



## OPTIMIZING ACCURACY OF ABRASIVE WATERJET CUTTING SYSTEM: A COMPREHENSIVE STUDY ON MECHANICAL AND SOFTWARE COMPENSATION STRATEGIES ON 5-AXIS CNC WATERJET CUTTING MACHINE AND ANALYSIS USING DESIGN FMEA

Uğur ŞİMŞİR<sup>1\*</sup>


<sup>1</sup>Milteksan CNC A.S., Basibuyuk Mh. Sureyyapasa Basibuyuk Yolu Sk. No:4/1, 34854, Istanbul Türkiye

**Abstract:** Abrasive Waterjet cutting technology is more environmentally friendly than other methods like plasma and laser cutting. As a cold cutting method, it does not use flammable gases. Furthermore, it uses water and natural garnet as abrasive materials. The amount of waste is significantly lower compared to other methods. This study aims to increase cutting speed and efficiency while lowering carbon emissions by controlling the taper angle and enhancing machine sensitivity. The primary objective of this study was to optimize errors caused by the kerf and the taper angle in a waterjet cutting machine, both mechanically and during the cutting process. The goal of increasing precision was achieved successfully. A 10 mm thick SS314 Steel was processed using a CNC waterjet cutting device. Mechanical compensation was performed using a laser-based algorithm that measured and compensated for values at 10 mm intervals along the X and Y axes. After determining the waterjet's taper angle, the 5-axis cutting head was aligned perpendicular to the edge to ensure accuracy. The most efficient cutting parameters were found to be a pressure of 3.750 bar, an abrasive flow rate of 0.4 kg/m, a 1.02 mm nozzle, a 0.35 mm orifice, and 80 mesh garnet abrasive. The cutting speed was set at 300 mm/min. The taper angle was 1 degree, and the 5-axis machining head was positioned perpendicularly to the material's edge. Cutting was performed by tilting the head by 1 degree to effectively eliminate the taper angle effect. Design FMEA, as defined by the FMEA Tables of the IATF 16949 Automotive Standard, is typically used to identify the most critical characteristics. The patent for this original study is registered with the Patent Office (Patent no: TR 2018 20101). The improvements in cutting angle and precision have increased machine efficiency, which in turn has led to higher cutting speeds and reduced carbon emissions. By controlling the cutting angle, a thinner kerf is created, which leads to a reduction in waste.

**Keywords:** Taper angle, High precision, Compensation, Sustainability, Automation, Machine design

\*Corresponding author: Milteksan CNC A.S., Basibuyuk Mh. Sureyyapasa Basibuyuk Yolu Sk. No:4/1, 34854, Istanbul Türkiye

E mail: ugur.simsir@milteksan.com (U. SIMSİR)

Ugur ŞİMŞİR  <https://orcid.org/0000-0002-0948-6364>

Received: April 30, 2025

Accepted: August 16, 2025

Published: September 15, 2025

**Cite as:** Şimsir U. 2025. Optimizing accuracy of abrasive waterjet cutting system: A comprehensive study on mechanical and software compensation strategies on 5-axis CNC waterjet cutting machine and analysis using design FMEA. BSJ Eng Sci, 8(5): 1493-1503.

### 1. Introduction

Automation and digital transformation are critical for the success of manufacturing firms. Accordingly, the formulation of strategies and the translation of strategic objectives into concrete, actionable initiatives in this domain are of paramount importance (Oner et al., 2024). Using the abrasive water-jet cutting method, all types of materials can be cut efficiently without thermal effects or the need for equipment changes (Ibaraki and Knapp, 2014). The cutting process is performed by directing water, with or without an abrasive onto the material (Rajamani et al., 2022). The water is pressurised by pumps to create high-pressure jet. During the cutting process, a consistent standoff distance between the nozzle and the material, as well as a stable feed rate,

must be maintained. For this purpose, cutting machines are integrated with automatic feed rate control.

Water jet technology was invented in the late 1800s for pressure-water washing in the mining industry. In the early 1900s, experiments were conducted to cut rocks using pressurized water which led to the development of a 500 bar waterjet. Equipment capable of generating 2.750 bar pressure was developed in the 1970s, leading to the first industrial waterjet cutting system in 1972.

Since then, abrasive material has been added to the pressurized water stream, allowing for more efficient cutting of hard materials. Today, waterjet systems can generate pressures up to 6.200 bar.

There are two major cutting methods in waterjet technology: pure waterjet and abrasive waterjet. In pure waterjet cutting, no additional material is added to the



water sprayed onto the workpiece. This method is used for cutting materials with relatively low hardness. In abrasive waterjet cutting, an abrasive (such as garnet) is added to the water stream. This method is used for very hard and thick materials, including titanium-tungsten alloys, carbon steels, and stainless steels. It is also used for soft metals like aluminum, copper, brass, and lead, as well as for hard non-metallic materials like glass, marble, granite, and ceramic.

The physical mechanisms of the abrasive waterjet cutting (AWJ) process are largely imperceptible to operators. In brief, AWJ flow theory describes the jet as a supersonic, three-phase flow consisting of a fluid (water and air).

During cutting and drilling operations, this complex flow is rapidly confined to a limited area. Boundary conditions fluctuate, and pressurized water exhibits approximately 15% compressibility at 400 MPa.

The hydraulic power ( $P$ ), required to produce a high-velocity water jet through an orifice is directly proportional to the product of the pressure ( $p$ ) and the flow rate ( $Q$ ) (equation 1):

$$P = p \frac{Q}{c} \quad (1)$$

In this equation,  $c$  is a constant equal to 60, where  $P$  is expressed in kilowatts (kW),  $p$  in megapascals (MPa), and  $Q$  in liters per minute (l/min). For instance, the flow rate can be determined using Bernoulli's principle. The flow rate through an orifice with cross-sectional area  $A$  is given by equation 2.

$$Q = c_d A \sqrt{2p/\rho} \quad (2)$$

Where  $A = d^2/4$ , with  $d$  denoting the orifice diameter,  $c_d$  representing the discharge coefficient (typically 0.65),  $p$  indicating the pressure, and  $\rho$  denoting the density of water.

According to the KMT waterjet manual and operation book, waterjet technology can meet the cutting needs of various industries and handle complex shapes. Abrasive waterjet cutting systems integrated with CNC (computer numerical controlled) systems can easily perform cutting procedures involving numerous zigzags, sharp edges, very narrow angles, and very small diameters (Hsu and Lei, 2003; Lianjun et al., 2014; Polzer et al., 2014; Lu et al., 2020)

The primary benefit of waterjet technology is its nature as a cold cutting process. Because the material experiences neither mechanical nor thermal stress during cutting, no post-cutting stress relief treatments are required. Any material can be cut without creating a thermal effect (Ibaraki and Ota, 2014). Thus, undesired hardening, burns, deformations, any droplets, molten metal residues and poisonous gas on the material are eliminated. Moreover, the sandwich materials and the material pairs with different combustion points or melting temperatures, which cannot be cut using laser or plasma technology, can be cut with a water jet (Rajamani et al., 2022).

Krajcarz D. discusses the comparison of waterjet cutting with laser and plasma cutting. This reference could be cited in the introduction when discussing environmental benefits and cold cutting nature of waterjet technology are mentioned, particularly in sentences like "Metal Cutting technology with Abrasive Waterjet is more environmentally friendly than other plasma and laser cutting methods and is a cold cutting method that does not use flammable gases." or "Moreover, the sandwich materials and the material pairs with different combustion points or melting temperatures, which cannot be cut using laser or plasma technology, can be cut with a water jet". (Krajcarz D. 2014)

Wan L et al. examine the sustainable and clean applications of steel slag for abrasive waterjet machining. It could be cited when discussing the study's objectives related to sustainability and carbon emission reduction. Relevant sentences include "With this study, it is aimed to increase the cutting speed by controlling the taper angle and increasing the machine sensitivity, and to increase efficiency and lower carbon emission." or "Since there is an improvement in cutting angle and cutting precision, the machine efficiency is increased, therefore the cutting speed is increased and carbon emission is reduced. (Wan et al., 2023)

Yun et al. (2019) focus on optimizing energy consumption with hybrid laser-waterjet. It can be cited when discussing energy efficiency and carbon emission reduction in your paper. A suitable place for this reference would be in the conclusion, for example, after the sentence "Thanks to patented mechanical improvements and software compensation with taper angle control to achieve the desired precision edge cutting quality, the cutting speed has been increased by approximately 30%, which means that the work is done 30% faster and 30% energy is saved."

Wang et al. (2019) present a comprehensive kinematic error compensation strategy for a double swivel head used in five-axis abrasive water jet (AWJ) machining systems. The authors develop a mathematical model to quantify the kinematic errors resulting from rotary axis deviations and mechanical imperfections. Using a multi-body system approach, the compensation algorithm significantly improves the accuracy of the cutting head's motion, leading to enhanced cutting precision and surface quality. Experimental validation demonstrates that the proposed compensation method reduces angular errors and improves geometric consistency in complex 3D cutting operations.

Chen et al. (2019) address the persistent issue of shape inaccuracies in the cut-in and cut-out regions during abrasive water jet cutting processes. The paper proposes a correction model that accounts for dynamic changes in jet pressure and traverse speed at the beginning and end of the cutting path. By modifying toolpath parameters and introducing predictive error modeling, the authors successfully mitigate edge deformation and discontinuities. Experimental results confirm that the

approach reduces shape deviations, particularly in high-precision applications involving intricate geometries.

Lin et al., focus on the visualization and evaluation of spatial kinematic rotation errors in five-axis abrasive water jet cutting heads. Lin et al., propose a novel method using coordinate measuring techniques and spatial analysis to capture rotational inaccuracies in real time. The study highlights how cumulative errors in the A and C rotary axes affect the final tool orientation and cutting quality. The visualized error distributions provide critical insights for calibration and compensation strategies, ultimately leading to more accurate five-axis AWJ machining (Lin et al. 2021.)

Since no material combustion or melting occurs during the cutting process, no chemical pollution arises. Due to this advantage, there is no need for additional investment such as gas suction, treatment or filtration. The waterjet does not create any direct pressure effect on the material being cut. The mechanical reaction involved in the cutting process operates at a micro-molecular level, preventing material deformation and the formation of burrs, even though the "water jet" carries a high amount of "kinetic energy."

In the waterjet cutting process, the cutting edges have an astonishing clarity, eliminating the need for a deburring process, which would create an additional cost. Due to the very small cutting mark (max 1.1 mm), material losses are minimized. It is possible to cut narrow or sharp edges, depending on the diameter of the waterjet beam. Without any modifications, the same cutting tool can be used to cut materials of different thicknesses simply by changing the cutting speed.

Waterjet cutting experiments were conducted and the results on the response properties of the process variables and second order regression models were evaluated. In addition, statistical multiple response optimization was performed to improve the cutting quality properties.

FMEA is an important technique used by many quality systems. FMEA is a powerful analysis technique for preventing errors by estimating risks. It is based on the principle that the problem arising from the occurrence of the error is perceived as the customer. In FMEA studies, probability, severity and detectability are estimated for all identified faults. Accordingly, actions that should be taken, planned or ignored are evaluated.

Benefits of FMEA:

- It is an important opportunity to make fundamental changes successfully.
- It enhances the product or service's quality, reliability, and safety.
- It boosts the company's image and strengthens its competitiveness.
- It contributes to enhancing customer satisfaction.
- It cuts down on product development time and costs.
- Sets priorities in design development activities.
- Helps initiate corrective and preventive actions.

- Lists potential defects and their effects.

FMEA is performed by a team; it cannot be performed by a single person. This team may consist of 3 to 7 people (preferably 5), and an interdisciplinary approach is useful. The team is led by a team leader, who should be selected from individuals who have received FMEA training.

The FMEA team leader is a guarantor of the method's application and organizes the team's work, including setting the meeting agenda, guiding the meeting, preparing the meeting report, and ensuring the work continues.

Since the aim of FMEA is to prevent known or possible failures are identified before they reach the customer, some predictions need to be made. Prioritization is one of the most important points of the method and there are three criteria for this.

- Probability refers to the frequency at which failure occurs.
- Severity is the seriousness and effects of the error
- Detectability: The ability to detect the error before it reaches the customer

The value of these criteria ranges from 1 to 10. Priority is established based on the Risk Priority Number(RPN), which is determined by evaluating these three criteria.

Once an FMEA is initiated, it becomes a living document. It should be updated whenever there are significant changes in the design and process. A design FMEA can be considered complete when it is approved for production and a start date is given. (Karacan et al., 2021)

A process FMEA is considered complete once all operations have been identified and evaluated, and critical and important features are incorporated into the quality control plans.

One of the most competitive sectors is the automotive industry. The FMEA technique is explained much more effectively in the IATF 16949 standard, which is the automotive quality management system. The basic functions of the product are defined as special characteristics in this standard and in this framework, special characteristics are definitely addressed in quality studies. In addition, unlike the classical FMEA, corrective action is not only initiated when the risk priority number is above 100, but also when the severity value is 8 and above. In order to calculate the probability severity and detectability values more accurately, the relevant tables have been prepared in a much more understandable way in this standard. For these reasons, the FMEA developed for the automotive quality standard was used in our study (Table 1).

**Table 1.** FMEA Scoring Criteria Based on IATF 16949

PFM	PFE	Sev	Prob.	Det.	RPN	Prevention/Action	Responsible/ Termin	Sev.	Prob.	Det.	RPN
Taper angel	All products can be scrap.	8	5	4	160	A axis mut be calibrated	U. Simsir	8	2	4	64
Cutting edge quality	Some products can be scrap.	8	4	4	128	Cutting pressure and sutting speed must be calirated	U. Simsir, Software developer, R&D Manager	8	2	4	64
Nozzle Size	Cutting velocity may be low	7	4	4	112	Nozzle size must be checked and changed	U. Simsir, R&D Manager	7	2	4	56
Orifice size	Cutting pressure may be low	7	3	3	63	Orifice failure must be checked and may need to change orifice	No Corrective action is required				
Abrasive grade	Cost may be increased	6	3	3	54	Abrasive mesh size must be checked and may need to change abrasive	No Corrective action is required				

PFM= potential failure mode, PFE= potential failure effect

The flowchart of the developed system is as follows:

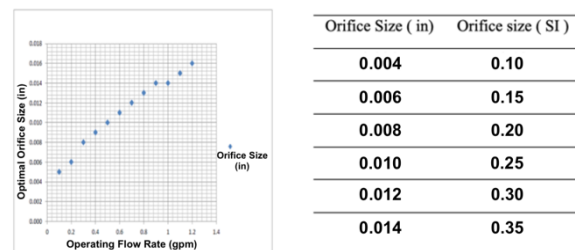
1. The distance of the material from the nozzle is measured.
2. The material type and thickness are input into the CNC software.
3. The software selects the cutting pressure and cutting speed.
4. The taper angle is calculated based on the distance from the nozzle tip to the lowest surface of the material.
5. The taper angle is compensated with the 5th axis (A-axis) to be perpendicular to the material's cut surface.
6. Mechanical errors in the X and Y axes are compensated by the volumetric error software.
7. While the X, Y, and C axes work interpolated during cutting, the A-axis is automatically positioned to remain perpendicular to the cut material surface.

The patent of this original study is registered by the Turkish Patent Office (Simsir and Bicer, 2018 Patent no: TR 2018 20101)

## 2. Operating principles and functionality of water jet cutting systems

In water jet cutting systems, a high-pressure pump produces a water pressure of 4.000-6.000 bar. This pressurized water is driven through steel pipes to the pneumatic valve on the machine's cutting head. Using the valve control, the pressurized water in the cutting head is transferred through the orifice and mixed with the abrasive. The material is then cut by the water exiting the nozzle. This process is referred to as a "water jet" because the flow rate of water passing through the orifice is increased to approximately three times the speed of sound. The inner diameter of the orifice typically ranges from 0.20 mm to 0.35 mm. The selection of the orifice size, which depends on the pump capacity and the

amount of water transferred, is a crucial factor for ensuring a high-quality cutting process (Figure 1). Garnet abrasive with a particle size of 80–120 mesh is transported along with the water exiting the orifice through the nozzle, which has an inner diameter typically ranging from 0.60 mm to 1.02 mm. Approximately 2–3 mm is maintained between the tip of the nozzle and the material, and cutting is performed by the water exiting the nozzle. Abrasive carrying systems are used not only to transport the abrasive by creating a vacuum with the waterjet exiting the orifice but also to achieve a more uniform flow. The abrasive-infused waterjet, propelled by the nozzle, must move forward at a consistent speed.



**Figure 1.** The connection between orifice diameter and flow rate min (Liu et al., 1998).

## 3. Materials and Methods

### 3.1. Nozzle Movability and Cutting Speed Selection Experiments

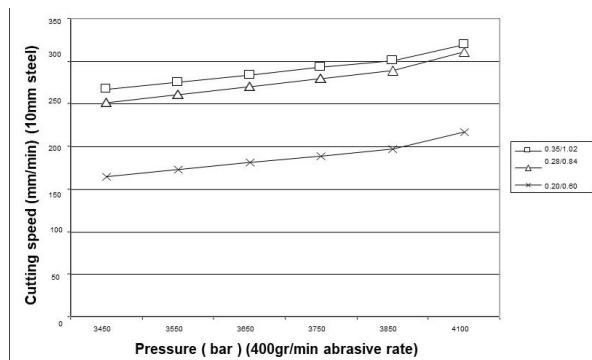
During the cutting process, if the surface geometry is uneven — for example, if there are dips or bulges — the distance between the nozzle and the surface changes. This variation negatively affects the cutting quality, as consistent stand-off distance is critical for optimal results. Ideally, the nozzle should maintain a constant distance of 2–3 mm from the surface. To address this, an obstacle detection sensor is used to detect any

irregularities in the surface. When a hole or a raised area is detected, the sensor adjusts the nozzle's position accordingly to maintain a consistent cutting height.

The nozzle ability to move adaptively offers several advantages, including:

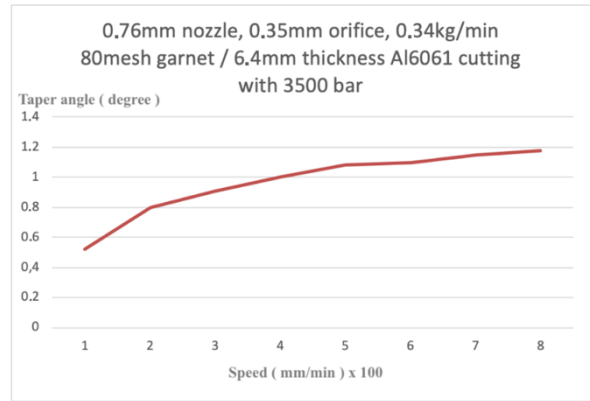
- i. improved quality of cutting
- ii. reduces costs (e.g. lower garnet and energy consumption)
- iii. reduced time loss
- iv. increased the cutting speed

Cutting speed varies depending on the purpose. In this case, because the cutting surface quality of the material was not a primary concern and the goal was simply to separate the parts, the fastest cutting speed (rough cut) was selected. In addition, the nozzle-orifice combination and abrasive flow rate were selected based on experiments conducted for this application to achieve the optimum cutting speed. Nozzles with inner diameter of 0.60-1.02mm and sapphire orifices with a beam diameter of 0.20mm (0.008") and 0.35mm (0.014") were used. Tests conducted at different pressures and flow rates determined that the optimum cutting process is achieved with a pressure of 3.750 bar and abrasive flow rate 0.4kg/m a 1.02mm nozzle diameter and 0.35mm orifice beam diameter. The results are presented in Figure 2.



**Figure 2:** Cutting speeds for 10mm thickness of SS314 steel with different orifice-nozzle combinations and abrasive flow rates.

The waterjet's taper angle was aligned perpendicular to the cutting edge using a 5-axis cutting head. The conic angle of the waterjet was set to 1 degree and oriented to compensate for taper formation. By tilting the cutting head precisely by 1 degree, the taper effect was effectively eliminated, resulting in a perpendicular cut profile. Accordingly, the most efficient cutting level parameters are shown in Figure 3.



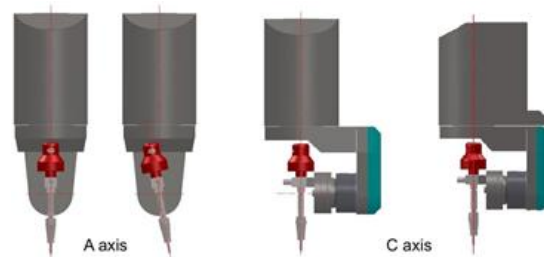
**Figure 3.** Taper angle as a function of speed.

**3.2. Volumetric Compensation**

In the aviation industry, automation systems are required for the high accuracy of the workpieces and machines. Complex 5-axis machines are used for machining large workpieces and their accuracy depends on the tolerances determined during production and the temperature created during the machining. Volumetric compensation has become a fundamental necessity in especially in the aviation industry to meet these demands (Bohez, 2001; Givi and Mayer, 2014).

**3.3. Improvement of Machining Tolerances**

Machining errors can occur for many reasons, including design, kinematics, and assembly-related features. Volumetric compensation is a feature used for correcting machining errors (Ibaraki et al., 2012; Harnicarova et al., 2013;). The best way to correct some of these errors is to design the machine correctly, but this is not always possible or economical. The C and A axes of the machine designed to adjust the taper angle are shown in Figure 4.



**Figure 4.** Nozzle structure C and A axes.

Figure 5 shows that the taper angle is adjusted to be perpendicular to the material's edge using these axes.

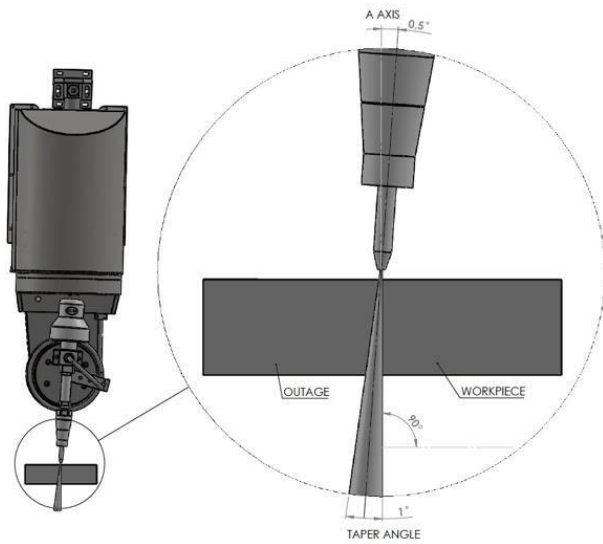


Figure 5. Tapering angle of Nozzle and correction.

### 3.4. Easy Volumetric Compensation

Machining errors may have various reasons related to design, kinematics or assembly. "Easy Volumetric Compensation" allows the correction of machining errors. The best way to fix some of these errors is to design the machine correctly. However, sometimes this may not always be possible or economical. It may also be impossible or unacceptably expensive to increase the hardness for medium- or large-sized machines. For this reason, classical error correction instruments should be integrated with more advanced instruments that allow adaptation to different machine designs, even with certain kinematics. In general, "Easy Volumetric Compensation" requires the identification of 25 control points per axis. For this reason, the calibration of points and their integration into the CNC are much faster. This process improves machine accuracy and is more economical because it eliminates the need to hire third-party companies to perform the measurements. Much more accurate and reliable improvements are achieved in many machines, even without achieving the accuracy levels of Standard Volumetric Compensation (Figure 9) (Lianjun et al., 2014). The waterjet cutting machine used in this study is shown in Figure 6. Since the appropriate temperature conditions were selected, no temperature compensation was performed.

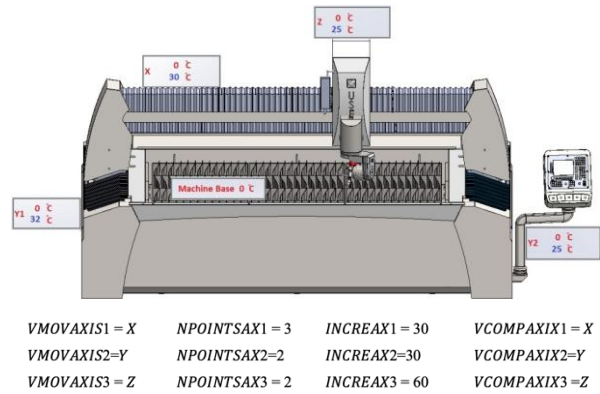


Figure 6. Waterjet Cutting machine structure and axes.

Volumetric compensation is a method used to improve the accuracy of CNC machine tools by correcting geometric and kinematic deviations. Depending on the machine size and required precision, different levels of volumetric compensation can be applied. This section classifies them as small, medium, and large, based on the complexity and calibration requirements.

### 3.5. Small-Scale Volumetric Compensation

This is the simplest and fastest volumetric compensation method, generally applied to compact machines or systems requiring basic correction. The compensation volume is defined using 25 control points.

It corrects basic transmission and alignment errors.

Compensation tables are generated by the calibration software and can be manually edited on the CNC interface. The machine manufacturer must define essential parameters such as the axes involved, position ranges, and compensation limits.

Although this method allows for rapid calibration, it is less accurate than more advanced techniques.

### 3.6 Medium-Scale Volumetric Compensation (Volumes >10 m<sup>3</sup>)

Medium-scale compensation is suitable for larger machines where more complex geometric deviations must be corrected.

It targets compensation volumes larger than 10 m<sup>3</sup>.

A total of 21 geometric errors are corrected, including axis misalignments, squareness deviations, and rotational shifts.

Compensation tables are generated by the calibration software but cannot be edited through the CNC interface.

The manufacturer only needs to specify the relevant axes. All other parameters are embedded within the calibration application by default.

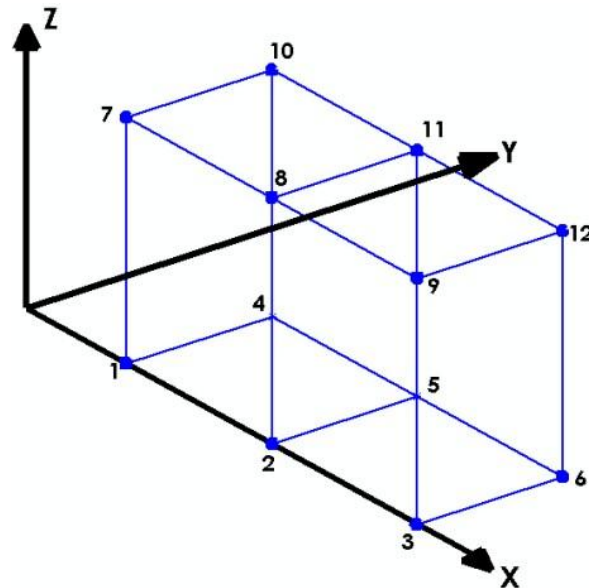
This method provides higher accuracy compared to small-scale compensation but requires a longer calibration process.

### 3.7. Large-Scale Volumetric Compensation (Very Large Machines)

Large-scale compensation follows the same methodology as medium-scale, but is applied to machines with significantly greater working volumes and higher demands on geometric accuracy.

Applicable for machine volumes well above 10 m<sup>3</sup>, typically in aerospace or heavy-industry equipment. All 21 geometric error types are compensated in greater detail. Calibration and compensation are fully handled through dedicated software, with minimal manual input on the CNC side. Provides the highest level of geometric correction, but requires the most time-intensive calibration process. Volumetric compensation, when properly implemented, allows manufacturers to significantly reduce geometric deviations. By classifying compensation methods based

on machine size and complexity, users can select the most appropriate approach to ensure optimal machine accuracy. The main reasons for the lack of “accuracy”, and “precision” in a “machine tool” might arise from the geometrical failures that might originate from production montage, misuse and the production machine wearing that might cause early deformation (Liu et al., 1998; Liu et al., 2014; Öner et al., 2024). Volumetric compensation chart is shown in Figure 7.



	1	2	3	4	5	6	7	8	9	10	11	12
ErrorX	-0.1684	-0.1440	-0.1550	-0.1632	-0.1646	-0,1860	-0.1780	-0.1566	-0.1577	-0.0018	-0.0091	-0.0285
ErrorY	0.0703	0.0932	0.0964	0.0904	0.0952	0.0993	0.0090	0.1150	0.1145	0.0542	0.0930	0.0895
ErrorZ	-0.0135	-0.0109	-0.0009	-0.0028	0.0080	-0.2230	0.0013	0.0012	0.0012	-0.0235	-0.0162	-0.0033

Figure 7. Volumetric compensation chart.

"Volumetric compensation" corrects geometric errors by enhancing the accuracy and reproducibility of machines. The volume to be compensated is defined as a point cluster, where the error is measured and corrected at each point. Standard volumetric compensation can be costly, as it requires significant time and expensive equipment to collect and apply all the compensation points to the CNC. While this cost is generally accepted for large, high-performance machine tools, it is harder to justify for smaller or medium-sized machines. Therefore, depending on the end-user's needs, a simpler volumetric compensation algorithm can be employed. This algorithm simplifies the process and is adaptable to any machine (Lianjun et al., 2014; Patel and Ehman, 1997; Polzer et al., 2014; Veldhuis and Elbestawi, 1995). Easy volumetric compensation limits the number of points to 25 per axis, making calibration faster and integration into the CC simpler. It significantly improves

accuracy while being quicker and easier to implement, which is a key benefit for small-sized machines. Although traditional ball screw and cavity compensation can still be used, it is not necessary, as volumetric compensation already addresses these errors. However, these compensation tables can also be used and automatically integrated into the CNC compensation matrix (Liu et al., 1998).

#### 4. Modeling the volumetric errors

In modeling the volumetric errors of machines, a "1 error twist" is applied to each "component error" of the driver. After aligning the twist bends with the working space, they are processed through the "kinematic model" of the "five-axis machine" as described in (Bohez, 2001; Bohez 2002; Feng et al., 2019; Rajamani et al., 2022).

##### 4.1.Measurement and representation of geometric error for 5 axis machine

An unconstrained rigid object has six degrees of freedom (DOF) in space. As a result, any linear axis is associated

with six error components corresponding to these six DOFs. For example, a linear motion along the X-axis can cause three translational errors ( $\delta x(x)$ ,  $\delta y(x)$ ,  $\delta z(x)$ ) in the X, Y, and Z directions and three rotational errors ( $\epsilon x(x)$ ,  $\epsilon y(x)$ ,  $\epsilon z(x)$ ) about the X, Y, and Z axes. In the 5-axis machine, the rotation of the Cutting Head about z is defined as the C-axis, and the rotation about x and y axes is defined as the B-axis as shown in Figure 8. These six errors are also called position-dependent geometric errors (PDGEs). As a result, for a typical 5-axis machine tool, there are a total of twenty-one geometric error components, including three squareness errors (Sxy, Sxz, Syz) caused by assembly imperfections between the three linear axes. These errors are measured and compensated by the software. The three squareness errors are constant regardless of the command position of the linear axes and are also called position-independent geometric errors (PIGEs) (Hsu and Wang, 2007; Polzer et al., 2014; Okafor and Ertekin, 2000; Ramesh et al., 2000; Schwenke et al., 2008).

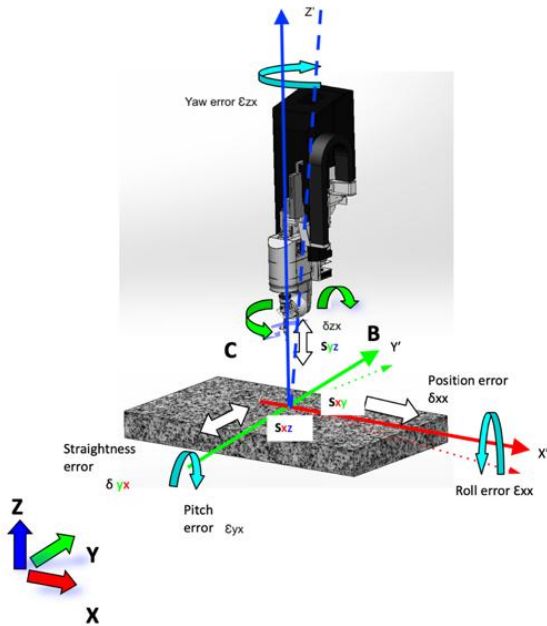


Figure.8. Position-dependent geometric errors (PDGEs).

4.2. Error twists ( $\xi_e$ )

The "geometric errors" between the "tool-tip" and the "workpiece" are represented using error twists. Consider that the "machine" is ideally positioned at "Pd" and that the "linear axis" or "rotational axis" of the "rotary drive" is ideally directed along " $\omega_d$ ." However, the "actual machine position" might be at "P," and the "axis" could exhibit an "angular error" denoted by " $\theta_e$ ." The "ideal twist" ( $\xi_d$ ) and the "actual twist" ( $\xi$ ), which include these geometric errors, are expressed as follows equation 3:

$$\xi_d = [q_d \times \omega_d \omega_d]^T, \xi = [q \times \omega \omega]^T \tag{3}$$

The conversion from the "ideal twist"  $\xi_d$  to the "actual twist"  $\xi$  results from an "error motion" represented by ( $e^\wedge \xi \theta_e$ ). The "error twist" ( $\xi_e$ ) includes both an "angular geometric error" ( $\theta_e$ ), which lies along the "common perpendicular" between the "ideal" and "actual axis lines," and a "linear positional error" ( $d$ ) of the "axis." The "error twist" is expressed as  $\xi_e = [v_e \ \omega_e]^T$  (equations 4 and 5).

$$\omega_e = \frac{\omega_d \times \omega}{\sin \theta_e}, h_e = \frac{d}{\theta_e} = \frac{|q - q_d|}{\theta_e} \tag{4}$$

$$v_e = \frac{q \times (\omega_d \times \omega)}{\sin \theta_e} + h_e \frac{\omega_d \times \omega}{\sin \theta_e} = \frac{q_d \times q}{d} + \frac{q - q_d}{\theta_e} \tag{5}$$

If the "angular geometric error" is zero ( $\theta_e = 0$ ), the "actual axis line" becomes parallel to the "ideal axis line." As a result, the "twist vector" consists solely of "translational errors" ( $\omega_e = 0$ ,  $h_e = \infty$ ), and the "error twist" simplifies to equation 6:

$$\xi_e = \begin{bmatrix} \frac{q - q_d}{d} \\ 0 \end{bmatrix} \tag{6}$$

When " $d = 0$ ," meaning " $h_e = 0$ " and " $q = q_d$ ," the "actual" and "ideal axes" align perfectly with no "linear errors." Consequently, the "error twist" reflects only "angular errors." (equation 7).

$$\xi = \begin{bmatrix} v_e \\ \omega_e \end{bmatrix} = \begin{bmatrix} q \times (\omega_d \times \omega) \\ \omega_d \times \omega \end{bmatrix}^T \tag{7}$$

For example, if the "rotary drive" (C) has a "tilt error" ( $\epsilon_{yc}$ ) about the "Y-axis," the "coordinate system" of the "C drive" is defined in the "MCS" as " $q = [O_x \ O_y \ O_z]^T$ ." According to equation 8, " $\omega_e$ " and " $v_e$ " are given as follows:

$$\omega_e = [0 \ 1 \ 0]^T$$

$$v_e = q \times \omega_e = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{8}$$

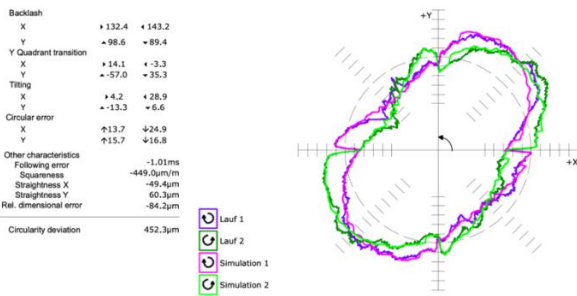
As a result, this leads to the formation of the "error twist" expressed as " $\xi_e = [v_e \ \omega_e]^T$ ." The "geometric errors" for all "axes" can be described similarly Table 2 presents the "geometric errors" of the "AC axis table" that provides tilting for the "five-axis machine tool". (Xiang and Altintas, 2016; Yang and Altintas, 2013; Yang et al., 2015).

**Table 2.** Error twist components for the AC axis of the five-axis rotary table

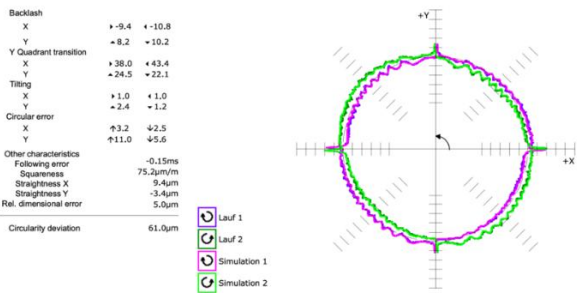
X- axis	Y- axis	Z- axis	Squareness Error	A- axis	C- axis	PIGEs Rotary axes
$\delta_{xx}$	$\delta_{yy}$	$\epsilon_{zz}$	$S_{xy}$	$\delta_{xa}$	$\Delta_{xc}$	$\delta_{xoc}$ $S_{boa}$
$\delta_{yx}$	$\delta_{xy}$	$\epsilon_{xz}$	$S_{yz}$	$\delta_{ya}$	$\Delta_{yc}$	$\delta_{yoc}$ $S_{coa}$
$\delta_{zx}$	$\delta_{zy}$	$\epsilon_{yz}$	$S_{xz}$	$\delta_{za}$	$\Delta_{zc}$	$\delta_{zoc}$ $S_{coca}$
$\epsilon_{xx}$	$\epsilon_{xy}$	$\epsilon_{xz}$		$\epsilon_{xa}$	$\epsilon_{xc}$	$\Delta_{zoa}$ $S_{boc}$
$\epsilon_{yx}$	$\epsilon_{yy}$	$\epsilon_{yz}$		$\epsilon_{ya}$	$\epsilon_{yc}$	(ISO 230-7)
$\epsilon_{zx}$	$\epsilon_{zy}$	$\epsilon_{zz}$		$\epsilon_{za}$	$\epsilon_{zc}$	

**5. Results**

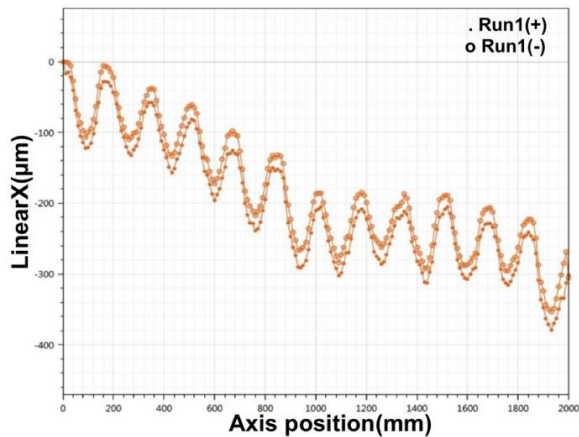
The squareness measurement results made before and after the improvements are shown in Figure 9 and Figure 10 and. The measurement and software precision improvement results made before and after the improvements are shown in Figures 11, 12, 13 and 14.



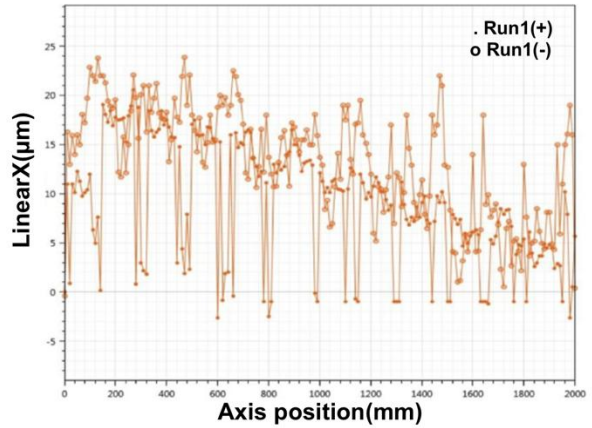
**Figure 9.** Squareness test, before compensation.



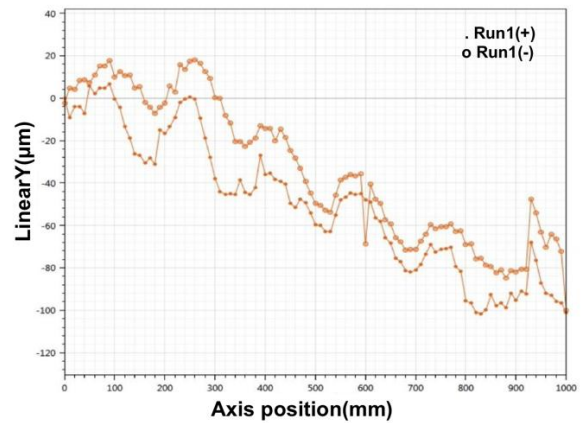
**Figure 10.** Squareness test, after compensation.



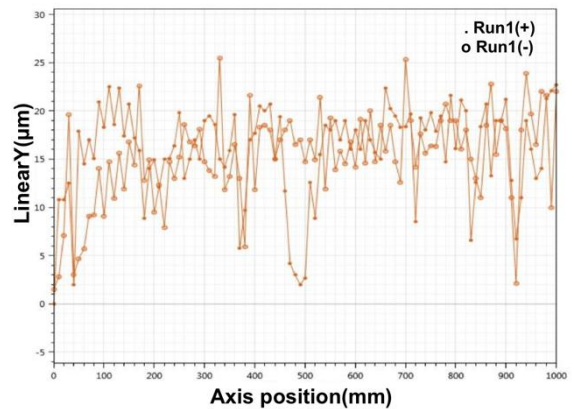
**Figure 11.** X axis, before software compensation.



**Figure 12.** X axis, after software compensation.



**Figure 13.** Y axis, before software compensation.



**Figure 14.** Y axis, after software compensation.

Scanning Electron Micrograph Experiments showed that for higher cut surface quality with abrasive waterjet, there is a need to reduce the cutting speed of the material with more precise positioning. In this study, the results were compared by making the tapering angle perpendicular to the cutting edge on a 5-axis CNC waterjet machine. This machine was chosen as the workpiece material and its mechanical sensitivity was increased by compensation.

Cuts were made on a sample in three different ways: with taper and without compensation, with taper and with compensation, and with vertical and with compensation. The optimum control parameter settings were: water pressure = 3.500 bar, abrasive flow rate = 400 g/min, nozzle inner size = 1.01 mm, and orifice = 0.35 mm. Scanning electron microscopic (SEM) results of the cutting surface are shown in Figure 15. The experimental results definitively demonstrate the success of this application, yielding a significantly improved surface quality, with Ra values improving from Ra=400 µm (a) to Ra=25 µm (c). A more precise measurement tolerance was achieved, leading to superior edge cutting quality at higher speeds and a notable increase in machine efficiency.

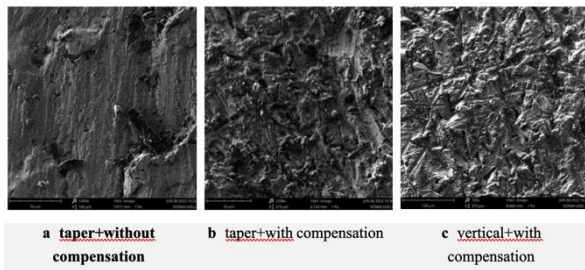


Figure 15. SEM photographs of the “cutting surface”.

From the scanning electron micrographs, it is understood that when the abrasive hits the hard steel workpiece with high pressure, brittle fracture occurs and the surface is not fully worn during the angled and inaccurate positioning cut. It has been observed that fracture and abrasion are better in cutting by making perpendicular and precise positioning to the cutting surface.

## 6. Conclusion

The widespread adoption of waterjet cutting machines globally is often hindered by inefficiencies stemming from the waterjet’s inherent taper angle. This taper angle significantly compromises cutting surface quality as cutting speed increases. This study successfully addressed these limitations by implementing a novel approach: orienting the conical water jet exiting the nozzle perpendicularly to the cutting surface and incorporating mechanical compensation to enhance machining tolerances in waterjet systems.

The experimental results definitively demonstrate the success of this application, yielding a significantly improved surface quality. A more precise measurement

tolerance was achieved, leading to superior edge cutting quality at higher speeds and a notable increase in machine efficiency. The utilization of a 5-axis nozzle, coupled with axis compensation, proved highly effective, reducing the cutting tolerance from 400µm to a remarkable 25µm. This breakthrough in achieving more precise cutting at elevated speeds significantly expands the application areas for waterjet cutting machines.

Furthermore, the patented mechanical improvements and software compensation with taper angle control, which enable the desired precise edge cutting quality, have resulted in approximately a 30% increase in cutting speed. This translates directly to a 30% reduction in processing time and a corresponding 30% energy saving. Additionally, the reduction in kerf due to controlled cutting angles has led to a decrease in waste and material savings. As a future endeavor, further studies could explore optimizing process duration by experimenting with various taper angle values under pressures up to 6.000 bar.

## Author Contributions

The percentages of the author’ contributions are presented below. The author reviewed and approved the final version of the manuscript.

	U.Ş.
C	100
D	100
S	100
DCP	100
DAI	100
L	100
W	100
CR	100
SR	100
PM	100
FA	100

C=Concept, D= design, S= supervision, DCP= data collection and/or processing, DAI= data analysis and/or interpretation, L= literature search, W= writing, CR= critical review, SR= submission and revision, PM= project management, FA= funding acquisition.

## Conflict of Interest

The author declared that there is no conflict of interest.

## Acknowledgements

The author would like to thank Prof. Dr. Ufuk Cebeci for his valuable comments and support.

## Ethical Consideration

Ethics committee approval was not required for this study because of there was no study on animals or humans.

References

- Bohez ELJ. 2001. Compensating for systematic errors in 5-axis NC machining. *Comput-Aided Des*, 34: 391-403.
- Bohez ELJ. 2002. Five-axis milling machine tool kinematic chain design and analysis. *Int J Mach Tools Manuf*. 42(4): 505-520.
- Chen, et al., 2019. Correcting shape error located in cut in/cut out region in abrasive water jet cutting process. *Int J Adv Manuf Technol*, 2019: 102.
- Feng S, Huang C, et al., 2019. Surface quality evaluation of single crystal 4H-SiC wafer machined by hybrid laser-waterjet: Comparing with laser machining. *Mater Sci Semicond Process*, 93: 238-251.
- Givi M, Mayer JRR. 2014. Validation of volumetric error compensation for a five-axis machine using surface mismatch producing tests and on-machine touch probing. *Int J Mach Tools Manuf*, 87: 89-95.
- Harničárová M, Valčíček J, et al., 2013. Comparison of non-traditional technologies for material cutting from the point of view of surface roughness. *Int J Adv Manuf Technol*, 69: 81-91.
- Hsu YY, Lei WT. 2003. Accuracy enhancement of five-axis CNC machines through real-time error compensation. *Int J Mach Tools Manuf*, 43: 871-877.
- Hsu YY, Wang SS. 2007. A new compensation method for geometry errors of five-axis machine tools. *Int J Mach Tools Manuf*, 47: 352-360.
- Ibaraki S, Iritani T, Matsushita T. 2012. Calibration of location errors of rotary axes on five-axis machine tools by on-the-machine measurement using a touch-trigger probe. *Int J Mach Tools Manuf*, 58: 44-53.
- Ibaraki S, Knapp W. 2012. Indirect measurement of volumetric accuracy for three-axis and five-axis machine tools: a review. *Int J Autom Technol*, 6(2): 110-124.
- Ibaraki S, Ota Y. 2014. A machining test to calibrate rotary axis error motions of five-axis machine tools and its application to thermal deformation test. *Int J Mach Tools Manuf*, 86: 81-88.
- Karacan İ, Erdoğan İ, İğdil M, Cebeci U. 2021. Machine vision supported quality control applications in rotary switch production by using both process FMEA and design FMEA. *Nat Appl Sci J*, 4(2): 16-31.
- KMT Waterjet Manual and Operation Books. UK, London, UK, pp: 25-26.
- Krajcarz D. 2014. Comparison metal water jet cutting with laser and plasma cutting. *Procedia Eng*, 69: 838-843.
- Lianjun Z, Chunli H, Guangjun C. 2014. Application of tool compensation in CNC machining. *Mater Sci Forum*, 800-801: 435-439.
- Lin W, Lei Y, Zhang S, Wu Z. 2021. Visualization and evaluation of the spatial kinematic rotation error of a five axis abrasive water jet cutting head. *Int J Adv Manuf Technol*, 114: 3217-3228.
- Liu HT, Miles P, Veenhuizen SD. 1998. CFD and physical modeling of UHP AWJ. Jenny Stanford Publishing, Singapore, pp: 121-137.
- Liu ZF, Li DD, Liu ZZ. 2014. Gantry machining tool assembly method and predictive optimization based on multi-body system. *Comput Integr Manuf Syst*, 20: 394-400.
- Lu H, Cheng Q, Zhang X, Liu Q, Qiao Y, Zhang Y. 2020. A novel geometric error compensation method for gantry-moving CNC machine regarding dominant errors. *Processes*, 8(8): 906.
- Okafor AC, Ertekin YM. 2000. Derivation of machine tool error models and error compensation procedure for three axes vertical machining center using rigid body kinematics. *Int J Mach Tools Manuf*, 40: 1199-1213.
- Öner M, Cebeci U, Doğan O. 2024. BSC-based digital transformation strategy selection and sensitivity analysis. *Mathematics*, 12(2): 225.
- Patel AJ, Ehman KF. 1997. Volumetric error analysis of a Stewart platform-based machine tool. *Ann CIRP*. 46(1).
- Polzer A, Piska M, Dufkova K. 2014. On the modern CNC milling with a compensation of cutting tools deflections. *DAAAM Int Sci Book*, 2014: 311-322.
- Rajamani D, Balasubramanian E, Dilli Babu G, Ananthakumar K. 2022. Experimental investigations on high precision abrasive waterjet cutting of natural fibre reinforced nano clay filled green composites. *J Ind Text*, 51(3\_suppl): 3786-3810.
- Ramesh R, Mannan MA, Poo AN. 2000. Error compensation in machine tools: a review. *Int J Mach Tools Manuf*, 40(9): 1235-1256.
- Schwenke H, Knapp W, Haitjema H, Weckenmann A, Schmitt R, Delbressine F. 2008. Geometric error measurement and compensation of machines: an update. *CIRP Ann*, 57: 660-675.
- Simsir U, Biçer K. 2018. Patent no: TR 2018 20101 B CNC waterjet cutting machine with six-axis movement capability. Turkish Patent and Trademark Office, Ankara, Türkiye, pp:15-25.
- Veldhuis SC, Elbestawi MA. 1995. A strategy for the compensation of errors in five-axis machining. *Ann CIRP*, 44(1): 373-377.
- Wan L, Xiong J, et al., 2023. Feasible study on the sustainable and clean application of steel slag for abrasive waterjet machining. *J Clean Prod*, 420: 138378.
- Wang, et al., 2019. Kinematic error compensation of a double swivel head in five axis abrasive water jet machine tool. *Int J Adv Manuf Technol*, 103: 2783-2793.
- Xiang S, Altintas Y. 2016. Modeling and compensation of volumetric errors for five-axis machine tools. *Int J Mach Tools Manuf*, 101: 65-78.
- Yang J, Altintas Y. 2013. Generalized kinematics of five-axis serial machines with non-singular tool path generation. *Int J Mach Tools Manuf*, 75: 119-132.
- Yang J, Mayer JRR, Altintas Y. 2015. A position independent geometric errors identification and correction method for five-axis serial machines based on screw theory. *Int J Mach Tools Manuf*, 95: 52-66.
- Yun H, Zou B, Wang J, Huang C, Li S. 2019. Optimization of energy consumption in coating removal for recycling scrap coated cemented carbide tools using hybrid laser-waterjet. *J Clean Prod*, 229: 104-114.