Bilge International Journal of Science and Technology Research

ISSN: 2651-401X
e-ISSN: 2651-4028
Research Article

2025, Volume: 9, Issue: 2, 53-63
Received: 03.04.2025; Accepted: 07.05.2025
DOI: 10.30516/bilgesci.1669552

THE NOVEL HEWMA EXPONENTIAL TYPE MEAN ESTIMATOR UNDER RANKED SET SAMPLING

EDA GİZEM KOÇYİĞİT *[1]

Abstract: This study introduces a novel HEWMA-based exponential estimator for Ranked Set Sampling (RSS). The proposed estimator integrates HEWMA control chart statistics with the exponential ratio estimator to enhance efficiency. By incorporating control chart statistics, memory-type estimators improve estimation accuracy by utilizing not only the mean of the current sample but also historical means, if available. This approach enables using time-dependent repeated survey data or data collected from the same population at different time points. Given that the only existing estimator for the RSS method in the literature is the ratio estimator using EWMA, the proposed estimator offers a more efficient alternative. Its efficiency is evaluated through simulation studies using synthetic datasets with varying correlation coefficients to simulate diverse scenarios, as well as an empirical study employing real-world data with a distinct structure. The results demonstrate that incorporating at least one old sample mean value enhances efficiency. Additionally, the estimator's effectiveness improves as correlation and the number of old means used (T) increase. The selection of HEWMA weight parameters is crucial, depending on sample size and correlation. The proposed estimator performs optimally at low to medium correlation levels in the simulation studies and consistently outperforms alternatives in the real data analysis.

Keywords: Ranked set sampling, HEWMA, Memory type, Estimator, Simulation.

[1]**Address:** Dokuz Eylül University, Tınaztepe Campus, Faculty of Science, Department of Statistics, İzmir, Türkiye.

***Corresponding author:** eda.kocyigit@deu.edu.tr

Citation: Koçyiğit, E., (2025). The Novel HEWMA Exponential Type Mean Estimator under Ranked Set Sampling. Bilge International Journal of Science and Technology Research, 9(2): 53-63.

1. INTRODUCTION

Ranked Set Sampling (RSS), originally proposed by McIntyre (1952), offers a more efficient alternative to Simple Random Sampling (SRS), especially in scenarios where measuring the study variable is costly, time-consuming, or destructive. RSS improves estimator precision by incorporating rankings before actual measurements, thus utilizing available information more effectively. In parallel, memory-type estimators like the Exponentially Weighted Moving Average (EWMA) chart by Roberts et al. (1959) and its extension, the Hybrid Exponentially Weighted Moving Average (HEWMA) chart introduced by Haq (2013), are widely used for monitoring processes that evolve over time. These methods integrate historical and recent data, giving more weight to recent observations while still considering past trends. HEWMA enhances its flexibility by allowing a dynamic weighting scheme to capture shifts in the underlying process better. Arslan et al. (2023) demonstrated that the hybrid HEWMA control chart is more sensitive than existing control charts, including the classical EWMA, in detecting early shifts in process parameters. This feature makes HEWMA particularly suitable in the RSS context, where the efficient use of auxiliary and temporal information is crucial. Combining RSS with HEWMA can further improve estimation accuracy.

Aslam et al. (2020) introduced memory-type ratio and product-type mean estimators utilizing EWMA in stratified and rank-based sampling techniques. Shahzad et al. (2022) proposed EWMA-type memory-type estimators in two-stage sampling. Alomaie and Iftikhar (2024) introduced calibrated EWMA estimators for time-dependent survey data to the literature with various applications. Aslam et al. (2024) introduced a novel memory-based ratio estimator for survey sampling, whereas Kumar and Bhushan (2025) proposed a logarithmic memory-type estimator for time-dependent studies. Kumar et al. (2024) presented a class of memory-type general variance estimators under SRS. Singh et al. (2024) enhanced the accuracy of memory-type mean estimators, and Sharma et al. (2024) designed procedures for the EWMA mean estimator in time-dependent studies. Recently, Kumari et al. (2025) proposed a memory-type estimator using two auxiliary variables, and Koçyiğit (2025) introduced a new and effective HEWMA-type estimator under SRS. This method has been frequently studied and developed differently in recent years (Singh et al. 2021, Bhushan et al. 2022, Shahzad et al. 2022, Yadav et al. 2023, Aslam et al. 2023, Alomair and Shahzad 2023, Qureshi 2024, Tariq et al. 2024, Kumar et al. 2024). However, no study has proposed a estimator using HEWMA under RSS in the literature. Studies have shown that both the RSS method and HEWMA control chart, proposed as alternatives, produce more effective results than the SRS method and EWMA control chart.

This study evaluates the effectiveness of the HEWMA exponential type estimator proposed for the RSS method, comparing it to other estimators in the literature through simulation studies using synthetic datasets and real data from Türkiye. As a result, a new estimator for the RSS method is developed, improving estimation accuracy by integrating historical sample means.

2. MEAN ESTIMATORS UNDER RSS

Cingi and Kadılar (2009) generally classified the mean estimators as basic, ratio/product and regression type. However, these estimators have expanded with the development of technology and the expansion of literature, including exponential type, logarithmic type estimators, estimator families, etc. Aslam et al. (2020) included memory-type estimators among these estimator groups. This section presents the basic mean, ratio, regression type, and memory-type mean estimators for RSS, followed by introducing the new proposed estimator.

2.1. Estimators in the literature

In RSS, the mean estimation is primarily performed using the formula in Equation (1), where Y represents the study variable of interest and X serves as the auxiliary variable.

$$\mu_{RSS} = \sum_{j=1}^{c}\sum_{i=1}^{s} y_{(i;j)} \Big/ sc \qquad (1)$$

Here, s is the RSS set size, and c is the number of repetitions/ cycles. This estimator does not use any auxiliary variable information. Equation (2) presents the ratio estimator in RSS.

$$\mu_2 = \left(\mu_{RSS} / \bar{x}_{SKÖ}\right)\bar{X}, \tag{2}$$

where $\bar{x}_{RSS} = \sum_{j=1}^{c}\sum_{i=1}^{s} x_{(i:j)} \Big/ sc$ is the sample mean of the auxiliary variable drawn with RSS. RSS's exponential ratio and regression type estimators are as in Equations (3) and (4), respectively.

$$\mu_3 = \mu_{RSS} \exp\left[\left(\bar{X} - \bar{x}_{RSS}\right) / \left(\bar{X} + \bar{x}_{RSS}\right)\right] \tag{3}$$

$$\mu_4 = \mu_{RSS} + b\left(\bar{X} - \bar{x}_{RSS}\right) \tag{4}$$

In Equation (4), b denotes the regression coefficient, which can be estimated using $\hat{b} = \rho\left(s_y / \bar{y}\right)\left(s_x / \bar{x}\right)$, where s_y and s_x are the standard deviations of Y and X samples, respectively. ρ is the correlation coefficient between X and Y. In Equations (2), (3), and (4), $\bar{X}$ is the population mean of the auxiliary variable.

The following formula presents the EWMA-based memory-type ratio estimator in RSS proposed by Aslam et al. (2020).

$$\mu_{5T} = \left(\mu_{EWMAY} / \bar{x}_{EWMAX}\right)\bar{X} \tag{5}$$

Here, $\mu_{EWMAY(T)} = \alpha\mu_{RSS(T)} + (1-\alpha)\mu_{EWMAY(T-1)}$ and $\mu_{EWMAX(T)} = \alpha\bar{x}_{RSS(T)} + (1-\alpha)\mu_{EWMAX(T-1)}$ are the EWMA statistics for Y and X. α is the weight parameter of the EWMA statistic and chosen in $0 < \alpha \le 1$.

2.2. Proposed estimator

The following formulas calculate the HEWMA statistics for variables Y and X, respectively:

$$\mu_{HEWMAY(T)} = (1-\beta)\mu_{HEWMAY(T-1)} + \beta\mu_{EWMAY(T)} \tag{6}$$

$$\mu_{HEWMAX(T)} = (1-\beta)\mu_{HEWMAX(T-1)} + \beta\mu_{EWMAX(T)} \tag{7}$$

In this case, β is the weight parameter of the HEWMA statistic, and, similar to the EWMA weight parameter α, it should be chosen from the range $0 < \beta \le 1$. Both α and β determine the weight given to old mean(s) in the estimation. When T = 1, only the current sample data is used, and no old mean are included. According to Koçyiğit (2025), the initial value of the EWMA statistic should be the oldest mean value.

Inspired by the estimators from Aslam et al. (2020) and Koçyiğit (2025), the HEWMA type exponential proportional estimator is proposed as in Equation (8):

$$\mu_{PROT} = \mu_{HEWMAY(T)} \, exp\left[\frac{\overline{X} - \mu_{HEWMAX(T)}}{\overline{X} + \mu_{HEWMAX(T)}}\right] \qquad (8)$$

3. SIMULATION STUDY

The simulation study was conducted using the R program. The populations were derived from the bivariate normal distribution with ρ_{xy} = 0.6, 0.7, and 0.8, with N(3,1) random parameters and a size of N = 3600. For RSS, we consider the set size as s = 3, 4, and 5 and the number of cycles as c = 1, 2, and 3. For EWMA and HEWMA statistics, we used T = 2, α = 0.1, 0.3, 0.5, 0.7, and 0.9, and β = 0.3, 0.5, 0.7, and 0.9. Koçyiğit (2025) emphasized that the β coefficient should be selected higher than 0.5, but since that study was carried out under SRS, we also tried β = 0.3 for RSS in this simulation study. The simulation draws 100,000 samples from the populations defined using RSS and calculates the corresponding estimator values. The flowchart in Figure 1 summarizes the process for calculating the memory type estimators in simulation. In both EWMA and HEWMA, only sample means were utilized.

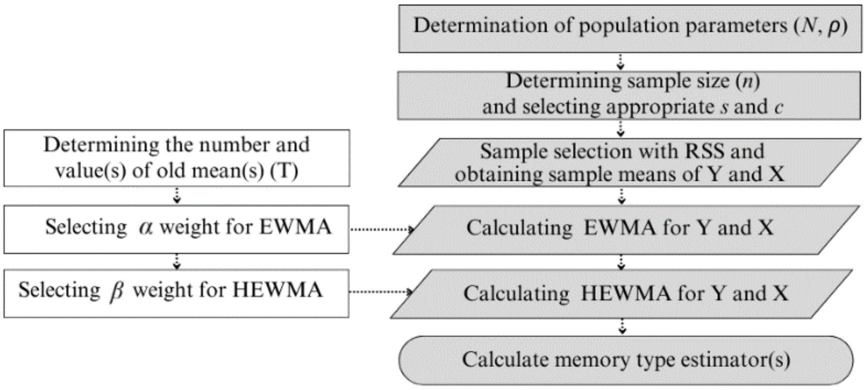

Figure 1. Flowchart of the estimation process using memory type estimators under RSS

Equation (9) calculated the estimators' mean square error (MSE) values , and Equation (10) calculated the relative Efficiency (RE) values . The results are given in Tables 1, 2, and 3. The highest RE value in each s, c combination is written in bold.

$$MSE\left(\mu_j\right) = \frac{1}{100000}\sum_{i=1}^{100000}\left(\mu_{j\_i} - \overline{Y}\right)^2, \quad j = RSS, 2, 3, 4, 5T, PROT \qquad (9)$$

$$RE_v = \frac{MSE\left(\mu_{RSS}\right)}{MSE\left(\mu_v\right)}, v = 2, 3, 4, 5T, PROT \qquad (10)$$

Table 1 shows that the proposed μ_{PROT} estimator produces the most effective results for each s, c combination. The estimator achieves its highest RE value for the low correlation case, (approximately 2.59) with α=0.3, β=0.5, s=3, c=3). At a correlation of 0.7, the estimator reaches the maximum RE value of roughly 2.91 for α=0.1, β=0.5, s=3, c=3.

As shown in Table 2, an increase in correlation leads to a rise in the RE value of the proposed estimator. However, at 0.8 correlation, the best estimator is μ_{5T}. The highest RE value of μ_{5T} is calculated as approximately 3.64 in the case of α=0.5, s= 3, c=3. The highest RE value of the proposed estimator (Approximately 3.42) is obtained from the case of α=0.1, β =0.5, s= 3, c=3.

The simulation study results indicate that memory-type estimators (μ_{5T} and μ_{PROT}) outperform simple, ratio, and regression-type estimators across all conditions. The efficiency of the estimators is significantly influenced by the sample size and selected weight parameters. The proposed estimator does not yield the best result when β=0.3.

Table 1. Simulation results for different values of α, β, s and c when ρ_{xy} =0.6 and 0.7

		s	3	4	5	3	4	5	3	4	5
T=2 N=3600 *N(3,1)* ρ_{xy} =0.6		*c*	*1*	*1*	*1*	*2*	*2*	*2*	*3*	*3*	*3*
		RE_2	1.03827	1.05589	1.06075	1.09609	1.08367	1.08518	1.10936	1.09328	1.08272
		RE_3	1.25847	1.21574	1.18709	1.27761	1.22552	1.19435	1.28021	1.22719	1.19083
		RE_4	1.28753	1.23531	1.17885	1.22488	1.20972	1.20183	1.28686	1.23867	1.19462
α=0.1		RE_{5T}	1.30157	1.30544	1.30144	1.32685	1.31298	1.34863	1.35661	1.33690	1.32216
	β =0.3	RE_{PROT}	2.21469	2.10293	2.04153	2.20185	2.09172	2.09546	2.22210	2.12243	2.05426
	β =0.5	RE_{PROT}	**2.56756**	2.43533	2.36323	**2.56853**	2.45429	2.40789	2.57041	2.45682	2.38237
	β =0.7	RE_{PROT}	2.17749	2.10191	2.05036	2.20117	2.12996	2.07416	2.20654	2.12532	2.04804
	β =0.9	RE_{PROT}	1.54458	1.49053	1.44977	1.55480	1.49701	1.45651	1.55909	1.49974	1.45835
α=0.3		RE_{5T}	1.87045	1.85555	1.83616	1.90366	1.88491	1.86952	1.92829	1.92067	1.87710
	β =0.3	RE_{PROT}	2.19198	2.10680	2.03030	2.19716	2.10755	2.05241	2.21036	2.13958	2.05861
	β =0.5	RE_{PROT}	2.54347	2.45101	2.38094	2.54243	2.40403	2.40403	**2.57889**	**2.47459**	**2.40235**
	β =0.7	RE_{5T}	1.87116	1.85823	1.85992	1.91629	1.89130	1.88297	1.93606	1.91757	1.87419
	β =0.9	RE_{PROT}	1.54298	1.48893	1.45378	1.55632	1.49288	1.45563	1.55607	1.49787	1.45944
α=0.5		RE_{5T}	2.17735	2.18594	2.16045	2.21952	2.17630	2.18858	2.24010	2.22500	2.20119
	β =0.3	RE_{PROT}	2.19621	2.12283	2.05344	2.19055	2.10245	2.06332	2.19892	2.14335	2.08463
	β =0.5	RE_{PROT}	2.53754	**2.46909**	2.34902	2.55542	2.44093	**2.41278**	2.57101	2.45728	2.39840
	β =0.7	RE_{PROT}	2.18246	2.08871	2.05187	2.19277	2.12178	2.05250	2.22485	2.11155	2.04398
	β =0.9	RE_{PROT}	1.54087	1.48638	1.44718	1.54645	1.49748	1.45324	1.55940	1.50007	1.45801
α=0.7		RE_{5T}	1.86006	1.87156	1.85224	1.90002	1.90263	1.87861	1.92958	1.89252	1.85782
	β =0.3	RE_{PROT}	2.17519	2.12388	2.03756	2.18077	2.13098	2.04782	2.21263	2.11167	2.05253
	β =0.5	RE_{PROT}	2.54876	2.46418	2.37546	2.55920	**2.45890**	2.38594	2.57237	2.46256	2.37859
	β =0.7	RE_{PROT}	2.18511	2.11256	2.03706	2.19553	2.11828	2.06539	2.20901	2.13174	2.05784
	β =0.9	RE_{PROT}	1.53587	1.48573	1.44864	1.54803	1.49401	1.45977	1.56293	1.49580	1.45798
α=0.9		RE_{5T}	1.28819	1.30350	1.29526	1.34522	1.32956	1.32866	1.35068	1.34160	1.33102
	β =0.3	RE_{PROT}	2.19947	2.10816	2.04508	2.20933	2.10412	2.05324	2.19172	2.12957	2.09137
	β =0.5	RE_{PROT}	2.52177	2.45059	**2.38297**	2.54296	2.45711	2.40780	2.57742	2.45623	2.39420
	β =0.7	RE_{PROT}	2.18838	2.10840	2.04936	2.19557	2.11775	2.06097	2.20296	2.12245	2.06982
	β =0.9	RE_{PROT}	1.54127	1.48337	1.44640	1.55225	1.48995	1.45576	1.55453	1.50515	1.45537
ρ_{xy} =0.7		RE_2	1.28524	1.26409	1.22942	1.33026	1.28897	1.24427	1.35187	1.30136	1.25375
		RE_3	1.42334	1.35649	1.30386	1.43916	1.36387	1.30863	1.44499	1.36879	1.31182
		RE_4	0.60014	1.27526	1.28698	1.41231	1.37369	1.32275	1.46180	1.39479	1.33560
α=0.1		RE_{5T}	1.58427	1.54658	1.52623	1.63093	1.59357	1.51809	1.66136	1.59525	1.52967
	β =0.3	RE_{PROT}	2.47321	2.34226	2.27931	2.46763	2.36939	2.24809	2.50271	2.37281	2.26240
	β =0.5	RE_{PROT}	2.88072	2.71107	2.63621	2.88851	2.74342	2.63976	**2.91038**	2.74050	2.63173
	β =0.7	RE_{PROT}	2.47092	2.35134	2.26681	2.50079	2.35965	2.27702	2.51112	2.36703	2.26626
	β =0.9	RE_{PROT}	1.74579	1.66187	1.60074	1.75841	1.66942	1.60817	1.76944	1.66421	1.60211
α=0.3		RE_{5T}	2.30083	2.24105	2.15997	2.33875	2.25400	2.16533	2.33605	2.24250	2.17058
	β =0.3	RE_{PROT}	2.48145	2.37561	2.26321	2.50040	2.35665	2.26556	2.48271	2.34634	2.26420
	β =0.5	RE_{PROT}	**2.88967**	2.73033	2.62868	2.89617	2.72913	2.64257	2.89261	2.75599	2.61064
	β =0.7	RE_{PROT}	2.47138	2.34616	2.25145	2.48208	2.35887	2.27150	2.49077	2.37263	2.26508
	β =0.9	RE_{PROT}	1.74693	1.65682	1.59410	1.75640	1.67228	1.60721	1.76531	1.67352	1.60265
α=0.5		RE_{5T}	2.69737	2.58348	2.49598	2.74704	2.61925	2.50818	2.74215	2.62053	2.52764
	β =0.3	RE_{PROT}	2.49311	2.34712	2.25342	2.51752	2.36419	2.24776	2.49824	2.36048	2.26732

	β =0.5	RE_{PROT}	2.85309	**2.73925**	2.64020	2.88351	2.73319	2.63591	2.91015	2.73432	**2.63693**
	β =0.7	RE_{PROT}	2.47308	2.35013	2.27472	2.49945	2.35813	2.26392	2.49802	2.36501	2.27307
	β =0.9	RE_{PROT}	1.74299	1.65974	1.59630	1.75930	1.66637	1.59996	1.76675	1.66896	1.60455
α=0.7		RE_{5T}	2.27744	2.21252	2.16571	2.32792	2.24118	2.18926	2.36342	2.25382	2.18927
	β =0.3	RE_{PROT}	2.48019	2.35108	2.27082	2.48714	2.35460	2.28343	2.50964	2.37472	2.27329
	β =0.5	RE_{PROT}	2.87813	2.73222	**2.64701**	2.90152	**2.76709**	**2.64577**	2.88916	**2.76265**	2.62201
	β =0.7	RE_{PROT}	2.46954	2.36128	2.26442	2.48437	2.35985	2.26196	2.49180	2.35276	2.27840
	β =0.9	RE_{PROT}	1.74608	1.66120	1.60053	1.76175	1.66982	1.59997	1.76455	1.67369	1.60522
α=0.9		RE_{5T}	1.57434	1.54638	1.51720	1.66016	1.59715	1.53764	1.63648	1.59688	1.54721
	β =0.3	RE_{PROT}	2.46035	2.34457	2.27855	2.51528	2.38194	2.26781	2.49001	2.36282	2.28444
	β =0.5	RE_{PROT}	2.88813	2.72990	2.64183	**2.90448**	2.73948	2.63637	2.90833	2.75378	2.60945
	β =0.7	RE_{PROT}	2.46711	2.36513	2.25458	2.50194	2.36655	2.26593	2.48736	2.36286	2.27238
	β =0.9	RE_{PROT}	1.75000	1.66460	1.59557	1.76381	1.66726	1.59706	1.76706	1.67426	1.60151

Table 2. Simulation results for different values of α, β, s and c when ρ_{xy} =0.8

		s	*3*	*4*	*5*	*3*	*4*	*5*	*3*	*4*	*5*
T=2 N=3600 *N(3,1)* ρ_{xy} =0.8		*c*	*1*	*1*	*1*	*2*	*2*	*2*	*3*	*3*	*3*
		RE_2	1.71283	1.60795	1.53905	1.76213	1.63640	1.55552	1.79236	1.65120	1.58990
		RE_3	1.67328	1.56290	1.49231	1.67976	1.57294	1.49022	1.69276	1.57388	1.50565
		RE_4	0.71195	1.43372	1.52706	1.75356	1.67324	1.59336	1.84294	1.70553	1.62942
α=0.1		RE_{5T}	2.13754	1.97525	1.88501	2.18123	1.99022	1.89744	2.18478	2.03953	1.93649
	β =0.3	RE_{PROT}	2.94202	2.70619	2.55047	2.92495	2.69915	2.57440	2.92997	2.72245	2.61241
	β =0.5	RE_{PROT}	3.36626	3.17044	2.99930	3.39494	3.17055	2.99719	3.42350	3.12073	2.99669
	β =0.7	RE_{PROT}	2.89584	2.70702	2.58512	2.94066	2.74096	2.58620	2.93127	2.73304	2.58188
	β =0.9	RE_{PROT}	2.04247	1.91750	1.82074	2.05255	1.92256	1.81501	2.06474	1.93335	1.82418
α=0.3		RE_{5T}	3.10842	2.852734	2.70097	3.13675	2.84618	2.72654	3.12179	2.87843	2.73210
	β =0.3	RE_{PROT}	2.93512	2.722804	2.58991	2.94951	2.71453	2.60093	2.93741	2.72597	2.58702
	β =0.5	RE_{PROT}	3.38653	3.14022	2.98365	3.39932	3.16130	2.98032	3.40011	3.18136	3.01476
	β =0.7	RE_{PROT}	2.89534	2.70602	2.58168	2.91648	2.72416	2.59685	2.92061	2.72599	2.57661
	β =0.9	RE_{PROT}	2.05459	1.91103	1.81754	2.06473	1.91969	1.82450	2.06604	1.92622	1.82251
α=0.5		RE_{5T}	**3.56380**	**3.31011**	**3.09262**	**3.62782**	**3.34520**	**3.13156**	**3.63738**	**3.38155**	**3.14342**
	β =0.3	RE_{PROT}	2.91470	2.72525	2.54972	2.93477	2.74195	2.57426	2.93844	2.75333	2.58317
	β =0.5	RE_{PROT}	3.36035	3.18900	2.98337	3.37095	3.14516	2.98391	3.40109	3.14007	3.01310
	β =0.7	RE_{PROT}	2.91205	2.71697	2.57419	2.90798	2.69810	2.58530	2.92061	2.74156	2.58744
	β =0.9	RE_{PROT}	2.05261	1.91254	1.81332	2.06334	1.93418	1.83043	2.06785	1.92910	1.82562
α=0.7		RE_{5T}	3.05964	2.85001	2.67018	3.10072	2.89379	2.69481	3.09449	2.88060	2.71351
	β =0.3	RE_{PROT}	2.92198	2.72438	2.56504	2.91465	2.73783	2.59153	2.89156	2.76326	2.58293
	β =0.5	RE_{PROT}	3.37407	3.15686	3.00465	3.39558	3.17473	2.99640	3.40294	3.14645	2.99577
	β =0.7	RE_{PROT}	2.89909	2.70666	2.58938	2.92225	2.71378	2.58114	2.90881	2.73945	2.57079
	β =0.9	RE_{PROT}	2.04912	1.91097	1.81518	2.06123	1.91906	1.82088	2.07428	1.92487	1.83668
α=0.9		RE_{5T}	2.11918	1.98502	1.90494	2.19624	2.01012	1.90776	2.18161	2.03832	1.90143
	β =0.3	RE_{PROT}	2.87322	2.73399	2.57816	2.94079	2.69731	2.57410	2.91609	2.72483	2.56854
	β =0.5	RE_{PROT}	3.38836	3.16588	2.98610	3.38081	3.18038	3.00487	3.41528	3.17518	3.00461
	β =0.7	RE_{PROT}	2.91836	2.72530	2.57491	2.92054	2.70879	2.57103	2.93332	2.73264	2.58685
	β =0.9	RE_{PROT}	2.05199	1.90913	1.82096	2.06410	1.92280	1.829139	2.07381	1.92644	1.82675

4. REAL DATA

This section uses a dataset compiled from the highway statistics of the provinces in Turkey for the years 2024-2022, provided by TÜİK and the EGM Traffic Department. In this dataset, the variables include traffic accidents with death/injury in 2024 as Y (Y_{2024}) and registered motor vehicles in 2024 as X (X_{2024}). The population parameters are summarized in Table 3. The correlations between reveal strong positive relationships among all variables across the years. X_{2022} shows perfect correlation with both X_{2023} and X_{2024} (≈1.00), and strong correlations with Y_{2022} (0.88), Y_{2023} (0.87), and Y_{2024} (0.91). Similarly, X_{2023} is perfectly correlated with X_{2024} (≈1.00), and also shows high correlations with Y_{2022} (0.91), Y_{2023} (0.89), and Y_{2024} (0.93). X_{2024} maintains strong correlations with Y_{2022} (0.92), Y_{2023} (0.91), and Y_{2024} (0.94). Among the Y variables, Y_{2022} and Y_{2023} are perfectly correlated (≈1.00), and both have a very high correlation with Y_{2024} (0.98). It is noteworthy that, unlike the synthetic data sets derived from the bivariate normal distribution in the simulation study, the real data set is quite skewed and has a higher correlation.

Table 3. Population parameters

Variables	N	Min.	Max.	Mean	Std. Dev.	Skewness	Kurtosis
Y_{2024}	81	102	33622	3134.469	4756.553	3.8753	19.5151
X_{2024}	81	1355	651282	32084.15	76892.09	6.6006	49.2155
Y_{2023}	81	175	25622	2902.111	3888.364	3.3178	13.9347
X_{2023}	81	816	637591	28275.06	74762.91	6.8143	51.7037
Y_{2022}	81	129	22914	2435.321	3425.946	3.5234	15.4720
X_{2022}	81	354	412631	15677.93	47566.93	7.2630	57.0207

For the simulation number 100,000, samples were drawn from the abovementioned population with s = 3, 4, and 5, c = 1, 2, and 3. Based on the results obtained from the simulation study, α, β = 0.5, 0.7, and 0.9 were determined. In addition, it was observed how the effectiveness of the estimators changed when the old means number was increased by taking T = 2 and 3. The variable to be estimated belongs to the year 2024 for each T. In the case of T = 2, the variables belonging to the years 2024 and 2023, and in the case of T = 3, the variables belonging to the years 2024, 2023, and 2022 were included in the EWMA and HEWMA algorithms. All other calculations in this section were carried out similarly to the simulation study in Section 3. Table 4 presents the RE values of the estimators.

Table 4 shows the most efficient estimator, the proposed μ_{PROT} estimator. In the case of T=2, when the sample size is minimal, the estimator gives the best values when s=3, c=1 is selected as α=0.5, and in all other cases, α>0.5, and in all cases, β =0.9 should be chosen. For T=3, α>0.5 and β =0.9 should be selected. The efficiency of the estimator increases as the number of T increases. The highest efficiency values are also seen for T=2 and 3, s=3, c=1, while the max. RE is 10.45350 for T=2, α=0.5 and β =0.9, and the max. RE value of 13.75371 is obtained for T=3, α=0.9, and β =0.9.

Since the year of interest is 2024 and the correlation with previous years decreases as we move further back in history with an increase in T, choosing α=β=0.9 causes the oldest average to receive the lowest weight, while the years closer to the present are given more weight. Including the variables with the highest correlation in the estimator with weight positively impacts the efficiency.

Table 4. Simulation Results for Real Data

		s	3	4	5	3	4	5	3	4	5
		c	*1*	*1*	*1*	*2*	*2*	*2*	*3*	*3*	*3*
		RE_2	4.05980	3.76254	3.34796	3.54771	2.97535	2.50368	3.01664	2.46233	2.01799
		RE_3	4.71821	4.98895	5.18671	5.29068	5.68117	5.96377	5.74144	6.11419	6.32070
		RE_4	0.55629	3.13484	3.56586	3.80105	4.01457	3.93822	4.10733	4.01980	3.64466
T=2											
α=0.5		RE_{5T}	4.38971	3.97931	3.70130	1.25833	1.37386	1.45534	0.93964	0.91893	1.00415
	β =0.5	RE_{PROT}	8.51374	6.67309	5.56744	2.73485	2.86038	2.84855	1.89553	1.98091	2.10372
	β =0.7	RE_{PROT}	9.83821	7.73143	6.69895	3.57461	3.74212	3.71784	2.52595	2.61378	2.73821
	β =0.9	RE_{PROT}	**10.45350**	8.93427	7.83421	4.58751	4.72355	4.73417	3.33558	3.42663	3.57875
α=0.7		RE_{5T}	5.09934	4.65513	4.37873	2.21934	2.25129	2.22947	1.73000	1.60356	1.62076
	β =0.5	RE_{PROT}	9.84340	7.91933	6.75985	3.56202	3.70604	3.69892	2.53200	2.59311	2.72194
	β =0.7	RE_{PROT}	10.38918	9.11879	8.40121	5.06556	5.14851	5.22211	3.71435	3.83801	4.03975
	β =0.9	RE_{PROT}	9.64148	9.13816	8.83723	6.65741	6.78301	6.97192	5.30239	5.53439	5.73087
α=0.9		RE_{5T}	4.90734	4.51157	4.00190	3.56979	3.16447	2.74799	3.01612	2.49698	2.14923
	β =0.5	RE_{PROT}	10.36584	8.90518	7.73983	4.64662	4.67680	4.71657	3.36815	3.45453	3.57188
	β =0.7	RE_{PROT}	9.55911	**9.20972**	**8.85166**	6.65892	6.77114	6.93413	5.32073	5.52574	5.73164
	β =0.9	RE_{PROT}	7.28685	7.42025	7.61141	**7.16181**	**7.49348**	**7.89318**	**6.91536**	**7.27596**	**7.57546**
T=3											
α=0.5		RE_{5T}	3.43619	3.17004	2.93587	1.36932	1.42025	1.43193	1.05777	0.99389	1.02447
	β =0.5	RE_{PROT}	11.24214	8.94697	7.74689	4.33793	4.45208	4.49475	3.03944	3.13275	3.34494
	β =0.7	RE_{PROT}	13.45084	11.41015	10.00778	6.02409	6.08622	6.32640	4.31480	4.37793	4.73140
	β =0.9	RE_{PROT}	13.74341	12.19999	11.29841	7.39763	7.61135	7.74592	5.60554	5.75040	6.19978
α=0.7		RE_{5T}	4.68708	4.41981	4.05342	2.39704	2.33988	2.26811	1.91841	1.71608	1.69744
	β =0.5	RE_{PROT}	13.34960	11.44205	10.14812	5.91176	6.11431	6.31161	4.24942	4.35818	4.79750
	β =0.7	RE_{PROT}	13.31635	12.05169	11.41502	7.72355	8.02707	8.25253	5.90144	6.08667	6.45808
	β =0.9	RE_{PROT}	10.81900	10.44883	10.45433	**8.48030**	8.64490	**9.04991**	7.07788	7.36312	7.86653
α=0.9		RE_{5T}	4.91418	4.48625	4.01227	3.53111	3.13567	2.72548	3.00984	2.46611	2.16603
	β =0.5	RE_{PROT}	**13.75371**	**12.27952**	**11.42045**	7.26702	7.58168	7.80883	5.56629	5.67407	6.18815
	β =0.7	RE_{PROT}	10.72578	10.51414	10.62548	8.47284	**8.77406**	9.00512	7.05950	7.32109	7.76861
	β =0.9	RE_{PROT}	7.23490	7.54251	7.78024	7.47679	7.79107	8.15463	**7.34840**	**7.67025**	**8.13815**

5. CONCLUSIONS

In this study, a new HEWMA type memory exponential ratio estimator for RSS is introduced. Thus, a new alternative that gives more effective results than the only estimator in the literature is obtained.

The simulation studies found that the μ_{PROT} outperformed others at low and medium correlation values, as well as in the entire real data study. As the correlation and the number of old mean(s) used (T) increase, the effectiveness of the proposed estimator increases. The selection of HEWMA weight parameters is essential depending on the sample size and correlation. Like Koçyiğit (2025), it should be selected as $\beta \geq 0.5$.

Future research should focus on integrating the HEWMA control chart statistics, which have been demonstrated to outperform EWMA in certain contexts, with a broader set of estimators under RSS. Comparative simulation studies involving different memory-type estimators, such as adaptive EWMA variants or Bayesian-based memory estimators, could provide deeper insights into their relative efficiency. Additionally, rather than relying on arbitrarily chosen weight parameters in HEWMA, optimization techniques—such as grid search, cross-validation, or data-driven methods based on mean squared error minimization—should be employed to determine optimal weight values. Potential application areas include quality control in manufacturing, environmental monitoring, and medical studies where measurement costs are high but auxiliary information is available.

Acknowledgements
-

Ethics Committee Approval
N/A

Peer-review
Externally peer-reviewed.

Author Contributions
Conceptualization, Investigation, Material and Methodology, Visualization, Writing-Original Draft, Writing-review & Editing, Other:

Conflict of Interest
The author(s) have no conflicts of interest to declare.

Funding
The authors declared that this study has received no financial support.

REFERENCES

Alomair, M. A., & Shahzad, U. (2023). Compromised-imputation and EWMA-based memory-type mean estimators using quantile regression. *Symmetry*, *15*(10), 1888. https://doi.org/10.3390/sym15101888

Alomair, A. M., & Iftikhar, S. (2024). Calibrated EWMA estimators for time-scaled surveys with diverse applications. *Heliyon*, *10*(10). https://doi.org/10.1016/j.heliyon.2024.e31030

Aslam, I., Amin, M. N., Mahmood, A., & Sharma, P. (2024). New memory-based ratio estimator in survey sampling. *Natural and Applied Sciences International Journal (NASIJ)*, *5*(1), 168-181. https://doi.org/10.47264/idea.nasij/5.1.11

Aslam, I., Noor-ul-Amin, M., Hanif, M., & Sharma, P. (2023). Memory type ratio and product estimators under ranked-based sampling schemes. *Communications in Statistics-Theory and Methods*, *52*(4), 1155-1177. https://doi.org/10.1080/03610926.2021.1924784

Aslam, I., Noor-ul-Amin, M., Yasmeen, U., & Hanif, M. (2020). Memory type ratio and product estimators in stratified sampling. *Journal of Reliability and Statistical Studies*, 1-20. https://doi.org/10.13052/jrss0974-8024.1311

Bhushan, S., Kumar, A., Alrumayh, A., Khogeer, H. A., & Onyango, R. (2022). Evaluating the performance of memory type logarithmic estimators using simple random sampling. *Plos one*, *17*(12), e0278264. https://doi.org/10.1371/journal.pone.0278264

Cingi, H., & Kadilar, C. (2009). *Advances in sampling theory-ratio method of estimation*. Bentham Science Publishers. https://doi.org/10.2174/97816080501231090101

Barbak, A. (2023). Citizen-Centered Public Security in Turkey: Policy and Practice. In *Citizen-Centered Public Policy Making in Turkey* (pp. 447-469). Cham: Springer International Publishing. https://trafik.gov.tr/kurumlar/trafik.gov.tr/04-Istatistik/Aylik/202412/Aralik_2024.pdf (Accessed 03 April 2025).

Haq, A. (2013). A new hybrid exponentially weighted moving average control chart for monitoring process mean. *Quality and Reliability Engineering International*, *29*(7), 1015-1025. https://doi.org/10.1002/qre.1453

Kadilar, C., Unyazici, Y., & Cingi, H. (2009). Ratio estimator for the population mean using ranked set sampling. *Statistical Papers*, *50*(2), 301-309. https://doi.org/10.1007/s00362-007-0079-y

Koçyiğit, E. G. (2025). Using past sample means in exponential ratio and regression type estimators under a simple random sampling. *Soft Computing*, *29*(3), 1389-1406. https://doi.org/10.1007/s00500-025-10408-2

Kumar, S., Chhaparwal, P., Kumar, K., & Kumar, P. (2024). Generalized memory-type estimators for time-based surveys: simulation experience and empirical results with birth weight dataset. *Life Cycle Reliability and Safety Engineering*, *13*(1), 15-23. https://doi.org/10.1007/s41872-023-00239-1

Kumar, A., & Bhushan, S. (2025). Logarithmic imputation techniques for temporal surveys: a memory-based approach explored through simulation and real-life applications. *Quality & Quantity*, 1-25. https://doi.org/10.1007/s11135-025-02096-9

Kumari, M., Sharma, P., Singh, P., & Ozel, G. (2025). Enhanced Mean Estimation Using Memory Type Estimators with Dual Auxiliary Variables: Accepted-January 2025. *REVSTAT-Statistical Journal*. https://doi.org/00.00000/revstat.v00i0.000

Kumar, A., Emam, W., & Tashkandy, Y. (2024). Memory type general class of estimators for population variance under simple random sampling. *Heliyon*, *10*(16). https://doi.org/10.1016/j.heliyon.2024.e36090

McIntyre, G. (2005). A method for unbiased selective sampling, using ranked sets. *The American Statistician*, *59*(3), 230-232.

Shahzad, N., ZAIDI, A., Zia, S., Derasit, Z., & Shahzad, N. (2022). Memory Type Estimator Of Population Mean Using Exponentially Weighted Moving Averages In Two-Phase Sampling. *Journal of Positive School Psychology*, *6*(10).

Sharma, P., Singh, P., Kumari, M., Singh, R. (2024). Estimation Procedures for Population Mean using EWMA for Time Scaled Survey. *Sankhya B*, 1-26. https://doi.org/10.1007/s13571-024-00347-7

Singh, G. N., Bhattacharyya, D., Bandyopadhyay, A., & Khalid, M. (2021). Study of a memory type shrinkage estimator of population mean in quality control process. *IEEE Access*, *9*, 161555-161564. https://doi.org/161555-161564. 10.1109/ACCESS.2021.3132686

Singh, P., Sharma, P., & Maurya, P. (2024). Enhancing accuracy in population mean estimation with advanced memory type exponential estimators. *Journal of Reliability and Statistical Studies*, 417-434. https://doi.org/10.13052/jrss0974-8024.1728

Qureshi, M. N., Tariq, M. U., & Hanif, M. (2024). Memory-type ratio and product estimators for population variance using exponentially weighted moving averages for time-scaled surveys. *Communications in Statistics-Simulation and Computation*, *53*(3), 1484-1493. https://doi.org/10.1080/03610918.2022.2050390

Roberts, J. M., Arth, M. J., & Bush, R. R. (1959). Games in culture. *American anthropologist*, *61*(4), 597-605.

Tariq, M. U., Qureshi, M. N., Alamri, O. A., Iftikhar, S., Alsaedi, B. S., & Hanif, M. (2024). Variance estimation using memory type estimators based on EWMA statistic for time scaled surveys in stratified sampling. *Scientific Reports*, *14*(1), 26700. https://doi.org/10.1038/s41598-024-76953-2

Turkish Statistical Institute Motor Vehicle Statistics. (2025). Accessed Date: 28.03.2025. https://biruni.tuik.gov.tr/medas/?kn=89&locale=tr

Yadav, S. K., Vishwakarma, G. K., Varshney, R., & Pal, A. (2023). Improved memory type product estimator for population mean in stratified random sampling under linear cost function. *SN Computer Science*, *4*(3), 235. https://doi.org/10.1007/s42979-023-01673-9