International Journal of Multidisciplinary Studies and Innovative Technologies

e-ISSN: 2602-4888
dergipark.org.tr/en/pub/ijmsit
Research Article

2025, 9 (1), 108-117
DOI: 10.36287/ijmsit.9.1.15
Received: April 2, 2025; Accepted: April 21, 2025

Reconstruction of a Signal Sampled Slower than the Usual Nyquist Rate Using Complex Sampling

Hüseyin Canbolat[1*]

[1*]*Dept. of Electrical and Electronics Eng./Ankara Yildirim Beyazit University, Ankara, Türkiye*
huseyin.canbolat@aybu.edu.tr (ORCID: 0000-0002-2577-0517*)*

Abstract – In this paper, a modified sampling rate, which is smaller than the usual Nyquist rate, is determined for bandlimited signals using a method like single side band (SSB-like). SSB-like method is characterized using the base signal of single side band Amplitude Modulation (SSB-AM) instead of the actual AM signal. The sampling should be performed on the base SSB signal, which is a complex valued signal. That is, SSB-like sampling is performed on a complex valued signal with a corresponding Nyquist frequency which is half of the actual required sampling frequency. Also, a theoretical sampling method is proposed as a theorem that results in sampled values based on the mathematical properties of transformations. The proposed complex sampling method is novel in the context of sampling and valid for any band-limited real valued signal.

Keywords – complex sampling, amplitude modulation (AM), SSB-AM, Nyquist rate, signal reconstruction

Citation: Canbolat, H. (2025). Reconstruction of a Signal Sampled Slower than the Usual Nyquist Rate Using Complex Sampling. International Journal of Multidisciplinary Studies and Innovative Technologies, 9(1): 108-117.

I. INTRODUCTION

This paper presents a new method for data reconstruction of a signal from its samples which are taken slower than the Nyquist rate.

The reconstruction of a signal from the samples can be achieved if the samples of the signal are taken at a sampling frequency f_s which is larger than the bandwidth of the sampled signal. This rate is known as Shannon's sampling theorem in the communication community since it was introduced and applied to communication area by Shannon in 1949 [1].

The faster than Nyquist (FTN) is an approach, which samples a low-frequency signal $f_l(t)$ instead of a high frequency band limited signal $f_h(t)$, which has the spectrum of $f_l(t)$ at high frequency. Then $f_h(t)$ can be obtained from the samples of $f_l(t)$ via a frequency shift [5]. In FTN the signal is sampled at a lower sampling frequency than the one required by $f_h(t)$ based on the similarity of the spectrums of $f_l(t)$ and $f_h(t)$. For FTN, the high frequency signal should resemble the low frequency signal in terms of the spectrum. However, the proposed approach here can be applied to all band-limited signals.

Here, the proposed method has the potential to increase the sampling period $T = \frac{1}{f_s}$ up to twice the value proposed by the Nyquist theory, where f_s is the sampling frequency. In this paper, the following issues are presented: i) the reconstruction of the under sampled signal; ii) the conjecture that the samples of a signal can be obtained by adding enough AM signals at the center frequencies, $kw_s = 2\pi k f_s$, which are the integral multiples of the sampling frequency.

II. SAMPLING THEORY AND AM

A. Sampling theory

Let a band limited signal, $g(t)$, be given (Figure 1). The signal $g(t)$ has a bandwidth, $BW = 2f_{max}$, where f_{max} is the maximum frequency component of $g(t)$.

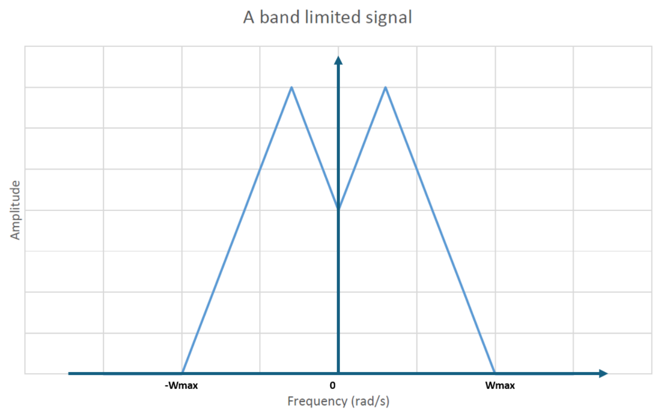

Fig. 1. Spectrum of a band limited signal

The signal $g(t)$ is desired to be stored in a digital system through the samples taken at every T second, such that the original signal, $g(t)$, can be reconstructed from the samples without a loss of information.

The sampling process is formulated by the use of the impulse train called Dirac comb [4] (Figure 2a) of period T,

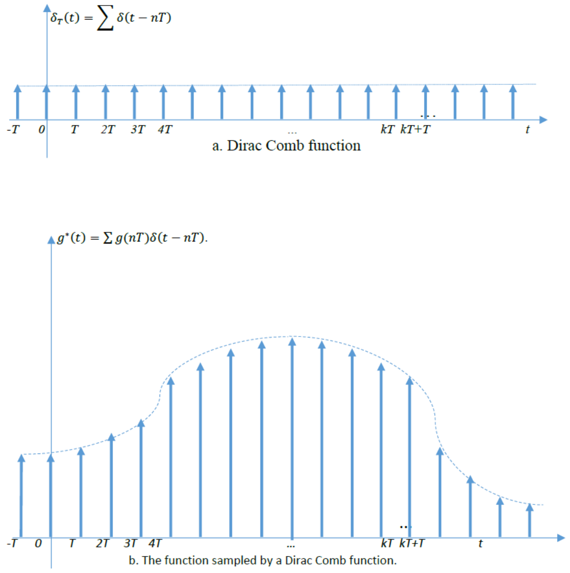

Fig. 2 (a) The Dirac (impulse train) comb function; (b) the samples g*(t) of a function with sampling frequency w_s=1/T

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT). \qquad (1)$$

Note that, the limits of summation will be the set of integers from -∞ to +∞ unless otherwise stated.

The sampled signal can be represented as follows [2]

$$g_T(t) = g(t)\delta_T(t) = \sum g(t)\delta(t - nT). \qquad (2)$$

Using the properties of impulse functions we have

$$g^*(t) = g_T(t) = \sum g(nT)\delta(t - nT). \qquad (3)$$

That is, we get a weighted impulse train by the weights equal to $g(nT)$ (Figure 2b).

For a causal system, the Laplace transformation of the Dirac Comb given in (1) is

$$\mathcal{L}(\delta_T(t)) = \int_0^{\infty} \delta_T(t)e^{-st}dt = \sum_0^{\infty} e^{-nTs}. \qquad (4)$$

Note that Laplace transform may be defined as two-sided and one-sided versions. For causal systems, both definitions are equivalent [6].

Similarly, the Laplace transform of $G^*(s)$ is

$$G^*(s) = \sum_0^{\infty} g(nT)e^{-nTs} = \sum_0^{\infty} g(nT)\,(e^{sT})^{-n}. \qquad (5)$$

In Figure 3, spectrum of $|G^*(jw)|$ is seen as a sampling frequency higher than Nyquist frequency, while a spectrum with aliasing is given in Figure 4. $G^*(s)$, the impulse sampled function, has some useful properties, listed below:

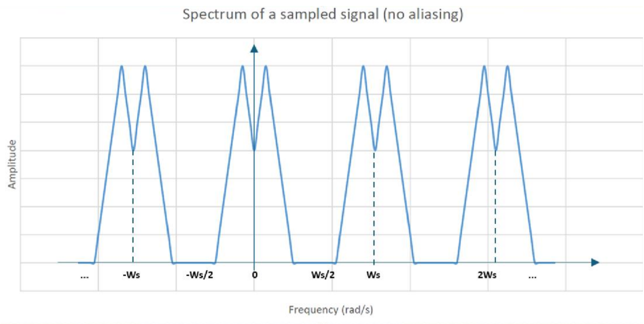

Fig. 3. Spectrum $|G^*(jw)|$ of a sampled signal (no aliasing), that is, $w_s > 2w_{max}$

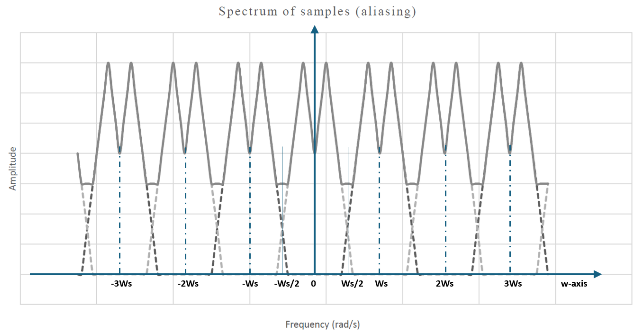

Fig. 4. The spectrum of an under-sampled signal ($|G^*(jw)|$ with $w_s < 2w_{max}$ (aliasing) Note: Only $\pm w_s/2$ is shown!

i. $G^*(s)$ is periodic in s with the period $j\frac{2\pi}{T} = jw_s$, that is (Figure 3)

$$G^*(s) = G^*\left(s + j\frac{2\pi}{T}k\right);\ \forall k \in \mathbb{Z}. \qquad (6)$$

ii. If $G(s)$ has a pole at $s=s_1$, then $G^*(s)$ has poles at

$s = s_1 + jkw_s;\ \forall k \in \mathbb{Z}$.

Note that this is not true for the zeros of $G(s)$ in general.

iii. $G^*(s)$ can be expressed as follows:

$$G^*(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} G(s + jkw_s). \qquad (7)$$

iv. In terms of Fourier Transformation, the frequency response $G^*(jw)$ is the repeated version of $\frac{G(jw)}{T}$ shifted by the integral multiples of sampling frequency, w_s along the jw- axis. If the sampling frequency satisfies the Nyquist rate, then the signal can be recovered from the samples.

v. Since the respective conditions $w_s > 2w_{max}$ and $w_c > w_{max}$ (see Figure 5) for sampling and AM are similar, one can conclude that $G^*(jw)$ includes the AM modulated versions of $g(t)$ infinitely many times with carrier frequencies of kjw_s, for all integer k. Note that, the AM condition should be same to satisfy the sampling conditions to extract AM signal from the spectrum of a sampled signal using a proper bandpass filter (Figure 3).

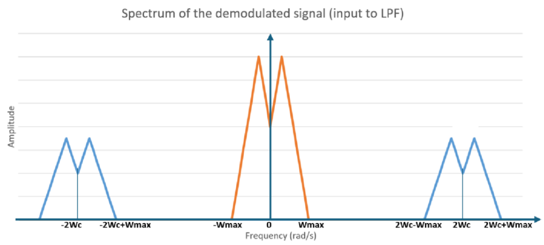

Fig. 5. The spectrum of a demodulated AM signal before LPF. Aliasing occurs when $w_c < w_{max}$.

vi. Therefore, the AM modulation of $g(t)$ with carrier frequency, kw_s for a given k, can be extracted from $G^*(s)$ using an appropriate bandpass filter with BW=w_s centered at kw_s.

The recovery of the signal is not possible due to the aliasing phenomenon if the sampling frequency is less than twice the maximum frequency of the signal according to the Nyquist theorem [3].

B. Amplitude Modulation

The basic AM signal, $m_a(t)$, can be expressed as the multiplication of the carrier signal with the message signal as follows:

$$m_a(t) = m(t)cos(w_c t) \tag{8}$$

where $m(t)$ is the band-limited message signal with a maximum frequency of w_{max}, $\cos(w_c t)$ is the carrier function with the carrier frequency, $w_c > 2w_{max}$.

Since the AM is based on frequency contents, the AM is analyzed using Fourier Transform. The Fourier Transform of $m_a(t)$ is given by (Figure 6)

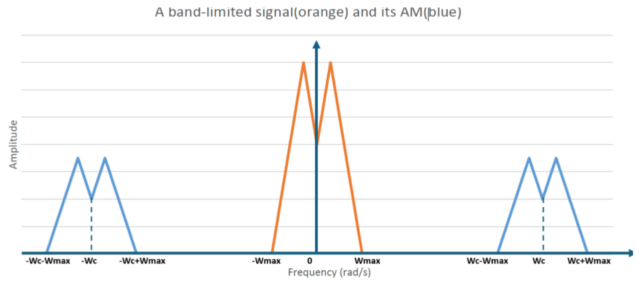

Fig. 6. A band limited signal (center) and its AM signal at w_c (left and right)

$$M_a(jw) = \mathcal{F}\{cos(w_c t)m(t)\} = \frac{1}{2}\mathcal{F}\{[e^{-jw_c t} + e^{jw_c t}]m(t)\}$$

$$M_a(jw) = \frac{1}{2}\big(M(jw + jw_c) + M(jw - jw_c)\big). \tag{9}$$

III. Reconstruction with AM

In this section, a novel method, which is a generalization of the G^{**} method given in [7-8], is offered to recover an under-sampled signal. Note that, AM can be formulated as a pole placement problem [9].

Let a sampling frequency, w_s, be chosen such that

$$Nw_s < w_{max} < (N+1)w_s$$

for some integer N. This means that the resulting sampled signal spectrum exhibits "aliasing" on the base spectrum of $G(s)$ by the first N terms in positive and negative directions. Figure 4 shows this for N=1. General case is not shown in a figure, since it becomes too messy even for N=2.

Let us assume that the $G(s \pm jkw_s)$ for $0<k<(N+1)$ are available. Let $G_N^{**}(s)$ be defined as

$$G_N^{**}(s) = G^*(s) + \frac{1}{T}\left[G(s) - \sum_{k=-N}^{N} G(s + jkw_s)\right]. \tag{10}$$

Then it is obvious that G_N^{**} is has the terms $G(s)$ and all other $G(s+jkw_s)$ except for $k \in \{-N, \dots, -1, 1, \dots, N\}$.

These shifted $G(s+kjw_s)$ terms are all aliased with each other including $G(s)$.

Now, let N=1. Let us assume that the frequency response of $g(t)$ is symmetric and $G^*(s)$ is the Laplace transform of the impulse sampled version with $w_{max} < w_s < 2w_{max}$. Due to non-Nyquist choice of sampling frequency, $G^*(jw)$ exhibits aliasing and $g(t)$ cannot be reconstructed from the samples at hand (see Figure 4).

Note that, $G^{**}(s) = G_1^{**}(s)$ is used for simplicity.

Let us assume that $G(s \pm jkw_s)$ and $G(s-jkw_s)$ are available. Let $G^{**}(s)$ be defined as

$$G^{**}(s) = G^*(s) - \frac{G(s+jw_s)+G(s-jw_s)}{T}. \tag{11}$$

Then $G^{**}(jw)$ has a non-aliased version of $G(jw)$ around w=0 because the parts, which cause aliasing, are removed (Figure 7). Then using an appropriate low pass filter, the original signal can be recovered from the samples even though $g(t)$ was sampled with a non-Nyquist sampling frequency.

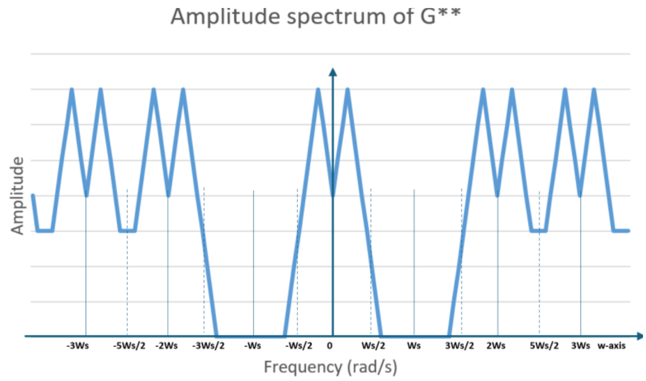

Fig. 7. The spectrum of $|G^{**}(jw)|$. Aliasing is removed

Note that, considering the Fourier transforms, we can write

$$G(s + jw_s) + G(s - jw_s) = 2G_a(s). \tag{12}$$

where $g_a(t)$ is the AM modulated $g(t)$. This means that, a weighted version of AM signal of $g(t)$ should be removed from $G^*(s)$. On the other hand, shifting $G(s)$ properly provides the same result.

Now, according to the above approach, it seems that we can remove all parts that result in aliasing for smaller sampling frequencies. Thus, we can recover the signal from very few samples. The following remarks should be noted:

i. It is possible to think as follows: If we have an AM modulated version of $g(t)$ at hand, we do not need to have samples of the signal for recovery. The AM wave can be used to recover it. Indeed, this is true if the AM carrier frequency is larger than the Nyquist rate. If the carrier has a frequency less than the Nyquist frequency, the signal cannot be obtained from AM without distortion. Therefore, for an under-sampled signal, the above procedure can be employed for reconstruction.

ii. On the other hand, if we think that enough AM versions are at hand, then the signal can be recovered even from a single sample. This clearly makes no sense. At least there should be a limit for recovery. The best-known limit is the Nyquist rate, which requires the sampling frequency to be twice the maximum frequency of $G(jw)$. Note that the AM condition and the Nyquist rate are same for a full recovery.

iii. However, the above approach suggests that there should be a method to recover the signal from its samples, which are taken slower than the Nyquist rate. The method should make it possible to remove the aliasing, thus, allowing the recovery without a need to have an AM version at hand.

The approach here suggests that the reconstruction of a signal from its samples may be achieved for a non-Nyquist sampling period. The study continues to devise such a method. The approach presented is the preliminary results for recovery of an under-sampled signal. The G^{**} method can be seen as a possibility that recovery can be achieved with a sampling frequency less than the Nyquist rate.

IV. OBTAINING SAMPLES USING AM

In this section, it is conjectured that the sampling of a signal can be achieved by addition of sufficiently many AM signals based on the above approach.

Let us define the k^{th} AM-harmonic function, $q_k(t)$, for $m(t)$ with $M(jw)=0$ for all $w>w_{max}$ as follows:

$$q_k(t) = m(t)\cos(kw_s t) \quad (13)$$

where k is an integer, $w_s=2\pi f_s=2\pi/T>2\pi f_{max}=2w_{max}$ is the sampling frequency determined by the sampling period T.

From (9), the Fourier Transform of $q_k(t)$ is

$$Q_k(jw) = \frac{1}{2}\left(M(jw + jkw_s) + M(jw - jkw_s)\right) \quad (14)$$

Indeed, $q_k(t)$ is the amplitude modulation of $m(t)$ performed at the carrier frequency of $w_{ck}=kw_s$.

From (8) and (14), (7) can be written as

$$G^*(jw) = \frac{1}{T}M(jw) + \frac{2}{T}\sum_{k=1}^{\infty} Q_k(jw) \quad (15).$$

From (15), one can predict that the samples of $m(t)$, which are taken at every T second, can be obtained.

(15) leads to the following theorem stated as [7]:

Theorem 1:

Let $m(t)$ be a band limited signal, whose spectrum is zero for all $w>w_{max}$. Then the samples of $m(t)$, which are taken at every T second that satisfies the Nyquist condition (that is, $1/T=f_s>2f_{max}$), can be obtained as a summation of sufficiently many AM modulations, $Q_k(jw)$ of $m(t)$ without using a sampling device.

The proof of the above theorem can be constructed based on (15). Let us consider the inverse Fourier transformation of (15)

$$g^*(t) = \frac{1}{T}m(t) + \frac{2}{T}\sum_{k=1}^{\infty} q_k(t) \quad (16).$$

Since $g^*(t)$ is defined as the shifted impulses by kT with weights, $m(kT)$, (16) can be written as

$$g^*(kT) = m(kT) = \frac{1}{T}m(kT) + \frac{2}{T}\sum_{k=1}^{\infty} q_k(kT) \quad (17).$$

Here the problem is to determine the limit, M, of summation in (17) to obtain a satisfactory list of samples taken at every sampling instant, kT. Due to this, the word "many" should be figured based on the parameters and properties of the sampled signal. The study is continuing on this.

V. RECONSTRUCTION VIA SSB AM

As mentioned in section III, the reconstruction of a signal from its non-Nyquist samples can be achieved via its AM modulation only in continuous time. G^{**} method can remove the distortion caused by aliasing in continuous time. However, it is not possible to remove the distortion in discrete time due to the cyclic nature of the discretization around the unit circle in z-plane. This can be explained by considering the starred transformation, which is the Laplace transformation of the signals that are formed by weighted impulses, which are shifted by the sampling period, because of the sampling model using Dirac comb given in (1). The z-transformation of the starred Laplace is obtained by substituting $z = e^{sT}$. This results in a mapping of strips of the Laplace domain, which are defined as

$$S_k = \left\{z \in \mathbb{C} \left| \left(k - \frac{1}{2}\right) w_{s2} < Im(s) < \left(k + \frac{1}{2}\right) w_{s2} \right.\right\}, \quad (18)$$

where $w_{s2} = \frac{w_s}{2} = \frac{\pi}{T}$ is the half of the sampling frequency. It can be shown that the $\mathcal{L}^* \rightarrow \mathcal{Z}$, where $\mathcal{Z}$ denotes the z-transformation obtained from starred Laplace transformation, maps each of the strips S_k as follows,

$$\begin{vmatrix} |z| < 1 & for\ Re(s) < 0 \\ |z| = 1 & for\ Re(s) = 0. \\ |z| > 1 & for\ Re(s) > 0 \end{vmatrix} \quad (19)$$

(19) is valid for each integer k in (18). This means that the removal of aliasing by G^{**} method is not possible since the aliased portions cannot be removed in discrete time as soon as the z-transformation is completed. This can be seen in Figure 5-4 (see Figure 8 for a visualization obtained in desmos.com). The actual mapping takes s=0 to z=1 and the center of the cylinder should be at the origin), since the imaginary axis part of the whole strip is mapped on the unit-circle in z-plane. This ends up the aliasing is preserved around z=-1. Therefore, the G** method cannot be used in discrete time.

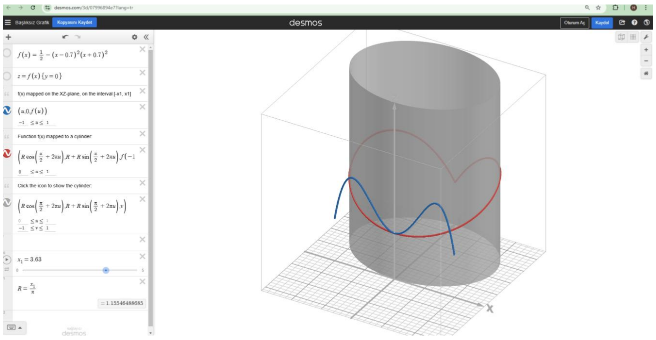

Fig. 8. The transformation of a function around a cylindrical surface (Note: Here s=0 mapped into z=0)

The basic question is whether there is a method to avoid aliasing in discrete time. The answer is yes as it is explained below.

Note that the Fourier transformation of real valued signals is conjugate-symmetric around w=0 [6]. That is, given $x(t)$,

$$\mathcal{F}(x(t)) = X(jw) \Rightarrow \mathcal{F}(x^*(t)) = X^*(-jw).$$

If $x(t)$ is real valued, $x(t)=x^*(t)$. Then

$$\mathcal{F}(x(t)) = \mathcal{F}(x^*(t)) \Rightarrow X(jw) = X^*(-jw).$$

$$\begin{cases} |X(jw)| = |X^*(-jw)| \\ \arg(X(jw) = -\arg(X^*(-jw)) \end{cases}. \quad (20).$$

(20) means that the Fourier transformation carries the whole information of the signal in positive (or negative) frequencies only for a real signal. This is the idea behind the SSB-AM. SSB-AM is based on the operation of eliminating the negative or positive frequency components of the signal and then recover it from these one sided information.

.In the sampling process, it is possible to eliminate the negative (or positive) frequency components of the signal before sampling then sample it according to the bandwidth of the eliminated signal to avoid aliasing. Since the bandwidth of the eliminated signal is half of the original signal, the sampling frequency can be selected lower than the sampling frequency required by the Nyquist criteria. This can be justified by the fact that the spectrum of real valued signals is symmetric in amplitude and in phase with respect to the vertical axis and the origin, respectively, as shown in (20). The principle of single side band AM (SSB-AM) is based on this fact. Here, the use of basic SSB signal is proposed for a slower sampling since the bandwidth of SSB signal is w_{max} instead of $2w_{max}$.

The classical AM uses the modulation of the whole signal. That is, the message signal is modulated for both negative and positive frequencies. In telecommunication, the reduction of energy is an important factor. Message signals generally have Fourier transforms, which are symmetric and antisymmetric in amplitude and phase of the signal, respectively, with respect to the frequency axis. That is, the amplitude of the Fourier transformation is the same for positive and negative frequencies. Therefore, sending AM wave requires at least twice the energy to send and demodulate the signal. SSB-AM method attenuates one of the side bands and sends the AM signal with a reduced power.

The SSB-AM can be formulated as follows:

The linear modulation scheme can be defined as

$$s_a(t) = s_I(t) + js_Q(t), \quad (21)$$

Where the subscripts I and Q denote the in-phase components and the quadrature components, respectively.

For a given message signal, $m(t)$, with the Hilbert transform, $\hat{m}(t)$, there are three cases:

i. The linear modulation if $s_I(t) = m(t)$ and $s_Q(t) = 0$.

ii. Let $s_I(t) = \frac{1}{2}m(t)$

   a. The upper side band (USB) modulation if $s_Q(t) = \frac{1}{2}\hat{m}(t)$.

   b. The lower side band (LSB) modulation if $s_Q(t) = \frac{-1}{2}\hat{m}(t)$.

Employing the above equations,

$$m(t) = Re\{s_a(t)\} = Re\{s(t) + j\hat{s}(t)\}. \quad (22)$$

It can be shown that the Fourier transform, $S_a(jw)$, of $s_a(t)$ is as follows [1]:

i. $S_a(jw) = M(jw)$, (23a)

ii. $S_a(jw) = \begin{cases} 0.5M(jw) & w \geq 0 \\ 0 & w < 0 \end{cases}$, (23b)

iii. $S_a(jw) = \begin{cases} 0 & w \geq 0 \\ 0.5M(jw) & w < 0 \end{cases}$, (23c)

Where $M(jw)$ is the Fourier transform of $m(t)$. Note that same relations are also satisfied by Laplace transforms.

The actual SSB-AM at the carrier frequency of w_0 can be formulated as follows:

Here only the upper (right) side band of the signal is considered. The same procedure can be applied to the lower (left) side band. Let a signal $m(t)$ with a Hilbert transform of $\hat{m}(t)$ is given. Then its SSB modulated signal $s_{ssb}(t)$ is defined as

$$s_{ssb}(t) = Re\{s_a(t)e^{jw_0t}\} \quad (24)$$

$$s_{ssb}(t) = Re\{[m(t) + j\hat{m}(t)]\,[\cos(w_0t) + j\sin(w_0t)]\}$$

$$s_{ssb}(t) = m(t)\cos(w_0t) - \hat{m}(t)\sin(w_0t), \quad (25)$$

where w_0 is the angular frequency of the carrier.

It is obvious from (23) and (24) that $s_{ssb}(t)$ is equal to Re$\{s_a(t)\}=m(t)$ when $w_0=0$.

The signal recovery for SSB can be carried out as in the case of double side bad (DSB) AM. That is, multiply the SSB signal by $\cos(w_0t)$ and apply an appropriate low pass filter to remove the double frequency components at $2w_0$.

It is evident that the USB and LSB include only positive and negative frequency components of $M(jw)$, respectively. Hence, the total bandwidth of USB and LSB are half of the message signal $m(t)$. Therefore, the spectrum has no aliasing. Since USB and LSB modulation preserve sufficient information for the reconstruction of the original signal $m(t)$, then same conclusion is valid for the complex sampling of $s_a(t)$ with a sampling period, T, related to the bandwidth, $BW_a=0.5BW=w_{max}$, where w_{max} is the maximum frequency component of $M(jw)$. The whole modulation-demodulation process, which is used for USB and LSB modulated signals, can be repeated for the sampling of $s_a(t)$.

Algorithm for sampling at half frequency of Nyquist rate:

Algorithm 1: (Complex sampling (CS))

1. Compute the Hilbert transformation, $\hat{m}(t)$.

2. Find USB $s_a(t)$ based on (21). Note that $s_a(t)$ is a complex function, from which $m(t)$ can be extracted using (22). The amplitude spectrum for such a signal is shown in Figure 9.

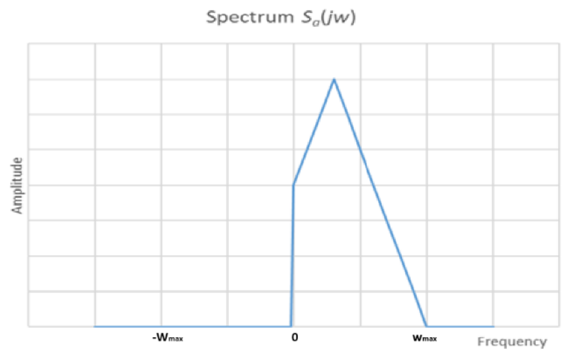

Fig. 9. Spectrum of USB-SSB base signal

3. Then sample $s_a(t)$ with the rate $w_s=w_{max}$ to get the sequence $s_a(nT)$, where T is the sampling period. Note that the number of samples for real and imaginary parts (or amplitude and phase) of $s_a(t)$ are same since it is a complex valued signal. In fact, the number of samples are the same for fast (real) and slow (complex) sampling process. In complex sampling one have more time to process the samples. The spectrum of the sequence is given in Figure 10.

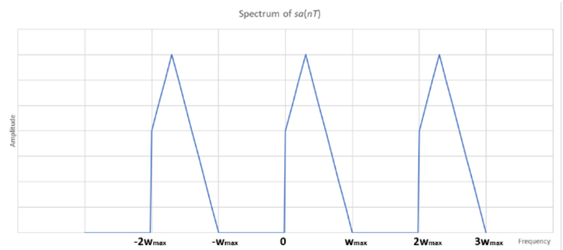

Fig. 10. Spectrum of the sampled signal

4. The recovery of $m(t)$ can be carried out as in case of SSB-AM signals [10,11,12]. Then the real part of the recovered signal will be $m(t)$. This conclusion is true

for the signals processed in the discrete time portion of the system as in the case of a control system, where the output may be different due to the processing of the signals. The detailed block diagram of complex sampling is still studied. It seems that the blocks given in [12] should be modified thoroughly for sampling. Therefore, no block diagram is given here.

Algrithm 1, can be described as follows in a simple way:

1. Given $m(t)$, find the Hilbert transform.
2. Compute $s_a(t)$ using (21), where $m(t)$ is included as in (22).
3. Sample $s_a(t)$, that is, use an analog to digital converter (ADC).
4. Process the signals according to the DSP rules.
5. Obtain the corresponding analog signal using a Digital to Analog converter (DAC).
6. Use an SSB-AM demodulator to get $m(t)$.

Remarks for Algoritm 1 (CS):

1. The number of samples in CS is same as sampling in real sampling (RS). However, the CS samples are taken for a longer period. The computational burden is expected to be less than for RS compared to the longer period for CS. The reason for the reduced burden is because any one has longer time for the same kind of process with some extra complex mathematical operations, for which powerful computing tools are available.
2. Even though CS provides a longer sampling period, it needs two slow samplers to get the data in terms of complex valued signal instead of one faster sampler.
3. Only the formulation of CS is presented here since the theoretical development of the technique is completed recently while some examples are performed to show the validity of the approach.

VI. Conclusion

A signal recovery method is proposed for under sampled signals using SSB-AM base signal of the signal that should be sampled. The proposed method is expected to be applied any band-limited signal contrary to the approach presented in [7-8], which has a very limited application since it is valid in continuous time where the samples are available as the weights of shifted impulses of a Dirac comb. The approach in [7] can remove the aliasing in continuous time. However, the aliasing cannot be avoided when the samples are considered as number sequences in discrete time due to the cyclic nature of Fourier transforms for discrete time sequences (Figure 8).

The proposed SSB-AM based complex sampling method in this study is a general method for any band limited signal. This method is expected to be applied to cases where very high frequency components should be handled but it is not possible due to the limitations of the available harware.

It was conjectured in [7] that the samples of the signal (taken according to the prescribed Nyquist rate) can be obtained using sufficiently many AM signals at integer multiples of the sampling frequency mathematically without a need to use sampling devices. Here the conjecture is repeated with a small modification.

The future study includes the application of the proposed SSB-AM approach to some signals and development and implementation of it. Similarly the study for conjecture 1 is continuing.

References

[1] C. E. Shannon, "Communication in the Presence of Noise," Proc. of IRE, vol. 37, pp. 10–21, Jan. 1949. Reprinted in Proc. of IEEE, vol.86, pp.447-457 no.2 Feb. 1998.

[2] C. L. Philips and H. T. Nagle, Digital Control System Analysis and Design, 3rd Ed. Prentice Hall, 1994.

[3] S. Haykin, and M. Moher, Communication Systems, 5th Ed. John Wiley and Sons, 2009.

[4] Fischer, Jens V. and Rudolf L. Stens. "On Inverses of the Dirac Comb." MDPI Mathematics, (2019). DOI:10.3390/math7121196

[5] J. B. Anderson, F. Rusek and V. Öwall, "Faster-Than-Nyquist Signaling," in Proceedings of the IEEE, vol. 101, no. 8, pp. 1817-1830, Aug. 2013, doi: 10.1109/JPROC.2012.2233451.

[6] Oppenheim, Alan V.; Willsky, Alan S. and Hamid, Nawab S., Signals and Systems, (Prentice-Hall series in Signal Processing) Pearson, 1996.

[7] Canbolat, H., (2024) "Reconstruction of a Band Limited Signal Sampled Slower than the Nyquist Rate Using AM Modulated Version of the Signal." J. of Engineering and Applied Sciences Technology. SRC/JEAST-394. DOI: doi.org/10.47363/JEAST/2024(6)276

[8] H. Canbolat, "A Modified Sampling Rate for a Continuous Sampling using Dirac Comb", ELECO2024 (Proc. National Conf. on EE and Biomedical Eng.), Bursa, Türkiye, November 2024. DOI: 10.1109/ELECO64362.2024.10847200

[9] H. Canbolat, "Amplitude Modulation as a Pole-placement Problem", TOK2024 Automatic Control National Meeting, Konya, Türkiye, pp. 801-805 September 2024.

[10] D. K. Weaver, "A Third Method of Generation and Detection of Single-Sideband Signals," in Proceedings of the IRE, vol. 44, no. 12, pp. 1703-1705, Dec. 1956, doi: 10.1109/JRPROC.1956.275061.

[11] K. Martin, "Complex Signal Processing is Not-Complex", available on https://www.mikrocontroller.net/attachment/151612/complex_signals.pdf

[12] P. Driessen, "ECE350 Textbook Fall 2020", available on https://mistic-lab.github.io/ece-communications-labs/_docs/pdriessen_textbook.pdf

Below the figures are given in a larger form!

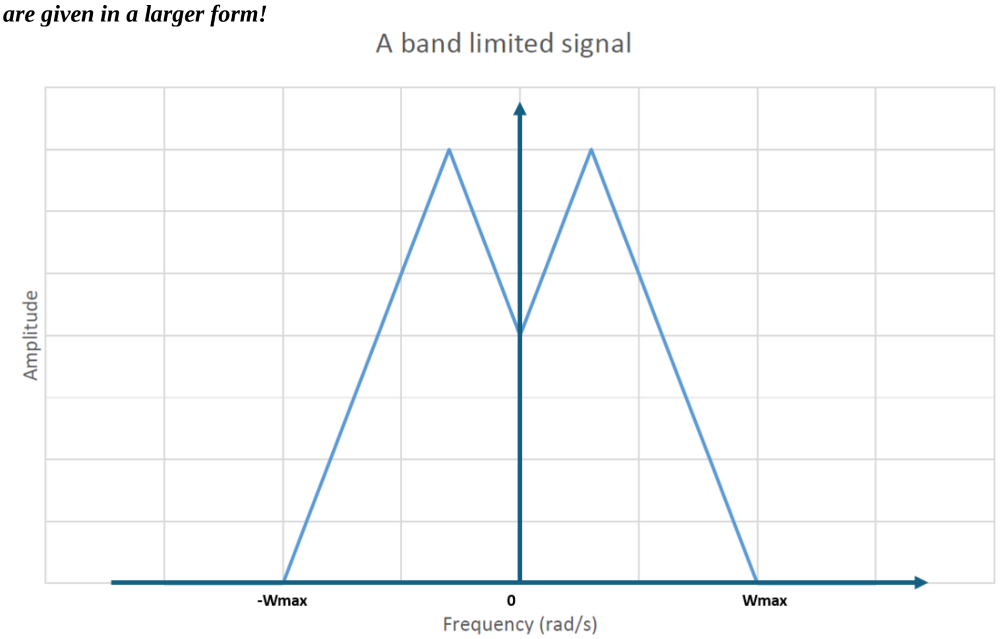

Figure 1 Spectrum of a band limited signal

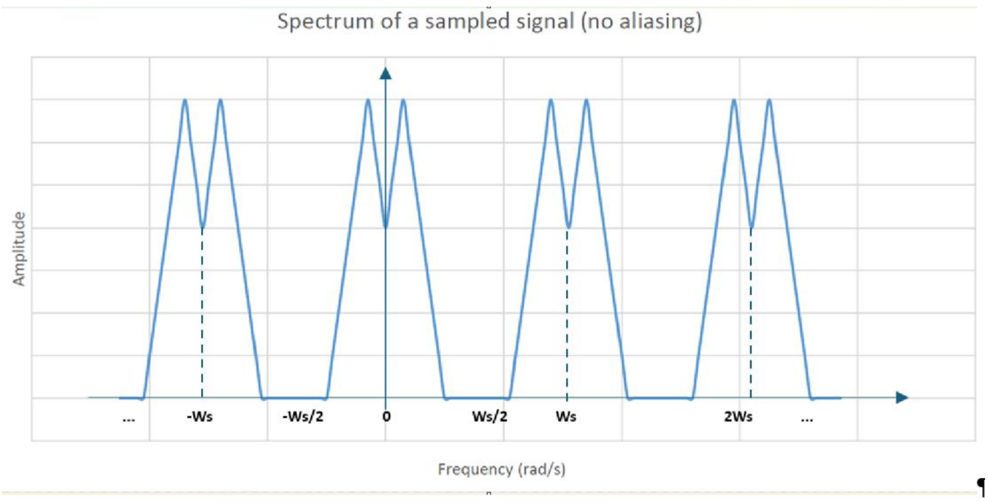

Figure 3 Spectrum |G(jw)| of a sampled signal (no aliasing), that is,* $w_s > 2w_{max}$

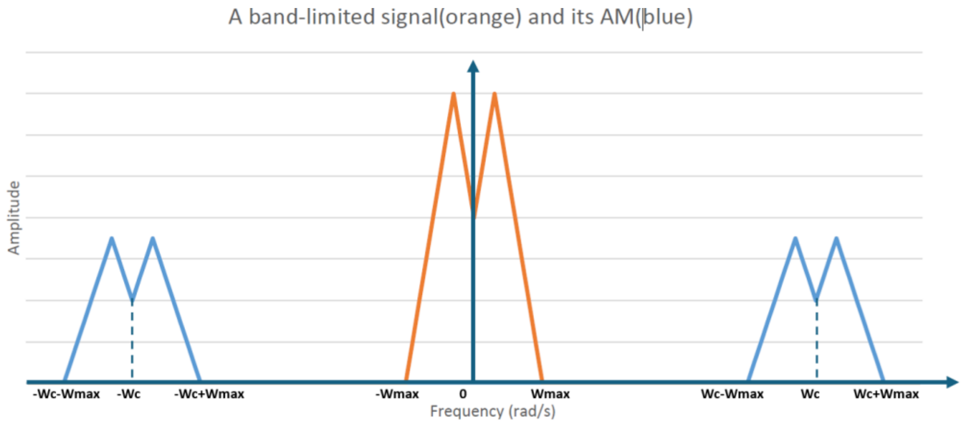

Figure 6 A band limited signal (center) and its AM signal at w_c *(left and right)*

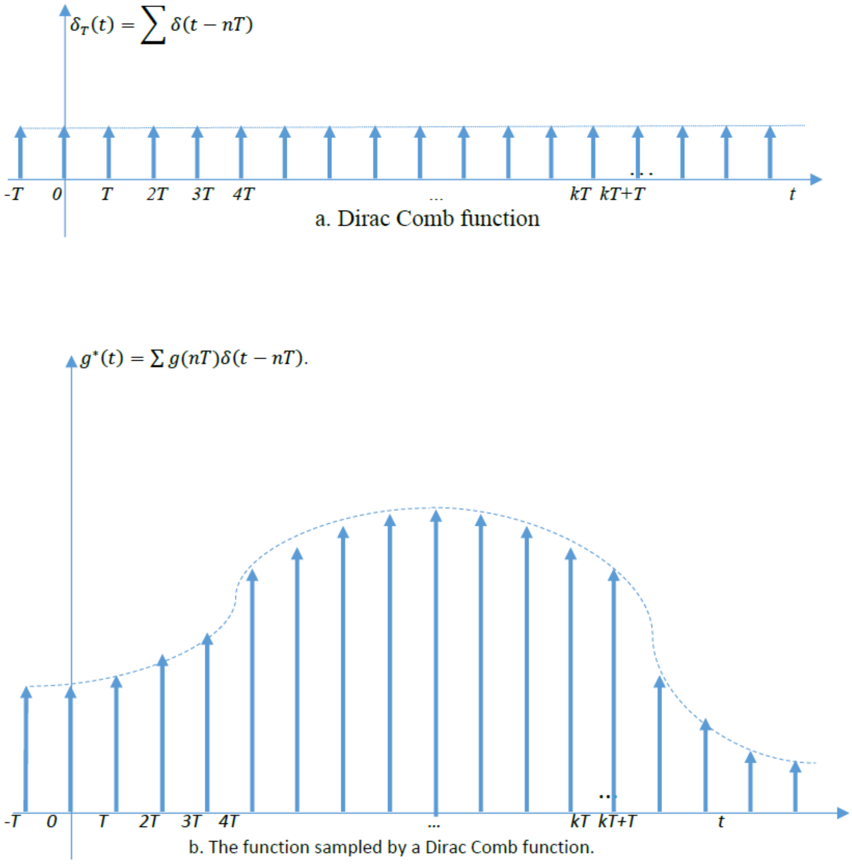

Figure 2 (a) The Dirac (impulse train) comb function; (b) the samples g(t) of a function with sampling frequency $w_s=1/T$*

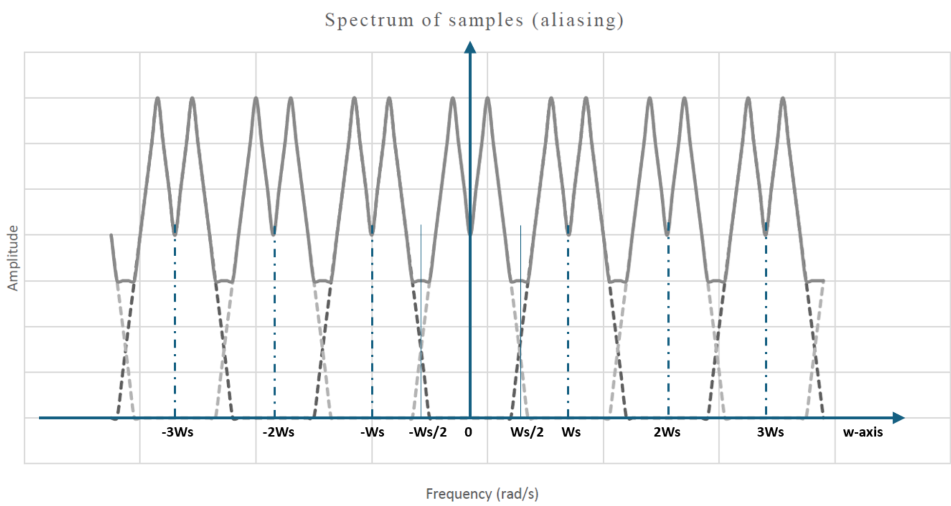

Figure 4 The spectrum of an under-sampled signal (|G(jw)| with $w_s<2w_{max}$ (aliasing) Note: Only $\pm w_s/2$ is shown!*

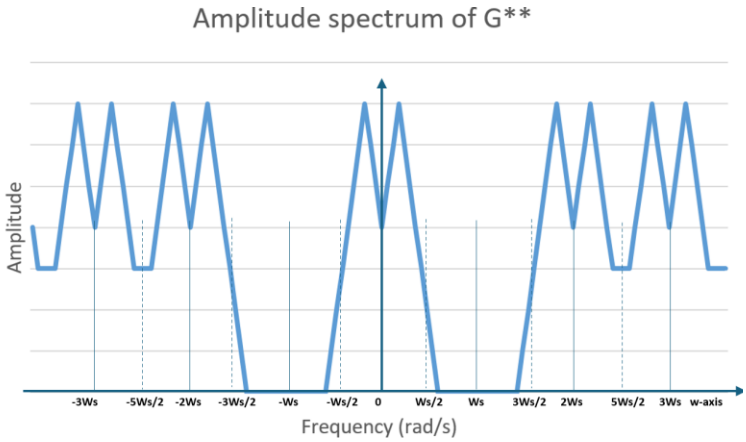

*Figure 7 The spectrum of |G**(jw)|. Aliasing is removed*

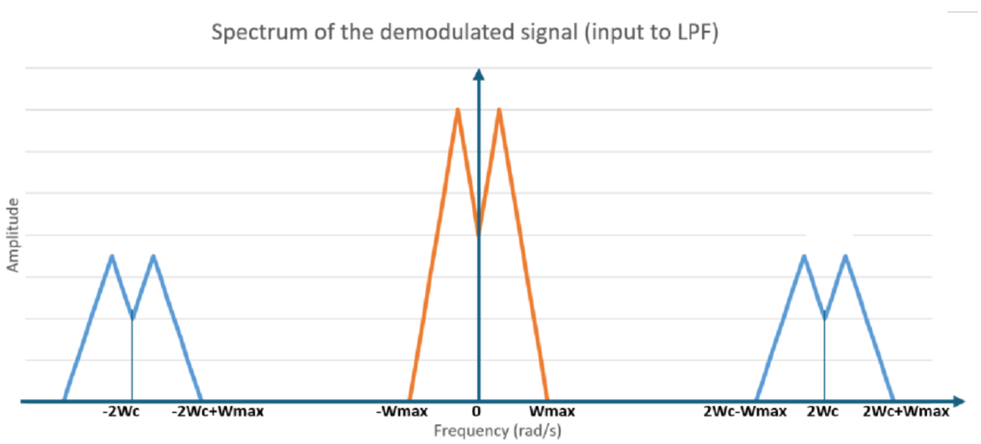

Figure 5 The spectrum of a demodulated AM signal before LPF. Aliasing occurs when $w_c < w_{max}$.

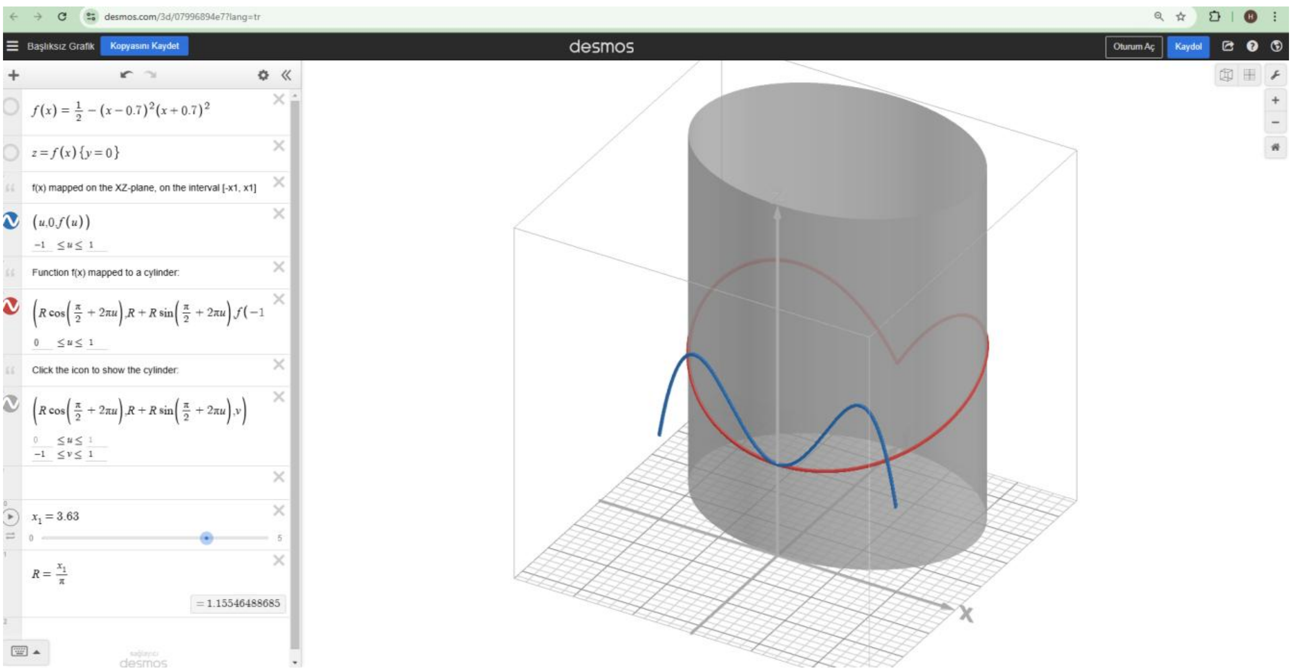

Figure 8 The transformation of a function around a cylindrical surface (Note: Here s=0 mapped into z=0)

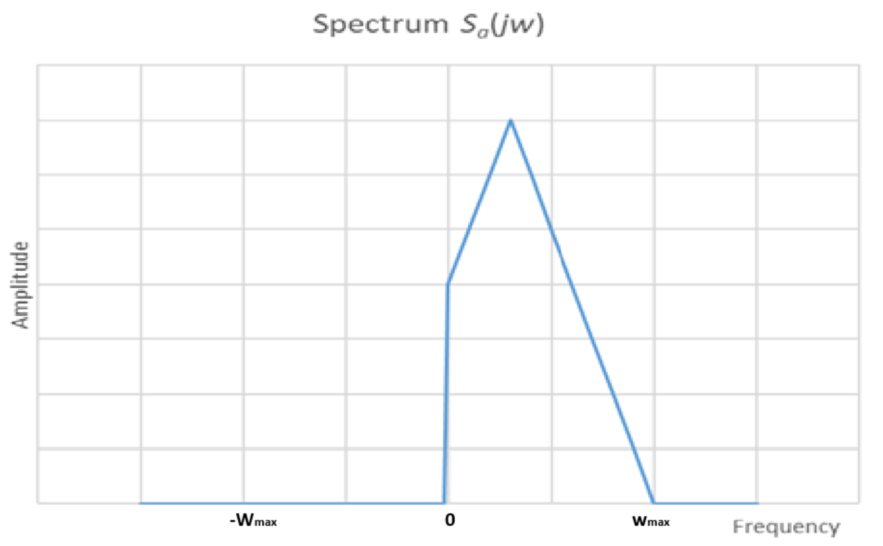

Figure 9 Spectrum of USB-SSB base signal

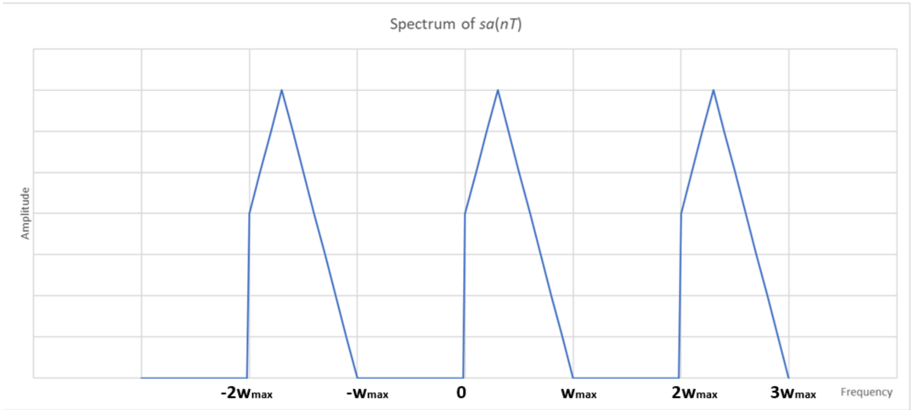

Figure 20 Spectrum of the sampled signal