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Investigation of Cutting Force Coefficient Variation in Milling

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ABSTRACT

This study investigates the variation of cutting force coefficients (CFCs) in milling operations and their impact on force predictions and stability analysis. While CFCs are often assumed constant for simplicity, they are inherently dependent on feed per tooth and cutting speed, introducing uncertainties in machining dynamics. Using an oblique transformation model, CFCs are predicted from an orthogonal cutting database, considering chip thickness and cutting speed. The results indicate that variations in CFCs significantly influence cutting force estimations, affecting stability predictions based on the stability lobe diagram (SLD). Assuming constant CFCs may lead to inaccurate force predictions, miscalculated stability limits, and increased risk of chatter. This can result in excessive forces, accelerated tool wear, poor surface quality, and potential scrap part. In this study, cutting forces are compared against predictions from both constant and variable CFC models, revealing the improved accuracy of the latter in dynamic milling conditions. The influence of cutting speed and feed per tooth on tangential, radial, and axial force components is systematically analysed. It is observed that higher cutting speeds tend to reduce CFC values due to thermal softening effects, whereas increased feed per tooth generally amplifies force coefficients because of increased chip loads. A stability model incorporating CFC variations may provide a more accurate representation of process dynamics. The study emphasises the necessity of adaptive force and stability models in machining simulations to enhance predictive accuracy. These findings offer critical insights for optimising process parameters, selecting stable cutting conditions, and designing chatter avoidance strategies in industrial applications.

Keywords: *Cutting force coefficient, Cutting force, Chatter stability, Uncertainty.*

Frezeleme İşleminde Kesme Kuvveti Katsayısı Değişiminin İncelenmesi

ÖZET

Bu çalışma, frezeleme işlemlerinde kesme kuvveti katsayılarının (CFC'ler) değişimini ve bunun kuvvet tahminleri ile kararlılık analizine etkisini incelemektedir. CFC'ler genellikle basitlik adına sabit varsayılmasına rağmen, gerçekte diş başına ilerleme ve kesme hızı gibi parametrelere bağlı olarak değişkenlik göstermektedir. Bu durum, talaşlı imalat dinamiklerinde belirsizliklere yol açmaktadır. Eğik dönüşüm modeli kullanılarak, CFC'ler talaş kalınlığı ve kesme hızı dikkate alınarak ortogonal kesme veritabanından tahmin edilmektedir. Sonuçlar, CFC'lerdeki değişimlerin kesme kuvveti tahminlerini önemli ölçüde etkilediğini ve bu etkinin kararlılık lobu diyagramına (SLD) dayalı kararlılık öngörülerini de etkilediğini göstermektedir. CFC'lerin sabit kabul edilmesi, yanlış kuvvet tahminlerine, hatalı kararlılık limitlerine ve artan titreşim (tırlama) riskiyle sonuçlanabilir. Bu durum, aşırı kesme kuvvetleri, hızlanan takım aşınması, zayıf yüzey kalitesi ve hurdaya ayrılabilecek parçalarla sonuçlanabilir. Bu çalışmada, sabit ve değişken CFC modelleri kullanılarak yapılan kuvvet tahminleri, kesme kuvveti tahminleriyle karşılaştırılmış ve değişken modellere dayalı tahminlerin dinamik frezeleme koşullarında daha isabetli sonuçlar verdiği görülmüştür. Kesme hızı ve diş başına ilerlemenin, teğetsel, radyal ve eksenel kuvvet bileşenlerine etkisi sistematik olarak analiz edilmiştir. Yüksek kesme hızlarının, termal yumuşama etkisi nedeniyle CFC değerlerini azalttığı, buna karşılık artan ilerlemenin talaş yükünü artırarak kuvvet katsayılarını yükselttiği gözlemlenmiştir. CFC değişimlerini dikkate alan bir kararlılık modeli, işlem dinamiklerini daha doğru temsil edebilir. Çalışma, talaşlı imalat simülasyonlarında uyarlanabilir kuvvet ve kararlılık modellerinin önemini vurgulamaktadır. Bu bulgular, süreç parametrelerinin optimizasyonu, kararlı kesme koşullarının seçimi ve endüstriyel uygulamalarda tırlama önleyici stratejilerin geliştirilmesi açısından kritik bilgiler sunmaktadır.

Anahtar Kelimeler: *Kesme kuvveti katsayısı, Kesme kuvveti, Tırlama titreşimi kararlılığı, Belirsizlik.*

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1. INTRODUCTION

As a fundamental manufacturing method, machining shapes raw materials into final components with precise geometries and surface finishes. The effectiveness and accuracy of this process are largely governed by how well the feed rate, cutting speed, and depth of cut are optimised based on the material being machined. Understanding the cutting forces in milling is fundamental for improving machining efficiency, tool life, and surface quality. Cutting force modeling plays a crucial role in predicting and optimising machining performance, where the cutting force coefficient (CFC) is a key parameter that links machining conditions to force generation. Traditionally, CFCs have been treated as constant values for a given workpiece material and tool geometry. However, experimental and analytical studies have revealed that CFCs exhibit significant variation depending on machining parameters such as feed per tooth and cutting speed, challenging the assumption of constant coefficients in force models [1, 2].

Building on the evidence that cutting force coefficients vary with feed per tooth and cutting speed [1, 2], milling is governed by dynamic tool and workpiece interactions; consequently, control strategies that regulate only the average cutting force struggle to track parameter dependent changes in the instantaneous tangential, radial, and axial forces, which limits performance. Instantaneous force estimation is therefore needed but is computationally expensive for real time control; to reduce latency, advanced model order reduction within a hybrid twin framework is promising [3-5]. To capture operating contexts that also modulate force and the coefficients such as lubrication and thermal effects. Force prediction frameworks should move beyond dry and flood cooling to include sustainability-oriented lubrication (MQL, nanofluid MQL, electrostatic MQL) and cryogenic strategies with sensitivity to tool and workpiece materials [6, 7]. Because force is a primary driver of residual stress, models should incorporate residual stress effects, which are rarely included despite their influence on accuracy [8-11]. Additionaly, additional damping and coolant solutions can also affect cutting forces [12, 13]. Accordingly, the generation of tangential, radial, and axial forces and their dependence on feed per tooth and cutting speed should be investigated in detail.

In milling operations, the cutting forces are influenced by material properties, tool geometry, and process parameters. Among these, feed per tooth and cutting speed are particularly influential in determining the specific energy required for chip formation. At low cutting speeds, material deformation mechanisms lead to higher energy consumption per unit volume of removed material, resulting in higher CFCs. Conversely, at higher speeds, thermal softening and strain rate effects may reduce the force coefficients. Similarly, increasing the feed per tooth alters the uncut chip thickness, which directly impacts the shearing mechanism, cutting edge engagement, and plowing effects, all contributing to the variability in force coefficients [14].

Beyond force prediction, CFC variation significantly influences the dynamic stability of milling operations, particularly the stability lobe diagram (SLD). The SLD is a critical tool for determining stable cutting conditions by mapping spindle speed and depth of cut against the onset of regenerative chatter [15]. Mechanistically, cutting force coefficients enter the regenerative stability formulation through the directional cutting coefficients that couple tool displacement to force. Variations in these coefficients rescale the cutting matrix in the characteristic equation, altering the phase and magnitude conditions for self excitation and thereby shifting the peaks and valleys of the stability lobe diagram across spindle speed. Since the SLD is directly dependent on cutting force coefficients, variations in CFCs lead to proportional shifts in the stability boundaries. When CFCs increase due to changes in feed or cutting speed, the cutting forces acting on the tool also increase, altering the dynamic stiffness and damping characteristics of the system. This directly affects the critical depth of cut and chatter stability limits, potentially reducing the stable operating regions predicted by the SLD. Conversely, a reduction in CFCs at higher cutting speeds may expand stable cutting zones, improving productivity by allowing higher depth of cut without inducing chatter [16]. Despite extensive research on cutting force modeling and stability analysis, the dynamic impact of CFC variation on SLD predictions remains underexplored. Many existing stability models assume constant force coefficients, which may lead to inaccuracies in high-speed milling and variable feed conditions. A more precise characterisation of CFC dependent SLD variations is necessary to improve machining process planning, optimise tool paths, and enhance chatter avoidance strategies.

This study aims to investigate the variation of CFCs as a function of cutting speed and feed per tooth in milling operations, with a particular focus on their impact on dynamic stability and SLD predictions. By applying analytical models, CFC variations are explored. The approach quantifies how coefficient changes rescale the cutting matrix in the regenerative model and shift the critical depth of cut across spindle speed, thereby revealing when and by how many stability lobes move as operating conditions vary. The framework is designed to be transparent and reproducible, provides clear sensitivity to process parameters and lubrication context, and supplies a practical route to update stability predictions without prohibitive computational cost. The resulting models deliver more accurate force and stability predictions that support toolpath optimisation, proactive chatter suppression, and the calibration of digital twin controllers. By linking measurable process inputs to coefficient variation and to stability outcomes, the work advances both scientific understanding and industrial practice, enabling condition aware planning for thin walled and productivity focused milling while laying a foundation for real time decision support in digital manufacturing.

2. MATERIAL AND METHOD

Analysis of milling force, force coefficients, and the stability calculations are briefly given in this section.

2.1. Prediction of cutting forces

In the analytical determination of cutting force coefficients, the average cutting force model is based on the principle that the instantaneous cutting force is directly correlated with the material removal rate at any given moment. By analysing a differential element of the milling cutter, the radial and tangential forces acting on each axial segment are assumed to be proportional to the instantaneous uncut chip area [16]. Cutting forces in milling are generally categorised into two primary components: the shearing force, arising from material deformation in the shear zone, and the cutting-edge force, caused by frictional interaction between the tool's flank face and the workpiece, as illustrated in Figure 1. The shearing force is typically modelled as a function of the tangential (K_{tc}), radial (K_{rc}), and axial (K_{ac}) force coefficients, in combination with the instantaneous shear area. In contrast, the cutting-edge force is characterised by the tangential (K_{te}), radial (K_{re}), and axial (K_{ae}) edge force coefficients, each scaled by the cutting width. Together, these coefficients K_{tc}, K_{rc}, K_{ac}, K_{te}, K_{re}, and K_{ae}, constitute the complete set of cutting force coefficients, which are fundamental to accurate force prediction in milling operations [16, 17]. The cutting force components are derived by resolving the resultant cutting force F_c into orthogonal directions. These components are formulated as functions of the shear yield stress (τ_s), the resultant force orientation angles (θ_n, θ_i), the oblique angle (i), and the oblique shear angles (ϕ_i, ϕ_n). The expression for the resultant cutting force F_c is presented in Eq. (1). The force components in the cutting speed direction (F_{tc}), the thrust direction (F_{fc}), and the normal direction (F_{rc}) are defined by the equations provided in Eq. (2).

$$F_c = \tau_s bh/[(\cos(\theta_n + \varphi_n) \cos \theta_i \cos \varphi_i + \sin \theta_i \sin \varphi_i) (\cos i \sin \varphi_n)] \quad (1)$$

where b and h can be defined as axial depth of cut and chip thickness, respectively.

$$\left.\begin{aligned}
F_{tc} &= F_c(\cos \theta_i \cos \theta_n \cos i + \sin \theta_i \sin i) = \tau_s bh(\cos \theta_n + \tan \theta_i \tan i)[\cos(\theta_n + \varphi_n) \cos \varphi_i + \tan \theta_i \sin \varphi_i] \sin \varphi_n,\\
F_{fc} &= F_c \cos \theta_i \sin \theta_n = \tau_s bh \sin \theta_n [\cos(\theta_n + \varphi_n) \cos \varphi_i + \tan \theta_i \sin \varphi_i] \cos i \sin \varphi_n,\\
F_{rc} &= F_c(\sin \theta_i \cos i - \cos \theta_i \cos \theta_n \sin i) = \tau_s bh(\tan \theta_i - \cos \theta_n \tan i)[\cos(\theta_n + \varphi_n) \cos \varphi_i + \tan \theta_i \sin \varphi_i] \sin\varphi_n.
\end{aligned}\right\} \quad (2)$$

The cutting forces can be expressed as:

$$\left.\begin{aligned}
F_t &= K_{tc}bh + K_{te}b,\\
F_f &= K_{fc}bh + K_{fe}b,\\
F_r &= K_{rc}bh + K_{re}b.
\end{aligned}\right\} \quad (3)$$

where the cutting coefficients are:

$$\left.\begin{aligned}
K_{tc} &= \tau_s(\cos \theta_n + \tan \theta_i \tan i)[\cos(\theta_n + \varphi_n) \cos \varphi_i + \tan \theta_i \sin \varphi_i] \sin\varphi_n,\\
K_{fc} &= \tau_s \sin \theta_n[\cos(\theta_n + \varphi_n) \cos \varphi_i + \tan \theta_i \sin \varphi_i] \cos i \sin \varphi_n,\\
K_{rc} &= \tau_s(\tan \theta_i - \cos \theta_n \tan i)[\cos(\theta_n + \varphi_n) \cos \varphi_i + \tan \theta_i \sin \varphi_i] \sin\varphi_n.
\end{aligned}\right\} \quad (4)$$

If Armarego's classical oblique model [18, 19] is applied, the force expressions can be reformulated using geometric relationships as follows.

$$\left.\begin{aligned}
F_{tc} &= bh[(\tau_s/\sin \varphi_n) \cos(\beta_n - \alpha_n) / (\sqrt{\cos^2(\varphi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2\beta_n})],\\
F_{fc} &= bh[(\tau_s/(\sin \varphi_n \cos i) (\cos(\beta_n - \alpha_n) + \tan i \tan \eta \sin\beta_n) / (\sqrt{\cos^2(\varphi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2\beta_n})],\\
F_{rc} &= bh[(\tau_s/\sin \varphi_n) (\cos(\beta_n - \alpha_n) \tan i - \tan \eta \sin\beta_n) / (\sqrt{\cos^2(\varphi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2\beta_n})].
\end{aligned}\right\} \quad (5)$$

Hence, the cutting coefficients are:

$$\left.\begin{aligned}
K_{tc} &= (\tau_s/\sin\varphi_n)\cos(\beta_n-\alpha_n)/(\sqrt{\cos^2(\varphi_n+\beta_n-\alpha_n)+\tan^2\eta\,\sin^2\beta_n}),\\
K_{fc} &= (\tau_s/(\sin\varphi_n\cos i)(\cos(\beta_n-\alpha_n)+\tan i\tan\eta\,\sin\beta_n)/(\sqrt{\cos^2(\varphi_n+\beta_n-\alpha_n)+\tan^2\eta\,\sin^2\beta_n}),\\
K_{rc} &= (\tau_s/\sin\varphi_n)(\cos(\beta_n-\alpha_n)\tan i-\tan\eta\,\sin\beta_n)/(\sqrt{\cos^2(\varphi_n+\beta_n-\alpha_n)+\tan^2\eta\,\sin^2\beta_n}).
\end{aligned}\right\}\quad(6)$$

2.2. Chatter Stability

The analysis of milling stability poses significant challenges due to the directional variation of cutting forces during the operation. Several analytical techniques have been introduced to tackle this problem [17, 18], with the zeroth-order approximation standing out for its balance between computational efficiency and accuracy in estimating stability limits. The main equations utilised in this work are outlined below; a comprehensive derivation is available in [17]. Given that the milling cutter often represents the most compliant component in the machining setup, its radial modes of vibration predominantly determine the stability threshold. The dynamic behavior of the milling process is described by the following characteristic equation:

$$\det([I]) + \Lambda[A_0][\Phi(i\omega_c)] = 0 \quad (7)$$

In this formulation, [I] denotes the identity matrix, and $[A_0]$ represents the average (zeroth order) component in the Fourier series representation of the time-dependent directional dynamic milling force coefficients. To account for the time-periodic behavior of the force coefficients, Budak and Altintas [17] employed a Fourier series expansion and simplified the model by considering only the zeroth-order term, resulting in a time-invariant approximation of the system. This approach facilitates analytical prediction of stability while maintaining an acceptable level of accuracy. In this formulation, Φ represents the system's frequency response function (FRF) matrix, and the eigenvalue corresponding to the characteristic equation is expressed as follows:

$$\Lambda = -(N/4\pi)\, bK_t(1-e^{-i\omega_c T}) \quad (8)$$

Here, N denotes the number of flutes, b is the axial depth of cut, K_t is the tangential specific cutting force coefficient, ω_c is the chatter frequency, and T is the time delay between successive tooth engagements. The stability limit can be expressed as:

$$b_{lim} = -(2\Lambda_R)/(NK_t)(1+\kappa^2) \quad (9)$$

Here κ can be expressed as:

$$\kappa = \Lambda_I/\Lambda_R = (\sin\omega_c T)/(1-\cos\omega_c T) \quad (10)$$

Λ_R and Λ_I represent the real and imaginary components of the eigenvalues, respectively. The chatter frequency and spindle speed are related through the equation $T=1/\omega_c(\epsilon+2k\pi)$, where ϵ is the phase shift and k corresponds to the number of undulations generated during cutting. Stability thresholds are most often examined in regions close to the system's inherent frequencies.

3. RESULTS AND DISCUSSION

The cutting force coefficients are calculated from the orthogonal parameters of the three materials listed in Table 1 [14], based on the oblique transformation model presented in [18,19].

Table 1. Orthogonal cutting data, h, uncut chip thickness [mm], Vc, cutting speed [m/min]

	AISI – 1045 Steel	Al 7050 – T7451	Al 6061 – T6
Shear stress τ_s (MPa)	$450.3+0.4V_c+227.5h$	$266.804+174.128h-0.043V_c+0.896\alpha_n$	$205.928+204.038h+0.016V_c-0.056\alpha_n$
Shear angle ϕ_n (deg.)	$\tan^{-1}\dfrac{(0.4+0.6h)\cos\alpha_n}{1-(0.4+0.6h)\sin\alpha_n}$	$19.4+42.017h+0.02V_c+0.384\,\alpha_n$	$13.866+62.929h+0.005V_c+0.259\alpha_n$
Friction angle β_a (deg.)	$26.8-0.031V_c+11.77h$	$25.877-1283h-0.007V_c+0.181\alpha_n$	$20.835-4.901h-0.007V_c+0.291\alpha_n$
K_{te} (N/mm)	$0.0103V_c^2+1.985V_c-55.431$	10	$50.94-0.004V_c+0.039\alpha_n$
K_{re} (N/mm)	$0.0983V_c^2+10.299V_c-294.96$	10	$94.255-0.018V_c-0.8451\alpha_n$

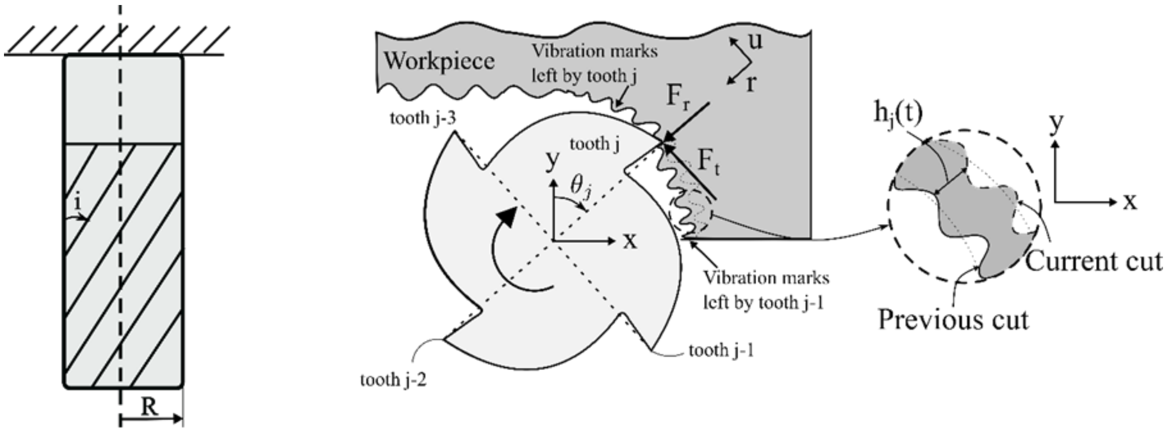
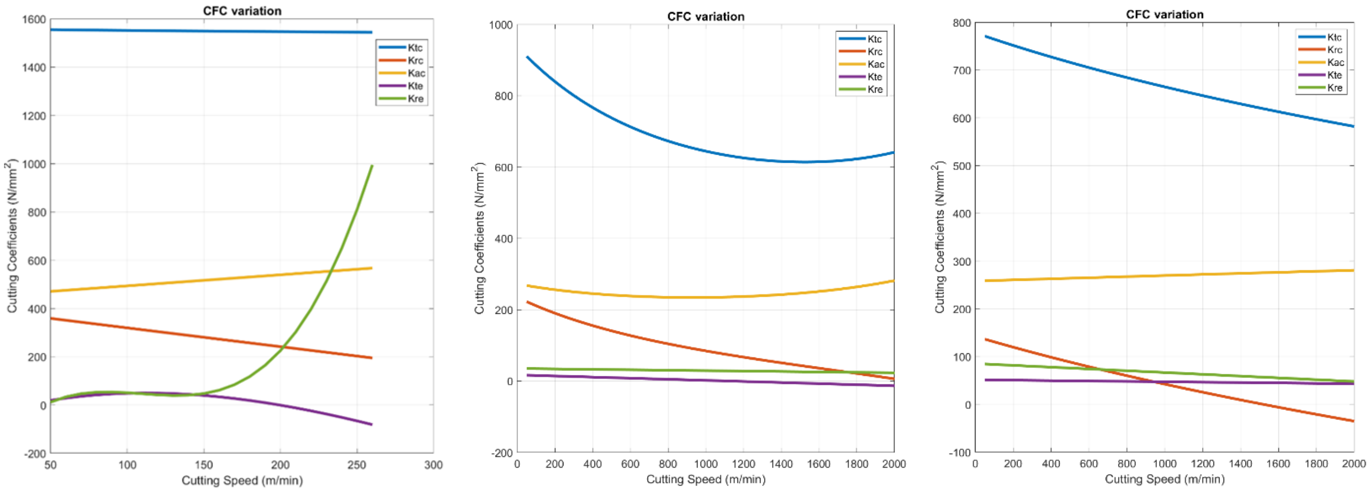


Figure 1. Cutting forces on a flat end mill

The variation in cutting coefficients is investigated for the materials, AISI 1045 steel, Al 7050–T7451, and Al 6061–T6. The MATLAB simulations are determined using the parameters, chip flow angle (helix angle of the cutting tool) 30°, normal rake angle 10.5°, average chip thickness 0.1 mm for the varying spindle speeds as illustrated in Figure 2. Cutting speeds are selected in a range of suggested values by the cutting tool manufacturers starting from 50 m/min for each material. CFC variation for varying average chip thickness is shown in Figure 3.

A table is created to compare the cutting coefficient change along the selected range of spindle speed and average chip thickness. The analysis of cutting force coefficient (CFC) variations with respect to spindle speed and chip thickness revealed significant material-dependent trends as shown in Table 2.



a) b) c)

Figure 2. CFC variation for varying spindle speed. a) AISI 1045 steel, b) Al 7050 – T7451, c) Al 6061 – T6

For varying spindle speeds (Figure 2), the edge force coefficients K_{te} and K_{re} exhibited the most substantial changes, particularly in AISI 1045 steel, where K_{te} increased by 265.09% and K_{re} by 98.84%. Aluminum alloys showed similarly high sensitivity, with K_{rc} in Al 6061-T6 increasing by 125.79% and K_{te} in Al 7050-T7451 increasing by 180.56%. In contrast, changes due to chip thickness variations (Figure 3) were generally more moderate. The most notable sensitivity to chip thickness was observed in Al 6061-T6, with increases of 17.21% in K_{tc} and 28.10% in K_{rc}. Across all materials, the edge force coefficients K_{te} and K_{re} remained nearly constant under varying chip thickness, suggesting that these coefficients are primarily influenced by spindle speed rather than chip load.

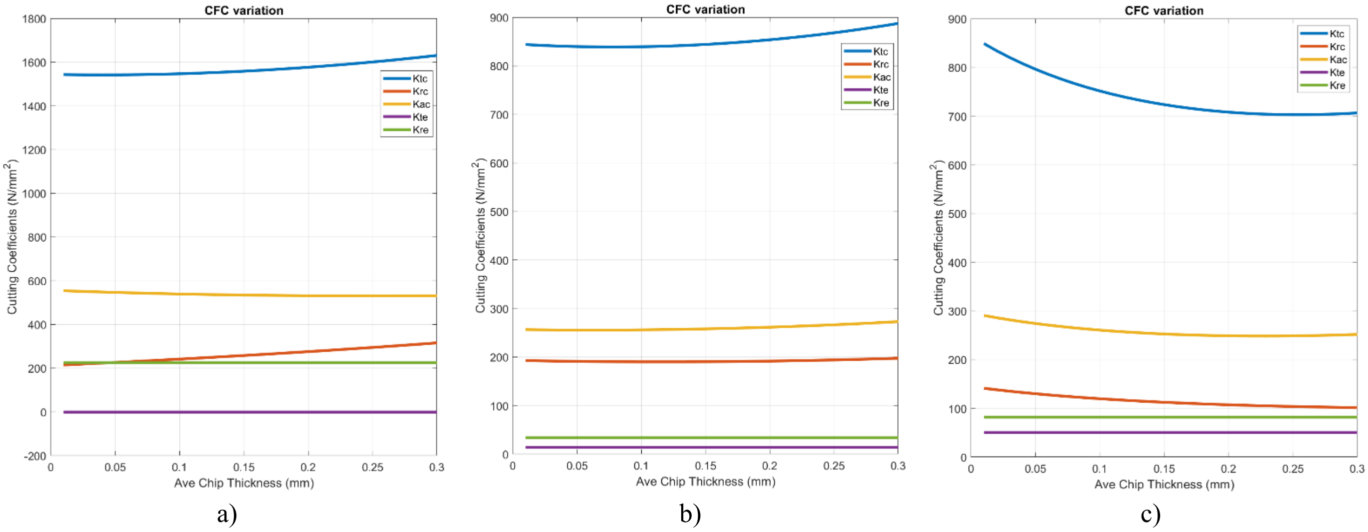


Figure 3. CFC variation for varying average chip thickness. a) AISI 1045 steel, b) Al 7050 – T7451, c) Al 6061 – T6

Table 2. Cutting coefficient change for varying spindle speed and average chip thickness

Material	**CFC change for varying spindle speeds (%)**				
	K_{tc}	K_{rc}	K_{ac}	K_{te}	K_{re}
AISI 1045 Steel	0.71	45.71	17	265.09	98.84
Al 7050 – T7451	32.57	97.23	16.82	180.56	33.4
Al 6061 – T6	24.53	125.79	7.81	15.58	43.24
	CFC change for varying chip thickness (%)				
AISI 1045 Steel	5.45	32.92	4.29	0	0
Al 7050 – T7451	5.47	3.7	6.36	0	0
Al 6061 – T6	17.21	28.1	14.53	0	0

3.1. Uncertainty in Cutting Forces and Stability Lobe Diagram

Traditionally, cutting force models assume constant CFC values for a given material and tool geometry, simplifying force estimations. However, this assumption leads to uncertainties in cutting force predictions, especially in variable feed and speed conditions. The orthogonal cutting data showed that, in some cases, the force predictions deviated by more than 20% when assuming constant CFC values, particularly at high feeds and speeds.

Even when chatter suppression techniques such as passive damping, tuned mass absorbers, or active control systems, are implemented to mitigate regenerative vibrations, CFC variation remains a non-negligible issue. The results indicate that substantial changes in coefficients such as K_{te} and K_{rc}, especially in AISI 1045 steel and Al 6061-T6, can persist over certain spindle speed ranges and chip thickness conditions. This variability persists even in robotic assisted operations [20-22] or actively controlled machining environments [23-25], suggesting that force model calibration across the operating envelope is essential.

Neglecting the effect of CFC variation may lead to inaccurate stability lobe diagrams, which can result in unexpected chatter onset due to actual cutting forces exceeding predicted limits. The implications include accelerated tool wear, increased energy consumption, degraded surface finish, and dimensional inaccuracies.

In high precision manufacturing, such discrepancies may lead to scrap parts and costly production interruptions. Therefore, incorporating CFC variability into force and stability models is critical to improving the reliability of chatter predictions and the effectiveness of both passive and active vibration suppression strategies [26, 27].

Since CFC values directly affect the dynamic cutting forces, their variation also influences stability limits in milling operations. The stability lobe diagram (SLD) is typically constructed using constant force coefficients, assuming stable regions remain fixed for a given tool workpiece system. However, the actual stability boundary shifts dynamically as CFCs change with speed and feed [28-30].

When compared the SLD obtained using constant CFCs vs. variable CFCs. The results demonstrate that:

- Using constant CFC values overpredicts stability at certain speeds, leading to unexpected chatter in operations.
- At higher cutting speeds, where CFCs tend to decrease, the stability lobes shift upward, increasing the maximum depth of cut for stable machining.
- At lower speeds, underestimated forces can lead to unexpected tool deflection and instabilities even in predicted stable zones.

3.2. Discussion

The findings highlight the importance of accounting for CFC variations in machining dynamics. The conventional approach of assuming constant force coefficients introduces significant uncertainties in force prediction and stability analysis, leading to inefficient process planning and potential machining failures. To mitigate these effects, the following strategies are recommended:

- Develop adaptive force models that incorporate real time CFC variations based on feed and speed conditions.
- Utilise variable CFC based SLDs for better chatter prediction and stability optimisation.
- Implement in process force monitoring to adjust cutting parameters dynamically and maintain stable cutting conditions.
- Use machine learning techniques to predict CFC variations and improve force modeling accuracy in digital twin environments.

By integrating CFC dependent force and stability models into machining simulations, tool life can be extended, productivity improved, and machining quality enhanced, leading to more reliable, cost-effective, productive and sustainable manufacturing operations [31, 32].

4. CONCLUSIONS

This study investigated the variation of cutting force coefficients (CFCs) as a function of feed per tooth and cutting speed in milling operations and analysed its impact on cutting force predictions and stability. The key findings and implications are summarised as follows:

- The numerical results demonstrated that cutting force coefficients change significantly with feed per tooth and cutting speed. At low feeds, plowing effects dominate, increasing CFC values, while at higher feeds, shearing effects reduce the force coefficients. Similarly, increasing cutting speed leads to reductions in CFCs due to thermal softening and strain rate effects.
- The traditional approach of assuming constant CFC values introduces errors in force estimations, particularly at varying feed rates and speeds. The deviations can result in underestimated or overestimated cutting forces, affecting process stability, tool wear, and surface quality.
- Since dynamic cutting forces directly depend on CFCs, their variations influence the stability limits in milling. The study showed that using constant CFC values can lead to mispredictions in chatter-free regions. A modified SLD incorporating variable CFCs provides a more accurate representation of stability boundaries.
- By integrating CFC dependent force and stability models, the accuracy of machining simulations can be significantly enhanced. Adaptive stability modeling, in process monitoring, and machine learning based predictions can help optimise cutting conditions, minimise chatter risks, improve tool life, and enhance surface quality.

Future research should focus on developing real time adaptive force models that incorporate CFC variations and exploring their implementation in digital twin environments for predictive machining stability. A more refined understanding of material behavior under high speed and high feed conditions can further improve the accuracy of force modeling and stability predictions in milling operations. Accordingly, the scope of the present numerical contribution has been clarified and a dynamometer based experimental plan has been outlined. Utilising a piezoelectric dynamometer to record triaxial cutting forces under controlled feed and speed settings, the planned validation is expected to corroborate the simulation predictions and further substantiate the reliability and applicability of the proposed modelling framework.

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