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Availability analysis of a consecutive three stages deteriorating standby system considering maintenance and replacement

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Abstract

This paper deals with the modelling and evaluation of availability of a system subject to three consecutive stages of deterioration: minor, medium and major deteriorations under minor and major maintenance, and replacement at deterioration and failure respectively. The system has three possible modes: working with full capacity, deterioration and failure mode. In this paper, probabilistic models have been developed to evaluate the relationship between availability and the performance of a standby deteriorating system. Various graphs have been plotted to discover the impact of the deterioration and failure on steady-state availability. The system is analysed using first order linear differential equations.

1. Introduction

The process industry comprises of large complex engineering systems, subsystems arranged in standby, series, parallel or a combination of them. For efficient and economical operation of a process plant, each system or the subsystem should work failure free under the existing operative plant conditions. However, once the system starts, potential fault conditions gradually accumulate leading to degraded performance. The study of a cold standby repairable deteriorating system reliability model is one important model application of reliability theory. In practical engineering applications, most repairable systems are deteriorative that system failure often cannot be as good as new, it is more reasonable for these deteriorating repairable systems to assume that the successive working times of the system after repair will become shorter and shorter while the consecutive repair times of the system after failure will become longer and longer. Most of these systems are subjected to random deterioration which can result in unexpected failures and disastrous effect on the system availability and the prospect of the economy. Therefore it is important to find a way to slow down the deterioration rate, and to prolong the equipment's life span. Maintenance policies are vital in the analysis of deterioration and deteriorating systems as they help in improving reliability and availability of the systems. Maintenance models can assume minor maintenance, major maintenance before system failure, perfect repair (as good as new), minimal repair (as bad as old), imperfect repair and replacement at system failure. For repairable system, availability is a very meaningful measure, and achieving a high or required level of availability is an essential requisite. Improving the availability of system, the production and associated profit will also increase. Increase in production lead to the increase of profit. This can be achieve be maintaining reliability and availability at highest order. To achieve high production and profit, the system should remain operative (availability) for maximum possible duration.
The concept of deterioration and its effect on the reliability and availability model of a repairable system is vital in the study of system performance. For this reason, several models on deteriorating systems under different conditions have been studied by several researchers such as [1], [2], [3], [5], [7], [9], [13], [14]. Analysis of reliability and availability model for deteriorating system have been studied under different conditions such as [4] who investigated reliability analysis of a deteriorating system with delayed vacation of repairman, [8]. A Reliability-based Opportunistic Predictive Maintenance Model for k-out-of-n Deteriorating Systems, [12] proposed the Bayesian reliability estimation for deteriorating systems with limited samples using the maximum entropy approach, [15]. Modelling the reliability and availability characteristics of a system with three stages of deterioration, [16] deal with the study of deteriorating cold standby repairable system with priority in use.

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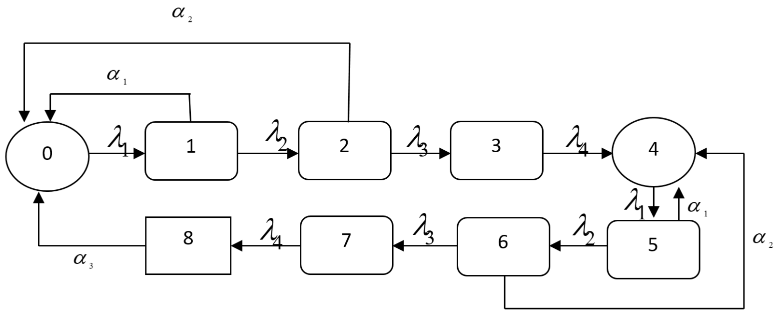

Figure 2.1: Transition diagram of the system with maintenance and replacement

State	Description
S_0	Initial state, one is working, the other unit is on standby. The system is working.
S_1	The working unit is in minor deterioration mode and is under online minor maintenance, the other unit is on standby. The system is working.
S_2	The working unit is in medium deterioration mode and is under online major maintenance, the other unit is on standby. The system is working.
S_3	The working unit is in major deterioration mode. The system is working.
S_4	The working unit has failed; the standby unit is switch to operation. The system is working.
S_5	One unit has failed; the working unit is in minor deterioration mode and is under online minor maintenance. The system is working.
S_6	One unit has failed: the working unit is in medium deterioration mode and is under online major maintenance. The system is working.
S_7	One unit has failed: the working unit is in major deterioration mode. The system is working.
S_8	Both units have failed. The system is inoperative and replaced with new one.

This paper considers a two unit cold standby system with three consecutive stages of deterioration before failure and derived its corresponding mathematical models. Furthermore, we study availability of the system using Kolmogorov's forward equation method. The focus of our analysis is primarily to capture the effect of minor, medium and major deterioration, failure and replacement rates on the availability.

The organization of the paper is as follows. Section 2 contains a description of the system under study. Section 3 presents formulations of the models. The results of our numerical simulations are presented in section 4. Finally, we make some concluding remarks in Section 5.

2. Description and States of the System

In this paper, two unit cold standby system is considered. It is assumed that the system most pass through consecutive stages of deterioration which are minor, medium and major deterioration before failure. It is also assumed that switching is perfect, e.g. never fails and never does any deterioration. Primary units are considered to be repairable. At early state of the system life, the operating unit is exposed to minor deterioration with rate λ_1 and this deterioration is rectified through minor maintenance α_1 which revert the unit to its earliest position before deterioration. If not maintained, the unit is allowed to continue operating under the condition of minor deterioration which later grows to medium deterioration with rate λ_1 . At this stage, the strength of the unit still strong that it can reverted to early state with major maintenance with rate α_2. However, the system can move to major deterioration stage with rate λ_3 where the strength of the unit has decreases to the extent that it cannot be reverted to its early state, neither that of minor nor medium deterioration stages. Here the unit is allowed to continue operation until it fails. Only one primary unit can be served at a time. Each of the primary units fails independently of the state of the others with parameter λ_4. Whenever one of these units fails; the standby unit is switched to operation and the three stages of deterioration which continue until the unit fails and the system is immediately replaced by with a new one with rate α_3.

3. Formulation of the Models

In order to analyse the system availability of the system, we define to be the probability that the system at is in state . Also let be the row vector of these probabilities at time . The initial condition for this problem is:

$$P(0) = [p_0(0), p_1(0), p_2(0), \ldots, p_8(0)] = [1,0,0,0,0,0,0,0,0]$$

We obtain the following differential equations:

$$\begin{aligned}
{p_0}'(t) &= -\lambda_1 p_0(t) + \alpha_1 p_1(t) + \alpha_2 p_2(t) + \alpha_3 p_8(t) \\
{p_1}'(t) &= -(\alpha_1 + \lambda_2) p_1(t) + \lambda_1 p_0(t) \\
{p_2}'(t) &= -(\alpha_2 + \lambda_3) p_2(t) + \lambda_2 p_1(t) \\
{p_3}'(t) &= -\lambda_4 p_3(t) + \lambda_3 p_2(t) \\
{p_4}'(t) &= -\lambda_1 p_4(t) + \lambda_4 p_3(t) + \alpha_1 p_5(t) + \alpha_2 p_6(t) \\
{p_5}'(t) &= -(\alpha_1 + \lambda_2) p_5(t) + \lambda_1 p_4(t) \\
{p_6}'(t) &= -(\alpha_2 + \lambda_3) p_6(t) + \lambda_2 p_5(t) \\
{p_7}'(t) &= -\lambda_4 p_7(t) + \lambda_3 p_6(t) \\
{p_8}'(t) &= -\alpha_3 p_8(t) + \lambda_4 p_7(t)
\end{aligned} \tag{3.1}$$

This can be written in the matrix form as

$$\dot{P} = MP \tag{3.2}$$

where

$$M = \begin{pmatrix}
-\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & \alpha_3 \\
\lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & -\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & -\alpha_3
\end{pmatrix}$$

equation (3.2) is expressed explicitly in the form

$$\begin{pmatrix} {p_0}'(t) \\ {p_1}'(t) \\ {p_2}'(t) \\ {p_3}'(t) \\ {p_4}'(t) \\ {p_5}'(t) \\ {p_6}'(t) \\ {p_7}'(t) \\ {p_8}'(t) \end{pmatrix} = \begin{pmatrix}
-\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & \alpha_3 \\
\lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & -\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & -\alpha_3
\end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \end{pmatrix}$$

The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) is given by

$$A_{V1}(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) \tag{3.3}$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (3.2) becomes

$$MP = 0 \tag{3.4}$$

this is in matrix form

$$\begin{pmatrix}
-\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & \alpha_3 \\
\lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & -\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & -\alpha_3
\end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Subject to following normalizing conditions:

$$p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_4(\infty) + \ldots + p_8(\infty) = 1 \tag{3.5}$$

Following [10] and [11] we substitute equation (3.5) in the last row of equation (3.4) to compute the steady-state probabilities.

$$\begin{pmatrix}
-\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & \alpha_3 \\
\lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & -\lambda_1 & \alpha_1 & \alpha_2 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_1 & -(\alpha_1+\lambda_2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_2 & -(\alpha_2+\lambda_3) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & -\lambda_4 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Solving equation (3.4), we obtain the steady-state probabilities

$$p_0(\infty),\, p_1(\infty), p_2(\infty), p_3(\infty), \ldots, p_8(\infty)$$

The expressions for the steady-state availability involving minor and major maintenance and replacement given in equations (3.3) above is given by

$$A_{V1}(\infty) = \frac{2\alpha_3\lambda_4(\alpha_2\lambda_2+\lambda_2\lambda_3+\alpha_1\lambda_3+\alpha_1\alpha_2)+2\alpha_3\lambda_1\lambda_4(\alpha_2+\lambda_3)+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3}{\lambda_1\lambda_2\lambda_3\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_3\lambda_4+2\alpha_2\alpha_3\lambda_1\lambda_4+2\alpha_3\lambda_2\lambda_3\lambda_4+2\alpha_2\alpha_3\lambda_2\lambda_4+2\alpha_1\alpha_3\lambda_3\lambda_4+2\alpha_1\alpha_2\alpha_3\lambda_4}$$

Special case:

(i) System with replacement only at failure

$$A_{V2}(\infty) = \frac{\alpha_3\lambda_2\lambda_3\lambda_4+2\alpha_3\lambda_1\lambda_3\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3}{\lambda_1\lambda_2\lambda_3\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_3\lambda_4+2\alpha_3\lambda_2\lambda_3\lambda_4}$$

(ii) System with minor maintenance and replacement

$$A_{V3}(\infty) = \frac{2\alpha_3\lambda_3\lambda_4(\alpha_1+\lambda_2)+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3+2\alpha_3\lambda_1\lambda_3\lambda_4}{\lambda_1\lambda_2\lambda_3\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_3\lambda_4+2\alpha_3\lambda_2\lambda_3\lambda_4+2\alpha_3\lambda_2\lambda_3\lambda_4+2\alpha_1\alpha_3\lambda_3\lambda_4}$$

(iii) System with major maintenance and replacement

$$A_{V4}(\infty) = \frac{2\alpha_3\lambda_1\lambda_4(\alpha_2+\lambda_3)+2\alpha_3\lambda_2\lambda_4(\alpha_2+\lambda_3)+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3}{\lambda_1\lambda_2\lambda_3\lambda_4+2\alpha_3\lambda_1\lambda_2\lambda_3+2\alpha_3\lambda_1\lambda_2\lambda_4+2\alpha_3\lambda_1\lambda_3\lambda_4+2\alpha_2\alpha_3\lambda_1\lambda_4+2\alpha_3\lambda_2\lambda_3\lambda_4+2\alpha_2\alpha_3\lambda_2\lambda_4}$$

4. Numerical Examples

Numerical examples are presented to demonstrate the impact of deterioration, failure and replacement rates on steady-state availability based on given values of the parameters. For the purpose of numerical example, the following sets of parameter values are used:

Case 1: We fix $\alpha_1 = 0.3$, $\alpha_2 = 0.3$, $\alpha_3 = 0.04$, $\lambda_2 = 0.5$, $\lambda_3 = 0.9$, $\lambda_4 = 0.2$ and vary λ_1 between 0 and 1 as in Figure 4.1.
Case 2: We fix $\alpha_1 = 0.3$, $\alpha_2 = 0.3$, $\alpha_3 = 0.04$, $\lambda_1 = 0.1$, $\lambda_3 = 0.09$, $\lambda_4 = 0.2$ and vary λ_2 between 0 and 1 as in Figure 4.2.
Case 3: We fix $\alpha_1 = 0.3$, $\alpha_2 = 0.3$, $\alpha_3 = 0.04$, $\lambda_1 = 0.1$, $\lambda_2 = 0.5$, $\lambda_4 = 0.2$ and vary λ_3 between 0 and 1 as in Figure 4.3.
Case 4: We fix $\alpha_1 = 0.3$, $\alpha_2 = 0.3$, $\alpha_3 = 0.04$, $\lambda_1 = 0.1$, $\lambda_2 = 0.5$, $\lambda_3 = 0.9$ and vary λ_4 between 0 and 1 as in Figure 4.4.
Case 5: We fix $\alpha_1 = 0.93$, $\alpha_2 = 0.43$, $\lambda_1 = 0.6$, $\lambda_2 = 0.5$, $\lambda_3 = 0.39$, $\lambda_4 = 0.42$ and vary α_3 between 0 and 1 as in Figure 4.5.

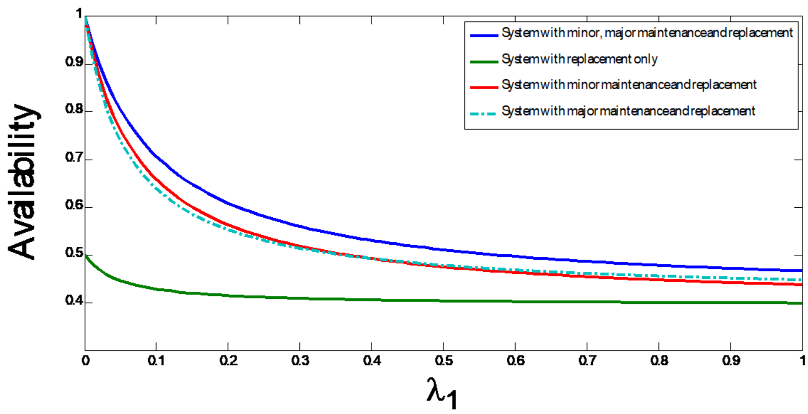

Figure 4.1: Relation between availability and minor deterioration

The results which compare the steady state availability with respect to λ_1 (minor deterioration rate), λ_2 (medium deterioration rate), λ_3 (major deterioration rate), λ_4 (failure rate) and α_3 (replacement rate) for all the four systems considered are depicted in Figures 4.1 to 4.5. Figure 4.1 show that system availability decrease as increases for any system. Furthermore, system with maintenance and replacement seems to be most effective and reliable among all the four systems. This is as a result of regular maintenance (both minor and major) and replacement at invoked whenever the system is in either minor or medium deterioration or failed states. It is shown that system with both maintenance and replacement produces more availability than the other systems. It is evident from Figures 4.2 – 4.5 that system with maintenance and replacement is more reliable and than the systems with minor maintenance and replacement, major maintenance with replacement and replacement only at failure. Thus, system with maintenance and replacement is the optimal configuration in this study. This shows that maintenance and replacement could make a significant difference while system is in minor, medium or major deterioration state.

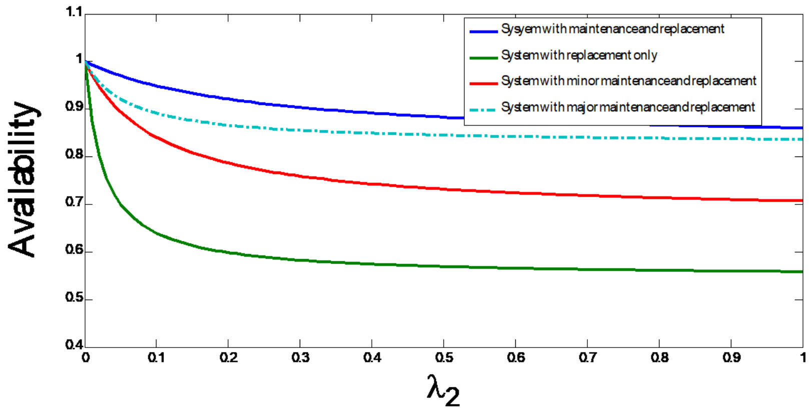

Figure 4.2: Relation between availability and medium deterioration

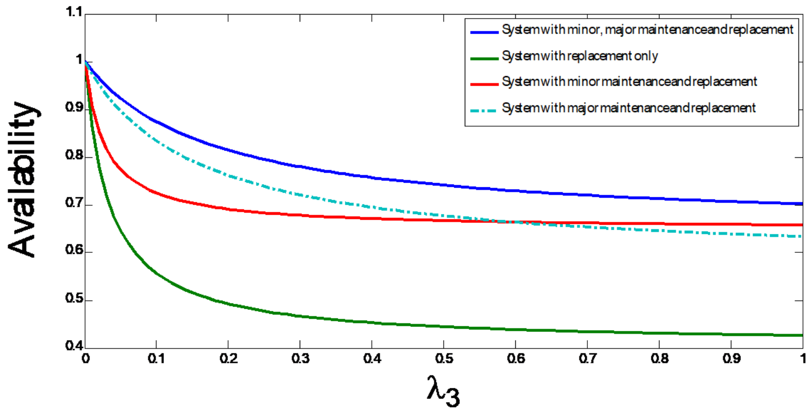

Figure 4.3: Relation between availability and major deterioration

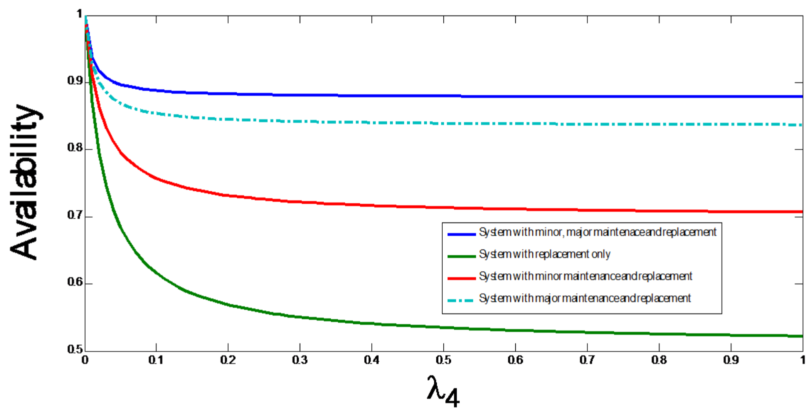

Figure 4.4: Relation between availability and system failure rate

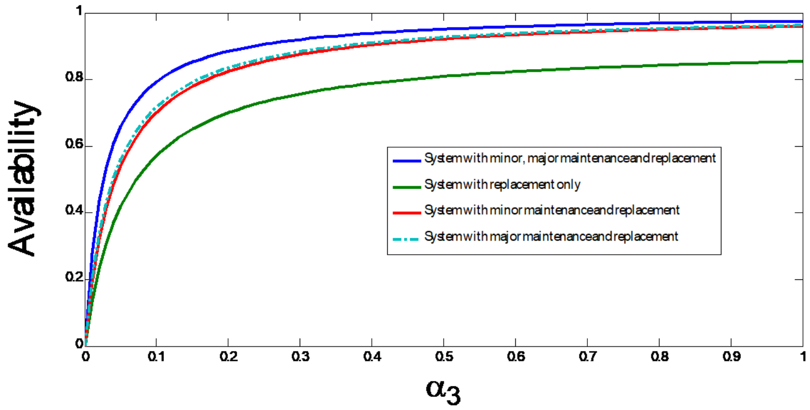

Figure 4.5: Relation between availability and replacement rate

5. Conclusion

This paper studied a two unit cold standby with three consecutive stages of deterioration before failure. Explicit expression for the steady-state availability was derived. The numerical simulations presented in Figures 4.1 – 4.5 provide a description of the effect of the deterioration rates, failure rate and replacement rate on steady-state availability. On the basis of the numerical results obtained for particular cases, it is suggested that the system availability can be improved significantly by:

(i) Adding more cold standby units.
(ii) Increasing the replacement rate.
(iii) Reducing the failure rate of the system by hot or cold duplication method.
(iv) Incorporating minor and major maintenance to system at minor and medium deterioration stages.
(v) Exchange the system at major deterioration with new one before.

The system can further be developed into system with multiple standbys in solving reliability and availability problems.

Conflict of Interests

The authors declare that there is no conflict of interests.

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