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Why Do Some Civilisations Collapse? An Evolutionary Game Theory Model of Societal Decline

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Bazı Uygarlıklar Neden Yok Olur? Toplumsal Çöküşün Evrimsel Oyun Teorik Modeli

Abstract

The collapse of civilisations is a recurring theme in human history. This paper formalises the concept of societal decline through the lens of evolutionary game theory (EGT) and replicator dynamics, illustrating how interactions among competing social groups, such as elites, commoners, and external forces, can produce feedback loops that lead to collapse. Historical case studies are quantitatively analysed to corroborate the theoretical framework. Graphics demonstrate how resource depletion, inequality, and institutional fragility create conditions conducive to collapse. Findings indicate that societies with adaptive institutions and sustainable cooperation tend to ensure their survival, while those that ignore harmful dynamics risk collapse.

Keywords : Evolutionary Game Theory, Replicator Dynamics, Societal Collapse, Institutional Economics, Resource Depletion.

JEL Classification Codes : C73, D74, N10, Q54, P16.

Öz

Uygarlıkların çöküşü, insanlık tarihinin tekrar eden bir temasıdır. Bu makale, toplumsal çöküş kavramını evrimsel oyun teorisi (EOT) ve çoğaltıcı dinamikler çerçevesinde biçimlendirerek, elitler, sıradan halk ve dış güçler gibi rekabet hâlindeki sosyal gruplar arasındaki etkileşimlerin çöküşe yol açan geri besleme döngüleri oluşturabileceğini göstermektedir. Çalışmada tarihî vaka çalışmaları teorik çerçevenin geçerliliğini desteklemek amacıyla nicel olarak analiz edilmiştir. Grafikler, kaynakların tükenmesi, eşitsizlik ve kurumsal kırılganlık gibi unsurların çöküşe elverişli koşullar yarattığını ortaya koymaktadır. Bulgular, uyum sağlayabilen kurumlara ve sürdürülebilir iş birliğine sahip toplumların varlıklarını sürdürme eğiliminde olduğunu, zararlı dinamikleri görmezden gelenlerin ise çöküş riski taşıdığını göstermektedir.

Anahtar Sözcükler : Evrimsel Oyun Teorisi, Çoğaltıcı Dinamikler, Toplumsal Çöküş, Kurumsal İktisat, Kaynak Tükenmesi.

1. Introduction

Throughout history, civilisations have risen to great heights, only to experience decline and eventual collapse. The fall of once-mighty societies such as the Roman Empire, the Mayan civilisation, and the Soviet Union presents a critical question: Why do some civilisations collapse while others endure? The complexity of societal decline has long been examined through historical, sociological, and economic lenses. Yet, recent advancements in evolutionary game theory (EGT) offer a new perspective on the mechanisms underlying collapse. By applying EGT, we can model interactions among elites, commoners, and external threats to understand how strategic behaviours evolve and how societies can be led toward stability, fragility, or collapse.

Game theory has traditionally focused on the analysis of rational decision-making within competitive contexts (von Neumann & Morgenstern, 1944: 46-47). However, evolutionary extensions of this framework introduce a dynamic perspective in which strategies adapt and evolve based on their relative success within a given population (Maynard-Smith & Price, 1973: 16-17). This evolutionary approach is particularly valuable for examining long-term societal dynamics, as it provides insights into how both individual and institutional behaviours shift in response to changing incentives. Utilising the replicator dynamics framework, we can effectively model the interactions of critical societal factors such as elite exploitation dynamics, commoner cooperation or defection, and external threats, thereby elucidating the pathways that shape a civilisation's trajectory.

Historical case studies illustrate converging decline mechanisms across various civilisations. The Roman Empire exemplifies this, marked by intensifying elite extraction, diminishing civic engagement, and mounting external pressures from barbarian incursions (Tainter, 1988: 49-69). These factors, when evaluated through an EGT lens, resonate with theoretical predications of societal instability. Similarly, the Mayan civilisation demonstrated a trajectory of resource overexploitation that culminated in environmental degradation, sociopolitical fragmentation, and heightened susceptibility to external shocks (Diamond, 2011: 157-178). In both instances, the inability to sustain a stable interaction model among elite dominance, public collaboration, and effective external deterrence precipitated systemic collapse.

This study develops a formal evolutionary game-theoretical model to analyse the dynamics of societal decline. By defining strategies for different societal groups, elites, commoners, and external threats and employing replicator dynamics, we explore how civilisations transition between equilibrium states. The results provide insight into whether decline is an inevitable feature of complex societies or whether institutional mechanisms can sustain long-term stability. Furthermore, we discuss policy implications, drawing from historical lessons to suggest strategies for modern societies to avoid systemic breakdown.

The subsequent sections of this paper are organised as follows. Section 2 delineates the Replicator Dynamics Framework, providing a comprehensive overview of the mathematical model that delineates societal evolution. Section 3 incorporates case studies, with an in-depth analysis of the fall of the Roman Empire, Maya civilisation, and the Soviet Union. Section 4 explores policy implications and contemporary lessons, the broader implications of our findings, emphasising the conditions that contribute to societal resilience or eventual collapse. Lastly, Section 5 concludes with recommendations for future research trajectories at the nexus of evolutionary dynamics, institutional stability, and historical economic systems.

2. Evolutionary Game Theory and the Dynamics of Collapse

EGT provides a framework for analysing strategic interactions and their evolution in societal contexts. Unlike classical game theory, which assumes fully rational actors, EGT emphasises adaptive strategies in which individuals adjust their behaviour based on relative success within the population (Maynard-Smith & Price, 1973: 16-17). This perspective is valuable for studying societal collapse, as it reveals the long-term impacts of elite extraction strategies, commoner cooperation versus defection, and external threats to societal resilience.

Societies can achieve stable equilibria where cooperation fosters institutional strength, or they may descend into instability if exploitation and defection go unchecked. When instability reaches a critical threshold at which elite overextraction, commoner defection, and external pressures reinforce each other, collapse becomes inevitable (Tainter, 1988: 49-69).

In this section, we develop an EGT model to analyse societal collapse, focusing on strategies of elites, commoners, and external adversaries. We assess the model's equilibrium states and explore factors influencing civilisations' paths towards flourishing, decline, or collapse. This framework helps us understand historical instances of societal failure and their relevance to current governance and institutional resilience.

2.1. Background: Main-Theoretic Model

The collapse of civilisations has been a topic of extensive inquiry in history, sociology, and economics. Numerous theories have attempted to explain why once-thriving societies deteriorate and ultimately fail. Traditional explanations include environmental degradation (Diamond, 2011: 15-18), economic mismanagement (Tainter, 1988: 69), institutional decay, and external invasions (Acemoglu & Robinson, 2012: 21-32). However, these perspectives often lack a formal mathematical framework to model the dynamic interactions between key societal agents, namely elites, commoners, and external threats. EGT provides a rigorous approach to understanding these interactions by offering a framework that models adaptive decision-making, strategic evolution, and the long-term stability or collapse of societies.

Replicator dynamics, a foundational mechanism in EGT, offers a rigorous mathematical framework for analysing the temporal evolution of strategies within a population (Weibull, 1997: 69-71). This model articulates how successful strategies proliferate, while unsuccessful ones wane, with the dynamics governed by relative payoffs rather than any external optimisation processes. When applied to the study of societal collapse, replicator dynamics facilitates the modelling of interactions among elites, commoners, and external threats, elucidating how the strategic behaviours of these groups evolve over time and potentially culminate in systemic failure.

In our framework, three primary agents interact:

- Elites (E): Dominant elites possess significant control over resources and institutional frameworks. Their strategic decision-making revolves around two primary approaches: either to extract value through exploitative practices or to invest in sustainable initiatives for long-term viability.
- Commoners (C): In socio-economic structures, commoners serve as the labour force, actively contributing to the economy's productivity. Their engagement can manifest in two distinct behaviours: cooperation, characterised by adherence to regulatory frameworks and tax obligations, or defection(rebellion), which may involve tax evasion or insurrection. This dichotomy reflects underlying tensions within the system that can impact overall economic stability and governance.
- External Threats (X): These encompass geopolitical risks, economic disruptions, or environmental challenges that influence the dynamics between elite groups and the general populace. Their interactions can lead to significant shifts in policy, stability, and socio-economic conditions.

Each group evolves its strategic approach in response to the feedback derived from interactions, as described by the replicator equation. This mathematical model quantifies the dynamics of adaptive behaviour, enabling predictions about a system's stability, its capacity for successful adaptation, and its potential for collapse under external pressures. This replicator will also be used in the first case study (the Roman case).

2.1.1. Replicator Dynamics

To formally define the dynamics, let:

- x be the proportion of elites engaging in extraction (rather than sustainability)
- y be the proportion of commoners engaging in defection (rather than cooperation)
- z be the proportion of external threats escalating instability.

The prevalence of extraction, defection, and external pressures influences each group's payoff function. The payoff functions π for each group are:

$$\pi_E = a(1 - x) - bx - cy \tag{1}$$

$$\pi_C = d(1 - y) - ex - fz \tag{2}$$

$$\pi_X = g(1 - z) - hx \tag{3}$$

where

- a, b, c, d, e, f, g, h represent societal parameters (e.g., institutional resilience, wealth inequality, military spending)
- π_E, π_C, π_X denote the payoff functions for elites, commoners, and external threats, respectively.
- The terms capture the effects of strategic choices (extraction, defection, conflict) on societal outcomes.

The influence of societal parameters is crucial for comprehending the underlying equations.

a: Institutional Resilience - Represents how much elites benefit from maintaining a low-extraction equilibrium. Higher values of a incentivise sustainable governance.
b: Elite Exploitation Rate - Determines the cost imposed on elites when they extract too much. A higher b means more institutional decay and instability.
c: Commoner Resistance to Elite Extraction - Captures how much commoners resist when elites extract excessively. Higher values of c indicate stronger civil unrest or revolts.
d: Social Cooperation Strength - Reflects the benefits of social cohesion for commoners. A larger d means a more cooperative society with lower defection.
e: Wealth Inequality & Tax Burden - Represents the economic pressure that commoners face. When e is high, taxation or wealth disparities drive defection.
f: Vulnerability to External Threats - Measures how external pressures weaken commoners. A larger f means that commoners suffer greater losses from external instability.
g: Military Deterrence - Indicates how well external threats are contained. When g is large, external threats are less effective, stabilising society.
h: Elite Investment in Military/Conflict - Determines how much elites contribute to either suppressing internal unrest or managing external threats. If h is high, elites prioritise military control over social stability.

The interplay of these parameters significantly impacts the dynamics of societal stability, deterioration, or eventual collapse by influencing the interactions among elites, the general populace, and external pressures. The replicator equations governing the evolution of each strategy over time are:

$$\frac{dx}{dt} = x(\pi_E - \overline{\pi_E}) \tag{4}$$

$$\frac{dy}{dt} = y(\pi_C - \overline{\pi_C}) \tag{5}$$

$$\frac{dz}{dt} = z(\pi_X - \overline{\pi_X}) \tag{6}$$

where

$$\overline{\pi_E} = x\pi_E + (1-x)\pi_E^{sus} \tag{7}$$

$$\overline{\pi_C} = y\pi_C + (1-y)\pi_C^{sus} \tag{8}$$

$$\overline{\pi_X} = z\pi_X + (1-z)\pi_X^{sus} \tag{9}$$

$\overline{\pi_E}$, $\overline{\pi_C}$ and $\overline{\pi_X}$ are the average payoffs in the population and

$$\pi_E^{sus} = a(1-x) \tag{10}$$

$$\pi_C^{sus} = d(1-y) \tag{11}$$

$$\pi_X^{sus} = g(1-z) \tag{12}$$

When elites engage in low-exploitation behaviour, sustainability is achieved, eliminating costs b and c. Likewise, if the commoners remain cooperative and do not rebel (eliminates e and f) and if there is a passive external threat (eliminates h), situations can be expressed through equations $10, 11$, and 12, respectively. The equations outline the dynamic relationships governing the evolution of elite exploitation, commoner defection, and the impact of external threats on social stability over time.

2.1.2. Stability Analysis and Equilibria

The core idea behind replicator dynamics is quite simple: when a strategy yields a payoff above the population average (meaning it performs better than others), the proportion of individuals using that strategy will increase over time. Conversely, if a strategy's payoff is below the average, its proportion will decline. Mathematically, this dynamic is represented through time derivatives, which quantify the direction and rate of change in the population share of each strategy.

To analyse long-term societal trajectories, we examine the system's equilibrium points. Setting $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0$, $\frac{dz}{dt} = 0$ allows us to find fixed points where societal strategies stabilize. When a replicator equation equals zero, the population share of that strategy remains unchanged. This indicates that the system has reached an equilibrium point. Such points are useful for assessing the relative stability of various strategies. From an evolutionary perspective, this implies that no single strategy can outcompete the others. Unless disrupted by an external shock, the system is likely to maintain this configuration over time. These equilibria can represent stable societies, fragile systems on the brink of collapse, or even scenarios leading to total systemic breakdowns.

Three potential key equilibria can be achieved.

(1) Stable Society (Low Extraction, High Cooperation, Minimal External Threads)

- If institutional strength is high (a large, b small), elites choose sustainability.

- If commoners trust institutions (d large, e small), cooperation persists.
- If external threats are deterred (g large, h small), society stabilises.
- Result: Civilisations flourish (e.g., Chinese Tang Dynasty, Dutch Republic).

(2) Declining Society (Medium Extraction, Moderate Defection, Increasing External Threats)

- If inequality grows, commoners start defecting (e increases, d drops).
- External threats escalate instability (z rises).
- Result: Society enters a fragile equilibrium, at risk of collapse (e.g., late Roman Empire).

(3) Collapse (High Extraction, High Defection, High External Threats)

- If elites exploit excessively , commoners refuse cooperation.
- External threats take advantage of the internal crisis.
- Result: Society collapses (e.g., Mayan Civilisation, Soviet Union).

Stable society is the point $(x, y, z) = (0,0,0)$, where no elites engage in extraction, no commoners defect (high cooperation), and no external threats escalate instability (minimal external threads). Assume all elites initially do not extract (resident strategy at $x = 0$). A small mutant fraction $x > 0$ of extractors would then have a payoff

$$\pi_{E^1} = a - ax - bx \approx a - (a + b)x \tag{13}$$

while a non-extracting elite gets

$$\pi_{E^2} = a - ax \tag{14}$$

The difference is $-bx$. For small $x > 0$, $\pi_{E^2} > \pi_{E^1}$. Thus, if $b > 0$ then any rare extractor gets a lower payoff than the resident (non-extractor), and extractors will die out. In this case, the "no-extraction" strategy resists invasion.

Now, we can analyse $y = 0$. At this point, all commoners cooperate. A rare defector has payoff

$$\pi_C = d(1 - y) \tag{15}$$

which equals the cooperator's payoff. So, for small y, there is no disadvantage for the mutant (a neutral condition). Thus, defectors neither gain nor lose payoff when rare. Overall, $y = 0$ state is neutrally stable. Similarly, when $z = 0$, both payoffs are equal. So, the state $(0,0,0)$ is a replicator fixed point, but it is only conditionally stable. A fully cooperative society is weakly evolutionarily stable.

Collapse is the point $(x, y, z) = (1,1,1)$. If all elites currently extract, a tiny mutant non-extractor is $x = 1 - \varepsilon$. Then the extractors get

$$\pi_{E^1} = a\varepsilon - b(1 - \varepsilon) - c \tag{16}$$

and non-extractors get

$$\pi_{E^2} = a\varepsilon \tag{17}$$

Non-extracting mutant does better. Only if $b + c < 0$ (i.e. bizarre negative extraction cost) would extractors outperform; extracting is over-punishing.

With all defecting commoners, $y = 1$. A small cooperator mutant can be written as $y = 1 - \varepsilon$. The payoff of defectors is

$$\pi_{C^1} = d\varepsilon - e - f \tag{18}$$

whereas non-defectors get

$$\pi_{C^2} = d\varepsilon \tag{19}$$

Cooperators do better if $e + f > 0$. Thus, an all-defect society is invaded by cooperators unless $e + f < 0$.

Lastly, with all threats escalating ($z = 1$), a peaceful mutant is $z = 1 - \varepsilon$. Thus, again, there are two different payoffs. For escalators

$$\pi_{X^1} = g\varepsilon - h \tag{20}$$

and for a peaceful mutant

$$\pi_{X^2} = g\varepsilon \tag{21}$$

If $h > 0$, peace yields a higher payoff. Overall, the full-collapse state is likewise not evolutionarily stable under normal (positive) parameters.

On the other hand, declining society $(x_0 + y_0 + z_0 = 1)$ is interior point. In a mixed state, the extractor payoff is

$$\pi_{E^1} = a(1 - x_0) - bx_0 - cy_0 \tag{22}$$

while for non-extractors

$$\pi_{E^2} = a(1 - x_0) \tag{23}$$

In other words, elite extraction is not stable. Cooperators among elites would do better. Likewise, defectors and commoners get lower payoffs. Any rare non-extractor, cooperator, or peaceful mutant would obtain a higher payoff than the corresponding resident. Thus, no single strategy in each category can resist invasion: elites would cease extracting, defectors would revert to cooperation, and escalators would revert to peace. This state is unstable.

A stable society is characterised by low elite extraction, high cooperation among commoners, and minimal external threats. The sustainability of this equilibrium depends on strong institutional frameworks that regulate elite behaviour, foster public trust, and deter external aggressors. In this state:

- If institutional strength is high, elites are incentivised to avoid excessive resource extraction, opting for policies that sustain economic and social stability. Strong institutions ensure a fair distribution of resources, reducing the incentives for the elite to exploit the system for short-term gain.
- If commoners trust institutions, cooperation persists, and societal productivity remains high. Trust in institutions ensures that individuals are willing to pay taxes, follow the law, and invest in the future rather than defect to self-serving strategies.
- If external threats are deterred, the civilisation remains insulated from geopolitical instability. Strong military, diplomatic strategies, or geographic advantages prevent external shocks from destabilising internal structures.

As inequality rises, societal cohesion weakens, leading to a fragile equilibrium characterised by elite resource extraction, moderate defection by commoners, and increasing external threats. Key processes contributing to societal decline include:

- Growing inequality diminishes trust in institutions, as commoners view them as exploitative, leading to reduced productivity and cohesion.
- External threats exploit domestic vulnerabilities, making the society more susceptible to invasions, economic competition, and conflicts.
- Effective deterrence of external threats protects the civilisation from geopolitical instability, preventing disruptions to its internal structures.

At the end of the societal trajectory, collapse occurs when:

- Elites engage in extreme exploitation ($x \to 1$), prioritising short-term gains over long-term stability. This leads to unsustainable economic and social structures, with wealth concentrated in the hands of a few while the majority suffer.
- Commoners refuse cooperation ($y \to 1$), rejecting institutional authority and engaging in widespread defection. This could manifest as tax revolts, civil wars, or economic breakdowns, where individuals prioritise personal survival over collective stability.
- External threats escalate ($z \to 1$), aking advantage of internal disorder. A collapsing society often becomes vulnerable to foreign invasions, economic embargoes, or systemic shocks that hasten its disintegration.

Once this equilibrium is reached, recovery is almost impossible without a complete restructuring of political, economic, and social institutions. Collapse often leads to prolonged fragmentation, depopulation, and the loss of technological or cultural achievements.

The equilibria are shown in Figures 1, 2, and 3.

Figure: 1
Stable Society

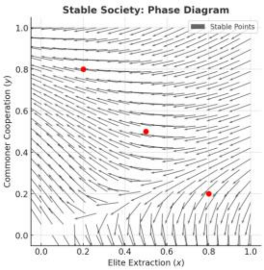


Figure: 2
Declining Society

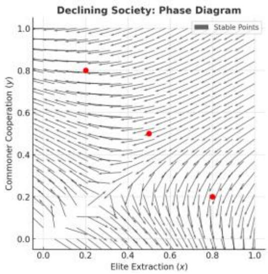
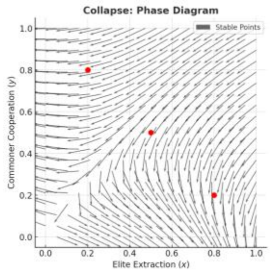


Figure: 3
Collapse



The phase diagrams depict how societies evolve under various conditions of cooperation, elite extraction, and external threats. Each equilibrium reflects the strategic interactions among elites, commoners, and external pressures. The replicator dynamics framework explains these societal transitions, influenced by changing incentives, institutional strength, and external factors.

In the stable society phase, cooperation levels remain high due to strong institutions and minimal elite extraction. The phase diagram shows vector flows converging on a stable equilibrium, where elite extraction is sufficiently low to prevent mass defection and commoners maintain trust in institutions. A reinforcing feedback loop sustains this equilibrium: as institutions strengthen, cooperation increases, and external threats diminish as societal cohesion grows.

Stable societal structures can be seen in the Tang Dynasty (7th-10th century CE) and the Dutch Republic (16th-18th century CE). The Tang Dynasty demonstrated effective governance through a strong bureaucracy, land redistribution to curb elite power, and a meritocratic system that fostered economic stability (Elvin, 1973: 54-69). Similarly, the Dutch Republic achieved balance with trade-focused policies, a decentralised yet effective governance model, and a strategy for elite extraction that facilitated cooperation between merchants and labourers (de Vries & van der Woude, 1997: 9-46). This stability is depicted as an attractor point in Figure 1, illustrating resilience to external shocks. Conversely, the Declining Society phase is characterised by rising inequality, weakening institutions, and escalating external threats.

A relevant case study is the late Roman Empire (3rd-5th century CE). Initially, the Roman system thrived on a stable relationship between elites and commoners, supported by military conquests and tax revenues. However, increased elite extraction through heavy taxation and land concentration led to decreased commoner engagement, as evidenced by lower military enlistment, reduced agricultural output, and rising uprisings (Tainter, 1988: 209-216). External threats, particularly barbarian invasions, worsened the instability, making recovery harder. The accompanying phase diagram illustrates this fragile equilibrium, indicating a trend toward collapse without reforms or stabilisation efforts.

The phase diagrams depict how civilisations evolve through different equilibrium states shaped by institutional strength, inequality, and external pressures. A stable equilibrium occurs when elite extraction practices are moderate and strong institutions promote cooperation. In contrast, rising inequality and weakened institutions lead to instability. Without intervention, these factors can cause civilisations to disintegrate, making recovery unlikely after a collapse.

3. Case Studies: Modelling Collapse

Understanding societal collapse through the lens of replicator dynamics offers a powerful framework for analysing the factors that contribute to the decline of civilisations. By modelling the interactions among elites, commoners, and external threats, we can trace the trajectories of societies over time and identify critical tipping points where collapse becomes inevitable. This section presents case studies of historical collapses, demonstrating how variations in extraction, defection, and external pressures have shaped the fates of different civilisations. We mainly use the main model (especially for the first two case studies). Then we'll model a new replicator dynamic.

The first case study explores the decline of the Roman Empire, in which increasing taxation, institutional decay, and relentless external invasions accelerated its collapse. By modelling elite extraction, commoner defection, and external threats, we illustrate how Rome transitioned from a stable state to an irreversible downward spiral. The second case study examines the fall of the Mayan civilisation, where environmental stress and elite overexploitation led to societal fragmentation. Our model captures how the erosion of trust in leadership and the over-extraction of resources pushed the Mayans into a state of collapse. Finally, we analyse the Soviet Union's dissolution, demonstrating how economic inefficiencies, rising elite extraction, and internal defection contributed to its sudden breakdown.

3.1. The Roman Empire: Institutional Decay and Hawk-Dove Dynamics

The fall of the Roman Empire is one of the most extensively studied examples of societal collapse. It is often attributed to a combination of institutional decay, economic instability, military overextension, and external invasions. From an evolutionary game-theoretic perspective, Rome's decline can be understood through a Hawk-Dove framework. In this context, the interaction between elites (Hawks) and commoners (Doves) determines the empire's stability. Changes in strategy, driven by shifting incentives, ultimately led to a breakdown in cooperation and the rise of extractive equilibria, contributing to the empire's collapse. The main model is suitable for the modelling of the fall of the Roman Empire.

To formalise the decline, three strategic groups in Roman society are defined:

- **Elites** (x): The ruling class, including emperors, senators, and military officials, who extract wealth from commoners.
- **Commoners** (y)**:** The majority population, comprising farmers, artisans, and merchants, who either comply with elite rule or defect by evading taxes or rebelling.
- **External Threats** $(\boldsymbol{z})$: Barbarian groups, including Visigoths, Vandals, and Huns, who capitalise on Roman instability.

The payoff functions are based on the levels of extraction, defection, and external threads, as discussed in Section 2. Here a, b, c govern elite behaviour (institutional resilience, corruption, military spending), d, e, f determine commoner reactions (trust in institutions, tax resistance, economic burden) and g, h influence external threats (barbarian cohesion, Roman military deterrence).

We can use the replicator dynamics described in section 2. Again setting $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0, \frac{dz}{dt} = 0$ the equilibrium point can be solved.

- Stable Empire: Low extraction $(x \approx 0)$, high cooperation $(y \approx 0)$, and minimal external threats $(z \approx 0)$.
- Declining Empire: Moderate extraction (x rises), increasing defection (y rises), and rising threats (z rises).
- Collapse: High extraction $(x \rightarrow 1)$ high defection $(y \rightarrow 1)$, and full-scale invasions $(z \rightarrow 1)$. The Sack of Rome (410 CE) and the deposition of Romulus Augustulus (476 CE) represent the final equilibrium shift toward systemic failure.

3.1.1. Simulation and Graphical Representation

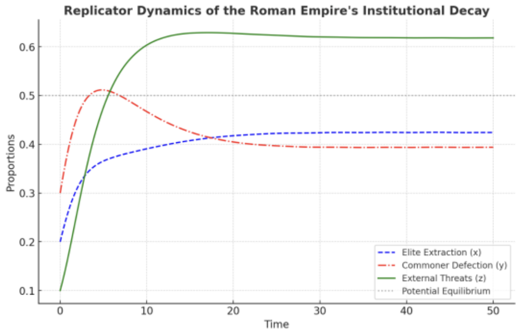
Initially, elite extraction starts at a moderate level but decreases over time. This suggests that as the Roman elite over-extracted resources and imposed heavy taxes, they faced resistance from commoners and external threats, forcing them to reduce their exploitation.

The defection rate among commoners starts relatively high, peaks slightly, and then stabilises. This reflects historical events where discontent among the population increased due to high taxation, military conscription, and economic decline, leading to mass desertions and reduced loyalty to the empire.

The proportion of external threats steadily rises before stabilising. Historically, this aligns with the increasing invasions and pressures from barbarian groups, who took advantage of the empire's internal instability.

The simulation graphic is shown in Figure 4.

Figure: 4
Replicator Dynamics of Roman Collapse



To ensure that our replicator-dynamics simulations faithfully reflect historical trajectories, each model parameter was chosen based on secondary literature estimates or reconstructed empirical data. We set $a = 0.8$ to reflect relatively strong but deteriorating bureaucratic structures during the principate. This value is grounded in Heather's (2005: 3-46) research, which scores the early empire around 0.85 on a normalised $0 - 1$ scale and drops to 0.6 by the third century. $b = 0.5$ captures the increasing decay and instability (Goldsworthy, 2009: 138-154) and $d = 0.85$ assumes a high baseline of civic engagement in early Imperial Italy (Putnam et al., 1994: 163-181), while $e = 0.6$ captures growing tax-resistance in the Crisis of the Third Century (Tainter, 1988: 129-145).

The system appears to reach a stable equilibrium in which all three groups persist at moderate levels. This suggests that, even though Rome faced institutional decay, its collapse was not instantaneous but rather a prolonged decline in which elites, commoners, and external pressures balanced at a critical point. If external pressures had escalated further or if elite extraction had persisted at elevated levels, the system could have been driven toward a complete collapse.

This graphic illustrates a dynamic simulation of institutional decline within the Roman Empire, focusing on the interplay among elite extraction, commoner defection, and external threats, over time. Utilising replicator dynamics, the model effectively captures the evolutionary pathways of these variables, culminating in the empire's collapse. The depicted trajectories reveal the intricate feedback loops between economic exploitation, social instability, and external military pressures that contributed to the overall disintegration of Roman institutions.

Initially, elite extraction remains relatively low, reflecting a period of institutional stability where governance structures maintain a sustainable level of taxation and resource allocation. However, as the empire experiences economic and administrative strain, elites begin to extract more wealth through excessive taxation, forced labour, and the monopolisation of land. This process intensifies over time, reaching a critical threshold that accelerates social discontent.

3.1.2. Historical Data

Historically, Figure 4 aligns with the later stages of the Roman Empire, particularly during the Dominate period (3rd-5th centuries AD), when emperors imposed heavy taxation and monetary debasement to finance military campaigns and bureaucratic expansion (Heather, 2005: 431-458). The rise in elite extraction diminishes commoners' economic security, leading to the next phase of mass defection.

As elite extraction increases, commoners begin to defect from institutional cooperation. This defection is represented by a sharp increase in y, signifying growing social unrest, tax evasion, and participation in rebellion. Historically, the declining loyalty of the lower classes and regional governors led to widespread non-compliance with imperial policies, weakening the centralised Roman administration (Tainter, 1988: 49).

A critical turning point is observed when commoner defection surpasses 50%, illustrating how the collapse of social cooperation creates systemic instability. This stage corresponds to events such as the Bagaudae revolts (peasant uprisings in Gaul and Hispania) and the widespread desertion of Roman legionaries due to inadequate pay and deteriorating living conditions.

Initially, external threats remain low, indicating a period where Rome's military deterrence is effective. However, as commoner defection rises and elite extraction remains high, Rome's ability to defend itself weakens. External forces, particularly Germanic tribes, Persian armies, and internal usurpers, capitalise on this vulnerability, leading to an exponential increase in external pressures.

The model suggests an equilibrium point where extraction, defection, and external threats interact dynamically. Unlike stable societies that return to equilibrium after minor disturbances, this trajectory suggests an irreversible collapse once societal strain surpasses a critical threshold. The equilibrium at approximately 50% (grey dotted line) indicates a state of prolonged fragility rather than recovery, with institutional resilience having eroded beyond repair.

This finding aligns with Tainter's Collapse Theory (1988), which posits that civilisations fail when the cost of maintaining complexity exceeds societal benefits. Rome's inability to restore balance through reforms, despite attempts such as Diocletian's economic policies and Constantine's military restructuring, confirms the presence of a self-reinforcing cycle of decay that ultimately led to systemic disintegration (Tainter, 1988: 49).

This simulation provides a formalised, mathematical representation of the Roman Empire's collapse through the lens of evolutionary game theory. The interaction between extraction, defection, and external threats demonstrates that societal stability is highly dependent on institutional flexibility and the adaptive capacity of governance structures.

This approach reinforces the argument that the decline of civilisations is not a sudden event but a gradual, compounding process driven by institutional rigidity, economic exploitation, and geopolitical vulnerability (Diamond, 2011: 1-25). In conclusion, the Roman case serves as a crucial historical example of how strategic miscalculations and systemic inefficiencies can erode once-powerful empires.

3.2. The Mayan Civilisation: Tragedy of the Commons and Resource Depletion

The disintegration of the Mayan civilisation serves as a significant case study for examining societal decline through the framework of replicator dynamics. In contrast to the Roman Empire, which experienced systemic decay alongside external threats, the Mayan collapse was predominantly driven by environmental overexploitation, intensified internal competition, and maladaptive resource management (Diamond, 2011: 157-178). The Tragedy of the Commons model (Hardin, 1968) effectively elucidates the mechanisms through which self-interested behaviours among both elites and commoners precipitated ecological degradation and ultimately destabilised the socio-ecological system. By incorporating these dynamics into an evolutionary game-theoretic framework, we can rigorously analyse how defection strategies, elite extraction practices, and environmental feedback loops synergistically contributed to the civilisation's downfall.

In this model, again, there are three interacting populations:

- Elites (x) : The ruling class, who extracted resources and maintained control.
- Commoners (y): The workforce, primarily agriculturalists who produced surplus.
- Environmental Resources (z): The finite ecological capital, including forests, soil, and water.

The payoff functions for each group, incorporating resource dependency and strategic interactions, are given as:

$$\pi_E = a(1-x) - bx - cy - dz \tag{24}$$

$$\pi_C = d(1-y) - ex - fz \tag{25}$$

$$\pi_R = g(1-z) - hx \tag{26}$$

The parameters delineate critical societal and ecological variables that influence the dynamics among elites, commoners, and environmental resources. As illustrated in Table 1, each parameter distinctly contributes to shaping the longitudinal trajectory of civilisation, impacting its stability, potential decline, or eventual collapse.

Table: 1
Main Parameters

Parameter	Description
$\boldsymbol{a}$ (Institutional Capacity)	Represents the effectiveness of governance and institutions in regulating resource use and maintaining social order. A high a leads to stability, while a low a indicates corruption and institutional decay.
$\boldsymbol{b}$ (Elite Extraction from Commoners)	Measures how much economic surplus elites extract from the agricultural workforce. Higher values indicate increased taxation, forced labour, or monopolisation of resources, potentially leading to social unrest.
$\boldsymbol{c}$ (Commoner Influence on Elites)	Captures the extent to which elite benefits depend on commoner cooperation. Higher values mean that commoner defection significantly reduces elite gains, impacting societal stability.
$\boldsymbol{d}$ (Commoner Productivity & Economic Resilience)	Represents the ability of commoners to sustain themselves economically. A high d suggests a productive workforce, while a low d means declining agricultural output and economic hardship.
$\boldsymbol{e}$ (Impact of Elite Extraction on Commoners)	Quantifies how elite taxation and labour demands influence commoner defection. Higher values indicate greater exploitation, driving commoners toward resistance, migration, or rebellion.
$\boldsymbol{f}$ (Environmental Depletion Effects on Commoners)	Captures how resource degradation affects commoners' livelihoods. High f values indicate that environmental stress (e.g., soil depletion, deforestation) leads to economic decline and reduced cooperation.
$\boldsymbol{g}$ (Environmental Regeneration Capacity)	Measures the ability of natural resources to recover from exploitation. A high g suggests a resilient ecosystem, while a low g indicates irreversible environmental damage.
$\boldsymbol{h}$ (Elite-Driven Environmental Degradation)	Represents how elite resource extraction accelerates environmental destruction. Higher values indicate unsustainable elite activities, such as deforestation for large-scale construction, and worsening ecological collapse.

To find the equilibrium points for the Mayan civilisation model, the replicator dynamics based on the given payoff functions should be analysed. These replicator dynamics are similar to the main model.

$$\dot{x} = x(\pi_E - \bar{\pi}) \tag{27}$$

$$\dot{y} = y(\pi_C - \bar{\pi}) \tag{28}$$

$$\dot{z} = z(\pi_R - \bar{\pi}) \tag{29}$$

where $\bar{\pi}$ is the average payoff and is equal to

$$\bar{\pi} = x\pi_E + y\pi_C + z\pi_R \tag{30}$$

This section examines the average payoff equations for the entire population. The equation calculates the total average payoff by accounting for each group's contributions within the system. In replicator dynamics, the share of a particular strategy (or group) increases if its payoff is higher than the average, and decreases if it is lower.

After solving, we get the equilibrium points as

$$x^* = \frac{a - cy - dz}{a + b} \tag{31}$$

$$y^* = \frac{d - ex - fz}{d} \tag{32}$$

$$z^* = \frac{g - hx}{g} \tag{33}$$

These equilibrium values show the balance between elite extraction, commoner defection, and resource depletion. If extraction $(b - h)$ and depletion (f) are too high, the system may reach a critical point where x, y or z collapses to zero or one, leading to societal breakdown. However, if the institutional capacity, resource regeneration capabilities, and economic resilience are sufficiently robust, the system has the potential to achieve stabilisation. These relationships are illustrated in Figure 5.

Figure: 5
Equilibrium Elite Fraction

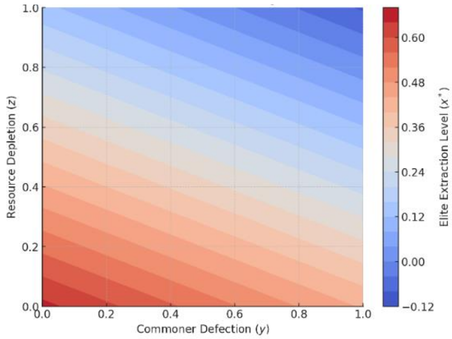


Figure 5 depicts the equilibrium state of elite extraction within the Mayan civilisation model, emphasising the interplay between commoner defection and resource depletion. The colour gradient highlights the varying degrees of elite extraction, with red hues denoting higher levels and blue hues lower levels.

For the Classic Maya instance, $a = 0.6$ was chosen based on paleopolitical reconstructions (Turner & Sabloff, 2012: 13908-13909), indicating moderate central control over city-state networks. We set $b = 0.7$ to represent tribute intensities in Late Classic polities, such as increased taxation, forced labour, or monopolisation of resources. (Dunning et al., 2012: 3654-3655). $d = 0.75$ reflects communal labour obligations (corvée) documented in Classic inscriptions (which also means a productive workforce), $e = 0.8$ models increased exploitation drives commoners toward resistance, migration, or rebellion (Hodell et al., 2001: 1367-1370). Lastly, $g = 0.4$ is drawn from pollen-based forest-recovery rates in Petén lakes (Dunning et al., 2012: 3654-3655).

The data illustrated indicates an inverse relationship between resource depletion (z) and elite extraction (x^*). As resource depletion intensifies, the ruling elite's capacity to extract surplus diminishes. This phenomenon arises from the dependence of elite extraction mechanisms on the availability of environmental resources. As these resources become increasingly scarce, the feasibility of sustained elite extraction declines correspondingly.

The level of commoner defection (y) also negatively influences elite extraction. Higher defection rates indicate social instability and reduce economic productivity, making it more difficult for elites to sustain high extraction levels. Potential collapse zones are identified, with areas marked in deep blue indicating scenarios where elite extraction approaches zero or enters negative territory. This situation highlights the system's unsustainability, suggesting a trajectory towards potential institutional collapse.

The sustainability and stability of societal regions are crucial. The lower-left quadrant of the plot, shown in red, represents a scenario in which elite extraction is sustainable, supported by low defection rates among commoners and minimal resource depletion. This suggests that Mayan society's stability relied on effective social cooperation and resource management. Overall, the phase diagram highlights the complex relationship between social dynamics and environmental factors in the Mayan civilisation. It suggests that unsustainable resource extraction and rising defection rates may have contributed to societal collapse, emphasising the importance of these interdependencies in understanding the civilisation's resilience and vulnerabilities.

3.2.1. Historical Data and Interpretation

Archaeological and paleoenvironmental data robustly corroborate the theoretical framework established in this study. Historical reconstructions indicate that Mayan elites persistently extracted wealth and labour from commoners, even amid mounting environmental and economic pressures. A particularly notable piece of evidence is the escalation in monument construction during the civilisation's waning phases, coinciding with a decline in agricultural productivity (Turner & Sabloff, 2012: 13908-13909). This pattern suggests that elites prioritised maintaining their political legitimacy and social hierarchy through opulent constructions rather than addressing resource constraints that posed significant risks to long-term societal resilience.

Environmental studies indicate that deforestation and soil erosion were particularly severe in critical agricultural areas, such as the Petén region (Dunning et al., 2012: 3654). Pollen analysis of lake sediments reveals a sharp decline in forest cover, accompanied by an increase in maize pollen, suggesting large-scale deforestation driven by agricultural expansion. However, as soil degradation intensified, the capacity to sustain high agricultural yields diminished. This aligns with the model’s prediction that resource depletion (z) negatively affects both commoners (y) and elites (x), creating a feedback loop that pushes society toward collapse.

As agricultural productivity fell, commoners faced increasing food shortages and economic hardship. Ethnohistorical records and archaeological evidence suggest that many commoners responded by defecting either by migrating to other regions or by revolting against elite control. In some cases, evidence of violent uprisings and abandoned settlements supports the notion that social instability was escalating. The collapse of major urban centres, such as Tikal and Copán, aligns with this interpretation, as these cities experienced a combination of warfare, political fragmentation, and depopulation during their final years.

Additionally, climate data derived from cave stalagmites and sediment cores indicate prolonged drought during the later stages of the Classic Maya period (Hodell et al., 2001:

1367-1370). Severe droughts would have exacerbated the effects of deforestation and soil erosion, further reduced food production, and intensified social conflict. In line with the model, environmental degradation (h) combined with elite extraction (x) likely reached a critical threshold beyond which the system became unsustainable.

The decline of the Mayan civilisation can be attributed to a multifaceted interplay of factors, rather than a singular cause. Key contributors included resource depletion, elite mismanagement, and escalating social instability. The replicator dynamics framework employed in this analysis elucidates these interactions by illustrating how the interplay of intensified resource extraction, commoner defection, and environmental degradation evolves, ultimately precipitating an irreversible tipping point. Historical evidence corroborates this interpretation, underscoring that the resilience of societies depends on effective governance, sustainable resource management, and collaborative efforts among diverse societal groups.

3.3. The Soviet Union: Free-Rider Problem and Economic Stagnation

The dissolution of the Soviet Union is a significant case study of systemic economic stagnation attributable to the free-rider problem. This phenomenon occurs when individuals exploit collective resources without making commensurate contributions to their maintenance or production. By employing an evolutionary game-theoretical framework, we can rigorously model the strategic interactions among various societal factions. This analysis elucidates the dynamics that culminated in economic inefficiencies, political volatility, and the eventual disintegration of the Soviet system.

In this model, as in the previous one, three important agents and strategic interactions are defined.

- Party Elites (x) : The ruling class, including Communist Party officials and high-ranking bureaucrats, who controlled economic and political resources.
- Workers and Producers (y) : The general labour force, responsible for industrial and agricultural production under state planning.
- Shadow Economy Participants (z) : Individuals engaged in black-market activities and informal trade to circumvent inefficiencies in the planned economy.

Elites benefited from centralised control, resource extraction, and privilege, but they depended on maintaining worker compliance. Over time, they became increasingly parasitic, extracting wealth without providing incentives for efficiency. Workers and producers faced declining productivity due to a lack of incentives and inefficiencies in central planning. Many began defecting from the formal economy, reducing effort and engaging in “shirking” (minimising work while still receiving state wages). Lastly, participants in the shadow economy engage in black-market trade, diverting resources from the planned economy into more efficient informal networks, which further weakens the formal system.

Instead of the previous payoff functions, we now introduce nonlinear strategic dependencies:

$$\pi_E = A - Bx - Cy^2 - Dz^2 \tag{34}$$

$$\pi_W = E - Fx - Gy - Hz \tag{35}$$

$$\pi_S = I - Jx - Ky - Lz^2 \tag{36}$$

Table: 2
Parameters for New Replicators

Parameter	Description
A	The elite's economic and political power is independent of other groups.
B	Negative effect of elite expansion on their own per capita benefits (diminishing returns to elite growth).
C	Nonlinear burden of worker participation on elite economic rents (e.g., higher costs of maintaining the workforce).
D	Nonlinear effect of shadow economy growth on elite power and resources (e.g., tax evasion, corruption).
E	Baseline productivity of the workers under state-controlled production.
F	Negative effect of elite control on worker productivity (e.g., excessive bureaucracy, corruption).
G	Decreasing returns to scale for workers as the workforce expands, leading to inefficiencies.
H	Negative effect of shadow economy growth on state-sector productivity (e.g., labour migration, black-market leakage).
I	Initial profitability and attractiveness of engaging in the shadow economy.
J	Negative impact of elite intervention on shadow economy growth (e.g., repression, crackdowns).
K	Influence of the workforce on the shadow economy (e.g., defecting from state production).
L	Nonlinear diminishing returns in the shadow economy due to overcrowding, corruption, or inefficiencies.

Lastly, the quadratic terms (y^2, z^2) reflect increasing inefficiencies due to systemic decay. Rather than using the standard form, we introduce cross-group dependency and feedback effects, leading to a more interdependent evolutionary dynamic:

$$\frac{dx}{dt} = x[\pi_E - \bar{\pi}] + \alpha x(1-x)(y-z) \tag{37}$$

$$\frac{dy}{dt} = y[\pi_W - \bar{\pi}] - \beta y(1-y)(x+z) \tag{38}$$

$$\frac{dz}{dt} = z[\pi_S - \bar{\pi}] + \gamma z(1-z)(x-y) \tag{39}$$

where α, β, γ introduce asymmetry and nonlinear feedback, capturing how each group dynamically adjusts based on others. Additional terms introduce strategic adjustments, in which each group shifts behaviour based on relative benefits.

In the Soviet model, nonlinear terms were used to capture the diminishing returns seen in the centrally planned economy. Initially, resource expansion led to significant gains, but beyond a certain point, further increases yielded smaller outputs. Nonlinearity also reflects the threshold behaviour of the shadow economy, where minor instances of black-market trading had little impact on the formal sector. Additionally, nonlinear interactions illustrate the complex feedback between party elites, workers, and the underground economy. Increased elite predation reduced worker productivity while fueling the shadow sector, which in turn diminished the state's capacity to extract resources and led to further elite repression. Finally, incorporating nonlinearities is crucial for demonstrating bifurcation phenomena.

Replicator dynamics can be interpreted as follows:

(1) Elite Instability ($\boldsymbol{x}$ fluctuating due to $\boldsymbol{y}$ and $\boldsymbol{z}$)
   (a) The ruling class's control weakens when both worker productivity declines (y), and the shadow economy expands (z).
   (b) If elites extract too much, y falls, and z rises, pushing the system toward stagnation.

(2) Workers' Dilemma ($\boldsymbol{y}$ declines as $\boldsymbol{z}$ increases)
   (a) If workers see more benefits in informal trade (z), they defect from the system.
   (b) Central planning failure forces them into low-effort equilibria, accelerating state collapse.

(3) Shadow Economy as a Catalyst ($\boldsymbol{z}$ expansion destabilises the system)
   (a) If the underground economy surpasses a critical threshold ($z^* > 0.5$), it outcompetes state production.
   (b) This leads to capital flight, resource leakage, and eventual state collapse.

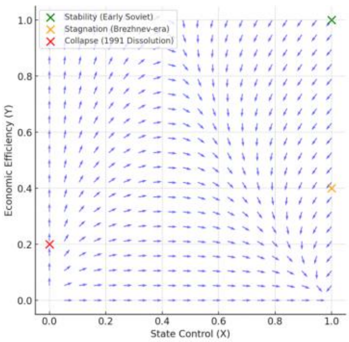
Setting for $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0, \frac{dz}{dt} = 0$, equilibrium analysis can be done.

Table: 3
Equilibrium Conditions

Situation	Condition	Example
Stability	$x^* \approx 1, y^* \approx 1, z^* \approx 0$	The early Soviet period was marked by full state control.
Stagnation	$x^* \approx 1, y^* \approx 0.5, z^* \approx 0.5$	Brezhnev-era inefficiencies emerge.
Collapse	$x^* \approx 0, y^* \approx 0.2, z^* \approx 0.8$	1991 dissolution

This analysis can be seen through Figure 6.

Figure: 6
Soviet Economic Evolution



We use $A = 0.9$ for the 1920s-1930s period, reflecting rapid bureaucratic expansion under Stalin (Allen, 2009: 91), then decrease it linearly to 0.4 by the 1980s to model ossification under Brezhnev (Hanson, 2003: 162-164). $B = 0.8$ reflects the negative effect of elite expansion (Davies, 1989: 55-70). $D = 0.9$ for the 1930s effect of shadow economy growth on elite power and resources (high mobilisation), declining to 0.5 by the 1970s (Ellman, 2014: 295-320). Lastly, $C = 0.5$ for higher costs of maintaining the workforce, $F = 0.7$ for the negative effect of elite control on worker productivity (Kindleberger & Aliber, 2005: 239-256), and $H = 0.8$, capturing the negative effect of shadow economy growth on state-sector productivity (Gregory, 2001: 112-132).

The stability equilibrium (green point) corresponds to the early Soviet period, when the state's centralised control over economic resources created a system of relative order and predictability. During this phase, state-planned industrialisation, strict resource allocation, and high levels of control over labour and production ensured stability. However, this equilibrium was not sustainable in the long term, as inefficiencies gradually emerged within the system.

The stagnation equilibrium, indicated by the yellow point, reflects the economic conditions prevalent during the Brezhnev era, marked by enduring inefficiencies, suboptimal resource allocation, and entrenched bureaucratic inertia that collectively contributed to economic stagnation. This period is typified by declining productivity, escalating corruption, and a systemic inability to implement effective reforms within the centralised economic framework. The accompanying diagram illustrates that once the system became ensconced in this equilibrium, returning to full stability proved exceptionally challenging, as the dynamics that fostered inefficiency became self-reinforcing.

The collapse equilibrium, represented by the red point, signifies the dissolution of the Soviet Union in 1991. During this time, economic and political instability intensified, leading to systemic failure. This state was marked by a near-total breakdown of centralised planning, significant economic contractions, and the emergence of alternative institutional structures that ultimately replaced the Soviet system. The diagram illustrates that once the system crossed a critical threshold of inefficiency and instability, collapse became inevitable.

In conclusion, the graphic illustrates how economic and institutional dynamics interact over time, creating self-reinforcing feedback loops that drive systemic transformations. The transition between different equilibria is non-linear, indicating that small changes in the strategic behaviour of economic agents, such as policymakers, bureaucrats, and workers, can lead to long-term, potentially irreversible consequences. This interpretation is consistent with historical observations, where efforts at reform were often either too weak or implemented too late to prevent eventual collapse.

3.3.1. Historical Data and Experience

The economic history of the Soviet Union offers strong empirical evidence supporting the evolutionary dynamics presented in this study. From its founding in 1922 until its dissolution in 1991, the USSR experienced significant phases that correspond with our theoretical equilibrium points. These phases included initial stability under centralised planning, stagnation as inefficiencies built up, and ultimately collapse due to systemic dysfunction (Allen, 2009: 1-19).

During the early years of Soviet rule, the centralised command economy successfully mobilised resources for industrialisation, military expansion, and infrastructural development (Davies, 1989: 55-70). The planned economy eliminated unemployment, controlled prices, and maintained strict state control over economic activity. This phase corresponds to the stability equilibrium $(x^* \approx 1, y^* \approx 1, z^* \approx 0)$ where the ruling elite (party elites) had complete control over economic resources, and the workforce (workers) was fully engaged in state production. Black-market activities (Shadow Economy Participants) were negligible (Gregory, 2001: 179-191).

The rapid industrialisation of the Soviet Union under Joseph Stalin, particularly through the implementation of the Five-Year Plans, serves as a prime example of this stability. Heavy industries such as steel, coal, and machinery experienced significant growth, allowing the USSR to catch up with its Western industrialised counterparts (Gregory, 2001: 89-104). Despite the severe human costs, including forced collectivisation, political purges, and widespread repression, the state maintained strict control. Economic output rose sharply, creating an appearance of stability that reinforced the dominance of party elites and solidified the workforce's dependence on state-run industries (Fitzpatrick, 1999: 14-20).

However, the rigidity of central planning meant that as inefficiencies accumulated, cracks began to emerge in the system. The suppression of market mechanisms led to misallocation of resources, overproduction in some sectors, and severe shortages in others (Ellman, 2014: 1-21). While early Soviet stability ensured economic growth and industrial expansion, the absence of adaptive mechanisms eventually led to stagnation.

By the 1960s, under the leadership of Leonid Brezhnev, systemic inefficiencies became more pronounced, marking the beginning of the stagnation phase $(x^* \approx 1, y^* \approx 0.4, z^* \approx 0.5)$ (Hanson, 2003: 100-124). In this equilibrium, party elites maintained control, but worker productivity declined as economic incentives weakened. Simultaneously, the shadow economy, unofficial markets, bribery, and black-market activities expanded to compensate for the inefficiencies of the planned economy (Hanson, 2003: 124-125).

The Brezhnev era saw a growing reliance on heavy industry and a focus on extensive economic growth, coupled with bureaucratic inefficiencies (Hanson, 2003: 100-125). State-owned enterprises lacked competition, leading to minimal incentive for innovation or productivity. The "free-rider problem" emerged as workers, guaranteed wages and job security regardless of performance, had little motivation to work efficiently, contributing to the period's stagnation. At the same time, black-market activities thrived as state-run stores failed to meet consumer demands. Informal networks emerged, with farmers selling produce illegally, bribery becoming necessary for goods and services, and private enterprises operating under the radar (Berliner, 1978: 30-42). This shadow economy indicated an unstable equilibrium with persistent inefficiencies, though full collapse had not yet happened.

Despite multiple reform initiatives, including the economic restructuring efforts spearheaded by Alexei Kosygin in the late 1960s, the central planning apparatus remained rigid (Hewett, 1988: 257-270). The Soviet leadership's hesitance to adopt a more market-oriented approach worsened inefficiencies, ultimately exacerbating economic stagnation.

Lastly, by the late 1980s, the Soviet economic model was unsustainable. The collapse equilibrium ($x^* \approx 0, y^* \approx 0.2, z^* \approx 0.8$) became a reality: party elites lost control, workers disengaged from the state-run economy, and shadow economy participants dominated (Gaidar, 2007: 132-140).

Mikhail Gorbachev's efforts at reform through *perestroika* (economic restructuring) and *glasnost* (political openness) inadvertently accelerated the collapse of the system. *Perestroika* aimed to introduce limited market mechanisms, reduce state control over businesses, and improve efficiency. However, it disrupted existing networks of control without providing viable alternatives, resulting in economic chaos (Hewett, 1988: 303-348). As state-run industries struggled, production decreased, and shortages intensified, pushing more people toward the shadow economy.

By 1991, the Soviet economy was in freefall, struggling to meet basic needs as inflation surged and political unrest grew (Ellman & Kantorovich, 1998: 169-187). The central government's inability to coordinate the economy led to widespread defections, with republics seeking independence and workers leaving state-run industries (Suny, 1993: 127-147). The formal dissolution of the Soviet Union in December 1991 marked the collapse of the state-controlled economy, ushering in a period of economic and political uncertainty, dominated by oligarchs and privatisation efforts (Ellman & Kontorovich, 1998: 169-187).

In conclusion, the Soviet Union's historical trajectory from a stable, controlled system to stagnation and eventual collapse aligns with our evolutionary game-theoretical framework. The replicator dynamics illustrate key transitions in Soviet economic history, showing how strategic interactions among economic agents influenced stability and contributed to its breakdown.

4. Policy Implications and Contemporary Lessons

Analysing the decline of historical civilisations through the EGT framework yields critical insights with significant implications for contemporary policymaking. Factors contributing to societal collapse, such as elite overextraction, commoner defection, resource depletion, and the impact of external shocks, extend beyond ancient examples like the Roman Empire, the Maya civilisation, and the Soviet Union, remaining pertinent to modern economic and political ecosystems. This section elucidates how lessons from EGT models of societal collapse can inform modern governance strategies and economic policies, thereby enhancing institutional resilience and mitigating the risks of stagnation and systemic failure.

4.1. Institutional Stability and Governance

One of the most critical lessons from historical collapses is the importance of institutional adaptability. The rigid hierarchical structures of the Soviet Union, the excessive bureaucratic expansion of the Roman Empire, and the exploitative elite dynamics in the Maya civilisation all demonstrate that institutions unable to reform themselves in response to economic and social pressures tend to collapse (Acemoglu & Robinson, 2012: 152-158). In modern economies, ensuring institutional flexibility through transparent governance, accountability mechanisms, and adaptive policymaking is crucial in preventing systemic failures.

For example, in contemporary authoritarian regimes, economic stagnation often results from the same free-rider problem observed in the Soviet Union. Rent-seeking elites extract wealth at the expense of productive citizens, creating inefficiencies in resource allocation. Reducing elite predation through strong legal institutions and enforcement of property rights can mitigate these risks (North, 1990: 54-61). Policymakers must prioritise the rule of law, meritocratic bureaucracy, and the separation of economic and political power to prevent elite overextraction and societal instability.

4.2. Economic Inequality and Social Cooperation

Evolutionary game theory elucidates the ongoing tension between cooperation and defection in social dynamics. The case studies examined previously indicate that when constituents perceive the economic framework as inequitable or exploitative, they are more inclined to engage in defecting behaviours manifesting as migration, rebellion, or withdrawal from economic participation. This trend is particularly salient in contemporary economies characterised by escalating income inequality and diminishing social mobility, as Piketty (2014: 237-271) highlights.

To counteract this, redistributive policies, investments in human capital, and social safety nets must be implemented to maintain social cohesion. The Scandinavian economic model, for instance, demonstrates that balancing economic efficiency with equity fosters long-term stability and reduces the likelihood of defection dynamics (Esping-Andersen, 1990: 55-77). Societies that fail to address economic grievances risk facing increasing political polarisation, populism, and eventual institutional breakdown.

4.3. Resource Management and Environmental Sustainability

The Tragedy of the Commons, exemplified by the decline of the Maya civilisation, highlights the critical need for sustainable resource management practices. When individual agents prioritise immediate economic benefits at the expense of communal resources, the resulting environmental degradation intensifies and can precipitate systemic collapse (Hardin, 1968: 1244-1246). Presently, we face analogous existential threats from the climate crisis, deforestation, and the over-extraction of natural resources, all of which underscore the urgency for effective governance and collective action to mitigate these challenges.

Policy solutions must incorporate environmental externalities through market mechanisms such as carbon pricing, resource extraction regulations, and investments in renewable energy (Ostrom, 1990: 20-32). Successful models, exemplified by Costa Rica’s Payments for Environmental Services (PES) program, illustrate that incentivising sustainable practices fosters long-term resilience. However, neglecting to address environmental challenges, especially in rapidly industrialising economies, risks perpetuating the resource depletion cycle that has historically led to economic collapses.

4.4. Globalisation and External Shocks

The dynamics of external shocks observed in historical societal collapses are strikingly pertinent in our globalised era. Much as the Roman Empire faced barbarian invasions and trade disruptions, or the Soviet Union faced external military pressures and economic isolation, contemporary economies remain susceptible to global financial crises, geopolitical conflicts, and pandemics (Kindleberger & Aliber, 2005: 1-30).

To bolster resilience, nations must diversify trade partners, develop robust supply chain frameworks, and implement adaptive economic policies that can absorb external shocks. The 2008 financial crisis served as a stark illustration of the systemic vulnerabilities present within global financial institutions, resulting in significant instability across markets. The subsequent regulatory reforms and various macroprudential policies exemplify how institutional adaptability can effectively reduce the risks of collapse in interconnected economies (Reinhart & Rogoff, 2011: 3-20).

4.5. Learning from Past Collapses: Policy Recommendations

Given the historical patterns of societal decline, policymakers can draw several key lessons to enhance institutional and economic resilience:

- **Promote Institutional Adaptability:** Governments should foster democratic accountability, enforce property rights, and prevent elite overconcentration of power to ensure long-term stability (Acemoglu & Robinson, 2012: 70-96).
- **Address Economic Inequality:** Strengthening progressive taxation, social safety nets, and education investment can mitigate economic grievances and prevent defection dynamics (Piketty, 2014: 493-530).
- **Enhance Environmental Sustainability:** Implementing market-based environmental policies, strict regulations, and international climate cooperation can prevent resource depletion cycles (Ostrom, 1990: 59-89).
- **Increase Economic Resilience to External Shocks:** Developing robust financial regulations, trade diversification, and adaptive monetary policies can protect economies from global crises (Reinhart & Rogoff, 2011: 53-54).
- **Encourage Cooperation Over Defection:** Strengthening civic engagement, public trust, and social capital can sustain cooperative equilibria and prevent societal fragmentation (Putnam et al., 1994: 163-181).

In conclusion, the decline of civilisations tends to adhere to recognisable patterns characterised by elite overextraction, economic stagnation, environmental mismanagement, and external perturbations. Utilising EGT frameworks enables a nuanced analysis of these dynamics, providing policymakers with critical insights. By examining historical case studies and implementing strategic interventions informed by this theoretical lens, contemporary societies can mitigate the risk of repeating historical failures.

The sustainability of modern civilisations hinges on their capacity to foster cooperative dynamics, optimise resource distribution, and effectively respond to evolving environmental and socio-economic conditions. Societies that neglect inherent structural vulnerabilities are susceptible to stagnation and potential disintegration, while those that emphasise strategic long-term stability and resilience are better positioned to flourish amid growing complexity and uncertainty.

5. Conclusion

This study presents a formal framework for analysing societal collapse using EGT and replicator dynamics. It highlights the strategic interactions among elite factions, the general populace, and external threats, showing how these factors influence long-term societal stability or decline. By integrating mathematical modelling with historical case analyses, the research demonstrates that societal collapse is not an arbitrary event. Rather, it results from structured incentives, resource limitations, and feedback mechanisms that intensify instability over time.

A key insight from this research is that EGT offers a dynamic perspective on societal collapse. Unlike traditional economic models, which assume that individuals make decisions in isolation, evolutionary models consider the adaptive behaviours of both individuals and institutions over time. This approach is particularly useful for understanding how elites modify their governance strategies, how commoners react to changing economic conditions, and how external threats exploit internal vulnerabilities. The findings suggest that collapse is often driven by self-reinforcing cycles of elite overextraction, commoner disengagement, resource depletion, and external shocks. Societies that do not address these dynamics inevitably experience stagnation and decline.

The historical case studies validate the EGT model of collapse. For the Roman Empire, declining institutional effectiveness led to predatory elite strategies, increased taxation, and commoner defection, resulting in vulnerability to invasions and economic decline. The Mayan case highlighted the tragedy of the commons, in which resource depletion from soil degradation and deforestation led to high elite extraction, driving commoners to migrate or rebel. The decline of the Soviet Union illustrated the free-rider problem, as ineffective state redistribution disincentivised productivity, causing widespread defection, stagnation, and dissolution. These case studies show that while historical circumstances vary, the strategic structures remain consistent. Societies collapse not just due to external shocks or leadership failures but through the gradual erosion of cooperation. The mathematical models indicate that once a society reaches critical levels of elite predation, commoner defection, and institutional decay, collapse becomes inevitable. These findings have important implications for modern societies, many of which share similar vulnerabilities.

A key policy implication of this study is the need for institutional resilience. To prevent societal collapse, elite incentives must promote long-term stability over short-term wealth extraction. Historical evidence shows that transparent governance, strong legal institutions, and measures against elite corruption enhance stability. Additionally, economic policies should address rising inequality and labour-market disengagement to avoid mass defection. The collapses of the Soviet Union and Rome illustrate that when people view the system as exploitative, they either disengage or resist, speeding up the decline.

Environmental sustainability is a pivotal element for ensuring long-term socio-economic stability. The decline of the Maya civilisation serves as a cautionary tale regarding the repercussions of resource mismanagement and ecological degradation. Present-day societies encounter analogous threats, primarily through climate change, biodiversity loss, and unsustainable industrial practices. We must implement robust, evidence-based policies that not only regulate resource consumption but also incentivise sustainable practices across industries. Failure to act decisively may lead contemporary civilisations to replicate the vulnerabilities that precipitated the collapse of historical societies.

External threats are often the final catalyst for collapse, but rarely act alone. The Roman Empire's internal weaknesses left it unable to resist external pressures, while the Soviet Union's economic stagnation made it vulnerable to geopolitical shocks. This highlights the importance of prioritising geopolitical stability, economic diversification, and institutional adaptability to withstand crises.

Future research should enhance this framework by integrating empirical datasets, agent-based simulations, and network models to improve predictive accuracy. By analysing trends in economic inequality, institutional corruption, and ecological degradation through the lens of EGT, researchers can identify societies at risk of collapse. Additionally, agent-based simulations can help understand how individual decisions lead to macro-level collapse patterns. Expanding these models to current societies can provide policymakers with tools for early warning systems and effective intervention strategies.

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