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Mathematical Competency Conceptions and Professional Needs of Primary School Teachers in Rural Schools

Köy Okullarında Görev Yapan Sınıf Öğretmenlerinin Matematiksel Yetkinlik Algıları ve Mesleki İhtiyaçları

Kübra Çelikdemir[a*]

[a]*TED University, Ankara, Türkiye*

Abstract
Developing mathematically competent individuals is achievable with competent teachers in teaching mathematics. In this context, understanding how teachers conceptualize mathematical competency and identifying their related professional needs is crucial. This study investigates how primary school teachers working in rural schools in Türkiye define a mathematically competent student and the professional needs they have regarding mathematics instruction to nurture such students. A total of 167 primary school teachers, within the first five years of their careers and working in different rural schools, participated in the study. Data were collected via an open-ended questionnaire and analyzed using content analysis and Chi-square analysis. Findings indicate that teachers define mathematical competency based on the mathematical competencies outlined in the literature as well as additional attributes that may serve as prerequisites for these competencies. Moreover, the teachers' professional needs extend beyond the professional competencies for mathematics instruction to include distinct needs specific to the rural school context. Chi-square analysis revealed a significant relationship between teachers' definitions of mathematical competency and their professional needs. These results suggest that teachers are substantially influenced by the conditions of rural schools. Accordingly, the findings highlight the importance of considering rural school conditions in the design and interpretation of teacher support and development practices.

Keywords: Mathematical competence, novice teachers, professional competence, professional needs, rural schools

Öz
Matematiksel yetkin bireylerin yetiştirilmesi, matematik öğretiminde yetkin öğretmenlerle mümkündür. Bu bağlamda, öğretmenlerin matematiksel yetkinliği nasıl algıladıklarını ve buna yönelik ihtiyaçlarının neler olduğunu belirlemek önem taşımaktadır. Bu çalışmada, Türkiye'deki köy okullarında görev yapan sınıf öğretmenlerinin matematiksel yetkin bir öğrenciyi nasıl tanımladıkları ve bu öğrencileri yetiştirmek için matematik öğretimine yönelik mesleki ihtiyaçları incelenmiştir. Araştırmaya, mesleğinin ilk beş yılında olan ve farklı köy okullarında görev yapan 167 sınıf öğretmeni katılmıştır. Veriler, açık uçlu sorulardan oluşan bir anketle toplanmış ve içerik analizi ile Chi-square analizi kullanılarak incelenmiştir. Bulgular, öğretmenlerin matematiksel yetkinliği, alanyazındaki matematiksel yeterlilikler ile bu yeterlilikler için önkoşul olabilecek diğer nitelikler üzerinden tanımladıklarını göstermektedir. Ayrıca, öğretmenlerin mesleki ihtiyaçlarının, matematik öğretimine yönelik profesyonel yeterliklerin ötesinde, köy okulu bağlamına özgü farklı unsurları içeren diğer ihtiyaçlar üzerine yoğunlaştığı belirlenmiştir. Chi-square analizi, matematiksel yetkinlik tanımları ile mesleki ihtiyaçlar arasında anlamlı bir ilişki olduğunu ortaya koymaktadır. Bu öğretmenlerin köy okullarının koşullarından önemli ölçüde etkilendiği sonucuna varılmıştır. Bu durum, köy okulu bağlamının öğretmenlere yönelik destek uygulamalarında ve mesleki gelişim çalışmalarında dikkate alınması gerektiğini göstermektedir.

Anahtar Kelimeler: Matematiksel yetkinlik, sınıf öğretmenleri, mesleki yetkinlik, mesleki ihtiyaçlar, köy okulları

[*]*ADDRESS FOR CORRESPONDENCE: Kübra Çelikdemir, Department of Primary Education, Faculty of Education, TED University, Ankara, Türkiye. E-mail address: kubra.celikdemir@ted.edu.tr,ORCID ID: 0000-0003-2428-9964.*
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1. Introduction

Developing mathematically competent individuals is achievable with competent teachers in teaching mathematics (Niss & Højgaard, 2019). In this context, providing teachers with the professional development they need to enhance their competencies is of utmost importance (Çevikbaş et al., 2024). Effective professional development should address teachers' needs, concerns, missions, and school environments, ensuring it is relevant and practical (Sancar et al., 2021). In other words, to nurture mathematically competent students and ensure that teachers are skilled in mathematics instruction, it is essential to identify the specific types of training and support teachers require. Only professional development programs tailored to teachers' needs can effectively enhance their practice.

Professional development is especially vital for early-career teachers since their flexible and changing knowledge base can be improved with well-structured support (Kaiser & König, 2019). In Türkiye, newly graduated teachers are often assigned to rural schools as part of national policy (Eurydice, 2022). These teachers face several contextual challenges, including harsh living conditions, the need for cultural adaptation, increased administrative duties, and limited social opportunities (Rural Schools Transformation Network [KODA], 2019). Students in these schools often come from families that struggle to support their education due to biases against education, cultural and language barriers, low parental education levels, heavy workloads, large family sizes, and a lack of educational materials and study space at home (KODA, 2023). These challenges are not unique to Türkiye; rural schools in OECD countries also face similar issues, including low socioeconomic status, low test scores, inadequate educational resources, high teacher turnover, and a higher proportion of new teachers (OECD, 2021).

Understanding how teachers conceptualize their needs is also important (Çevikbaş et al., 2024; Sancar et al., 2021). When investigating teachers' needs related to mathematical competence, it is crucial to understand how they conceptualize this need. In mathematics education, conceptions include beliefs, meanings, concepts, propositions, rules, mental images, and preferences (Philipp, 2007). Teachers' conceptions of mathematically competent students reflect their understanding of the specific competencies these students should possess. Developing productive conceptions in teachers is a complex, context-specific process (Huntly, 2008). In supportive environments, with well-designed instructional materials, effective supervision, administrative support, and strong community and parental backing, teachers tend to have more positive conceptions of their students and themselves. This positivity leads to effective instructional strategies, a positive learning environment, and constructive relationships with students (Manizade et al., 2023). Conversely, the lack of such support systems creates challenges for teachers, weakens their commitment, and can influence their decision to remain in the profession (Rhinesmith et al., 2023). Thus, it can be concluded that teachers' conceptions and their needs are interconnected.

Accordingly, this study investigates the professional needs of novice primary school teachers working in rural schools in Türkiye to improve their mathematics teaching competence. However, to understand their professional needs, it is necessary to first comprehend how they define mathematical competence. Therefore, before focusing on their professional needs, this study examines their conceptions of mathematical competence. This approach allows for a clearer understanding of the relationship between teachers' conceptions of mathematical competence and their professional needs. By investigating the conceptions of mathematical competence and professional needs among primary school teachers in rural schools in Türkiye, this study aims to better understand the challenges these teachers face. The findings will offer valuable insights into the specific professional development areas where rural teachers may require support and may contribute to the design of targeted programs to address the unique challenges faced in these contexts.

The subsequent section clarifies and discusses competence, mathematical competence, and professional competence. It also explains the theoretical framework of this study. Therefore, the meanings of these terms, the rationale for choosing that theoretical framework, and how it guided the understanding of mathematical and professional competence, and their associations, considering teachers' needs and conceptions, could be explained.

1.1. Competence, Mathematical Competence, and Professional Competence: The Theoretical Framework

Although there is no universally agreed-upon definition of competence due to diverse scientific interpretations, Weinert (1999) suggested viewing competence as a learned, demand-specific performance disposition. Consistent with this, competence is characterized as the ability to master the essential aspects and demands of a particular field and act effectively within it based on well-founded judgment (Niss, 2003; Niss & Højgaard, 2019). It is defined as *"insightful readiness to act appropriately in response to the challenges of given situations"* (Niss & Højgaard, 2019, p.12).

There are different frameworks for defining mathematical competence and professional competence. However, the only framework that integrates these two concepts from a unified perspective is the Danish KOM project (KOM, which stands for "Competencies and the Learning of Mathematics" in Danish) (Niss & Højgaard, 2019). Since this

study focused on the association between teachers' needs for professional competence and their conceptions of the mathematical competencies that they aim to develop in their students, the KOM framework will be utilized to provide a comprehensive perspective on both mathematical and professional competence. The professional competence for teachers in the KOM framework was designed with reference to students' mathematical competence. Therefore, the following sections will first explain mathematical competence and then discuss professional competence.

1.1.1. Mathematical Competence

Mathematical competence applies in situations where the challenges are mathematical. These situations can generate mathematical challenges both within and beyond mathematics, where mathematics can be applied to answer questions, solve problems, understand phenomena, relationships, and mechanisms, or inform decision-making (Niss & Højgaard, 2019). Accordingly, mathematical competence is defined as having "*the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical* contexts *and situations in which mathematics plays or could play a role*" (Niss, 2003, p. 7). Mathematical competence is distinct from content knowledge and procedural skills (Niss & Højgaard, 2019). Although they are interrelated, content knowledge and procedural skills are centered on knowing information and executing tasks, whereas competencies focus on the active application of mathematical principles.

Eight mathematical competencies were proposed in the KOM project. The first four focus on what it means to pose and answer questions in and by means of mathematics (Niss & Jankvist, 2023). The activation of these four competencies in the first set requires possessing the four competencies in the second set, categorized under "handling the language, constructs, and tools of mathematics" (Niss & Højgaard, 2019). Each mathematical competency is explained in Table 1.

Table 1
Mathematical competencies within the KOM framework

Competency sets	Competencies	Definition
Posing and answering questions	1. Mathematical thinking	Formulating fundamental mathematical questions, understanding potential answers, and recognizing diverse mathematical contexts. It also includes abstract thinking and generalization.
	2. Mathematical problem handling	Identifying, defining, and solving mathematical problems across various domains, including the ability to create problem-solving strategies.
	3. Mathematical modelling	Using mathematical models to address non-mathematical questions involves creating, analyzing, and evaluating these models.
	4. Mathematical reasoning	Analyzing and constructing logical arguments to provide a basis for mathematical statements, including providing logical reasoning and evaluating explanations.
Handling the language, constructs, and tools of mathematics	1. Mathematical representation	Manipulating and understanding diverse mathematical representations and selecting the most suitable ones for different contexts.
	2. Mathematical symbols and formalism	Deciphering and employing mathematical symbols and formalisms effectively.
	3. Mathematical communication	Effectively conveying mathematical concepts through various means.
	4. Mathematical aids and tools	Using material and digital tools in mathematical tasks and critically evaluating their capabilities and limitations.

The KOM framework serves three main purposes: (a) as a tool to describe how mathematics is taught and learned, (b) as a means to establish clear educational objectives for students in mathematics classes, and (c) as a valuable

resource for both teachers and students to monitor their academic progress (Kilpatrick, 2020; Niss & Højgaard, 2019). Consistent with these purposes, the competencies identified in the KOM project were used in diverse settings. They are encompassed within the realm of mathematical processes as delineated in the mathematical literacy framework of the Program for International Student Assessment [PISA] (Niss, 2015). They are also central in the reform of the Danish mathematics curriculum (Højgaard & Sølberg, 2023). In this study, the mathematical competencies defined in the KOM framework will be employed to analyze the conceptions of novice primary school teachers regarding the mathematical competencies they aim to develop in their students.

1.1.2. Professional Competence

To become competent in teaching mathematics, teachers require various professional qualities, including cognitive abilities (e.g., knowledge of mathematics and pedagogy), a positive attitude toward the subject, motivational factors (e.g., enthusiasm), and personal factors (e.g., well-being and stress resilience) (Buchholtz et al., 2023). A prominent framework in mathematics education defines professional competence as the integration of cognitive, affective-motivational, and situated perspectives (Blömeke et al., 2015). As such, teacher professional competence is regarded as a continuum, ranging from teachers' dispositions, such as professional knowledge and beliefs, to their actual performance. Cognitive skills specific to certain situations, such as perception, interpretation, and decision-making, act as intermediaries between the dispositions and performance. The enactment of these skills is closely tied to teaching situations, bridging the gap between teachers' dispositions and actual teaching practices (Buchholtz et al., 2023). Santagata and Yeh (2016) emphasized the bidirectional relationship between cognitive and situational perspectives, arguing that deliberate attention to and interpretation of practice, along with decision-making that generates new knowledge and beliefs, can lead to changes in competence.

Blömeke et al.'s (2015) model encompasses situation-specific cognitive skills, such as the ability to perceive and interpret classroom dynamics and make informed decisions. Consequently, research adopting this model delves into teachers' noticing skills and their interplay with knowledge and beliefs (Buchholtz et al., 2023). Since this study presents professional competence through the teachers' needs, the focus is not on practical action. In other words, the investigation of situation-specific skills is beyond the scope of this study. Nevertheless, the study contends that teachers' practices are pivotal components of professional competence, intricately tied to the contexts in which teachers operate (Blömeke et al., 2020). It also asserts that students' mathematical competencies depend on their teachers' professional competencies (Yang & Kaiser, 2022). Thus, while the Blömeke et al. (2015) model may not serve as an analytical framework, it aligns with this study's perspective.

The researchers involved in the KOM project argue that a competent mathematics teacher can effectively nurture students' mathematical competencies (Niss & Højgaard, 2019). This requires teachers to possess mathematical competencies themselves, at least at the level corresponding to the content they teach. However, this is not sufficient on its own. Teachers must also have didactical and pedagogical competencies specifically tailored to mathematics (Niss & Højgaard, 2011). These competencies highlight the diverse skills necessary for effective mathematics teaching, including understanding of the curriculum, effective teaching practices, student engagement, assessment, collaboration, and continuous professional development (see Table 2). Each competency is crucial in creating and enriching a learning environment and fostering students' competencies in mathematics (Niss & Jankvist, 2023).

Table 2
Mathematics teacher professional competencies within the KOM framework

Competencies	Definition
Curriculum	Analyzing, relating to, and implementing mathematics curricula and syllabi; constructing (part of) new curricula.
Teaching	Devising, planning, organizing, and conducting mathematics teaching; creating diverse teaching/learning situations; selecting and creating teaching materials; inspiring and motivating students.
Uncovering learning	Uncovering, interpreting, and analyzing students' learning of mathematics; understanding students' notions, beliefs, and attitudes towards mathematics.
Assessment	Identifying, characterizing, and assessing students' learning outcomes and mathematical competencies; utilizing various assessment modes for formative and summative purposes.
Collaboration	Collaborating with colleagues within and outside mathematics; working with parents, authorities, and others on matters related to mathematics education.
Professional development	Developing one's own competency as a teacher; participating in professional development activities, reflecting on teaching, and staying abreast of new developments in research and practice.

In this study, the professional competencies defined in the KOM framework will be employed to analyze the professional needs of novice primary school teachers to enhance their competence in teaching mathematics.

1.2. This study

The purpose of this study is to explore the professional needs of novice primary school teachers working in rural schools in Türkiye to enhance their competence in mathematics teaching. Additionally, it seeks to investigate their conceptions of the mathematical competencies they aim to foster in their students and to examine potential associations between these needs and their conceptions of mathematical competence.
This study aims to answer the following research questions:

(1) What conceptions do novice primary school teachers working in rural schools have regarding the attributes of mathematically competent students?
(2) What are the professional needs of novice primary school teachers to enhance their competence in teaching mathematics?
(3) Are there any significant associations between the teachers' professional needs and conceptions of mathematical competence?

This study is significant in that it provides a deeper understanding of the professional needs and conceptions of novice primary school teachers working in rural schools in Türkiye, particularly in the context of mathematics education. Although previous research on rural teachers has primarily focused on general pedagogical practices, working conditions, school resources, and daily life experiences (e.g., Bayrak & Öztürk, 2024; Kaşkaya et al., 2025; Şahin et al., 2022), a review of the existing literature indicates that no prior studies have directly examined rural primary school teachers' professional needs specifically in relation to mathematics teaching. By focusing on mathematics education, this study addresses subject-specific professional needs that are often overlooked in broader investigations.

Moreover, the study is grounded in the KOM framework, which provides a comprehensive conceptualization of both students' mathematical competencies and teachers' professional competencies. The framework has previously been used to describe and analyze students' mathematical understanding and reasoning in various contexts (e.g., Højgaard, 2025; Rocha & Babo, 2024), and it has also been applied to examine teachers' professional competencies, particularly didactico-pedagogical skills. Recent research has further extended the use of this framework to define teachers' competencies in the digital age (Geraniou et al., 2024; Jankvist et al., 2022). However, research specifically investigating mathematics teaching in rural school contexts remains scarce, and the application of the KOM framework in this area has largely been limited to international settings rather than Türkiye.

By examining both teachers' professional competencies and their conceptions of students' mathematical competencies in rural schools, this study is expected to contribute to a better understanding of mathematics teaching practices in rural contexts. The findings may offer implications for teacher education programs and inform the design of professional development initiatives that are more responsive to the needs of rural primary school teachers. In addition, the results may provide evidence for discussions of educational practices and policies addressing the specific challenges of rural schooling in Türkiye. Ultimately, this study seeks to contribute to ongoing efforts to enhance the quality of mathematics teaching in rural areas.

2. Method

2.1. Study Design

This study adopted a qualitatively driven mixed-methods design, in which qualitative data obtained through open-ended questions were systematically transformed into categorical variables for quantitative analysis (Hall & Ryan, 2011). Teachers' written responses were first analyzed using qualitative content analysis based on the KOM framework and inductive coding. The resulting categories were then converted into variables and analyzed quantitatively using Chi-square tests to examine the relationships between teachers' professional needs and their conceptions of mathematical competence.

2.2. Context and Participants

Primary schools in Türkiye provide four years of education, covering Grades 1-4 and serving students aged approximately 6-9. Teachers are responsible for delivering instruction across various subjects, with five hours per

week allocated to mathematics in all four years. Primary school teachers complete a four-year undergraduate program. To work in public schools, they must pass a centralized national exam to apply for available teaching positions. New teachers are assigned to rural schools, where they face several challenges, such as limited resources, cultural and language barriers, and harsh living conditions (KODA, 2019, 2023).

Since most new teachers are female and the challenges in rural schools are particularly difficult for them, non-governmental organizations and the government collaborate to support these teachers' professional development and adaptation to these contexts. Special emphasis is placed on providing training and support systems to help female teachers overcome the unique challenges they face in rural settings. The author engaged with participants from one of these professional development programs, inviting approximately 250 teachers to participate in the study. Of these, 193 teachers consented to participate, and 167 completed the data collection tool, becoming participants in this study. Table 3 presents the distribution of these teachers by teaching experience, class size, number of colleagues at their schools, and whether they teach in multi-grade or single-grade classes.

Table 3
Descriptive statistics for the teachers' background variables

Variables & Categories	*f*	%
Teaching experience		
• 1 year	42	25.1
• 2 years	59	35.3
• 3 years	66	39.5
Number of students in their class		
• 5-10	32	19.2
• 11-20	114	68.3
• 20-30	21	12.6
Number of colleagues (primary school teacher) in the school		
• None	65	38.9
• 1-3	84	50.3
• 4-6	18	10.8
Teaching in … class		
• Multi-grade	93	55.7
• Single grade	74	44.3

2.3. Data Collection

The study data were collected using a questionnaire developed specifically for this study, which comprised three parts (see Appendix 1). The first section collected demographic information to provide insights into teachers' professional needs and their understanding of mathematical competence. The second and third sections featured open-ended questions aimed at clarifying teachers' professional needs for becoming competent mathematics teachers and their conceptions of the mathematical competencies they aim to develop. To obtain more comprehensive data, teachers were asked to provide detailed responses to three specific prompts in these sections.

2.4. Data Analysis

2.4.1. Analysis of the Conceptions and Professional Needs Regarding Mathematical Competence

Qualitative content analysis, incorporating both concept-driven and data-driven categories, was used for data analysis (Kuckartz, 2019). To ensure a robust analysis, a collaborator with expertise in qualitative data analysis was engaged. Initially, we employed a concept-driven approach, coding responses from a random sample of 20 teachers, guided by the KOM framework, as shown in Tables 1 and 2. During this process, we noticed that some responses did not fit neatly into the predefined categories of the KOM framework. As a result, we decided to conduct an inductive analysis, creating data-driven categories to capture these outlying responses. We held thorough discussions to establish potential categories and provide detailed explanations. To finalize the data-driven categories, we coded responses from an additional random sample of 40 teachers and determined these categories through collaborative discussion.

Finally, the remaining data were coded individually for each coder, and full consensus was reached at the end of the process.

We labelled the data-driven categories as "OTHER" because they did not align with the predefined mathematical competencies for students or the professional competencies for teachers. This was anticipated, as teachers' interpretations of the terms mathematical competence and professional competence may differ from those outlined in the KOM framework. Table 4 presents the data-driven categories under the "OTHER" themes, along with sample quotations.

Table 4
Categories of OTHER themes and sample quotations for mathematical competence and professional needs

OTHER theme for…	**Data-driven categories**	**Sample quotations**
		"The mathematically competent student (or I want my students to….)
Mathematical competence	*Character traits*: specific aspects of a student's character, including qualities such as creativity, curiosity, determination, fairness, industriousness, persistence, and analogous descriptive adjectives.	*…is curious to learn and work hard to be successful"*
	Productive disposition: specific attributes including the possession of positive attitudes, intrinsic motivation, the appreciation of mathematics, and self-efficacy in mathematical contexts as representative of productive dispositions.	*… has a natural love for mathematics and a deep appreciation for the subject"*
	Metacognitive competencies encompass a set of skills, including but not limited to goal setting, active listening, note-taking, research skills, self-monitoring, evaluation, and related habits such as effective time management.	*… is aware of what she can or cannot do and asks for help"*
	Content knowledge and procedural skills: attributes related to students' knowledge and application of procedures in particular mathematical concepts, such as mental calculation, skip counting, and rounding.	*… can do basic operations without help"*
		"I need …
Professional needs	*Character traits*: specific aspects of teachers' characters, such as creativity, dynamism, empathy, patience, and analogous descriptive adjectives for teachers' needs.	*… to be more dynamic in the class and patient toward those students who have learning difficulties"*
	Contextual needs: characteristics of the context in which the teachers work such as school and classrooms facilities, students and families' sociocultural backgrounds and compensation policies for the teachers.	*… to have manipulatives and effective materials to teach mathematics"*

Under the OTHER theme for a competent student, the focus shifts away from specific mathematical competencies. Instead, the four data-driven categories highlight potential prerequisites or advantageous attributes that may aid the development of mathematical competence. Two categories, character traits and metacognitive competencies, are not exclusive to mathematics. Similarly, the OTHER theme for competent teachers (professional competence) emphasizes the prerequisites for becoming competent mathematics teachers. These prerequisites are divided into two categories: individual (character traits) and contextual (contextual needs).

The teachers sometimes mentioned two or three of the categories in the same response. To address this, we concentrated on the salient point in the response to determine the competency category. For example, the following

response emphasized students' reasoning and problem-handling competencies. Still, the primary emphasis was on problem handling: "... can solve problems and reason about the alternative strategies in solving the problem" (T008).

2.4.2. Analysis of the Association Between Conceptions of Mathematical Competence and Professional Needs

The third research question explores the associations between teachers' conceptions of mathematical competencies and their professional needs. For this investigation, we needed a consolidated response for both professional needs and conceptions of mathematical competence.

To achieve this, we first assessed whether the three responses to each question exhibited thematic consistency or divergence. Consistency means that the themes across all three responses were identical. For example, if a teacher defined a mathematically competent student through the student's reasoning, thinking, and representation competencies, which belong to the KOM theme, we coded the consolidated response for this teacher as KOM. This indicates that the teacher defined a mathematically competent student solely in terms of the mathematical competencies outlined in the KOM framework. We applied the same procedure for the responses regarding professional needs.

For responses that deviated from this consistency, when one of the three responses fell under the KOM or OTHER themes, carefully re-read them. We then determined the focal point for the three responses and consolidated them under either the KOM or OTHER theme. As a result, we independently identified one theme (KOM or OTHER) for both conceptions of mathematical competence and professional needs. A second coder also worked through this process. The author and the second coder then compared their coding of the consolidated responses for each teacher, discussed any disagreements, and ultimately reached full consensus.

A series of chi-square tests was conducted to examine the relationship between teachers' conceptions of mathematical competencies and professional needs, and to explore whether this relationship varies by teachers' background indicators and school context. These analyses were conducted using SPSS 26, with a significance level (alpha) set at 0.05.

2.5. Trustworthiness of the Analysis

To enhance the trustworthiness of the analysis, a second researcher with experience in qualitative research independently coded the data. Prior to coding, the KOM framework was introduced to the second coder. The two coders reviewed and coded the data independently and then compared their coding. Discrepancies were discussed until full agreement was reached. This consensus-based coding process was used to strengthen the credibility and reliability of the findings.

2.6. Ethical Considerations

Ethical approval for this study was obtained from the TED University Human Research Ethics Committee (Approval No: 2024-42/06). All participants were informed about the purpose of the study, the voluntary nature of participation, and their right to withdraw at any time. Informed consent was obtained from all teachers prior to data collection. Participation was voluntary, no personal identifiers were collected, and all data were analyzed anonymously and stored securely.

3. Findings

First, the mathematical competencies that novice primary school teachers wish to develop in their students are reported (Research Question 1). Next, the professional needs of novice primary school teachers to enhance their competence in teaching mathematics are presented (Research Question 2). Finally, the associations between the teachers' conceptions of mathematical competencies and their professional needs are examined (Research Question 3).

3.1. Teachers' Conceptions of Mathematically Competent Students

Teachers' conceptions of competent students in mathematics are summarized in Table 5 with the frequencies and percentages of responses along with sample quotations for each category. The findings are categorized into two main themes: KOM and OTHER competencies, each providing valuable insights into the attributes associated with mathematically competent students. Notably, 52.50% of responses aligned with the mathematical competencies

outlined in the KOM framework. The remaining 47.50% fall under OTHER, which are outside the scope of the KOM competencies.

Table 5
Distribution of conceptions of mathematically competent students

Themes	Categories	f	%	Sample quotations *Competent students…*
KOM (52.50%)	1. Mathematical thinking	55	10.98	*…approach mathematical situations with a curious and open mind, and make relationships among the concepts.* (T028)
	2. Mathematical problem-handling	72	14.37	*…break down complex problems into smaller, manageable steps and find solutions systematically.* (T085)
	3. Mathematical modelling	1	0.20	*…apply mathematical concepts to real-world problems and create models to represent them.* (T003)
	4. Mathematical reasoning	52	10.38	*…explain their thought process and the logic behind their solutions clearly and coherently.* (T141)
	5. Mathematical representation	47	9.38	*…excel in working with different mathematical notations in doing mathematics.* (T022)
	6. Mathematical symbols and formalism	10	2.00	*…understand and use mathematical symbols and formal language appropriately to express mathematical ideas.* (T042)
	7. Mathematical communication	23	4.59	*…articulate their reasoning and explain solutions to others.* (T068)
	8. Mathematical aids and tools	3	0.60	*…use their hand span or rulers effectively to measure the length of the objects.* (T023)
OTHER (47.50%)	1. Character traits	87	17.37	*…displays perseverance, patience, and curiosity.* (T150)
	2. Productive disposition	50	9.98	*…eager to learn mathematics.* (T011)
	3. Metacognitive competencies	38	7.58	*…know their strengths and weaknesses and make an effort to be better in mathematics. (*T102)
	4. Content knowledge and procedural skills	63	12.57	*…are fluent in addition, subtraction, multiplication, and division.* (T057)
Total		501	100.00	

Among all of the responses, 35.93% were associated with the first set of the KOM mathematical competencies called "posing and answering various types of mathematics questions". In this set, except for modelling competency (0.20%), teachers valued mathematical thinking (10.98%), problem-handling (14.37%), and reasoning (10.38%) competencies highly. Although the modelling and problem-handling competencies appear to be interconnected, there is a notable disparity in the emphasis placed on each. The teachers tended to focus on students' aptitude for formulating and executing strategies for solving mathematical problems. In contrast, addressing challenges beyond mathematical contexts, a key aspect of the modelling competency did not emerge as a priority for teachers when considering mathematically competent students.

On the other hand, 16.57% of responses pertained to the second set, titled "handling the language, constructs, and tools of mathematics". The most valued competency in the second set was representation (9.38%), emphasizing students' capabilities to make transformations among the multiple representations of a concept. The remaining three competencies in this set, namely symbols and formalism, communication, and aids and tools competencies, were generally embedded within responses related to representation. For example: "Students should have multiple representation skills so that they can read and write fractions and show them through manipulatives and pictures"

(T002). Considering that teachers might have preferred to refer to representation competency as a more comprehensive attribute of a mathematically competent student.

The categories in the OTHER theme exhibited clearer boundaries. Attributes in this theme were expressed as prerequisites for achieving competence in mathematics. Productive character traits (17.37%), such as resilience, curiosity, and patience, were essential for teachers, as were content knowledge and procedural skills (12.57%). Moreover, positive dispositions toward mathematics (9.98%), such as a positive attitude, motivation, valuing mathematics, and enjoyment of learning mathematics, as well as students' ability to monitor and control their learning processes (7.58%), were identified as critical attributes of mathematical competence.

3.2. *Teachers' Professional Needs*

The professional needs are categorized into two main themes: KOM and OTHER. Table 6 shows a distribution of the professional need categories identified by teachers as essential for becoming competent mathematics teachers. While 45.31% of the responses aligned with the teacher competencies outlined in the KOM framework, 54.69% pertained to needs falling outside of teacher competencies, categorized as OTHER. Teaching competency (20.56%) from the KOM themes, along with character traits (21.56%) and contextual needs (33.13%) categories in the OTHER theme, dominated the professional needs of the teachers.

Table 6
Distribution of professional needs to become competent in teaching mathematics

Themes	Categories	f	%	Sample quotations *I need (to)…*
KOM 45.31%	1. Curriculum	28	5.59	*…a better understanding of the curriculum and how it aligns with the unique needs of our students. I want more guidance on how to adapt the curriculum to make it more relevant and engaging for my students.* (T003)
	2. Teaching	103	20.56	*…support in developing effective teaching strategies specific to mathematics.* (T132)
	3. Uncovering learning	40	7.98	*…learn how to help my students get a firm grasp mathematical concepts.* (T100)
	4. Assessment	1	0.20	*…guidance on creating fair and effective assessment tools in mathematics and applying them to identify areas where my students might be struggling in learning mathematics.* (T071).
	5. Collaboration	12	2.40	*…experienced colleagues since the other teachers in my school are like me (means novice).* (T015)
	6. Professional development	43	8.58	*…access to workshops, resources, and mentoring that can help me stay updated with the latest teaching methods, tools, and technology.* (T066)
OTHER 54.69%	1. Character traits	108	21.56	*…be patient and empathetic to connect better with my students and create a positive learning environment.* (T167)
	2. Contextual	166	33.13	*…hands-on manipulatives to teach math.* (T103)
Total		501	100.00	

In terms of teaching competency, teachers highlighted several needs: the ability to create effective learning environments, employ diverse teaching strategies, and develop instructional materials. A significant demand emerged for integrating game-based learning and problem-solving methodologies into teaching. One teacher stated, "I need to incorporate more fun games into my lessons to make learning math enjoyable" (T021). Another teacher expressed, "I aim to include more real-world problems in my lessons and emphasize practical problem-solving, but to do this, I need to find or develop resources focused on everyday challenges" (T032).

The categories in the OTHER theme carry greater weight than those in the KOM framework. Contextual needs accounted for the largest share at 33.13%. This highlights the considerable influence of teachers' working conditions on their identified needs, reflecting a strong desire to improve these environments. Beyond the absence of hands-on manipulatives (see Table 6), teachers also emphasized the need for campaigns to incentivize teaching in rural areas. For example, one teacher noted, "Teaching in a remote mountain village with no social scene around is tough. We need better rights as teachers here to stay motivated and perform well in this challenging environment" (T007). Additionally, there was concern about unmotivated students and parents, with a call for interventions to boost motivation: "I need the parents to be more supportive of education, and I want my students to be more eager to learn" (T95).

The needs related to character traits underscore the importance of personal qualities and attributes to become competent in teaching mathematics. Emphasizing both the working environment and teaching at the primary school level, the most sought-after character traits were patience and optimism: "I teach kids from first to fourth grade all in one class. It's a bit tough, but I just need to stay patient, think positive, and believe in myself to be able to teach mathematics effectively" (T160).

3.3. Association Between Teachers' Conceptions of Mathematical Competencies and Their Professional Needs

The cross-tabulation presented in Table 7 illustrates the distributions of teachers (n=167) by the themes of the mathematical competencies they aim to develop and the themes of professional needs they expressed to become competent teachers. Accordingly, four teacher groups emerged: KOM-KOM (n=45), KOM-OTHER (n=24), OTHER-KOM (n=45), OTHER-OTHER (n=53).

Table 7
Crosstabulation of professional needs and mathematical competencies

		Teachers' professional needs		Total	
		KOM	OTHER	f	%
Students' mathematical competencies	KOM	45	45	90	53.89
	OTHER	24	53	77	46.11
Total	f	69	98	167	100
	%	39.52	60.48	100	

Based on crosstabulation, the Chi-square test identified a statistically significant relationship between teachers' professional needs and their conceptions of mathematically competent students (χ^2 (1, N=167) = 6.07, p = 0.014). However, the standardized residuals did not exceed ±1.96 for any category, indicating that no single subgroup disproportionately drives this relationship. This suggests that while an association exists, it is not strongly influenced by a specific teacher group but rather reflects a broader trend across all categories.

Table 8
Chi-square test results for background and school context variables

Variable	χ^2	df	N	p
Teaching experience	7.77	6	167	.26
Number of students in the class	7.60	6	167	.27
Number of colleagues in the school	8.77	6	167	.19
Classroom structure (multi-/single grade)	1.10	3	167	.78

The identification of a significant relationship prompted an exploration of whether this relationship varies with teachers' background indicators and the school context (see Table 8). Subsequently, a series of Chi-square tests was performed. The individual analyses found no statistically significant correlation between teaching experiences (χ2 (6, N=167)=7.77, p=0.26), the number of students in the class (χ2 (6, N=167)=7.60, p=0.27), the number of colleagues in the school (χ2 (6, N=167)=8.77, p=0.19), and teaching in multi-grade or single-grade classrooms (χ2 (3, N=167)=1.10, p=0.78) and the allocation to specific groups in the crosstabulation. This means that differences in teaching experiences and school-related factors, including the number of students, colleagues, and classroom structure (multi-grade or single-grade), do not account for the relationship between teachers' conceptions of mathematically competent students and their professional needs to become competent teachers.

4. Conclusion and Discussion

The present study, grounded in the theoretical frameworks of the KOM project, focused on the mathematical competencies teachers aim to develop in their students (referred to as conceptions of mathematical competence), the professional competencies required of teachers (referred to as professional needs), and the association between these conceptions and needs. The KOM project has deepened the understanding of what it means to apply and teach mathematics by introducing mathematical competencies for students and professional competencies for mathematics teachers, which were adopted in this study (Niss & Højgaard, 2011; Niss & Højgaard, 2019; Niss & Jankvist, 2023). However, teachers' conceptions and professional needs extend beyond the competencies defined in the KOM project; these conceptions and needs were coded inductively and grouped under the "OTHER" theme.

Students in rural schools face challenges, including coming from families with low socioeconomic and sociocultural status, and they tend to have lower test scores and lower motivation (Echazarra & Radinger, 2019; KODA, 2023). In addition, studies focusing on teachers' daily life experiences in rural schools and their views on physical and infrastructural inadequacies consistently emphasize the challenging conditions of rural contexts (Bayrak & Öztürk, 2024; Kaşkaya et al., 2025). Within the scope of the present study, teachers' tendency to focus on areas beyond the mathematical competencies outlined in the KOM project, when considering the mathematical competencies they aim to develop, may indicate a desire to address these broader challenges. Therefore, the additional data-driven categories, including productive dispositions, character traits, metacognitive competencies, content knowledge, and procedural skills, may be viewed as prerequisites for developing mathematical competency in their students.

On the other hand, highlighting productive disposition as an additional competency may relate to the KOM project's focus, which is primarily on the cognitive aspects of mathematical competencies (Niss & Højgaard, 2011). Radisic (2023) argues that acknowledging the importance of dispositions in defining mathematical competence could lead to a more comprehensive understanding of mathematical proficiency, potentially resulting in new methods for measuring competence and a greater emphasis on noncognitive learning outcomes. This perspective is reflected in other competency frameworks for mathematics education, which define competence as encompassing affective, volitional, and cognitive dimensions (Kilpatrick, 2020). In addition, the teachers in this study conceived character traits, metacognitive competencies, content knowledge, and procedural skills as other categories of mathematical competence. Although the literature lacks studies on the influence of character traits and metacognition on mathematical competence, research indicates their predictive value for mathematical performance (Desoete & De Craene, 2019). Therefore, the relationship between students' character traits and metacognitive competencies and the acquisition of mathematical competence warrants further exploration. Having content knowledge and procedural skills as prerequisites for developing mathematical competency was also discussed within the KOM framework (Niss & Højgaard, 2019). They stated that active application of mathematics requires the utilization of procedural skills coupled with factual knowledge (Højgaard & Sølberg, 2023).

The findings of this study also confirmed the interdependence between these two sets of mathematical competencies within the KOM framework. Accordingly, the first set of mathematical competencies for students revolves around posing and answering questions, recognized as the core of mathematical competence (Niss & Jankvist, 2023). The second set emphasizes the ability to handle the language, constructs, and tools of mathematics, seen as a requirement for effectively posing and answering questions (Niss & Højgaard, 2019). The results underscore the heightened importance of competencies in questioning and answering, identifying them as key attributes of mathematically competent students. The competencies in the second set were perceived as less central by the teachers in this study.

The findings on teachers' professional needs indicated that, among the competencies outlined in the KOM project, teaching competency was the most frequently cited need among teachers. Teaching competency includes developing and planning mathematics lessons, organizing and conducting effective teaching sessions, creating a variety of learning activities, choosing and designing appropriate teaching materials, and inspiring and motivating students, and it requires didactical and pedagogical expertise (Niss & Højgaard, 2011). Although teaching experience does not guarantee expertise (Stigler & Miller, 2018), novice teachers generally have difficulties in these processes (Cevikbas et al., 2024). Therefore, this is a reasonable need for novice primary school teachers in this study. Recognizing these needs provides a foundation for designing targeted professional development programs for novice teachers, improving teachers' knowledge and skills in teaching mathematics (Lee & Santagata, 2020), and ultimately supporting students' mathematical competence (Niss & Jankvist, 2023).

Teacher character traits, referred to as pre-existing teacher characteristics that play a pivotal role in attaining competence in teaching, are considered one of the individual factors contributing to competence development in mathematics education (Manizade et al., 2023). However, contextual influences on character traits were evident in this study. By referring to the contextual conditions in which they teach, the teachers expressed needs related to specific character traits such as patience, perseverance, energy, and resilience. While teachers reported that these traits

were essential for achieving competence in mathematics teaching, the underlying need extends to ensuring their well-being in rural schools (OECD, 2021).

A supportive context regarding materials, school and district conditions, supervision and administration, and the local community enhances the potential of teachers' competencies (Çevikbas et al., 2024; Manizade et al., 2023). Our study found that nearly one-third of responses regarding teachers' professional needs were related to contextual factors. This finding supports the argument that most rural areas lack support systems, thus creating many additional challenges for novice teachers (Echazarra & Radinger, 2019; KODA, 2019; OECD, 2021). Since the contextual needs are offline variables, i.e., they are beyond the control of teachers, interventions for supportive context generally do not alleviate the need in the short term (Çelikdemir, 2020). Teachers' expectations for more motivated and supportive parents support this argument because it requires a cultural transformation in rural areas. Nevertheless, contextual interventions such as providing materials for teaching mathematics and campaigns to incentivize teaching in rural areas (identified as the salient contextual needs for teachers) can be implemented more easily and be considered as targeted interventions for teachers in terms of supporting their well-being and encouraging them to stay on in rural schools (Çelikdemir, 2020; Rhinesmith et al., 2023).

The relationship between teachers' conceptions of mathematically competent students and their professional needs was statistically significant. However, no specific subgroup of teachers disproportionately influenced this association, indicating a broader trend across all teachers. Understanding complex relationships requires consideration of the mediating factors here (Manizade et al., 2023). The school-related factors, including the number of students, colleagues, classroom structure (multi-grade or single-grade), and teaching experience ranging from 1 to 3 years, did not contribute significantly to elucidating this relationship. Therefore, the nature of the relationship warrants further exploration in light of the contextual challenges. Greater contextual challenges in rural areas, like harsh living conditions, the need for cultural adaptation, increased administrative duties, and limited social opportunities (KODA, 2019; Rhinesmith et al., 2023) ought to be considered. However, exploring these factors requires engaging with a limited number of participants in extended studies (Manizade et al., 2023), a task that falls outside the scope of this study and warrants consideration in future studies.

The findings of this study are limited to the novice primary school teachers working in rural schools in Türkiye. Exploring other groups, such as preservice teachers, experienced teachers, mathematics teachers working in elementary or secondary schools, or those in urban schools, could reveal other attributes or professional needs. Additionally, relying solely on teachers' self-reports may limit responses, as they may focus on attributes crucial to students and personal needs rather than providing a holistic perspective. Augmenting self-reports with alternative data collection methods, such as classroom observations and individual interviews, would offer a more nuanced understanding of their needs, conceptions and instructional practices regarding mathematical competence. Future studies should address these limitations by examining diverse teacher groups and employing various analytical approaches to present a more comprehensive portrayal of mathematical and professional competence.

This study is significant because it sheds light on novice primary school teachers' professional needs, their conceptions of mathematical competence, and the relationship between these two constructs within rural schools in Türkiye. It highlights the need to expand the KOM framework by incorporating dispositional aspects and recognizing character traits, metacognitive competencies, and content knowledge as potential prerequisites for developing mathematical competence. The study also underscores the importance of considering teachers' character traits and the teaching context in discussions of didactic and pedagogical competencies in mathematics education, particularly for teachers working in challenging rural contexts. These insights are essential for developing education policies that address the specific challenges encountered in rural schools. By using these findings, policymakers and teacher educators can develop targeted strategies and support systems tailored to the challenges of rural schools to build professional competence and enhance mathematical competence.

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Appendix 1
Data Collection Tool

1. Background Questions
School District:
Your total experience as a teacher:
Number of students in your school:
Number of students in your classroom:
How many primary school teachers are working in your school?
Are you teaching in a single-grade classroom or a multi-grade classroom?

- Single-grade
- Multi-grade

If you are teaching in a multigrade class, please state the grades that you are teaching:
……

2. Competent Teacher

Identify and describe three key professional needs that you require to become a competent mathematics teacher.

1) ……

2) ……

3) ……

3. Competent Student

Identify and describe the three key mathematical competencies that you aim to develop in your students.

1) ……

2) ……

3) ……