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Multitask Assignment and Path Planning for Heterogenous UAVs with Flight Dynamics

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Uçuş Dinamiklerine Sahip Heterojen İHA'lar için Çoklu Görev Atama ve Rota Optimizasyonu

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Abstract
This study addresses the cooperative multitask assignment and path planning of heterogenous UAVs with aircraft kinematics. The purpose of this study is to minimize the maximum mission time and total distance traveled by UAV fleet which execute multi-tasks in a military theatre composed of targets with air defense threat circle as well as no-fly zones. This study aims to optimize multi-objectives with structural and timing constraints. The study introduces a complex and well-structured mixed integer linear program, which successfully optimize path planning and multitask allocation of UAVs simultaneously. Contrary to common fashion in literature, the proposed model encompasses most of the real-life military requirements for UAV tactical operations, rather than leaving them as assumptions or integrating some of them by the results of commercial simulators. In terms of path planning phase, the study takes into account Dubin's distances complying with the flight dynamics of attack and surveillance UAVs, and no-fly zones in the theater. Regarding task allocation, the model in the study satisfies task order requirements of targets such as classification, attack and verification in the respective order by avoiding deadlock caused by waiting cycles over target. Targets are threats to UAVs, so the study also considers the probability of UAV loss during the mission scenario by developing chance constraints integrated in to the model. The model is coded in Matlab, and solved by Gurobi global optimal solver successfully under predefined realistic operative scenarios.

Keywords: *Heterogeneous UAV; path planning and task assignment; Dubin's distances; CMTAP; MILP*

Öz
Bu çalışma, uçak kinematiğine sahip heterojen İHA'ların birlikte çalışabilirliğine dayalı çoklu görev ataması ve rota planlamasını ele almaktadır. Çalışmanın amacı, hava savunma tehdit çemberleri ve uçuşa yasak bölgelerle çevrili hedeflerden oluşan askeri harekat alanında keşif ve imha görevleri icra eden İHA filosunun azami görev süresini ve toplam kat edilen mesafeyi minimize etmektir. Bu çalışma aynı zamanda yapısal ve zaman kısıtları altında birden fazla amaç fonksiyonun optimize edilmesini amaçlamaktadır. Çalışmada; İHA'ların rota planlaması ve çoklu görev atamasını eş zamanlı olarak başarıyla optimize edebilen, karmaşık ve iyi yapılandırılmış bir karışık tam sayılı doğrusal programlama modeli önerilmektedir. Probleme yönelik literatürde yer alan çalışma yöntem ve varsayımlarının aksine, bu çalışma, mevcut askeri İHA taktik operasyonlarına yönelik gereksinimlerin çoğunu varsayım olarak bırakmak ve/veya haricen elde edilen edilen simülasyon sonuçlarının çözüme sonradan entegre etmek yerine, doğrudan matematiksel model ile ele alarak global optimal çözümler elde edebilmiştir. Çalışma, rota planlama aşamasında taarruz ve keşif İHA'larının farklı uçuş dinamik ve hedefe yaklaşma yöntemlerini Dubin's mesafe ve manevralarına uygun olarak aynı zamanda hava sahasındaki uçuşa yasak bölgeleri de dikkate alarak hesaplamaktadır. Görev ataması açısından ise model; hedeflerin tespit, taarruz ve doğrulama gibi görev sıralamasını, hedef üzerinde bekleme döngülerini önleyerek, etkin şekilde gerçekleştirmektedir. Hedefler aynı zamanda İHA'lar için tehdit oluşturduğundan, bu çalışmada görev senaryosu sırasında İHA vurulma olasılığını da dikkate alarak modele entegre edilmiş olasılık kısıtları geliştirmiştir. Model, gerçekçi ve önceden tanımlanmış operasyonel senaryolar altında Matlab ile kodlanmış ve Gurobi global optimizasyon çözücüsü kullanılarak başarıyla çözülmüştür.

Anahtar Kelimeler: *Heterojen İHA; rota planlama ve görev atama; Dubin's mesafeleri; CMTAP; MILP.*

1. Introduction

In modern warfare and surveillance operations, the optimization of multiple unmanned aerial vehicles (UAVs) for missions involving target surveillance and destruction is critical. The complexity of these operations increases when multiple UAVs are to be allocated with different roles — such as classification, attack, and verification — while respecting various operational constraints. The optimization problem becomes even more challenging when considering the dynamic interactions among a fleet of UAVs tasked with completing these objectives in a time sensitive, coordinated manner. Cooperative multiple task assignment problem (CMTAP) is a subproblem of general task assignment problem, in which agents are assigned to

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the targets with multi tasks; and moreover, the path plan and routing of the agents are also considered in terms of multi visits to obtain best objective function value. In the earlier studies, UAVs are considered as identical (isomorphic) operative agents which have exact specifications and capabilities (Schumacher *et al.* 2002, Alighanbari *et al.* 2005, Eun and Bang 2007), while the most recent studies considered heterogeneity of UAVs in terms of capabilities (Chen *et al.* 2018, Cao *et al.* 2019, Zhang et al. 2022). As the parameters and variables increase, and assumptions are relaxed in the CMTAP, the combinatorial problem structure becomes more complex and demanding to solve for exact optimal solutions by deterministic algorithms. Therefore, CMTAP is a type of combinatorial problem with a large solution landscape, which aims to optimize single or multiple objectives by satisfying the time constraints, flight path requirements and task order on the targets by considering the UAV capabilities (Chen *et al.* 2018). Song *et al.* (2023) describe the relationship between UAV and its respective ground control unit as centralized (offline), decentralized (online), and mixed (hierarchical). The centralized task allocation is the most common approached practiced in the literature to attain exact solution (Zhen *et al.* 2018). In centralized architecture all of the system information (assignment plan, plan modifications according to the changing situations, task execution success, overall reward, and cost of the system etc.) is examined by the main controller stationed in the ground control unit (Wang *et al.* 2020). For small scale problems, the centralized task allocation is preferable due to its ability of obtaining exact solution; however, when the search landscape of the problem increases, the stability and resilience of the system might fail due to the amassed processing load on the ground control unit.

The solution methods for CMTAP can be grouped under two categories such as exact algorithms and heuristics. Centralized problems with relatively smaller scopes can effectively be handled by exact solution methods; however, when the scope becomes larger, or the problem architecture is distributed then heuristics/meta heuristics might perform better by their feasible computational time at the expense of optimality in the solution. Seminal paper from Zollars *et al.* (2023) deals with calculating the optimal trajectories for simultaneous target attack in the presence of the no-fly zones (obstacles). Using direct orthogonal collocation methods, it transcribes the two-point boundary value optimal control problem into a nonlinear programming problem. In similar fashion, but now taking in account the presence of wind as additional constraint, Luo *et al.* (2018) address the integrated optimization of UAV task allocation and path planning, taking in account presence of wind. It extends the Vehicle Routing Problem (VRP) model to a Variable-Speed Dubin's Path VRP (VS-DP-VRP) model, aiming to minimize flight time using the genetic algorithm. Oh *et al.* (2011) propose a coordinated road network search algorithm for multiple heterogeneous UAVs. The problem is formulated as a Multichoice Multidimensional Knapsack Problem (MMKP), aiming to minimize flight time while considering UAVs' physical constraints. Dubin's path planning is utilized to produce the shortest and flyable paths that adhere to these constraints. On the other hand, Zahrádka *et al.* (2019) introduce the Dubin's Team Orienteering Problem with Neighborhoods (DTOPN), a variant of the Orienteering Problem tailored for multiple curvature-constrained vehicles like fixed-wing UAVs. The objective is to maximize collected rewards from target locations within a limited travel budget, considering the vehicles' turning radius constraints. The authors propose a Greedy Randomized Adaptive Search Procedure (GRASP) with Path Relinking to solve the problem. As an extension of Oh *et al.* (2011), Oh *et al.* (2015) propose the framework which combines mission planning (task allocation) with path planning (trajectory generation) using Dubin's paths. This integration ensures that UAVs follow flyable paths that respect their kinematic constraints. The authors develop an algorithm for efficient patrolling of road networks by multiple UAVs. This method ensures that every road segment is covered while optimizing the overall mission efficiency.

The focus of this study is the assignment of heterogeneous UAVs with different capabilities to the targets which require classification, attack, and verification tasks in order. UAVs are assumed to travel with Dubin's turns and maneuvers, so path optimization and trajectory calculations are also considered as a major issue to be resolved together with the assignment problem. The UAVs operate under a set of constraints. The distances between operational nodes (targets, airports, etc.) are modeled using Dubin's distances, which are account for the motion constraints of UAVs, particularly when operating in a constrained environment with fixed turning radii. The goal of this optimization problem is to minimize the overall mission cost, which includes travel time, operational delays, and the total distance covered by the UAVs, while ensuring all constraints are met and the objectives are successfully completed. One of the most important assumptions made is the fact that the target threat circle is divided to discrete points to make the problem tractable. In this case, we use 8 points for the division of the circle, with the incremental angle of $\frac{\pi}{4}$. By applying most of the operational complexities into the proposed model, we aim to find globally optimal UAV paths that optimize the completion of surveillance, attack, and verification tasks in a coordinated and time-efficient manner, showcasing the power of optimization techniques in managing complex UAV operations carried out at front bases within tactical level, which is a rank of UAV operations carried out by four or five UAVs against three or four targets under several operational restrictions and risks.

2. Materials and Methods

In this study, fixed-wing UAVs are regarded as Dubin's cars with kinematic features such as minimum turn rate and head angles. These flight dynamics are calculated by linear algebra and basic geometric calculations expressed in this study.

- Surveillance Tasks (Classification and Verification): UAVs travel around $(3/4)^{th}$ of the perimeter of the target safety circle (buffer zone) and accomplish surveillance tasks. The target location is assumed to be center location and r^j_{THREAT} is assumed to be a circular threat area which represents the air defense zone of the target. Since the probability of kill is rather high in the threat zone of target, another zone above the target threat area with coefficient $\lambda > 1$ is determined as safety circle to the target λr^j_{THREAT}. UAVs assigned to surveillance tasks enter the target safety circle to minimize the probability of kill and to accomplish the surveillance task by traveling a circular path around the $(3/4)^{th}$ of the perimeter of the target's safety circle. Partially traveled circular rotation around the target $[(3/4)(2\pi r)]$ is assumed to be enough to have a satisfactory surveillance task performance. After the required travel around the target, assigned UAVs head to the new assignment.
- Attack Tasks: UAVs directly fly to the center location of the target by intruding the target threat zone. The target location is assumed to be center location. Combat UAVs head to the target center point, and the arrival on the center coordinates of the target stands for the accomplished attack task unless the UAV is shot. After successful attack task, UAV heads to the new assignment if it survives.
- No-Fly Zone (NFZ): There are several restricted areas in the theater where the flight is prohibited due to safety reasons, such as electronic warfare, jamming, high air defense threat etc. In this study, no-fly zones are assumed as circular threat zones for the UAVs, and the formulations for UAVs to traverse around the no-fly zones are calculated.

2.1 No-fly zones

In this section we discuss the logic behind handling the problem of no-fly zones. We represent the no-fly zones as circular spatial prohibitions, and we treat these zones as targets with the task of surveillance without the requirement to visit $(3/4)^{th}$ of the perimeter of the whole threat circle. Thus, if the NFZs are placed within the path between two nodes that are to be visited, then we treat them by recursively calculating the tangents to the NFZ from both nodes. By summing these distances up, we can find the minimal Dubin's path distance in the presence of NFZ. This idea is represented in Figure 1 for the starting and ending angular directions.

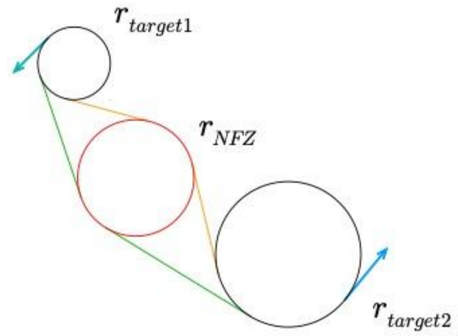

Figure 1. Dubin's path with a no-fly zone.

There are two sets of tangents that need to be analyzed, with green tangents covering NFZ from below and orange tangents covering NFZ from above. After calculating the tangents, we can easily calculate the point at which the UAV enters the circle, and the entry point is fundamental to calculating the exit point which differs for surveillance and attack tasks. Tangent calculations also let us calculate the leaving angular direction, providing us with the possibility to calculate the total circular path covered. The coordinates for a generic entry point reads as $P_{ENTRY} = [O_x + \lambda r^1_j cos\theta_i,\ O_y + \lambda r^1_j sin\theta_i]$, and is depicted in Figure 2. Here, r^1_j is the radius of threat j, $r^2_j = \lambda r^1_j$ is the radius of the safe circle for threat j, and $\lambda > 1$ is the safety coefficient. The details regarding Dubin's path calculations are presented with details in Appendix A, B, and C.

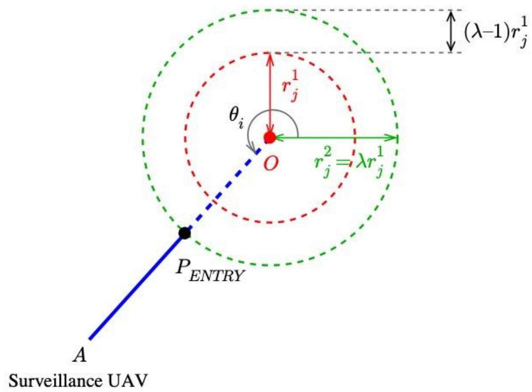

Figure 2. Entry point on target.

2.2 Problem definition and assumptions

The CMTAP problem examined in this study is a two-dimensional military theater, in which targets have their own air-defense systems, so that targets can be a threat for UAVs during the operation. UAVs can take off and land on various airports, and they can be shot during their operations. UAVs are heterogeneous with different capabilities. The model architecture is constructed in four parts, which are introduced in the order of objective function, structural constraints, chance constraints and time constraints. Table 1 summarizes the notation we use followed by further related explanations.

Table 1. Notation

Sets	
N	Set of nodes for airports, targets and no-fly zone centers
T	Set of target nodes
H	Set of airport nodes
NFZ	Set of no-fly zone centers
U	Set of UAVs
U_{combat}	Set of combat UAVs
$U_{surv.}$	Set of surveillance UAVs
K	Set of tasks
K_{attack}	Set of attack tasks, {2,3}
$K_{surv.}$	Set of surveillance tasks, { 1,4 }
Φ	Set of all possible head angles
B	Set of angular directions
Indices	
i	Index of the node departed
j	Index of the node arrived
h	Airport node index
u	UAV index
k_1	Index of the previous task
k_2	Index of the next task
ϕ_1	Angle index of the previous task
ϕ_2	Angle index of the next task
b_1	Angular direction index of the previous task
b_2	Angular direction index of the next task
Parameters	
N_j^M	Total number of missions for each target j
N_h^{MUN}	Airport ammunition capacity for each airport h
N_u^{MUN}	UAV Ammunition capacity for each UAV u
G_j^{MUN}	Minimum ammunition required to destroy each target j
$Velocity_{u.}$	Vector of velocity values for each UAV u
L_d	Dubin's paths for the pairs of input nodes, tasks, angles and angular directions
$cost_1$	Cost of traded off operational readiness per $hour$
$cost_2$	Cost of UAV attrition per km
$Big\ M$	Very big number
Decision Variables	
$x_{i,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2}$	Binary variable taking a value of 1 if UAV u flying from node i after executing task k_1 is assigned to node j for task k_2, and 0 otherwise
$tstart_{i,j}^{u,k}$	Non-negative real variable representing the start time of mission k on target by UAV u flying from node i to node j
$\text{tend}_{i,j}^{u,k}$	Non-negative real variable representing the finish time of mission k on target by UAV u flying from node i to node j

- Considering the task set K, $k = 1$ corresponds to classification, $k = 2$ corresponds to first attack on the target, $k = 3$ corresponds to second attack on the target and $k = 4$ corresponds to the verification of the target destruction. In case the target needs to be hit only once, the attack tasks $k = 2$ and $k = 3$ correspond to the same task.
- The set of head angle ϕ has a variable cardinality that depends on the discretization. As an example, Figure 3 shows a set of head angles with cardinality eight.
-

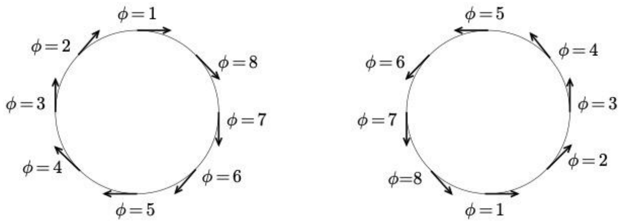

Figure 3. An example for the set of possible head angles with eight values.

- Set of angular directions Bhas two values: $b = 1$ corresponds to the clock-wise and $b = 2$ corresponds to the counter clock-wise directions as depicted in Figure 4.

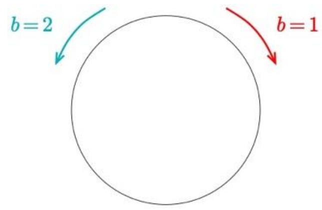

Figure 4. Possible angular directions.

2.3 Model formulation

This section introduces the objective function, structural, chance and timing constraints with definitions.

Objective functions:

The proposed model has two objective functions to minimize the maximum total mission time and cost of assigning UAVs to the missions. Equation (1) minimizes the maximum mission completion time for each target by summing the final mission completion time of each target with the total flight time of all assigned UAVs to each target. Here, it is important to note that final mission for each target is represented by $k = 4$, which is defined as verification task in the study. Equation (2) minimizes the total distance traveled by each UAV until all tasks are completed on all targets.

Both Z_1 andZ_2 are minimization objectives; however, we apply $cost_1$ for Z_1 and $cost_2$ for Z_2 , respectively, to have the multi-objectives with same units by transforming them into weighted objective functions.

$$Z_1 = \min \max_{i\in T} \left\{ \sum_{u\in U} \sum_{j\in T} tend_{j,i}^{u,4} + \sum_{u\in U} \sum_{k_1,k_2\in K} \sum_{h\in H} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} \frac{x_{i,H,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} Ld_{i,H,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2}}{Velocity_u} \right\} \quad (1)$$

$$Z_2 = \min \sum_{u\in U} \sum_{k_1,k_2\in K} \sum_{i,j\in N} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{i,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2}\, Ld_{i,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} \quad (2)$$

In the proposed model, $cost_1$ is measured by $(\$/hours)$ and can be interpreted as the opportunity cost of having a UAV in the air per hour since the operational readiness might be affected due to the lack of a capable UAV at the front base. Similarly, $cost_2$is measured by $(\$/km)$ and can be interpreted as the cost of attrition (material, fuel, systems reliability, etc.) of a UAV per km. Thus, by applying the related costs to their respective objective functions, then we can simply relax Equations (1) and (2) into single objective function as stated in Equation (3).

$$Z = mi\,n(cost_1 Z_1 + cost_2 Z_2) \quad (3)$$

Structural constraints:

This sections introduces the structural constraints given in Equations (4)–(34).

Assignment constraint: All targets have at least three tasks to be executed, which are classification, attack, and verification, respectfully. Some targets might also have secondary attack tasks, which is called double attack task. In this case the number of tasks that should be executed for targets are four depending on the number of attack tasks assigned (either 1 or 2 attack tasks).

$$\sum_{u\in U} \sum_{k_1,k_2\in K} \sum_{i\in N} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{i,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} = N_j^M$$

$$j \in T \quad (4)$$

Flow constraint: Any UAV assigned for any task k on the node p is either shot by air defense system or departs the target after the task execution. The chance constraints are also developed to work as flexible flow constraints, the details of which are explained in this section.

$$\sum_{k_1,k_2\in K} \sum_{i\in N} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{i,p,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} - \sum_{k_1,k_2\in K} \sum_{j\in N} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{p,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} \le 1,$$

$$u \in U, p \in T \quad (5)$$

UAV ammunition constraint: It is assumed that one ammunition unit is enough to perform the attack task, so this constraint guarantees that total number of attack tasks of any combat UAV must be equal or lower than its ammunition capacity.

$$\sum_{k_1\in K} \sum_{k_2\in K_{attack}} \sum_{i,j\in N} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{i,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} \le N_u^{MUN},$$

$$u \in U \quad (6)$$

Airport ammunition constraint: The number of attack tasks assigned to any combat UAV positioned at any airport h should not exceed the ammunition capacity of that airport, so the ammunition stock of the airports should be enough to meet the attack task demand of their combat UAVs.

$$\sum_{k_1\in K} \sum_{k_2\in K_{attack}} \sum_{j\in T} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{h,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} \le N_h^{MUN},$$

$$u \in U,\ h \in H \quad (7)$$

Mission completion constraint: UAVs can take off from any airport h, execute tasks on the assigned targets j and complete the scenario mission by landing on any airport unless they are shot during their mission.

$$\sum_{k_1,k_2\in K} \sum_{h\in H} \sum_{j\in T} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{h,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} + \sum_{k_1,k_2\in K} \sum_{i\in T} \sum_{h\in H} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{i,h,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} \le 2,$$

$$u \in U \quad (8)$$

Task execution constraint: All tasks must be executed only once on each target j.

$$\sum_{u\in U} \sum_{k_1\in K} \sum_{i\in N} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{i,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} = 1,$$

$$j \in T, k_2 \in K \quad (9)$$

Attack task ammunition requirement constraint: Since all targets must be assigned with attack task, the number of combat UAV should be enough to meet the attack task requirements of the targets.

$$\sum_{u\in U} \sum_{k_1\in K} \sum_{k_2\in K_{attack}} \sum_{i\in N} \sum_{\phi_1,\phi_2\in \Phi} \sum_{b_1,b_2\in B} x_{i,j,\phi_1,\phi_2,b_1,b_2}^{u,k_1,k_2} \ge G_j^{MUN},$$

$$j \in T \quad (10)$$

Cycle breaking constraint: UAVs cannot execute the subsequent tasks on the targets, and each UAV must leave the target after mission execution.

$$x^{u,k_1,k_2}_{i,i,\phi_1,\phi_2,b_1,b_2} = 0, \quad u \in U,\ k_1,k_2 \in K,\ i \in N,\ \phi_1,\phi_2 \in \Phi,\ b \in B \qquad (11)$$

Angle consistency constraint: Each UAV arrives the assigned target with its departing angle ϕ as long as it survives during the previous mission.

$$\sum_{k_1\in K}\sum_{i\in N}\sum_{\phi_1\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_2,k_1}_{j,i,\phi_2,\phi_1,b_1,b_2} - \sum_{k_1\in K}\sum_{i\in N}\sum_{\phi_1\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \le 1, \quad u \in U, k_2 \in K, j \in T, \phi_2 \in \Phi \qquad (12)$$

Mission start constraint: Mission starting nodes of each assigned UAV must be one of the airports *h.*

$$\sum_{k_1,k_2\in K}\sum_{i,j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \le M \sum_{k_1,k_2\in K}\sum_{h\in H}\sum_{j\in T}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{h,j,\phi_1,\phi_2,b_1,b_2}, \quad u \in U \qquad (13)$$

Surveillance UAV constraint: Surveillance UAV cannot be assigned to attack tasks k_{attack}.

$$\sum_{k_1\in K}\sum_{k_2\in K_{attack}}\sum_{i\in N}\sum_{j\in T}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} = 0, \quad u \in U_{surv} \qquad (14)$$

Node visit constraints: Each UAV can depart from and arrive a node at most once.

$$\sum_{k_1,k_2\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \le 1, \quad \mathrm{u} \in U,\ \mathrm{i} \in N \qquad (15)$$

$$\sum_{k_1,k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \le 1, \quad \mathrm{u} \in U,\ \mathrm{j} \in N \qquad (16)$$

UAV task equality constraint: The task precedence of each UAV should be maintained unless the UAV gets destroyed.

$$\sum_{k_2\in K}\sum_{j\in T}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \le \sum_{k_2\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_2,k_1}_{i,j,\phi_1,\phi_2,b_1,b_2}, \quad u \in U, k_1 \in K, i \in N \qquad (17)$$

Airport usage constraints: Each UAV can depart from and arrive any airport maximum once.

$$\sum_{k_1,k_2\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{h,j,\phi_1,\phi_2,b_1,b_2} \le 1, \quad u \in U,\ h \in H \qquad (18)$$

$$\sum_{k_1,k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,h,\phi_1,\phi_2,b_1,b_2} \le 1, \quad u \in U,\ h \in H \qquad (19)$$

UAV take-off and landing constraints: Each UAV can only have one departure and landing airport.

$$\sum_{k_1,k_2\in K}\sum_{j\in T}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{h,j,\phi_1,\phi_2,b_1,b_2} \le 1, \quad u \in U,\ h \in H \qquad (20)$$

$$\sum_{k_1,k_2\in K}\sum_{j\in T}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{j,h,\phi_1,\phi_2,b_1,b_2} \le 1, \quad u \in U,\ h \in H \qquad (21)$$

Arrival and departure angle constraints: Angle direction of the leaving UAV must be equal to zero (ϕ = 1), while the angle of arriving UAV must be equal to π ((ϕ = 5) only for airports. This constraint facilitates the calculation of Dubin's maneuvers while UAVs are heading to or leaving from the airports.

$$\sum_{k_1,k_2\in K}\sum_{\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{h,j,\phi_1=1,\phi_2,b_1,b_2} \le M\left(1 - \sum_{k_1,k_2\in K}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{h,j,\phi_1,\phi_2,b_1,b_2}\right), \quad u \in U, j \in N, h \in H \qquad (22)$$

$$\sum_{k_1,k_2\in K}\sum_{\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,h,\phi_1,\phi_2=5,b_1,b_2} \le M\left(1 - \sum_{k_1,k_2\in K}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,h,\phi_1,\phi_2,b_1,b_2}\right), \quad u \in U, i \in N, h \in H \qquad (23)$$

Attack task arrival and departure angular constraints: Angular direction of u_{combat} must be equal to departing angle of the same UAV heading to another target.

$$\sum_{k_2\in K}\sum_{j\in N}\sum_{\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,2,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \le \sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,2}_{j,i,\phi_1,\phi_2,b_1,b_2}, \quad u \in U, i \in N, \phi_1 \in \Phi \qquad (24)$$

$$\sum_{k_2\in K}\sum_{j\in N}\sum_{\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,3,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \le \sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,3}_{j,i,\phi_1,\phi_2,b_1,b_2},$$
$$u \in U, i \in N, \phi_1 \in \Phi \quad (25)$$

Airport to airport travel prohibition constraint: The direct flight between any two airports is not allowed.

$$\sum_{u\in U}\sum_{k_1,k_2\in K}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{h_1,h_2,\phi_1,\phi_2,b_1,b_2} = 0,$$
$$h_1, h_2 \in H \quad (26)$$

Angular direction consistency constraint: The angular direction must be conserved between subsequent tasks of each UAV as long as it survives.

$$\sum_{k_1,k_2\in K}\sum_{i\in T}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \ge \sum_{k_1,k_2\in K}\sum_{i\in T}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_2\in B} x^{u,k_1,k_2}_{j,i,\phi_1,\phi_2,b_2,b_1},$$
$$u \in U, j \in T, b_1 \in B \quad (27)$$

Dynamic subtour elimination constraint: This constraint eliminates the subtours between the nodes.

$$\sum_{k_1,k_2\in K}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} + \sum_{k_1,k_2\in K}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{j,i,\phi_1,\phi_2,b_1,b_2} \le 1,$$
$$u \in U, (i,j) \in N \quad (28)$$

Double attack constraints: There is at least one attack task on each target (here denoted with k=2), while in the case of double attack, k=3 is activated.

$$\sum_{u\in U}\sum_{k_1\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,2}_{i,j,\phi_1,\phi_2,b_1,b_2} = 1,$$
$$j \in N \quad (29)$$

$$\sum_{u\in U}\sum_{k_1\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,3}_{i,j,\phi_1,\phi_2,b_1,b_2} \le 1,$$
$$j \in N \quad (30)$$

$$\sum_{u\in U}\sum_{k_1\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,K_{attack}}_{i,j,\phi_1,\phi_2,b_1,b_2} = N_j^M - 2,$$
$$j \in N \quad (31)$$

Chance constraints: The probability of destruction for a surveillance or a verification mission is equal to 0, whereas it is p during an attack task. Here, we assume that even if the chance of being destroyed for any UAV during a mission is zero, the random chance of failure or of being shot by the undefined/undetected enemy air defense system might still exist, which is the truth of contingency military operations. Therefore, with $p = 0$, we assume that there might be also chance of UAV loss in the military theater which can be called as operational casualty, so our formulation is able to give a solution for the worst-case scenario by optimizing the paths and task assignments considering the operative losses. However, if we are to make UAV operations in totally risk-free environment with failure free conditions, then our chance constraint can easily be transformed into absolute zero probability case by simply excluding +1 in Equation (32). Moreover, we can also modify the constraint equations for total destruction of attack UAVs by assigning p = 1. This will generate a special case for military decision makers, when they plan to use smart ammunition with UAV characteristics, or suicidal UAVs over enemy targets with high values. In this case, all suicidal UAVs (smart missiles) that execute attack tasks will be destroyed over their assigned targets.

$$\sum_{k_1\in K_{attack}}\sum_{k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{j,i,\phi_1,\phi_2,b_1,b_2} + 1 \ge (1 - prob)\sum_{k_1\in K}\sum_{k_2\in K_{attack}}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2},$$
$$u \in U, j \in T \quad (32)$$

$$\sum_{k_1\in K_{attack}}\sum_{k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{j,i,\phi_1,\phi_2,b_1,b_2} \le (1 - prob)\sum_{k_1\in K}\sum_{k_2\in K_{attack}}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2},$$
$$u \in U, j \in T \quad (33)$$

$$\sum_{k_1\in K_{Surv}}\sum_{k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{j,i,\phi_1,\phi_2,b_1,b_2} = \sum_{k_1\in K}\sum_{k_2\in K_{Surv}}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2}$$
$$u \in U, j \in T \quad (34)$$

Time constraints:

The proposed model also considers timings and scheduling. The related constraints are explained between Equations (35) and (42).

Starting time constraint: Starting time of a non-existing mission must be zero.

$$M\sum_{k_1\in K}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B} x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} \ge tstart^{u,k_2}_{i,j},$$
$$u \in U, k_2 \in K, i \in T, j \in T \quad (35)$$

Time segment constraint: The start time of the new mission on any target is the ending time of the previous mission of the assigned UAV to the task. Thus, ending time for a mission is calculated by the summation of starting time of the mission and the distance traveled between nodes. Here, $Ld^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2}$ is the cumulative distance-the sum of the Dubin's distances traveled between nodes and the surveillance flight around the target-if the task is surveillance. Otherwise, it only represents the Dubin's distance between the nodes, since the attack task is executed directly to the center of the target.

$$\mathrm{tstart}^{u,k_2}_{i,j} + \sum_{u\in U}\sum_{k_1\in K}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}\frac{x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2}Ld^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2}}{Velocity_u} = \mathrm{tend}^{u,k_2}_{i,j}, \quad u\in U, k_2\in K, i\in T, j\in T \tag{36}$$

Task sequence constraints: These constraints state that the tasks performed on target (j) must be in the following order: 1-(2-3)-4 (surveillance - attack - verification).

$$M\left(\sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,1}_{j,i,\phi_1,\phi_2,b_1,b_2} - \sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,2}_{j,i,\phi_1,\phi_2,b_1,b_2}\right) \geq \sum_{u\in U}\sum_{j\in N}tend^{u,1}_{j,i} - \sum_{u\in U}\sum_{j\in N}tstart^{u,2}_{j,i}, \quad i\in T \tag{37}$$

$$\mathrm{M}\left(\sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,3}_{j,i,\phi_1,\phi_2,b_1,b_2} - \sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,1}_{j,i,\phi_1,\phi_2,b_1,b_2}\right) \geq \sum_{u\in U}\sum_{j\in N}tend^{u,1}_{j,i} - \sum_{u\in U}\sum_{j\in N}tstart^{u,3}_{j,i}, \quad i\in T \tag{38}$$

$$\mathrm{M}\left(\sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,3}_{j,i,\phi_1,\phi_2,b_1,b_2} - \sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,2}_{j,i,\phi_1,\phi_2,b_1,b_2}\right) \geq \sum_{u\in U}\sum_{j\in N}tend^{u,2}_{j,i} - \sum_{u\in U}\sum_{j\in N}tstart^{u,3}_{j,i}, \quad i\in T \tag{39}$$

$$M\left(\sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,3}_{j,i,\phi_1,\phi_2,b_1,b_2} - \sum_{u\in U}\sum_{k_1\in K}\sum_{j\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,4}_{j,i,\phi_1,\phi_2,b_1,b_2}\right) \geq \sum_{u\in U}\sum_{j\in N}tend^{u,3}_{j,i} - \sum_{u\in U}\sum_{j\in N}tstart^{u,4}_{j,i}, \quad i\in T \tag{40}$$

UAV timing constraints: Previous mission ending time of any UAV must be equal to the starting time of the next mission for the same UAV, as long as the UAV is not destroyed during the attack task.

$$M\left(\sum_{k_1,k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} - \sum_{k_1,k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,k_2}_{j,i,\phi_1,\phi_2,b_1,b_2}\right) \geq \sum_{k_2\in K}\sum_{i\in N}tend^{u,k_2}_{i,j} - \sum_{k_2\in K}\sum_{i\in N}tstart^{u,k_2}_{j,i}, \quad u\in U, j\in T \tag{41}$$

$$-\mathrm{M}\left(\sum_{k_1,k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,k_2}_{i,j,\phi_1,\phi_2,b_1,b_2} - \sum_{k_1,k_2\in K}\sum_{i\in N}\sum_{\phi_1,\phi_2\in\Phi}\sum_{b_1,b_2\in B}x^{u,k_1,k_2}_{j,i,\phi_1,\phi_2,b_1,b_2}\right) \leq \sum_{k_2\in K}\sum_{i\in N}tend^{u,k_2}_{i,j} - \sum_{k_2\in K}\sum_{i\in N}tstart^{u,k_2}_{j,i}, \quad u\in U, j\in T \tag{42}$$

3. Results and Discussions

In this section, we present the results of solver performing on the developed mixed integer linear problem (MILP). The proposed model is able to successfully solve the test cases with combinations on the Intel(R) Core(TM) i5-6300U CPU @ 2.40GHz and 2.50 GHz processor, with 8 GB RAM. The model is coded in Matlab and solved by Gurobi global optimal solver for mixed-integers programs.

3.1 Instance generation

In this study, the military theater is generated similar to the operative area assigned to front bases to execute dispatched UAV operations in tactical level. In order to test the performance of the proposed optimization model and the trajectories of the UAVs, 14 cases were developed. The main structure of all cases assumes four or five heterogeneous UAVs, three targets with three or four tasks (second attack might be optional) and three airports, as well as four no-fly zones placed at known locations among targets and airports. The assumed units for distance, velocity and time are km, $km/hours$, and $hours$, respectively. The full dataset and the results of the test are attached in Tables 10-15 in Appendix D.

- **Four UAVs, single attack, probability of destruction equal to zero:** First UAV (red path) starts its mission from the airport 6, performs the classification on target 3, continues with the classification on the target 2 by avoiding the no-fly zone, again by missing the another no-fly zone it enters into the safety zone around target 1 and executes verification, then finishes operative mission by landing on airport 5. Second UAV (blue path) starts its mission from the airport 6, performs the verification task on the target 3 and finishes operative mission by landing airport 6. Third UAV (green path) starts its mission from the airport 5, performs the classification on target 1, continues with the attack on the target 3 by missing the no-fly zone, perform the verification on the target 2, then finishes the mission by returning to the airport 4. Fourth UAV (pink path) starts its mission from the airport 5, performs the attack task on target 1, continues with the attack on the target 2 by missing the no-fly zone, and it is destroyed by the air defense system on target 2. It is important to remark that even if the last mission is completed at 1.4872577, UAV 3 which executes the last task in this case land on airport 5 at 1.6817, which is the scenario completion time for the case. The results are shown in Figure 5 with details tabulated in Tables 2 and 3.

Table 2. Target, tasks, assigned UAVs, starting and ending times for the mission assignments of four UAVs (single attack, p=0)

Target	Tasks	Assigned UAV	$tstart$	$tend$
1	Classification	3	0.50867211	0.65067211
	Attack 1	N/A		
	Attack 2	4	0.69844790	0.82244790
	Verification	1	0.82244790	1.31144049
2	Classification	1	0.23442885	0.82244790
	Attack 1	N/A		
	Attack 2	4	0.82244790	1.11537123
	Verification	3	1.11537122	1.48725656
3	Classification	1	0	0.23442885
	Attack 1	N/A		
	Attack 2	3	0.65067211	1.11537122
	Verification	2	1.18267974	1.41710859

Table 3. Mission assignment details of the four UAVs (single attack, p=0)

UAV	Target sequence	Takeoff airport	Landing airport	Travel duration	Travel distance	Used munition
1	1-1-4	6	5	1.311440	188.6955900	0
2	4	6	6	1.417109	59.8847806	0
3	1-3-4	5	4	1.487257	234.6096600	1
4	3-3	5	N/A	1.115371	83.3846669	2
Total Distance Traveled:		**566.5747 km**				
Total Mission Time:		**1.6817 $hours$**				

- **Four UAVs, double attack, total risk-free environment:** This is a special case which assumes that airfield is totally risk free and there is no chance of losing UAV due to technical or electronic warfare issues as well. So, in this case we expect that all UAVs will survive and will be able to land on any airport after they execute their assigned tasks. The details about the total survive case are explained in chance constrains. The results are tabulated in Tables 4 and 5.

Table 4. Target, tasks, assigned UAVs, starting and ending times for the mission assignments of four UAVs (double attack, p=N/A)

Target	Tasks	Assigned UAV	$tstart$	$tend$
1	Classification	1	0.82244788	1.31144047
	Attack 1	4	1.31144047	1.42944047
	Attack 2	3	1.42944047	1.7223638
	Verification	2	1.7223638	1.94538633
2	Classification	1	0.23442885	0.82244790
	Attack 1	3	1.05650064	1.42944047
	Attack 2	4	1.42944047	1.7403638
	Verification	2	1.94538212	2.4510778
3	Classification	1	0	0.23442875
	Attack 1	3	0.91934635	1.05650064
	Attack 2	4	1.7403638	2.08760451
	Verification	2	2.45107359	3.02521325

Table 5. Mission assignment details of the four UAVs (double attack, p=N/A)

UAV	Target sequence	Takeoff airport	Landing airport	Travel duration	Travel distance	Used munition
1	1-1-1	6	5	1.311440	188.695590	0
2	4-4-4	5	6	3.025213	188.096287	0
3	2-2-3	6	5	1.722364	190.543311	3
4	2-3-3	5	6	2.087605	183.003470	3
Total Distance Traveled:		**750.3387 km**				
Total Mission Time:		**3.4287 $hours$**				

- **Five UAVs, third target double attack, others single attack, 20% probability of destruction:** Here, we test the scenario with five heterogeneous UAVs three of which have combat capability. The targets also require different numbers of attack tasks, so the combinations to search for global solution increases significantly. The results are tabulated in Tables 6 and 7.
- **Five UAVs, single attack, total destruction:** This is another special case which can be used in highly risky environment where the air defense systems are in actively in use, or against high priority ground targets where the loss of UAV is less important than the failure of an attack mission. In this case, decision maker can assign smart weapons to the targets as suicidal UAVs from a ground base or mother aircraft, which can also do surveillance by their image transmission equipment, and then execute single attack task to the target center location. The results are tabulated in Tables 8 and 9.

Table 6. Target, tasks, assigned UAVs, starting and ending times for the mission assignments of four UAVs (mix attack, p=0.2)

Target	Tasks	Assigned UAV	$tstart$	$tend$
1	Classification	1	0	0.22302241
	Attack 1	N/A		
	Attack 2	5	1.11976038	1.24420482
	Verification	3	1.24420482	1.6351037
2	Classification	1	0.22302156	0.74121721
	Attack 1	N/A		
	Attack 2	3	0.74121721	1.24420482
	Verification	5	1.24420482	1.58484267
3	Classification	2	0	0.28230403
	Attack 1	4	0.28230403	0.43310466
	Attack 2	3	0.45951684	0.7412172
	Verification	1	0.7412172	1.28869039

Table 7. Mission assignment details of the four UAVs (mix attack, p=0.2)

UAV	Target sequence	Takeoff airport	Landing airport	Travel duration	Travel distance	Used munition
1	1-1-4	6	5	1.577774	188.69559	0
2	4	6	6	1.616949	60.2695885	0
3	1-1-3	4	N/A	1.334645	200.196751	1
4	3	4	N/A	0.500762	38.0657021	1
5	3	5	N/A	0.857756	22.4	1
Total Distance Traveled:	$509.6276\ km$					
Total Mission Time:	$1.9373\ hours$					

4. Conclusions

The study presents a model to optimize UAV missions that involve surveillance, attack, and verification of the targets in a risky mission theater. The optimization problem extends the well-known Traveling Salesman Problem (TSP) with time constraints and Dubin's turns. The proposed model is especially designed for front bases which execute tactical UAV operations in a clustered and contingent military theater with risky and restricted airfield. This framework integrates wide range of constraints, including time synchronization, ammunition limits, and navigation around no-fly zones, as well as time constraints. These constraints ensure that solutions are both operationally feasible and optimal for mission success. By means of a detailed use-case analysis, the framework effectively delineates optimized UAV trajectories, illustrating task allocation across UAVs and demonstrating adherence to diverse operational constraints. The demonstration highlights the effectiveness of the model in a realistic setting with multiple targets, airports, and no-fly zones. This model is particularly valuable for military and surveillance operations, where coordination, time efficiency, and resource management are critical. Its ability to handle multiple UAV roles and prioritize tasks adds to its practical applicability. Overall, this study presents a comprehensive framework for addressing the multifaceted challenges of UAV mission planning. Future work may focus on enhancing computational efficiency and integrating real-time, adaptive mechanisms to respond to evolving operational scenarios. Furthermore, employing dynamic programming for real-time threat-aware fleet-to-target assignment and embedding collaborative paradigms—such as auction-based schemes, machine learning, and Q-learning—for fully autonomous UAV systems represents a promising direction for future research, particularly within the Cooperative Multi-UAV Task Assignment Problem (CMTAP) domain.

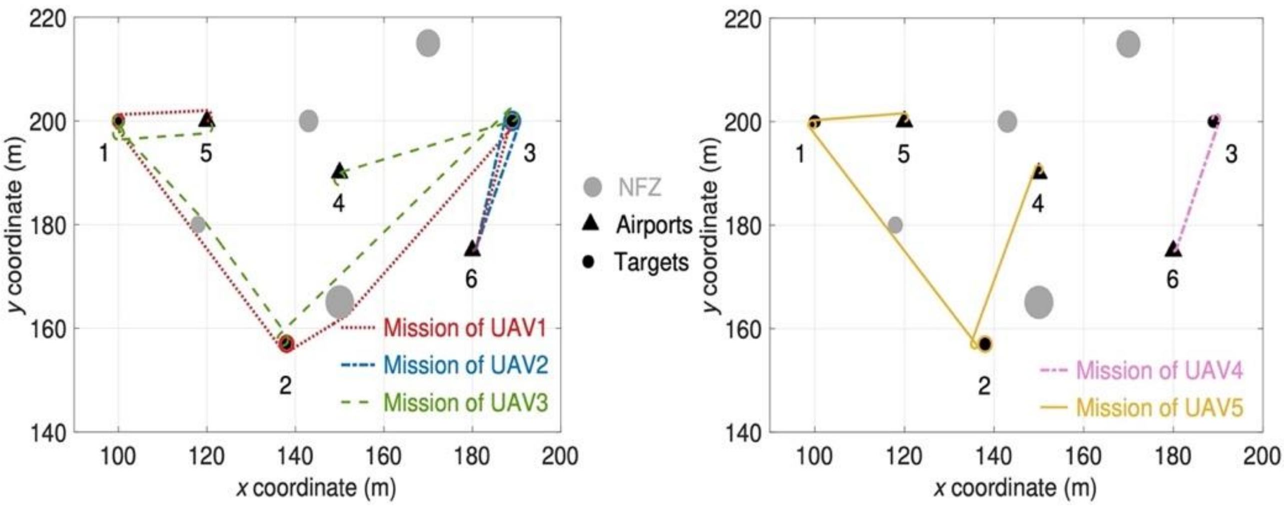

Figure 5. Mission assignments of four UAVs (single attack, p = 0)

Table 8. Target, tasks, assigned UAVs, starting and ending times for the mission assignments of four UAVs (single attack, p=1)

Target	Tasks	Assigned UAV	*tstart*	*tend*
1	Classification	3	0.28928575	0.73331167
	Attack 1	N/A		
	Attack 2	5	0.73331167	0.85775612
	Verification	1	1.08878092	1.57777351
2	Classification	3	0	0.28928575
	Attack 1	N/A		
	Attack 2	4	0.28928575	0.50076187
	Verification	1	0.50076187	1.08878092
3	Classification	1	0.26633302	0.50076187
	Attack 1	N/A		
	Attack 2	3	0.73331167	1.334645
	Verification	2	1.334645	1.61694904

Table 9. Mission assignment details of the four UAVs (single attack, p=1)

UAV	Target sequence	Takeoff airport	Landing airport	Travel duration	Travel distance	Used munition
1	1-4-4	6	5	1.577774	188.69559	0
2	4	6	6	1.616949	60.2695885	0
3	1-1-3	4	N/A	1.334645	200.196751	1
4	3	4	N/A	0.500762	38.0657021	1
5	3	5	N/A	0.857756	22.4	1
Total Distance Traveled:	509.6276 *km*					
Total Mission Time:	1.9373 *hours*					

Appendix A. Tangent equations for Dubin's turns

Dubin's turns in this study considers the most generic case of two target circles with radii r_1 and r_2, with starting angular positions ϕ_1 and ϕ_2, and with the angular directions b_1 and b_2. In this most generic case, threat radius of the target and the turn radius of the UAV are different, and in the worst-case scenario, the turn radius of fixed-wing UAV can be lower in the order of magnitude compared to the threat radius of the target. Thus, it is needed to create auxiliary circles to represent the turn circles of these UAVs, totaling in four different options, all of which are analyzed in this study. The equation of the tangent to both circles can be represented in the basic form as stated in Equation (43).

$$ax + by + c = 0 \quad (43)$$

In order for this line to be tangent, it needs to touch both circles with the following analytical Equations (44) and (45).

$$(x - x_{c1})^2 + (y - y_{c1})^2 = r_1^2 \quad (44)$$

$$(x - x_{c2})^2 + (y - y_{c2})^2 = r_2^2 \quad (45)$$

Combining Equations (44) and (45), we obtain a system of two non-linear equations given in Equations (46) and (47).

$$| ax_{c1} + by_{c1} + c | = r_{1\sqrt{a^2+b^2}} \quad (46)$$

$$| ax_{c2} + by_{c2} + c | = r_{2\sqrt{a^2+b^2}} \quad (47)$$

As there is one additional degree of freedom, we can set the coefficient b to be equal to any non-zero value (in this case set to 1 for easier normalization). Special cases when this assumption is not valid will be covered in Appendix B. By solving the set of equations, we can obtain four different tangents to cover for all possible combinations of angular directions, which are coded in this study for path optimization.

Appendix B. Edge cases for Dubin's turns

If the two circles are touching each other, we should consider the edge case in which we cannot assume that b=1. However, we can explicitly calculate the tangent equations using the image in Figure 9.

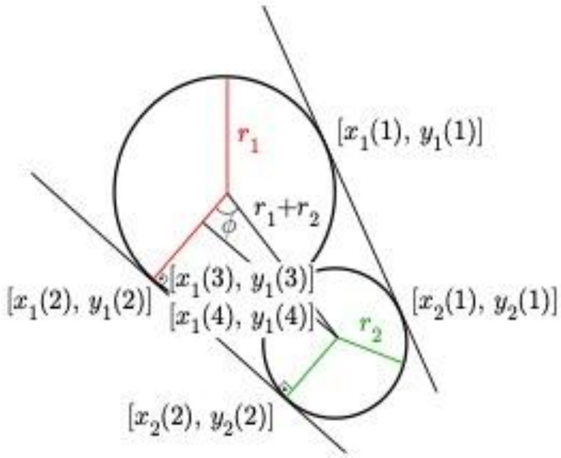

Figure 6. Edge case drawing.

We can now easily obtain the equations of the starting and ending points for each of the tangents. In this case, there are three different tangents, as there is only one tangent at the point where two circles meet, which is represented by $[x_1(3), y_1(3)]$ and $[x_1(4), y_1(4)]$. The edge case calculations are expressed in Equations (48)-(55).

$$x_1(1) = x_{c1} + r_1 cos(\phi) \quad (48)$$

$$y_1(1) = y_{c1} + r_1 sin(\phi) \quad (49)$$

$$x_1(2) = x_{c1} + r_1 cos(2\pi - \phi) \quad (50)$$

$$y_1(2) = y_{c1} + r_1 sin(2\pi - \phi) \quad (51)$$

$$x_1(3) = x_{c1} + (x_{c2} - x_{c1}) \frac{r_1}{(r_1 + r_2)} \quad (52)$$

$$y_1(3) = y_{c1} + (y_{c2} - y_{c1}) \frac{r_1}{(r_1 + r_2)} \quad (53)$$

$$x_1(4) = x_{c1} + (x_{c2} - x_{c1}) \frac{r_1}{(r_1 + r_2)} \quad (54)$$

$$y_1(4) = y_{c1} + (y_{c2} - y_{c1}) \frac{r_1}{(r_1 + r_2)} \quad (55)$$

Appendix C. Angular directions for Dubin's paths

After tangents calculations, it is important to calculate the circular path each UAV covers until it leaves the circle, depending on the angular direction of the UAV. It is also important to mention that, in general, there is a difference between the radius of the target threat and the turning radius of the UAV. Thus it is also important to check what is the smaller Dubin's distances between the four options, first where the Dubin's distances is calculated between only the two targets, second when the second circle is the turning radius of the UAV at the second target, third when the first circle is the turning radius of the UAV and the fourth when both of the circles are the turning radii of the UAV. This logic is implemented in Figure 10, where the green straight line represents the path the UAV should take for the given starting and ending angular direction. It is also important to mention that the primary circles in the second, third and fourth cases become the no-fly zones (NFZ).

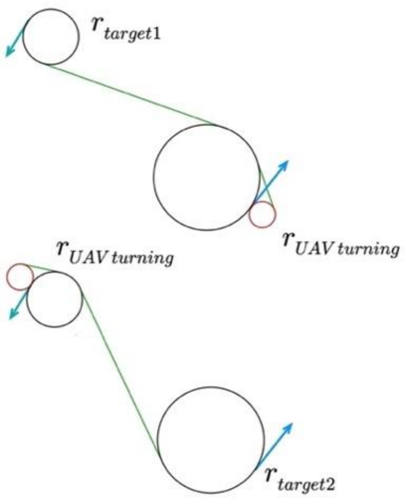

Figure 7 Third and fourth Dubin's path cases.

Appendix D. Data set for the scenario

The data for the scenarios are given in Tables 10-14.

Table 10. UAV data for cases with four UAVs

UAV	Type	Velocity (km/hr)	Ammo capacity	Turn rate (km)	Detection Radius (km)	Airport serial	$Cost_1$ ($/hr)	$Cost_2$ ($/km)
1	0	120	0	1	2	1	1	0.01
2	0	120	0	1	2	1	1	0.015
3	1	200	1	1.2	2	2	1	0.005
4	1	200	1	1.2	2	2	1	0.02

Table 11. UAV data for cases with five UAVs

UAV	Type	Velocity (km/hr)	Ammo capacity	Turn rate (km)	Detection Radius (km)	Airport serial	$Cost_1$ ($/hr)	$Cost_2$ ($/km)
1	0	120	0	1	2	1	1	0.01
2	0	120	0	1	2	1	1	0.015
3	1	200	1	1.2	2	2	1	0.005
4	1	200	1	1.2	2	2	1	0.02
5	1	180	2	0.8	2	2	1	0.02

Table 12. Airport data

Airport	Ammo capacity	x coordinate	y coordinate
1	1	150	190
2	2	120	200
3	1	180	175

Table 13. Target data

Target	Priority	Threat Radius (km)	Threat rate	x coordinate	y coordinate	Number of tasks
1	1	1.2	0.7	100	200	3-4
2	1	1.5	0.7	138	157	3-4
3	1	1.7	0.7	189	200	3-4

Table 14. Test results of the cases

Number of cases	Number of UAVs	Number of Tasks on targets	p *value*	Tota Mission timel	Total Traveled distance
1	4	3	0	1.6817	566.5747
2	4	3	0.2	1.6817	566.5747
3	4	3	0.5	1.8576	523.0246
4	4	4	0	3.2898	718.6281
5	4	4	0.2	3.2898	688.6883
6	4	(4-3-3)	0	1.8309	627.8139
7	4	(4-4-3)	0.2	3.111	620.7800
8	4	3	N/A	1.8321	566.7975
9	4	4	N/A	3.4287	750.3387
10	5	3	1	1.9373	509.6276
11	5	3	0	1.6140	563.8382
12	5	3	0.2	1.6140	563.8382
13	5	(3-3-4)	0.2	1.8879	606.7255
14	5	3	0.5	1.6140	534.6410

Declaration of Ethical Standards

The authors declare that they comply with all ethical standards.

Credit Authorship Contribution Statement

Author-1: Conceptualization, investigation, methodology and software, visualization and writing – original draft.
Author-2: Review and editing, supervision
Author-3: Supervision

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

All data generated or analyzed during this study are included in this published article. Datasets are available on request. The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

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