

Determination of Optimum Insulation Thickness Distribution for Refrigerators

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(Alınış / Received: 21.09.2017, Kabul / Accepted: 27.01.2018, Online Yayınlanma / Published Online: 29.03.2018)

Keywords

Insulation thickness,
Refrigerator,
Heat transfer,
Confined volume

Abstract: Most of electricity is consumed in either commercial or domestic refrigeration systems. Since the outer volume is determined, inner volume of a refrigeration system is important for a specified energy consumption. Therefore, the optimum distribution of insulation material according to inside and outside conditions for on and off time of a refrigerator is very important. Uniform distribution of insulation material is useful only convection and conduction resistances are the same for all sides and also on and off periods. In this study, a general solution of the optimum distribution of thermal insulation material for a given insulation material volume or given inner volume is suggested for refrigeration systems and also explained by a case study.

Soğutucular için Optimum Yalıtım Kalınlığı Dağılımının Belirlenmesi

Anahtar Kelimeler

Yalıtım kalınlığı,
Buzdolabı,
Isı transferi,
Kapalı hacim

Özet: Üretilen elektrik enerjisinin büyük bir kısmı ya ticari ya da ev tipi soğutma sistemlerinde tüketilmektedir. Dış hacim genelde belli olduğundan, bir soğutma sisteminin iç hacmi belirli bir enerji tüketimi değeri için önemlidir. Dolayısıyla, bir buzdolabının çalışma ve durma zamanında değişen iç ve dış şartlara göre yalıtım malzemesinin optimum dağılımı oldukça önemlidir. İzolasyon malzemesinin üniform dağılımı, tüm yüzeyler için sadece konveksiyon ve iletim dirençlerinin ve aynı zamanda buzdolabının çalışma ve durma periyotları için aynı değerde olması durumunda kullanılabilir. Bu çalışmada, soğutma sistemleri için belirli bir yalıtım malzemesi hacmi veya iç hacim için ısı yalıtım malzemesinin optimum dağılımının genel bir çözümü önerilmiş ve ayrıca bir örnek çalışma ile açıklanmıştır.

1. Introduction

Refrigeration systems have large portion of total energy consumption. One of the methods reducing the energy consumption is insulating the walls of a refrigeration system. Great effort is made on determining the optimum insulation thickness using thermo-economical models. Insulation thickness suggested by Christensen [1] and Dimitriyev [2] for refrigerators is between 100 – 150 mm for PU foam. Lee et al. [3] introduced a methodology for optimizing the insulation thickness for a given interior volume. Yoon et al. [4] also suggested an optimization strategy for insulation thickness of a refrigerator-freezer system.

Most of studies are related with the building envelopes and only a few deals with the wall orientation and different ambient conditions [5-23]. A literature review on the optimum economic thickness of the thermal insulation for a pipe or duct

investigated by Kaynaklı [24]. Wong et al. [25] investigated the heat transfer characteristics of an insulated tank. Usta and Ileri [26] studied the economic optimum values of refrigeration systems for industrial refrigeration systems. Wong and Chou [27] taken into account to variation of convection resistance due to increasing heat transfer surface area using a new regular polygon top solid wedge thermal resistance (RPSWT) model. Sofrata and Salmeen [28] developed a general mathematical model to select the best insulation thickness. Demir et al. [29] and Sevindir et al. [30] proposed a new analytical method for determining the optimum distribution of insulation material under steady state and transient conditions using degree day method for building envelopes and cold storage systems. Pramanick and Das [31] suggested an alternative calculus method for thermal insulation systems considering the variation of heat transfer coefficient. Their model based on increasing the conduction resistance where the convection resistance is lower (e.g. higher heat transfer coefficient). They also

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employed Bejan's method of intersecting asymptotes [32, 33] to find an order of magnitude for a ceiling value of the wall material. Although there are many studies, a comprehensive study dealing with the optimum distribution of insulation material. In this study, optimum distribution of insulation material considering operating conditions along with the technical parameters should be done.

In this study, optimum distribution of insulation material was studied for refrigerators. Effects of temperature differences, convection heat transfer coefficients and also run time ratio of a refrigerator on optimum distribution of a refrigerator have been investigated. Variation of convection heat transfer coefficients and temperature differences during on and off period of a refrigerator system were also included.

2. Material and Method

A typical refrigerator has finite number of walls having different thermal resistances which are in contact with different environments (e.g. compressor compartment, condenser space, kitchen environment etc.) at different temperatures (Figure 1). Amount of heat transfer per unit area through i^{th} wall which consists of different layers for 365 days period can be expressed as,

$$q_i = \tau \left[\frac{31536000 \Delta T_i}{\frac{1}{h_{in,i}} + \frac{1}{h_{out,i}} + \sum_{j=1}^n \frac{l_{j,i}}{k_{j,i}} + \frac{l_{ins,i}}{k_{ins}}} \right]_{on} + (1 - \tau) \left[\frac{31536000 \Delta T_i}{\frac{1}{h_{in,i}} + \frac{1}{h_{out,i}} + \sum_{j=1}^n \frac{l_{j,i}}{k_{j,i}} + \frac{l_{ins,i}}{k_{ins}}} \right]_{off} \quad (1)$$

Where τ is the run time ratio and can be expressed as;

$$\tau = \frac{\tau_{on}}{\tau_{on} + \tau_{off}} \quad (2)$$

Run time ratio depends on operating conditions of a refrigerator (room temperature, opening and closing door, new food input to the refrigerator etc.) and may be obtained from refrigerator manufacturers. If it is desired to rewrite the resistance of the wall except insulation material,

$$R_{other} = \frac{1}{h_{in,i}} + \frac{1}{h_{out,i}} + \sum_{j=1}^n \frac{l_{j,i}}{k_{j,i}} \quad (3)$$

$$q_i = \tau \left[\frac{31536000 \Delta T_i}{R_{other,i} + \frac{l_{ins,i}}{k_{ins}}} \right]_{on} + (1 - \tau) \left[\frac{31536000 \Delta T_i}{R_{other,i} + \frac{l_{ins,i}}{k_{ins}}} \right]_{off} \quad (4)$$

Also the derivative of q_i with respect to $l_{ins,i}$ is of the form,

$$\frac{dq_i}{dl} = 31536000 \left[\tau \left(- \frac{\frac{\Delta T_i}{k_{ins}}}{\left(R_{other,i} + \frac{l_{ins,i}}{k_{ins}} \right)^2} \right)_{on} + (1 - \tau) \left(- \frac{\frac{\Delta T_i}{k_{ins}}}{\left(R_{other,i} + \frac{l_{ins,i}}{k_{ins}} \right)^2} \right)_{off} \right] \quad (5)$$

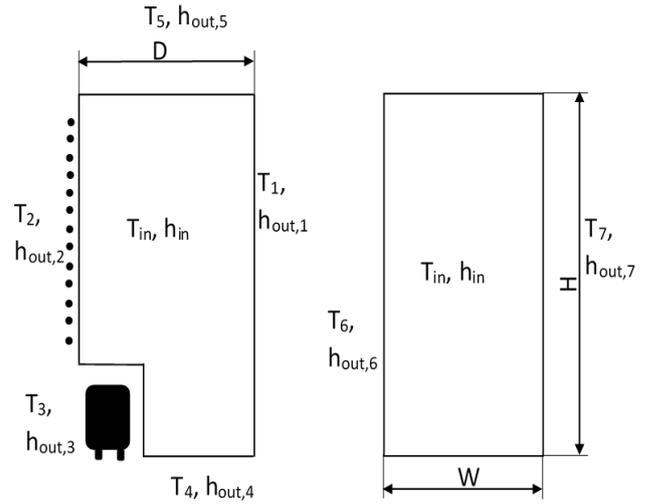


Figure 1. Refrigerator with finite number of walls having different thermal resistances which are in contact with different environments at different temperatures

For a given amount of insulation material volume (or inner refrigerator volume), increasing the insulation thickness of a wall results in decrease of other walls thickness and so the total insulation volume remains the same. Increasing thickness also decreases the amount of heat transfer from the corresponding surface. Since the variation of heat transfer rate by insulation thickness is not linear, the resulting total heat transfer from the surfaces decreases with increasing insulation thickness. It can be seen that the minimum total heat transfer rate is where dq/dl for all walls are equal to each other (Figure 2 and 3) and optimum insulation thicknesses can be determined by equalizing the dq/dl values for all walls.

Total insulation material volume is expressed as;

$$V_{ins} = V_{out} - V_{in} \cong \sum_{i=1}^N l_{ins,i} A_i \quad (6)$$

Equalizing the dq/dl for N walls,

$$\frac{dq_1}{dl} = \frac{dq_2}{dl} = \dots = \frac{dq_N}{dl} \quad (7)$$

it gives,

$$\begin{aligned}
 & 31536000 \left[\tau \left(-\frac{\frac{\Delta T_1}{k_{ins}}}{\left(R_{other,1} + \frac{l_{ins,1}}{k_{ins}}\right)^2} \right)_{on} \right. \\
 & \left. + (1 - \tau) \left(-\frac{\frac{\Delta T_1}{k_{ins}}}{\left(R_{other,1} + \frac{l_{ins,1}}{k_{ins}}\right)^2} \right)_{off} \right] \\
 & = 31536000 \left[\tau \left(-\frac{\frac{\Delta T_2}{k_{ins}}}{\left(R_{other,2} + \frac{l_{ins,2}}{k_{ins}}\right)^2} \right)_{on} \right. \\
 & \left. + (1 - \tau) \left(-\frac{\frac{\Delta T_2}{k_{ins}}}{\left(R_{other,2} + \frac{l_{ins,2}}{k_{ins}}\right)^2} \right)_{off} \right] = \dots \\
 & = 31536000 \left[\tau \left(-\frac{\frac{\Delta T_N}{k_{ins}}}{\left(R_{other,N} + \frac{l_{ins,N}}{k_{ins}}\right)^2} \right)_{on} \right. \\
 & \left. + (1 - \tau) \left(-\frac{\frac{\Delta T_N}{k_{ins}}}{\left(R_{other,N} + \frac{l_{ins,N}}{k_{ins}}\right)^2} \right)_{off} \right]
 \end{aligned} \tag{8}$$

and simplifying the Eq.(8) gives,

$$\begin{aligned}
 & \left(\frac{\tau \Delta T_1}{\left(k_{ins} R_{other,1} + l_{ins,1}\right)^2} \right)_{on} \\
 & + \left(\frac{(1 - \tau) \Delta T_1}{\left(k_{ins} R_{other,1} + l_{ins,1}\right)^2} \right)_{off} \\
 & = \left(\frac{\tau \Delta T_2}{\left(k_{ins} R_{other,2} + l_{ins,2}\right)^2} \right)_{on} \\
 & + \left(\frac{(1 - \tau) \Delta T_2}{\left(k_{ins} R_{other,2} + l_{ins,2}\right)^2} \right)_{off} \\
 & = \left(\frac{\tau \Delta T_N}{\left(k_{ins} R_{other,N} + l_{ins,N}\right)^2} \right)_{on} \\
 & + \left(\frac{(1 - \tau) \Delta T_N}{\left(k_{ins} R_{other,N} + l_{ins,N}\right)^2} \right)_{off}
 \end{aligned} \tag{9}$$

Since the resulting equations are implicit, they can be solved using graphical methods or specialized software (EES, MATLAB etc.).

Values in Table 1 were used for obtaining the variations and effects of different parameters. In Figure 2 and 3, two wall with unit area is considered. Walls (Wall #1 and Wall #2 which are chosen arbitrary) are in contact with the environments at different temperatures and also convection heat transfer coefficients are different.

Table 1. Calculation parameters

	ΔT_1 (°C)	$R_{other,1}$ (m ² °C /W)	ΔT_2 (°C)	$R_{other,2}$ (m ² °C /W)	k_{ins} (W/m K)
On period	40	0.15, 0.3	20	0.3	0.025, 0.35, 0.50
Off period	20	0.3	20	0.3	0.025, 0.35, 0.50

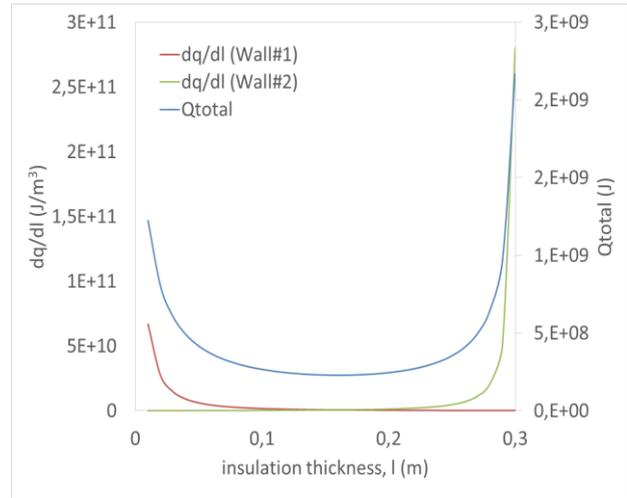


Figure 2. Variation of dq/dl and Q_{total} with the insulation thickness for a refrigerator wall

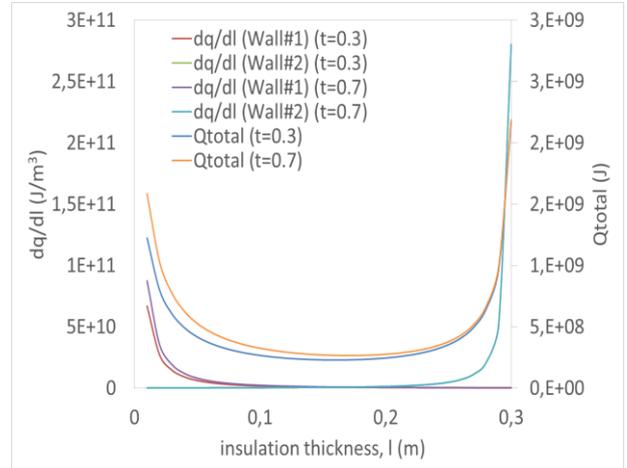
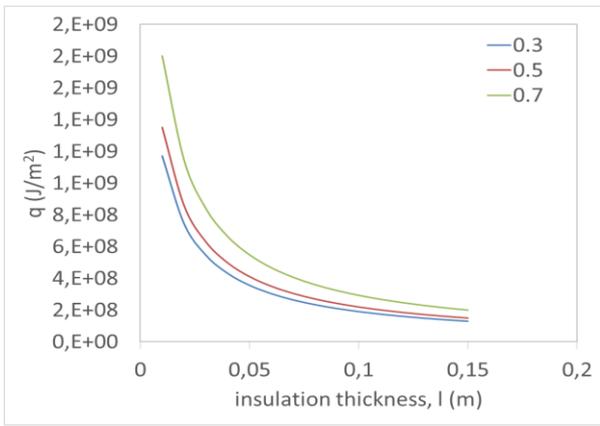


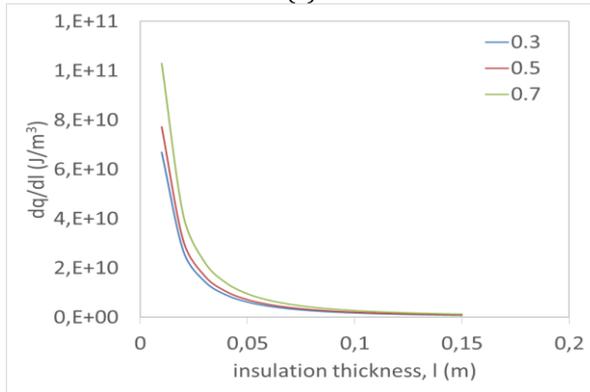
Figure 3. Variation of dq/dl and Q_{total} with the insulation thickness of a refrigerator wall for different run time ratios

Total insulation thickness is 0.3 m so the total insulation volume is 0.3 m³. The variation of the total amount of heat transfer as a function insulation thickness is shown in Figure 2.

Since the total insulation material volume is constant, increasing the insulation thickness of the first wall results in decrease the thickness of the second wall. Higher insulation thickness of a wall results decrease in total amount of heat transfer from the corresponding wall.

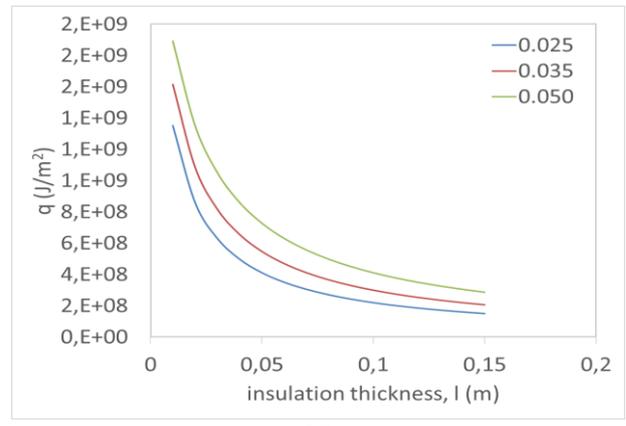


(a)

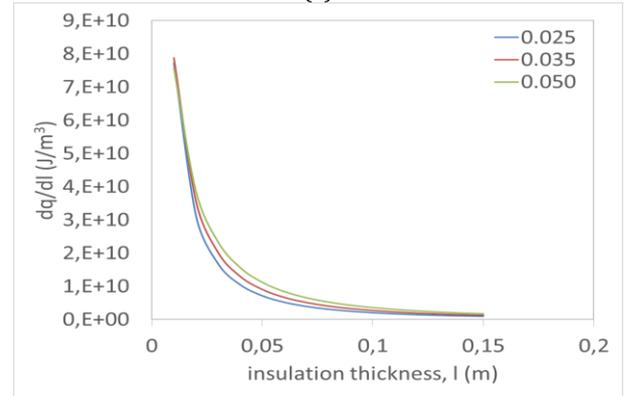


(b)

Figure 4. Effects of the run time ratio on (a) the amount of heat transfer and (b) dq/dl for Wall#1

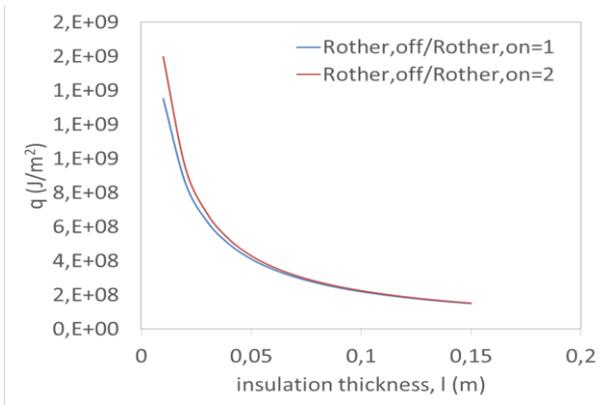


(a)

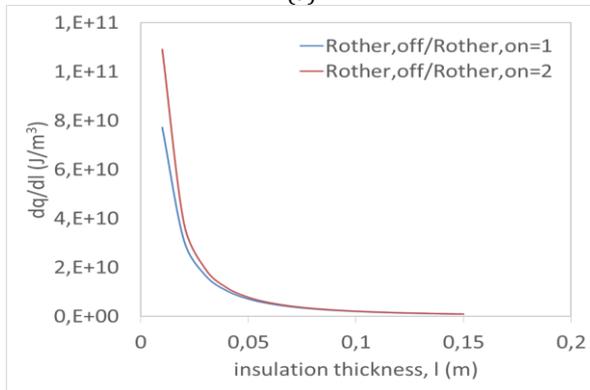


(b)

Figure 6. Effects of thermal conductivity of the insulation material on (a) the amount of heat transfer rate and (b) dq/dl for Wall#1



(a)



(b)

Figure 5. Effects of the ratio of wall resistances (excluding insulation material) on (a) the amount of heat transfer rate and (b) dq/dl for Wall#1

As seen in Figure 2, the total amount of heat transfer passes through a minimum point where the dq/dl values of the two walls are equal to each other and it is the optimum point. In Figure 3, the effects of the run time ratio on the total amount of heat transfer and dq/dl are also given.

Figure 4, 5 and 6 effects of the run time ratio, wall resistances ratio of on and off time period and thermal conductivity on the amount of heat transfer per unit area and dq/dl are represented respectively. The amount of heat transfer per unit area increases for bigger values of run time ratio due to higher temperature differences and convection heat transfer coefficients (Figure 4a) and therefore dq/dl increases as well (Figure 4b).

Convection heat transfer coefficient are higher inside the refrigerator cabin due to operation of a fan during the on period. Beside this, temperature differences are also higher for compressor compartment and condenser section during on period. Effects of thermal resistances ratio for on and off period on the amount of heat transfer and dq/dl are given in Figure 5.

Effects of thermal conductivity of the insulation material on the amount of heat transfer and dq/dl are given in Figure 6. The amount of heat transfer

Table 2. Design conditions for calculations

	T_1 (°C)	$h_{out,1}$ (W/m ² °C)	T_2 (°C)	$h_{out,2}$ (W/m ² °C)	T_3 (°C)	$h_{out,3}$ (W/m ² °C)	T_{in} (°C)	h_{in} (W/m ² °C)
On period	20	7	40	10	60	10	0	15
Off period	20	7	20	7	20	7	4	5

increases with higher thermal conductivity values as expected but has little effect on dq/dl up to insulation thickness of 0.02 m and higher than 0.14 m.

2.1. Case study

A typical refrigerator ($W \times D \times H = 700 \times 600 \times 1700$ mm) with its walls in contact with the environments at different temperatures are given in Figure 7.

As shown in Figure 7, front, bottom, top and side walls are assumed in contact with room (kitchen) air and the temperatures and convection heat transfer coefficients are the same for on and off periods (Table 2). Also, condenser spacing and compressor compartment conditions are also assumed the same as the kitchen air during the off period. For on and off periods, temperatures and corresponding convective heat transfer coefficients are given in Table 2 in detail.

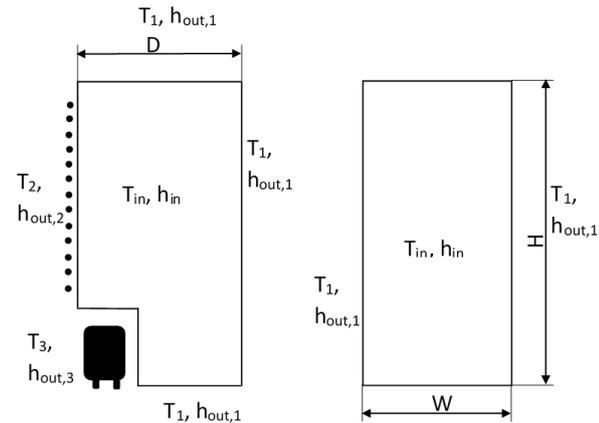


Figure 7. Refrigerator geometry used for case study and operating conditions

Calculation were carried out for the conditions given in Table 2. Since the resulting equations are implicit, graphical method used for determining the optimum insulation thicknesses and corresponding total amount of heat transfer. Results are summarized in following section.

3. Results

In Figure 8, 9, 10 and 11, insulation thickness ratios are given for run time ratio $\tau = 0.25, 0.50, 0.75$ and 1 respectively. Here, Wall#1 (walls in contact with room air: front, bottom, top and side walls are assumed as Wall #1 since corresponding temperature differences and convection heat transfer

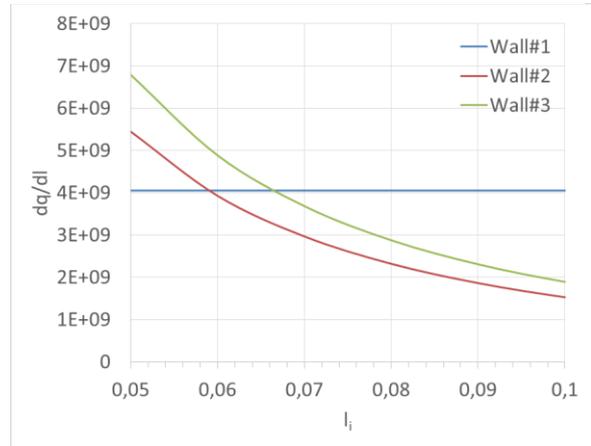


Figure 8. dq/dl vs l_i for $\tau=0.25$

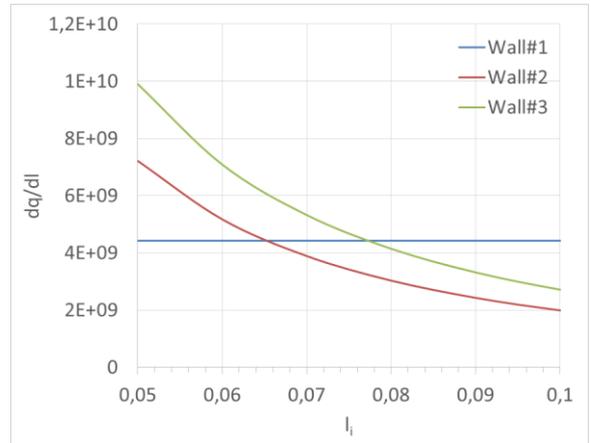


Figure 9. dq/dl vs l_i for $\tau=0.50$

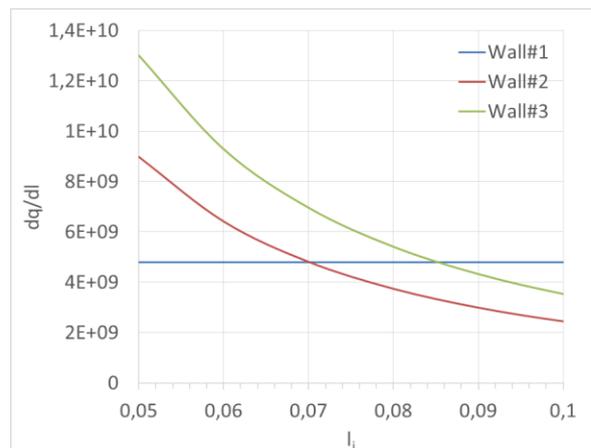


Figure 10. dq/dl vs l_i for $\tau=0.75$

coefficients are the same) is selected reference wall and dq/dl value is calculated for insulation thickness of 0.05 m. Then the variation of the dq/dl values for Wall#2 (back of refrigerator which is in contact with condenser) and Wall#3 (which is in contact with

Table 3. Calculated values

τ	l_1 (mm)	l_2 (mm)	l_3 (mm)	l_{uniform} (mm)	$Q_{\text{total,optimum}}$ (MJ/year)	$Q_{\text{total,uniform}}$ (MJ/year)	Difference (%)
0.25	50	59	66.5	53.1	1283	1294	0.8
0.50	50	65.3	77.2	55.2	1429	1461	2.2
0.75	50	70.1	85.3	56.7	1571	1628	3.5
1	50	74	91.5	58	1711	1796	4.8

compressor compartment) plotted on the same graph. Intersection points represents the equal dq/dl values which is desired to calculate l_2 and l_3 values. Also, uniform insulation thickness for the same insulation volume is calculated and results are compared with the optimum values. Detailed summary of calculated values is given in Table 3.

As seen in Table 3, maximum difference in total amount of heat transfer for optimum and uniform insulation thickness cases are up to 4.8% for the run time ratio of 1.

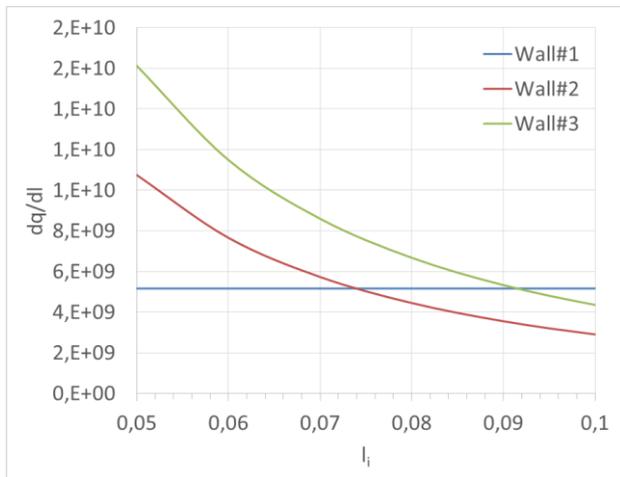


Figure 11. dq/dl vs l_i for $\tau=1$

4. Discussion and Conclusion

In this study, a general solution of the optimum distribution of thermal insulation material considering refrigeration systems are given. It is concluded that;

- Optimum distribution of insulation material using proposed method can be easily calculated. The results are the optimum values and new methodology eliminates iterative process,
- Maximum difference in total amount of heat transfer for the optimum and the uniform insulation thickness cases are up to 4.8% for to run time ratio of 1. A typical A+ refrigerator which has the 425 liter net volume consumes 0.92 kWh/24h and 335.6 kWh/year. Therefore, reduction in electricity consumption by about 2.2% (run time ratio 0.5) results in 7.38 kWh/year saving per single unit refrigerator.
- Effects of the economical parameters can be implemented the present methodology to introduce thermo-economic optimization strategy for

determination of optimum insulation thickness distribution.

Nomenclature

- A_i Surface area of i^{th} wall (m^2)
- $h_{in,i}$ Convection heat transfer coefficient of inside ambient of i^{th} wall ($\text{W}/\text{m}^2 \text{ } ^\circ\text{C}$)
- $h_{out,i}$ Convection heat transfer coefficient of outside ambient of i^{th} wall ($\text{W}/\text{m}^2 \text{ } ^\circ\text{C}$)
- $k_{j,i}$ Thermal conductivity of j^{th} layer of i^{th} wall ($\text{W}/\text{m } ^\circ\text{C}$)
- k_{ins} Thermal conductivity of insulation material ($\text{W}/\text{m } ^\circ\text{C}$)
- $l_{j,i}$ Thickness of j^{th} layer of i^{th} wall (m)
- $l_{ins,i}$ Thickness of insulation material of i^{th} wall (m)
- q_i Amount of heat transfer per unit area of i^{th} wall (J/m^2)
- Q_{total} Total amount of heat transfer per unit area (J)
- $R_{other,i}$ Total thermal resistance of i^{th} wall components excluding the insulation material ($\text{m}^2 \text{ } ^\circ\text{C}/\text{W}$)
- R_{ins} Thermal resistance of insulation material ($\text{m}^2 \text{ } ^\circ\text{C}/\text{W}$)
- ΔT_i Temperature difference of i^{th} wall ($^\circ\text{C}$)
- $T_{in,i}$ Inside ambient temperature of i^{th} wall ($^\circ\text{C}$)
- $T_{out,i}$ Outside ambient temperature of i^{th} wall ($^\circ\text{C}$)
- V_{ins} Total volume of insulation material (m^3)

References

- [1] Christensen, L. B. 1981. The insulation of freezers and refrigerators - how thick it should be? International Journal of Refrigeration, 4 (1981), 73-76.
- [2] Dimitriyev, V. I., 1984. Optimum insulation thicknesses for domestic refrigerators and freezers. International Journal of Refrigeration, 7(1984), 72-73.
- [3] Lee,T., Lee, W., Lee, Y., 2006. Optimization of the Insulation Wall Thickness of Refrigerator. International Refrigeration and Air Conditioning Conference, Paper 837.
- [4] Yoon, W., Seo, K., Kim, Y., 2013. Development of an optimization strategy for insulation thickness of a domestic refrigerator-freezer. International Journal of Refrigeration, 36(2013), 1162-1172.
- [5] Söylemez, M., Ünsal, M., 1999. Optimum insulation thickness for refrigeration

- applications, *Energy Conversion and Management*, 40(1999), 13-21.
- [6] Daouas, N., Hassen, Z., Aissia, H. B., 2010. Analytical periodic solution for the study of thermal performance and optimum insulation thickness of building walls in Tunisia. *Applied Thermal Engineering*, 30(2010), 319-326.
- [7] Kaynakli, O., 2011. Parametric investigation of optimum thermal insulation thickness for external walls, *Energies*, 4(2011), 913-927.
- [8] Özel, M., Pihtili, K., 2007. Optimum location and distribution of insulation layers on building walls with various orientations. *Building and Environment*, 42(2007), 3051-3059.
- [9] Al-Sanea, S. A., Zedan, M. F., 2011. Improving thermal performance of building walls by optimizing insulation layer distribution and thickness for same thermal mass. *Applied Energy*, 88(2011), 3113-3124.
- [10] Ekici ,B. B., Gulden, A. A., Aksoy, U. T., 2012. A study on the optimum insulation thicknesses of various types of external walls with respect to different materials, fuels and climate zones in Turkey. *Applied Energy*, 92(2012),211-217.
- [11] Axaopoulos, I., Axaopoulos, P., Gelegenis, J., 2014. Optimum insulation thickness for external walls on different orientations considering the speed and direction of the wind. *Applied Energy*, 117(2014), 167-175.
- [12] Ozel, M., 2011. Effect of wall orientation on the optimum insulation thickness by using a dynamic method. *Applied Energy*, 88(2011), 2429-2435.
- [13] Jinghua Yu, Liwei Tian, Changzhi Yang, Xinhua Xu, Jinbo Wang, 2011. Optimum insulation thickness of residential roof with respect to solar-air degree-hours in hot summer and cold winter zone of china. *Energy and Buildings*, 43(2011), 2304-2313.
- [14] Y.F. Li, W.K. Chow, 2005. Optimum insulation-thickness for thermal and freezing protection. *Applied Energy*, 80(2005), 23-33.
- [15] Sami A. Al-Sanea, M.F. Zedan, Saleh A. Al-Ajlan, 2005. Effect of electricity tariff on the optimum insulation-thickness in building walls as determined by a dynamic heat-transfer model. *Applied Energy*, 82(2005), 313-330.
- [16] Alireza Bahadori, Hari B. Vuthaluru, 2010. A simple method for the estimation of thermal insulation thickness. *Applied Energy*, 87(2010), 613-619.
- [17] Naouel Daouas, 2011. A study on optimum insulation thickness in walls and energy savings in Tunisian buildings based on analytical calculation of cooling and heating transmission loads. *Applied Energy*, 88(2011), 156-164.
- [18] Pan Dongmei, Chan Mingyin, Deng Shiming, Lin Zhongping, 2012. The effects of external wall insulation thickness on annual cooling and heating energy uses under different climates. *Applied Energy*, 97(2012), 313-318.
- [19] Afif Hasan, 1999. Optimizing insulation thickness for buildings using life cycle cost. *Applied Energy*, 63(1999), 115-124.
- [20] Jérôme Barrau, Manel Ibanez, Ferran Badia, 2014. Impact of the optimization criteria on the determination of the insulation thickness. *Energy and Buildings*, 76(2014), 459-469.
- [21] H. Asan, 1998. Effects of Wall's insulation thickness and position on time lag and decrement factor. *Energy and Buildings*, 28(1998), 299-305.
- [22] Muhammet Kayfeci, Ali Keçebaş, Engin Gedik, 2013. Determination of optimum insulation thickness of external walls with two different methods in cooling applications. *Applied Thermal Engineering*, 50(2013), 217-224.
- [23] Muhammet Kayfeci, İsmail Yabanova, Ali Keçebaş, 2014. The use of artificial neural network to evaluate insulation thickness and life cycle costs: Pipe insulation application. *Applied Thermal Engineering*, 63(2014), 370-378.
- [24] Ömer Kaynaklı, 2014. Economic thermal insulation thickness for pipes and ducts: A review study. *Renewable and Sustainable Energy Reviews*, 30(2014), 184-194.
- [25] King-Leung Wong, Huann-Ming Chou, Bing-Shyan Her, Huang-Ching Yeh, 2004. Complete heat transfer solutions of an insulated regular cubic tank with an SSWT model. *Energy Conversion and Management*, 45(2004), 2813-2831.
- [26] N. Usta, A. Ileri, 1999. Computerized economic optimization of refrigeration system design. *Energy Conversion and Management*, 40(1999), 1089-1109.
- [27] King-Leung Wong, Huann-Ming Chou, 2003. Heat transfer characteristics of an insulated regular polyhedron by using a regular polygon top solid wedge thermal resistance model. *Energy Conversion and Management*, 44(2003), 3015-3036.
- [28] H. Sofrata and B. Salmeen, 1993. Optimization of insulation thicknesses using micros. *Energy Conversion and Management*, 34(1993), 471-479.
- [29] H. Demir, M. K. Sevindir, Ö. Ağra, Ş. Ö. Atayılmaz, İ. Teke, 2015. Optimum distribution of thermal insulation material for constant insulation

- material volume or a given investment cost. *J. of Renewable and Sustainable Energy*, 7(2015), 063122.
- [30] M. K. Sevindir, H. Demir, Ö. Ağra, Ş. Ö. Atayılmaz, İ. Teke, 2017. Modelling the optimum distribution of insulation material. *Renewable Energy*, 113 (2017), 74-84.
- [31] A.K. Pramancik, P.K. Das, 2005. Heuristics as an alternative to variational calculus for optimization of a class of thermal insulation systems. *Int. Journal of Heat and Mass Transfer*, 48(2005), 1851-1857.
- [32] A. Bejan, 2000. *Shape and Structure, from Engineering to Nature*. Cambridge University Press, UK, 2000.
- [33] J. Lewins, 2002. Bejan's constructal theory of equal potential distribution. *Int. Journal of Heat and Mass Transfer*, 46(2002), 1541-1543.