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A Solow-Swan Type Growth Model of An Economy with Two Classes*

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Abstract

In this paper, a Solow-Swan type growth model is constructed for a two-class hypothetical economy with no government in order to highlight the links between group dynamics and aggregate dynamics. A hypothetical society with two classes (rich and poor) is assumed. They share the common technological knowledge but have different production functions. Rich group's production is capital intensive and the poor group has a labor intensive one. Each group consists of homogenous agents. It is algebraically shown that even if none of the groups are in their steady state, the economy under consideration could be in its steady state as a whole. Furthermore, it is shown that initial capital distribution between groups and their relative weights affect the steady state per capita levels of output and consumption. Finally, a python simulation is added in order to calibrate the model and the effects of an increase in the saving rate on steady state per capita capital and output levels of groups and aggregate economy are shown. Unlike the standard Solow-Swan framework, implicitly based on homogenous agents, this modified model is suitable for analyzing and explaining dynamics of growth for heterogeneous economic unions/groups such as EU and OECD.

Keywords: Agent-Based Modelling, Solow-Swan Growth Model, Steady State, Production Function, Income Distribution

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İki Sınıflı Bir Ekonominin Solow-Swan Tipi Büyüme Modeli

Taylan Akgül [1] **Zeynep Elitaş** [2]

Öz

Bu çalışmada, standart Solow-Swan modelinin aksine, grup dinamikleri ile toplam dinamikler arasındaki bağlantıları vurgulamak amacıyla, hükümetin bulunmadığı iki sınıflı varsayımsal bir ekonomi için Solow-Swan tipi bir büyüme modeli oluşturulmuştur. Bu modelde, zengin ve fakir olmak üzere iki sınıfın olduğu ve her iki sınıfın teknolojik bilgi düzeylerinin aynı fakat üretim fonksiyonlarının farklı olduğu varsayılmaktadır. Zengin sınıfın üretimi sermaye yoğun, fakir sınıfın üretimi ise emek yoğun olarak modellenmiştir. Her grup homojen ajanlardan oluşmaktadır. Grupların her biri durağan durumda olmasa da, ekonominin bir bütün olarak durağan durumda olabileceği cebirsel olarak gösterilmiştir. Ayrıca, gruplar arasındaki başlangıç sermaye dağılımı ve bunların göreceli ağırlıklarının, kişi başına üretim ve tüketimin durağan durum seviyelerini nasıl etkilediği de gösterilmiştir. Son olarak, modeli kalibre etmek ve tasarruf oranındaki artışın kişi başına düşen sermaye, grupların çıktı düzeyleri ve toplam ekonomi üzerindeki etkilerini göstermek amacıyla bir Python simülasyonu eklenmiştir. Bu modifiye edilmiş Solow-Swan modeli, ekonomik birliklerin büyüme dinamiklerini analiz etmek ve açıklamak için kullanılabilir. Gizil olarak homojen ajan tabanlı çerçevenin aksine bu modifiye model Avrupa Birliği veya OECD gibi heterojen kuruluşların ya da ekonomik bölgeler ve birliklerin büyüme dinamiklerini ampirik uzantılarla analiz etmek için uygundur.

Anahtar Kelimeler: Ajan Tabanlı Modelleme, Solow-Swan Büyüme Modeli, Durağan Durum, Üretim Fonksiyonu, Gelir Dağılımı.

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Introduction

Models of economic growth are mainly categorized into two as exogenous and endogenous growth models based on how they explain the sources and drivers of long-term economic growth. The basic principle of exogenous growth models assume that economic growth is driven by factors that are external to the economic system being modeled. These models outline how a steady state economic growth could be achieved as a result of changes in the three following economic forces: the population growth rate, the savings rate, and the rate of technological progress. They may also incorporate the role of policies and institutions affecting the economic performance of the system.

The Harrod–Domar model is the first to the exogenous growth model explaining the growth rate being determined by the savings ratio and the capital-output ratio (Harrod, 1939, p.14-33; Domar, 1946, p.137-147). In fact, Domar (1946) introduces the concept of what can be more precisely called potential social average productivity of investment and not the capital-output ratio, which later became common in Harrod-Domar-type growth applications. Model suggests that investment not only contributes to current aggregate demand but also enhances the economy's potential output in the future. Thus, over time, investment exerts both demand-side and supply-side effects. Full employment can last in the long run only if investment, along with other demand sources, grows enough to absorb the extra output from that investment. Harrod (1939) referred to this specific growth trajectory as the economy’s "warranted growth," a path in which the circular flow of the economy remains in equilibrium over time, given its technological parameters and savings behavior. Should aggregate demand fail to grow at the same pace as the expansion of output capacity, unemployment will inevitably increase.

The Harrod-Domar model encountered considerable criticism from neoclassical economists, who pointed to its inability to capture the complexities of long-term growth dynamics and its reliance on overly simplistic assumptions. One of its principal criticisms was its neglect of the contribution of technological change and human capital, which are basic drivers of long-run growth in real economies. The Solow-Swan model, however, put forward a more sophisticated model with its incorporation of diminishing returns to capital. This theory highlights the fact that even while capital accumulation can spur growth in the short run, in the long run its role is weakened unless backed up by technological progress (Solow, 1956, p.65; Swan, 1956, p.334-361). The model enunciates that the most effective determinant of economic growth is the capital accumulation driven by the rates of saving in the short-run. It also suggests that economic growth is exogenously determined by the technological progress rate and the population growth rate in the long term.

1. Related Literature

The Solow-Swan model was a revolutionary shift in the growth theory agenda in that it emphasized long-term economic growth. It maintained that technological progress as an exogenous variable holds the secret of explaining persistent increases in output. Moreover, population growth is included as it specifies the size of the labor force and, consequently, the productive potential of the economy. By incorporating these elements, the Solow-Swan model presented a more comprehensive and explanatory theory of economic growth that demonstrated economies will converge towards a steady-state equilibrium in which the growth of capital and labor is offset by technological progress.

Yet, for all its progress, the Solow-Swan model is not without its limitations. By assuming technological change and population growth as exogenous, it leaves the question of the determinants of innovation and the process of technological progress open. This opening gave the impetus for the development of endogenous growth models, which attempt to explain technological progress and innovation as endogenous results of economic activity, and not as external parameters.

The inability of the exogenous growth models to account for the dynamics of endogenous factors like innovation, human capital accumulation, or institutional change leads to a development of more general models which considers growth as an endogenous process. Endogenous growth models encompass the theoretical frameworks established in a sequence of seminal papers by Romer, in which he emphasizes the key role of human capital accumulation, research and development, and knowledge spillovers in the advancement of long-run economic growth (Romer, 1986, p.1006, 1990, p.95, 1994, p.9; Romer and Rivera-Batiz, 1991, p.536). These models violate the exogenous assumptions of earlier growth theories by demonstrating that economic growth can be generated by forces endogenous to the system itself, namely through the continuous accumulation and transmission of knowledge, and the diffusion of technological innovations. Romer's research accentuates the primary role of investments in intellectual capital, innovation, and education as fundamental drivers of long-run growth, with increasing returns to scale that generate persistent and self-reinforcing economic growth.

Romer's(1990) work on endogenous growth theory is a revolutionary departure from conventional growth specifications in the emphasis placed on internal economic mechanisms of inducing technological advance and innovation in the economy. At the core of his models, there is a presumption that knowledge, as opposed to physical capital, has increasing returns to scale. By this, he is saying that when effort goes into producing knowledge—through research and development (R&D), education, or the spread of ideas—the rewards rise disproportionately, triggering additional economic growth and inducing a virtuous circle. Romer (1986, p.1003) says that technological progress is not an exogenous or a random event but a consequence of conscious investment in R&D by individuals and firms. The investment encourages the development of new ideas, increases the productivity of capital, and creates knowledge spillovers that spill over industries and make the economy richer. The concept of knowledge spillovers is significant as it shows how innovation in an industry can spill over the economy so that growth can occur without succumbing to the diminishing returns to capital that traditional models predict. Romer (1990, p.74) advances this model one step further by adding the notion of "non-rival" goods, i.e., knowledge, which can be shared and utilized by numerous agents without the goods running out of stock. The characteristic of knowledge enables long-run and consistent growth that could not be explained by conventional models, founded on the accumulation of physical capital. By removing growth from the sphere of capital accumulation, Romer's theory extends the potential for constantly increasing productivity and output to economies. Moreover, Romer and Rivera-Batiz (1991, p.542) apply these principles at the global level, illustrating how human capital accumulation and knowledge spillovers can lead to convergence between economies, particularly if nations implement coordinated investment in technological innovation and education. By assuming knowledge and innovation to be endogenous—prompted by decisions within the economy itself—Romer's models emphasize the central role of policy decisions, i.e., spending on education and infrastructure, in long-run growth trajectories.

Lucas (1988) also contributes to endogenous growth theory through his elaboration on the concept of human capital externalities, noting that the accumulation of knowledge and skills not only yields a payoff to the owner but also positive externalities to the whole economy. In his model, human capital plays a pivotal role in raising the productivity of labor, which leads economic growth overall. In opposition to conventional frameworks stressing the accumulation of physical capital, Lucas's model contends that human ability and quality of workers—acquired through training, education, and experience—are the key drivers of sustained growth. One of the principal findings of Lucas's work is the phenomenon of declining returns to scale generated by human capital, a basic postulate of endogenous growth theory. When individuals invest in their own education and skill building, they contribute to the stock of knowledge, and thus the overall economy becomes more productive. Knowledge spillovers then enhance other individuals' productivity, and this makes the economic system more efficient and dynamic. This self-reinforcing feedback process must continue in order to maintain long-run growth since investment in human capital generates sustained innovation and technological advance, which in turn motivates even greater expenditures on schooling and the development of human skills. Lucas's approach also highlights the process of knowledge diffusion through which benefits of human capital accumulation are distributed among sectors and regions. This diffusion not only occurs through formal channels, such as research and development, but also through informal networks and contact between trained manpower. By their contact and exchange of ideas, these workers produce new knowledge and technology that enhances productivity and drives economic growth. Lucas also argues that the rhythm of human capital accumulation is established by economic incentives and institutions of policy. The policy environment, the labor market organization, and the availability of education can all support or hinder human capital accumulation. Therefore, policy decisions that increase access to schooling, reduce the costs of skill acquisition, and encourage innovation are the most effective drivers of long-run growth, as they directly shape the rate at which human capital is created and utilized.

Referral to works of Becker (1993), Acemoglu and Autor (2009), and Acemoglu and Pischke (2001) can also be done in terms of human capitals models. Schumpeterian models of growth concentrating on the role played by innovation and entrepreneurship in driving economic growth are available (Aghion et al., 2015, p.558). Setting the stage for future contributions of these books, a more richly detailed vision of economic growth is offered—of a one that focuses on the multi-dimension interaction of human capital, innovation, and entrepreneurship. Becker (1993) expanded the human capital tradition by thinking through how investment in education, training, and health into human capital not only benefits the individual's productivity but also for society's productivity in general. He argued that human capital accumulation is at the heart not only of individual well-being but also as a major source of aggregate societal productivity. Becker's framework combines human capital with wider social forces, such as family and societal investments, that are key to the determination of economic outcomes across generations. Acemoglu and Autor (2009) further developed the human capital theory by examining the relationship between technological progress and labor markets. They focused on the dynamic relationship between the supply of human capital and the demand for skill and argued that technological progress would generate new employment opportunities as it renders others obsolete. This change points to the need for robust education and training organizations, which must provide employees with the required capabilities to excel in a more technology-oriented economy. Acemoglu and Pischke (2001, p.114) contributed to this debate by underlining the role of some forms of human capital, i.e., vocational training and firm-specific knowledge, as being at the center of enhancing productivity in particular industries as well as adapting to the forces of an internationalized economy. In addition to this, Schumpeterian growth models and specifically in Aghion et al. (2015, p.569) offer a powerful theoretical setup of how innovation and entrepreneurship promote economic growth. According to the theories, it is not as much capital or accumulation of labor leading to economic growth but rather the impact of improvements in technology caused by entrepreneurship. Drawing on Schumpeter's "creative destruction," Aghion and co-authors describe how entrepreneurs, by bringing new products and production methods, disrupt settled industries and cause waves of innovation to produce long-term economic growth. The mechanism not only improves productivity but competition as well, thus causing further innovation and perpetuating a virtuous circle of sustained economic growth. Schumpeterian models also emphasize the important role played by institutions in facilitating innovation. For entrepreneurship to succeed and innovation to be a main growth driver, economies must have sound institutional arrangements that encourage research and development, protect intellectual property rights, and penalize taking risks. Public policy, particularly education, infrastructure, and regulation of markets, is an important determinant of producing an environment that encourages entrepreneurial ventures and technological innovation.

Agent-based modeling and the homogenous sector assumption limit the conventional Solow-Swan model in its ability to account for real evidence such as institutional complexity and labor and sector heterogeneity. Several enrichments have been proposed for those limitations that adds the possibility of a more realistic study of economic growth. Others have built growth models with sectoral disaggregation for the purpose (Uzawa, 1964, p.2; Kaldor, 1966, p.312; Pasinetti, 1981; Ngai and Pissarides, 2007, p.430; Acemoglu and Guerrieri, 2008, p.471). In a second strand of work, some models have been extended to accommodate heterogeneity among agents (Benhabib and Farmer, 1994, p.23; Aiyagari, 1994, p.665; Krueger and Perri (2006, p.170). Others add to the fundamental model by adding spatial inequalities (Barro and Sala-i-Martin, 1995; Lucas, 2001, p.247; Desmet and Rossi-Hansberg, 2014, p.1215). Innovation and structural change have also been added to the setup by Acemoglu (2002), and policy-led and institutional environments have been discussed by models in Glomm and Ravikumar (1997, p.184) and Bourguignon and Morrison (1998, p.235).

Collectively, these views explain that economic growth is not a simple product of capital accumulation but a dynamic, multi-faceted process led by the interface between human capital, technological innovation, and entrepreneurism. These endogenous models of growth hypothesize that growth in the long term relies on continuous investment in education, skills development, and innovation and on institutional arrangements meant to render new technologies spread and foster an entrepreneurial spirit. From this view, growth is an endogenous phenomenon—one induced by investment and decision within the economy, rather than by forces outside it.

Most growth models are based on assumption that individuals and/or firms are homogenous. This study, therefore, can be viewed as an attempt for constructing a growth framework based on heterogenous agents, which could be a suitable model for analyzing heterogenous economic unions and organizations such as EU or OECD.

In the sections that follow, we first present the structure of our model, followed by an analysis of the steady-state dynamics. Next, we introduce a Python extension to enhance the model's functionality. The final section concludes the paper.

2. Model

2.1. Structure

A Solow-Swan type growth model is being examined as it applies to a hypothetical society with no government, comprising two distinct groups: the poor (p) and the rich (r). Each group consists of homogeneous individuals (agents) with identical initial capital endowments. Since the effect of taxation, transfers and subsidies are beyond the scope of this analysis, a simple economy with no government is assumed. So, the total initial capital stocks for each group are defined as:

$$K_p(0) = L_p(0)k_p(0) \quad (1)$$

$$K_r(0) = L_r(0)k_r(0) \quad (2)$$

where $k_p(0)$ represents the initial capital stock of each poor agent and $k_r(0)$ denotes that of each rich agent. The distinction between rich and poor groups relies on the differences between saving rates, population stock and its' growth rate and production type (capital or labor intensive). Thus, there is a multidimensional difference between groups. $L_p(0)$ indicates the number of poor agents and $L_r(0)$ denotes number of rich agents at the outset. It is assumed that $L_p(0) > L_r(0)$ and $0<k_p(0)< k_r(0)$, implying a larger population in the poor group with lower per capita capital stock at time zero.

Then, the aggregate production functions of each group are defined as:

$$Y_p(t) = A(t)K_p(t)^{\alpha}L_p(t)^{1-\propto} \quad (3)$$

$$Y_r(t) = A(t)K_p(t)^{\beta}L_p(t)^{1-\beta} \quad (4)$$

Combining (3) and (4) the aggregate production function for whole economy will be obtained as:

$$Y(t) = Y_p(t) + Y_r(t) = A(t)K_p(t)^{\alpha}L_p(t)^{1-\propto} + A(t)K_p(t)^{\beta}L_p(t)^{1-\beta}$$
$$= A(t)\left[K_p(t)^{\alpha}L_p(t)^{1-\propto} + K_p(t)^{\beta}L_p(t)^{1-\beta}\right] \quad (5)$$

Furthermore, we posit that the rich group's production relies heavily on capital, whereas the poor group's production is labor-intensive, utilizing a shared technology. Each agent is assumed to contribute one unit of labor, measured in units rather than hours. Given these premises, we can express per capita output levels for the representative agents of the poor (p) and rich (r) groups at time "t" as follows, respectively:

$$y_p(t) = \frac{Y_p(t)}{A(t)L_p(t)} = k_p(t)^\alpha \quad (6)$$

$$y_r(t) = \frac{Y_r(t)}{A(t)L_r(t)} = k_r(t)^\beta \quad (7)$$

where 1>β>0.5>α>0. Marginal product of capital is positive for both types of agents since since $\frac{dy_p(t)}{dk_p(t)} = \alpha k_p(t)^{\alpha-1}$>0 for the poor and $\frac{dy_r(t)}{dk_r(t)} = \beta k_r(t)^{\beta-1} > 0$ for the rich ones respectively. Also, diminishing marginal returns holds since $\frac{d^2y_p(t)}{dk_p(t)^2} = \alpha(\alpha-1)k_p(t)^{\alpha-2} < 0$ and $\frac{d^2y_r(t)}{dk_r(t)^2} = \beta(\beta-1)k_r(t)^{\beta-2} < 0$.

2.2. Dynamics of the Steady State

Firstly, technology is assumed to grow at a constant rate "g". Thus:

$$A(t) = A(0)e^{gt} \quad (8)$$

Therefore, technical progress can be written as:

$$\dot{A}(t) = \frac{dA(t)}{dt} = gA(0)e^{gt} \quad (9)$$

So:

$$\frac{\frac{dA(t)}{dt}}{A(t)} = \frac{\dot{A}(t)}{A(t)} = \frac{gA(0)e^{gt}}{A(0)e^{gt}} = g \quad (10)$$

In line with the microeconomic theory, we assume that saving rate is higher for rich group (i.e., s_r>s_p) though capital depreciates at same rate (δ) for both groups. Additionally, we assume that savings are equal to investment for both groups. Hence, capital accumulates by net investment (investment-depreciation). Thus, change in capital stock for respective groups is defined as:

$$\dot{K}_p(t) = s_pY_p(t) - \delta K_p(t) \quad (11)$$

$$\dot{K}_r(t) = s_rY_r(t) - \delta K_r(t) \quad (12)$$

Additionally, we assume uniformity within each group regarding saving behavior, where the saving rate (s_r) is same for all rich agents and all poor agents share the same saving rate (s_p). Another assumption is that population growth rate of the poor group (n) is higher than the population growth rate of rich the poor group(m). With these assumptions we can establish the functions for population growth and per capita capital accumulation as follows:

$$L_p(t) = L_p(0)e^{nt} \quad (13)$$

$$L_r(t) = L_r(0)e^{mt} \quad (14)$$

$$\dot{k}_p(t) = \frac{d\left(\frac{K_p}{AL_p}\right)}{dt} = \frac{dK_p}{dt} - \frac{dA}{dt} - \frac{dL_p}{dt} = s_p y_p(t) - (\delta + n + g)k_p(t) \quad (15)$$

$$\dot{k}_r(t) = \frac{d\left(\frac{K_r}{AL_r}\right)}{dt} = \frac{dK_r}{dt} - \frac{dA}{dt} - \frac{dL_r}{dt} = s_r y_r(t) - (\delta + m + g)k_r(t) \quad (16)$$

where $L_p(0) > L_r(0)$ and $n > m$ due to assumptions about population dynamics and $sr > sp$ due to assumptions about savings.

Population growth of the poor group:

$$L_p\dot{(t)} = \frac{dL_p(t)}{dt} = nL_p(0)e^{nt} \quad (17)$$

Therefore:

$$\frac{L_p\dot{(t)}}{L_p(t)} = \frac{nL_p(0)e^{nt}}{L_p(0)e^{nt}} = n \quad (18)$$

By analogy, population growth rate of the rich group is obtained as:

$$\frac{L_r\dot{(t)}}{L_r(t)} = \frac{mL_r(0)e^{mt}}{L_r(0)e^{nt}} = m \quad (19)$$

Let's first derive the steady state per capita level for the poor group. At steady state per capita capital will remain constant. In other words, growth rate of per capita capital stock will be equal to zero. Thus:

$$\dot{K}_p(t) = s_p y_p(t) - (\delta + n + g)k_p(t) = s_p\, k_p(t)^\alpha - (\delta + n + g)k_p(t) = 0 \quad (20)$$

Hence:

$$s_p\, k_p(t)^\alpha = (\delta + n + g)k_p(t)$$

$$s_p/(\delta + n + g) = k_p(t)^{1-\alpha}$$

$$\left\{\frac{s_p}{\delta+n+g}\right\}^{\frac{1}{1-\alpha}} = k_p{}^* \quad (21)$$

where kp* denotes the steady state per capita capital stock for the poor group. From (18), it is evident that a higher the saving rate leads to a higher steady state per capita capital stock while higher technological improvement rate, higher depreciation rate and higher population growth rate leads to a lower steady state per capita capital stock. Additionally, a higher value of α results in a higher steady state per capita capital stock.

By analogy, steady state per capita capital stock for rich group (kr*) is derived as follows:

$$\left\{\frac{s_r}{\delta+m+g}\right\}^{\frac{1}{1-\beta}} = k_r{}^* \quad (22)$$

As the saving rate or β is higher, we observe a higher per capita steady state capital stock for rich group. Similarly, the lower technological improvement rate or the lower depreciation rate displays a higher per capita steady state capital stock for rich group.

Please note that since $s_r > s_p$, m<n and $\beta > \alpha$ we conclude that $k_r^* > k_p^*$. In essence, our assumptions regarding saving rates and capital intensity lead to a higher per capita steady state capital stock for the rich group.

Since $y_p(t) = k_p(t)^\alpha$, substituting $k_p{}^*$ for capital stock, the steady state per capita output for poor group is obtained as:

$$y_p^* = \left\{\frac{s_p}{\delta+n+g}\right\}^{\frac{\alpha}{1-\alpha}} \tag{23}$$

Therefore, similar to the steady-state capital stock, the steady-state per capita output level of the poor group rises with higher saving rates and declines with higher depreciation, technological improvement or population growth rates. Moreover, a higher value of α leads to a higher per capita steady-state output.

Since $y_r(t) = A(t)(k_r(t))^\beta$, substituting $k_r{}^*$ for capital stock, the steady state per capita output for rich group is obtained as:

$$y_r^* = \left\{\frac{s_r}{\delta+m+g}\right\}^{\frac{\beta}{1-\beta}} \tag{24}$$

Similarly, akin to the steady-state capital stock, the steady-state per capita output level of the rich group rises with higher saving rates and declines with higher depreciation, technological improvement or population growth rates. Additionally, a higher value of β results in a higher per capita steady-state output too.

It's important to note that steady-state per capita capital stocks and per capita outputs are independent of initial capital endowments for both the poor and rich groups. However, at the aggregate level, the situation may vary. In other words, the steady-state per capita capital stock and per capita output level could be influenced by the initial capital endowments and their distribution within the economy.

In order to clarify this point, we can start with reformulating the per capita capital stock for whole economy at time t as follows:

$$k(t) = \frac{A(t)k_p(t)L_p(t)+A(t)k_r(t)L_r(t)}{A(t)(L_p(t)+A(t)L_r(t))} = \frac{A(t)L_p(t)}{A(t)(L_p(t)+L_r(t))}k_p(t) + \frac{A(t)L_r(t)}{A(t)(L_p(t)+L_r(t))}k_r(t) \tag{25}$$

This equation can be rewritten by expressing per capita capital stock as a weighted average as:

$$k(t) = w_p(t)k_p(t) + w_r k_r(t) \tag{26}$$

where $w_p(t)$ denotes the population share of poor group at time t, and $w_r(t)$ denotes that of rich group. Note that $w_p(t)+w_r(t)=1$. Since the capital stock increase (dk/dt) is defined as the difference between saving and depreciation we can establish the following equation for the capital accumulation:

$$\frac{dk}{dt} = \dot{k}\,(t) = \left\{\frac{s_p y_p(t)L_p(t)+s_r y_r(t)L_r(t)}{A(t)L_p(t)+A(t)L_r(t)}\right\} - \delta\left\{\frac{k_p(t)L_p(t)+k_r(t)L_r(t)}{L_p(t)+L_r(t)}\right\} \tag{27}$$

Substituting weights and using the equations for per capita output functions, we can reduce equation (27) to:

$$\dot{k}(t) = \{w_p(t)s_pk_p(t)^{\alpha} + w_r(t)s_rk_p(t)^{\beta}\} - \delta\{w_p(t)k_p(t) + w_rk_r(t)\} \quad (28)$$

Since the per capita capital stock is constant ($\dot{k}(t) = 0$) at steady state:

$$\dot{k}(t) = \{w_p(t)s_pk_p(t)^{\alpha} + w_r(t)s_rk_p(t)^{\beta}\} - \delta\{w_p(t)k_p(t) + w_rk_r(t)\} = 0 \quad (29)$$

The last equation implicitly shows that there are two different cases:

Case 1: Both groups and whole economy are in their steady state. Formally:

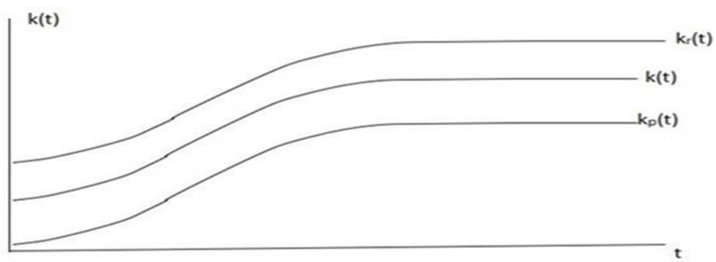
$$w_p(t)s_p(k_p(t))^{\alpha} - \delta w_p(t)k_p(t) = w_r(t)s_r(k_r(t))^{\beta} - \delta w_rk_r(t) = 0 \quad (30)$$

In this case, substituting Equations (21) and (22) for k_p and k_r the steady state per capita capital will be defined as:

$$w_p\left\{\frac{s_p}{\delta+n+g}\right\}^{\frac{1}{1-\alpha}} + w_r\left\{\frac{s_r}{\delta+m+g}\right\}^{\frac{1}{1-\beta}} = k^* \quad (31)$$

Since the steady-state per capita capital of the rich group exceeds that of the poor group, a larger share of the population being rich contributes more to a wealthier society. In simpler terms, a one percent increase in the share of the rich (wr) has a greater impact on steady-state per capita capital than a one percent increase in the share of the poor (wp). Figure 1 below illustrates Case 1 in which both groups and whole economy is in their steady state (d(k(t)/dt=0 for all).

Figure 1. Case 1 Illustration



Source: Authors' Own Work.

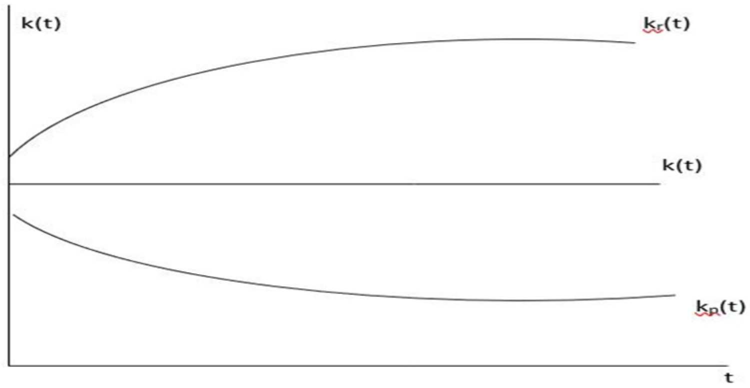
Case 2: Neither of groups is in its steady state but the whole economy is in its steady state. Formally:

$$\{w_p(t)s_pk_p(t)^{\alpha} - \delta w_p(t)k_p(t)\} = -\{w_r(t)s_rk_p(t)^{\beta} - \delta w_rk_r(t)\} \quad (32)$$

Equation (32) can be interpreted as a scenario where one group's savings exceed depreciation by an amount equivalent to the savings shortfall of the other group. This scenario is realistic only when the first group, whose savings exceed depreciation, is the rich one.

For both cases, Equation (29) shows that the steady state per capita capital for whole economy depends on the population weights. By intuition, we can state that the distribution of capital stock between groups determines the steady state per capita capital level, and hence per capita output and consumption. Also, by backward induction, we can also conclude that the initial per capita capital stock distribution affects the steady state levels of both capital stock and output. Since rich group has a higher marginal propensity to save, a worsen initial distribution will result in a higher steady state welfare. Figure 2 below illustrates Case 2 in which neither of groups are in their steady state but whole economy is its' steady state.

Figure 2. Case 2 Illustration

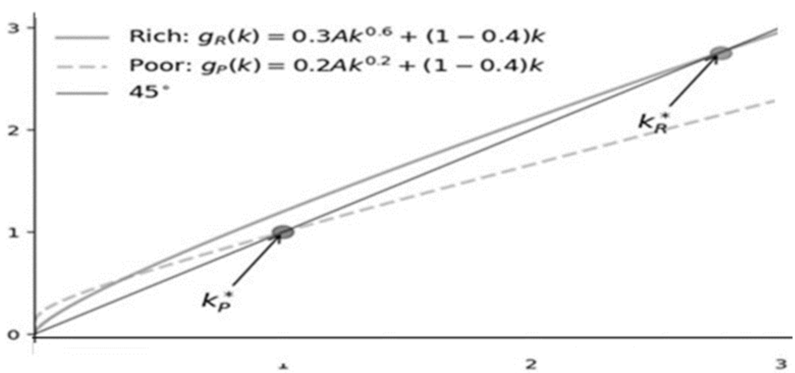


Source: Authors' Authors' Own Work.

3. Pyhton Simulation

In order to calibrate the model, we use a python simulation. The code for simulation is gathered from open-source website QuantEcon founded by Sargent and Stachurski (2023) and modified in line with the assumptions described in previous section. Python codes are available on demand.

Figure 3. Simulation of Steady States for Rich and Poor Groups with Different Saving Rates

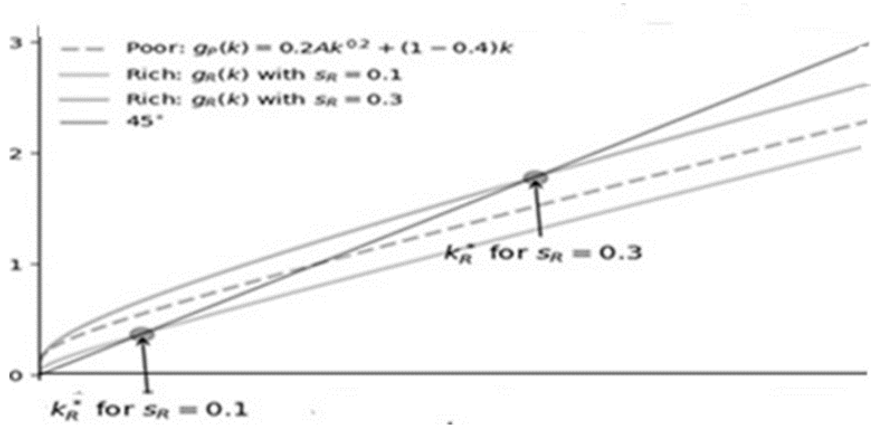


Source: Authors' Own Work.

Figure 3 above shows the case where the saving rate for rich group is 0.3, α=0.6 and depreciation rate (delta) is 0.4; and for the poor group these parameters are 0.2, 0.2 and 0.4 respectively. As can be seen, there is a gap between the steady state values of per capita capital of two groups.

In line with standard Solow-Swan model, an increase in the saving rate, of rich group for instance, results in a higher steady state per capita capital stock and hence higher steady state output. This case is shown in the Figure 4 below.

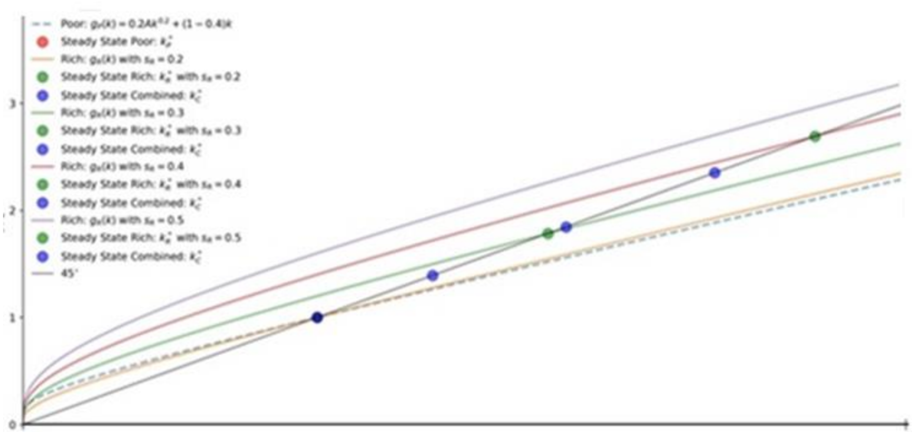
Figure 4. Effect of an Increase in Saving Rate of the Rich Group



Source: Authors' Own Work.

Finally, the figure below shows the steady state values of the whole economy and groups. Green dots show the steady state capital stock levels for rich group, red dots show that one for poor group and blue dots show the aggregate levels. As can be seen, an increase in saving rate of the rich group results in steady state levels of both rich group and whole economy.

Figure 5. Steady State Levels of Whole Economy and Effect of a Change in Saving Rate of the Rich Group



Source: Authors' Own Work.

Conclusion

Exogenous growth models, such as the Solow-Swan model, were constrained in their capacity to account for long-term economic growth, as they treat technological progress and other critical factors—such as innovation, human capital, and institutions—as external to the model. These frameworks posit that growth is propelled by forces that are independent of the economic system, thereby neglecting the intrinsic drivers of development within the system itself. This limitation created a notable void in explaining the disparity in sustained growth between different countries or regions.

Endogenous growth theory, yet designed by economists Robert Lucas and Paul Romer, overcomes these deficiencies since it posits economic growth is brought about by reasons which are endogenous to the economy, for instance, investment in human capital, technical advancements, and institutions. Romer's framework emphasizes how technological progress and idea generation can result in increasing returns to scale and thus, innovation and knowledge accumulation can facilitate sustained growth without encountering diminishing returns. Likewise, Lucas's framework emphasizes the central contribution of human capital accumulation towards ensuring productivity enhancement and sustained economic growth. These more specific, endogenous models give a more satisfactory explanation of the complex dynamics of growth since variables like technology and human capital are not merely exogenous shocks but a part of the economic system itself that evolve and interact with the system.

Unlike the standard Solow-Swan model, in this research, we apply a two-group growth framework in order to highlight the links between group dynamics and aggregate dynamics. This setting could be applied to analyze the growth dynamics of organizations or economic zones/unions such as EU or OECD countries with empirical extensions. The number of groups could be more than two, depending on the subdivisions regarding the development levels, or socio-economic similarities between countries or regions. For instance, a three-group framework including less developed, advanced and moderate economies would be appropriate for analyzing growth dynamics of regions such as Asia, East Asia or the EU. The model used in this study assumes that the technological knowledge is common among groups. However, in extended models, it might be assumed that advanced economies have improved technological knowledge, which is more realistic. Advanced technological knowledge results in increased productivity and hence higher growth rate. Therefore, advanced technology can compensate the diminishing returns of capital per capita for developed countries, which influences the growth patterns.

We also have shown algebraically that even none of the sub-groups are in their steady state, the economy under consideration could be in its steady state as a whole. This is possible if and only if the positive effect of one group on the aggregate growth rate is exactly equal to the negative effect of the other group. Thus, these affects will cancel out each other and hence the system will be in its steady state. Furthermore, the initial distribution of capital stock may have a deterministic effect on the steady state levels of per capita output and hence consumption.

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Çıkar Çatışması	Çıkar çatışması beyan edilmemiştir.
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Peer-Review	Double anonymized - Two External
Ethical Statement	* It is declared that scientific and ethical principles have been followed while carrying out and writing this study and that all the sources used have been properly cited.
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