrm{h} = -1, \mathrm{n} = 1$ ve $\beta = 1$.

x	y	$\boldsymbol{\beta = 0.55}$	$\boldsymbol{\beta = 0.7}$	$\boldsymbol{\beta = 0.85}$	$\boldsymbol{\beta = 1}$
0.1	0.1	0.9948813229	0.9911739307	0.9833801252	0.9726562500
	0.2	0.9996019982	0.9610542859	0.9287440177	0.9025921472
	0.3	0.8733974210	0.8490883192	0.8295375729	0.8144969526
	0.4	0.7922880413	0.7834470986	0.7764029998	0.7710498231
	0.5	0.7627814940	0.7600653593	0.7579067291	0.7562717784
0.2	0.1	0.9292778506	0.9567042375	0.9739894880	0.9826459680
	0.2	0.9948813229	0.9911739307	0.9833801252	0.9726562500
	0.3	0.9996019982	0.9610542859	0.9287440177	0.9025921472
	0.4	0.8733974210	0.8490883192	0.8295375729	0.8144969526
	0.5	0.7922880413	0.7834470986	0.7764029998	0.7710498231
0.3	0.1	0.9560231680	0.9725805137	0.9837508017	0.9902715758
	0.2	0.9292778506	0.9567042375	0.9739894880	0.9826459680
	0.3	0.9948813229	0.9911739307	0.9833801252	0.9726562500
	0.4	0.9996019982	0.9610542859	0.9287440177	0.9025921472
	0.5	0.8733974210	0.8490883192	0.8295375729	0.8144969526
0.4	0.1	0.9841787826	0.9900561747	0.9940764696	0.9964887783
	0.2	0.9560231680	0.9725805137	0.9837508017	0.9902715758
	0.3	0.9292778506	0.9567042375	0.9739894880	0.9826459680
	0.4	0.9948813229	0.9911739307	0.9833801252	0.9726562500
	0.5	0.9996019982	0.9610542859	0.9287440177	0.9025921472
0.5	0.1	0.9951577778	0.9969490505	0.9981788296	0.9989220154
	0.2	0.9841787826	0.9900561747	0.9940764696	0.9964887783
	0.3	0.9560231680	0.9725805137	0.9837508017	0.9902715758
	0.4	0.9292778506	0.9567042375	0.9739894880	0.9826459680
	0.5	0.9948813229	0.9911739307	0.9833801252	0.9726562500

5. Results and Discussion

Figure 1 presents the two-dimensional graphical representations of the solutions $u(x,y,t)$ and $v(x,y,t)$ obtained using the Atangana-Baleanu q-Elzaki Homotopy Analysis Transform Method (ABq-EHATM) for various values of the fractional parameter α, specifically $\alpha = 1$ and $\alpha = 0.6, \alpha = 0.7, \alpha = 0.8, \alpha = 0.9$, within the framework of the Atangana-Baleanu-Caputo Time-Fractional Coupled Burgers Equation (ABCTFCBE). Similarly, Figure 2 illustrates the two-dimensional graphs of the function $u(x,t)$, computed through ABq-EHATM for the same set of fractional values, demonstrating the influence of α on the solution behavior. Additionally, the numerical results corresponding to the computed solutions $u(x,y,t)$ and $v(x,y,t)$ for different values of α (i.e., $\alpha = 0.6, \alpha = 0.7, \alpha = 0.8, \alpha = 0.9,$ and $\alpha = 1$) are systematically tabulated in Tables 1 and 2, providing a comparative analysis of the obtained outcomes.

6. Conclusion

The ABq-EHATM exhibits significant potential as a viable methodology for the resolution of the ABCTFCBE. The method's capacity to decompose intricate equations into more manageable components, in conjunction with its precision and alignment with other numerical approaches, renders it a useful asset in the realm of mathematics. Additionally, the ABq-EHATM presents numerous benefits in comparison to conventional numerical methods, rendering its implementation more straightforward and yielding enhanced precision. The numerical solutions for ABCTFCBE have been rapidly and effectively acquired. Thus, it can be inferred that ABq-FHATM is highly efficient and resilient in generating numerical solutions for a wide range of Atangana-Baleanu fractional partial differential equations. Due to its considerable potential, it is anticipated that this approach will assume a progressively significant role in addressing nonlinear differential equations in the forthcoming years. Consequently, researchers are strongly encouraged to delve deeper into its prospective applications.

Statement of Conflict of Interest

The author declare that there is no conflict of interest.

Author's Contributions

The author declares that he has contributed 100% to the article.

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