

Shape and Stress Sensing of Thin Stiffened Composite Panels Using iFEM Methodology

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iFEM Metodolojisi Kullanılarak İnce Takviyeli Kompozit Panellerde Şekil ve Gerilim Algılama

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Abstract

Thin stiffened composite panels (TSCPs) are often used in the aerospace industry due to their low weight and high load-bearing capacity. However, harsh environmental conditions can lead to delamination and structural failure. Structural health monitoring (SHM) systems, especially for shape sensing, help to detect such failures in real time. The inverse finite element method (iFEM) is a robust shape sensing technique that minimizes a weighted least-squares function to match experimental strain measurements with theoretical predictions. In this study, the iFEM-iQS4 inverse shell element is applied to a TSCP to enable accurate shape and stress reconstruction from sparse sensor data. For this purpose, a rectangular stiffened multilayer laminate with symmetric cross-ply made of carbon fiber epoxy material is considered. Two models of sensor placement, "Full" and "Reduced", are evaluated. The "Full" model includes all elements with strain sensors, while the "Reduced" model uses only circumferential elements. The accuracy of the shape and stress sensing is evaluated by comparing the iFEM results with the FEM results. It is shown that iFEM-iQS4 effectively predicts deformation and stress fields despite the use of a limited number of sensors, demonstrating its potential for efficient SHM of TSCPs in real time.

Keywords: Structural health monitoring; Shape and stress sensing; Inverse finite element method; Thin stiffened composite panels

Öz

İnce takviyeli kompozit paneller (TSCP'ler), düşük ağırlıkları ve yüksek yük taşıma kapasiteleri nedeniyle havacılık endüstrisinde sıklıkla kullanılır. Ancak, zorlu çevre koşulları delaminasyona ve yapısal hasara yol açabilir. Özellikle şekil algılama için yapısal sağlık izleme (SHM) sistemleri, bu tür hasarları gerçek zamanlı olarak tespit etmeye yardımcı olur. Ters sonlu elemanlar yöntemi (iFEM), deneysel gerinim ölçümlerini teorik tahminlerle eşleştirmek için ağırlıklı en küçük kareler fonksiyonunu en aza indiren sağlam bir şekil algılama tekniğidir. Bu çalışmada, iFEM-iQS4 ters kabuk elemanı, seyrek sensör verilerinden doğru şekil ve gerilim yeniden yapılandırmasını sağlamak için bir TSCP'ye uygulanmıştır. Bu amaçla, karbon fiber epoksi malzemeden yapılmış simetrik çapraz katmanlı dikdörtgen takviyeli çok katmanlı bir laminat düşünülmüştür. "Tam" ve "İndirgenmiş" olmak üzere iki sensör yerleştirme modeli değerlendirilmiştir. "Tam" model, gerinim sensörlü tüm elemanları içerirken, "İndirgenmiş" model yalnızca çevresel elemanları kullanır. Şekil ve stres algılamanın doğruluğu, iFEM sonuçlarının FEM sonuçlarıyla karşılaştırılmasıyla değerlendirilir. iFEM-iQS4'ün sınırlı sayıda sensör kullanılmasına rağmen deformasyon ve stres alanlarını etkili bir şekilde tahmin ettiği gösterilerek, gerçek zamanlı olarak TSCP'lerin verimli SHM'si için potansiyeli ortaya konmuştur.

Anahtar Kelimeler: Yapısal sağlık izleme; Şekil ve stres algılama; Ters sonlu elemanlar yöntemi; İnce takviyeli kompozit paneller

1. Introduction

Thin Stiffened Composite Panels (TSCP) are non-separated parts in the manufacture of structural elements in the aerospace industry, including airplanes and launch vehicles. Low weight and high strength are the advantages of TSCPs over their counterparts. However, these components are also susceptible to severe environmental conditions, which can sometimes lead to degradation of TSCPs and even catastrophic failure of the entire structure. To detect such probable failures in these components while avoiding loss of life, environmental problems and economic costs, a proper structural health monitoring (SHM) mechanism is required to install a structure on board. "Shape Sensing" is an important part

of the SHM system, which is to rebuild the deformation field in real time using sensors. Various methods such as modal, analytical and curve fitting schemes have been proposed to solve the inverse shape sensing problem (Davis et al. 1996, Bogert et al. 2003, Kim et al. 2004, Kang et al. 2007). In addition, there are several other studies dealing with the experimental detection of the shape of beam structures using various numerical algorithms (Rapp et al. 2009, Nishio et al. 2010, Glaser et al. 2012). In addition, there is some other research that uses FEA modeling to determine the mechanical behavior of composite materials and thus the shape recognition of these structural components (Meng et al. 2015, Kirar et al. 2024).

The inverse finite element method (iFEM) is a comparative shape sensing technique that was first developed and implemented by Spangler and Tessler (2005) at NASA. In iFEM, a least squares function plays the main role in optimizing the error between the calculated and measured strains. To date, some iFEM elements have been developed for shape detection of beams, plates and shell-like components. Abdollahzadeh et al. (2020) conducted a comparative study between these inverse shell elements. Study showed the technical advantages of the iQS4 inverse shell element over others (i.e. iMIN3 and iCS8). The iQS4 element is a quadrilateral inverse plate/shell element with four nodes, each of which have three translational and three rotational degrees of freedom (DOF). It offers excellent performance for shape sensing of complex geometries and was first introduced by Kefal et al. (2016). This robust element has already been used to reconstruct deformation, strain and stress fields of multiple structural components (Abdollahzadeh et al. 2021, 2022, 2023, Belur et al. 2024, 2025).

The most important part of the iFEM analysis is to collect experimental strain measurements at appropriate locations of the structure. For this purpose, the configuration of the exact arrangement of the sensor positions is required. Different types of strain gauges, rosettes or FBG sensors can be used to obtain on-board strain data, which is then used in the iFEM formulation. In cases where it is not possible to perform an experimental study, FEM simulation can be used as an alternative. The FEM analysis can also be used as a reference for checking the iFEM results. For multilayered structures, strain values vary across the thickness and therefore it is necessary that iFEM is modified with layer-wised plate theories such as refined zigzag theory (RZT) to achieve an accurate result (Kefal et al. 2020, Abdollahzadeh et al. 2021). The iFEM-RZT method is especially crucial for thick and moderately thick multilayer plates. While this technique provides an accurate result, it requires the implementation of additional sensors across the entire thickness of the composite structure, which incurs additional cost and computational time.

So far, there have been many studies on shape sensing using iFEM for different types of materials and geometries, but a study on the shape sensing of TSCPs is lacking. This study attempts to fill this gap by showing that iFEM technology can provide good results for TSCPs without the need for coupling with more sophisticated techniques, resulting in time, energy and cost savings in calculations. In this work, the iFEM-iQS4 element for shape and stress detection of TSCPs is used to reconstruct highly accurate deformation and stress fields with only sparse sensor data. The mathematical formulation of

iFEM-iQS4 is discussed in Section 2, while a benchmark problem evaluated by iFEM-iQS4 analyses is selected in Section 3. The results obtained of iFEM are compared with high fidelity FEM solutions. These findings confirm the high accuracy and efficiency of iFEM-iQS4 for the shape and stress reconstruction of TSCPs and pave the way for SHM of other composite components using the iFEM methodology in the future. Finally, concluding remarks on this study are presented in Section 4.

2. Methods and Materials

2.1. iQS4 inverse plate/shell element

The iFEM is mathematically based on minimizing the errors between calculated and measured strain values. This task is solved with a weighted least squares function. To determine the analytical values of strains in a deformed plate/shell structure, iFEM uses the first shear deformation (FSDT) theory. Like FEM, iFEM also uses triangular and quadrilateral elements to discretize the entire domain of the body into finite sub-elements. In this paper, a four-node quadrilateral FSDT-based inverse element, iFEM-iQS4, is used which has six DOFs. Figure 1(a) shows the iQS4 element with its nodal DOFs including three translational, (u_i, v_i, w_i) and three rotational $(\theta_{xi}, \theta_{yi}, \theta_{zi})$ DOFs. Displacement vectors can be derived in terms of these translational and rotational DOFs. Then the analytical components of the in-plane and out-of-plane strains can be calculated as follows (Kefal et al. 2016):

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \mathbf{e} + z\boldsymbol{\kappa} = \mathbf{B}^e \mathbf{u}^e + z\mathbf{B}^k \mathbf{u}^e \quad (1)$$

$$\begin{Bmatrix} \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \equiv \mathbf{B}^s \mathbf{u}^e \quad (2)$$

In this context, \mathbf{e} , $\boldsymbol{\kappa}$ vectors represent the membrane, and bending portion of the strains and \mathbf{B}^e , \mathbf{B}^k , and \mathbf{B}^s are the first derivative matrices of interpolation functions. The detailed form of these functions is found in Kefal et al. (2016).

2.2. Extracting Input data from the FEA

To obtain the experimental corresponding of analytical strain measures, a series of strain gauges must be attached to the top and bottom of the structure in question as shown in Figure 1 (b). Based on these in-situ strain measurements the experimental counterparts of the membrane and bending strains, \mathbf{E} and \mathbf{K} , can be formulated as follows:

$$\mathbf{E} = \frac{1}{2}(\boldsymbol{\varepsilon}^+ + \boldsymbol{\varepsilon}^-) \quad (3)$$

$$\mathbf{K} = \frac{1}{2h}(\boldsymbol{\varepsilon}^+ - \boldsymbol{\varepsilon}^-) \quad (4)$$

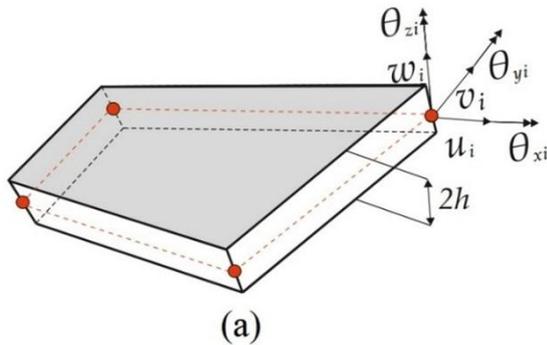
The superscripts “+” and “-” signify measurements obtained from sensors mounted on the top and bottom surfaces of the structure, as illustrated in Figure 1(b). When experimental sensors are not available, iFEM analysis can still be performed effectively by using a full finite element model (FEM) to simulate strain data. In this approach, a detailed FEM model of the stiffened plate is created under realistic loading and boundary conditions, and specific nodes or integration points are selected to serve as virtual sensor locations. Strain components such as ϵ_{xx} , ϵ_{yy} and γ_{xy} are extracted from these points and used as input for the iFEM reconstruction. This method allows validation of the iFEM formulation, evaluation of reconstruction accuracy and optimization of sensor placement strategies in a controlled and repeatable environment before applying the method to physical experiments.

2.3. Weighted least squares functional

Now to find the minimum error between these analytical and experimental strains, a least squares function is used in the following form:

$$\Phi_e = \alpha_e \|\mathbf{e} - \mathbf{E}\|^2 + \alpha_k \|\mathbf{k} - \mathbf{K}\|^2 \quad (5)$$

Herein the α coefficients are the penalty terms for the sensorless elements. These constants are chosen relatively small positive values to guarantee the integrity of the data. Taking the derivative of this function with



respect to the \mathbf{u}^e nodal displacement, yields in the final equation as below:

$$\frac{\partial \Phi_e}{\partial \mathbf{u}^e} = \mathbf{k}^e \mathbf{u}^e - \mathbf{f}^e = 0 \Rightarrow \mathbf{k}^e \mathbf{u}^e = \mathbf{f}^e \quad (6)$$

where \mathbf{k}^e stands for the stiffness matrix and \mathbf{f}^e for the load vector, which are in the local coordinate system. These local variables must be transformed into a global coordinate system using a suitable transformation matrix, \mathbf{T}^e .

$$\mathbf{K}\mathbf{U} = \mathbf{F}, \quad \mathbf{U} = \mathbf{K}^{-1}\mathbf{F} \quad (7)$$

$$\mathbf{K} = \bigcup_{e=1}^N [\mathbf{T}^{eT} \mathbf{k}^e \mathbf{T}^e], \quad \mathbf{Q} = \bigcup_{e=1}^N [\mathbf{T}^{eT} \mathbf{f}^e], \quad (8)$$

$$\mathbf{U} = \bigcup_{e=1}^N [\mathbf{T}^{eT} \mathbf{u}^e]$$

The summation symbol in above equations means the assembly procedure over N inverse elements. In this study, the accurate performance of iFEM-iQS4 analysis is checked by calculating the percent difference (PD) for the maximum values of total displacement, total rotation and equivalent stress components between iFEM and FEM analyses as follows:

$$\text{PD}(\%) = \left| \frac{\delta^{\text{iFEM}} - \delta^{\text{FEM}}}{\delta^{\text{FEM}}} \right| \times 100 \quad (9)$$

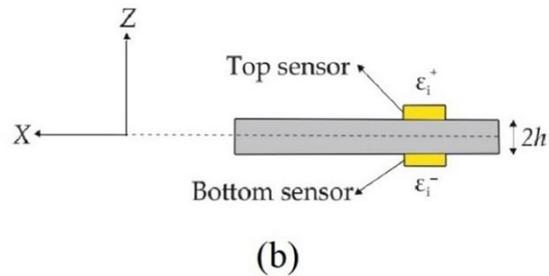


Figure 1. a) Geometry of the iQS4 element with nodal points showing translation, and rotation DOFs, b) Placement of strain gauges mounted at the top and bottom surfaces of each iQS4 element.

2.4. Modelling and materials

To demonstrate the practical suitability of iFEM-iQS4 for capturing the shape of TSCPs, a rectangular stiffened laminate with a length of 3 m and a width of 1 m is considered. Two transverse and four longitudinal stiffeners with a height of 150 mm each reinforce the laminate. Both the panel and the stiffeners have a uniform thickness of 25 mm and a symmetrical cross-ply arrangement with fiber angles of $(0^\circ/90^\circ/0^\circ/90^\circ/0^\circ)$. Each of these layers has a uniform thickness of 5 mm and is made of carbon epoxy material. The structure is subjected to self-weight loading, with all edges assumed to be fully

restrained. Figure 2 illustrates Boundary condition of the displacement, dimensions and stacking sequence of the laminates. The FEM analysis is performed to obtain simulated strain data. To determine the mesh resolution, a mesh dependency analysis is performed. Subsequently, a high-resolution FEM model with 1108 elements is used for this purpose. For the iFEM analysis, however, the structure is subdivided by only 118 elements. The number of subdivisions for FEM and iFEM analyses are chosen so that the required strain data for the center of each iFEM element can be extracted by a corresponding element of the FEM analyses. For the iFEM analysis, two sensor

arrangements with the designations "Full" and "Reduced" are considered. In the "Full" model, all elements are equipped with strain sensors and therefore have 118 elements that are provided with upper and lower strain rosettes. In the "Reduced" model, on the other hand,

there are only 30 sensors at the edge (perimeter) of the laminate. Figure 3 shows the FEM and iFEM mesh resolutions and the sensor arrangements of the iFEM analyses.

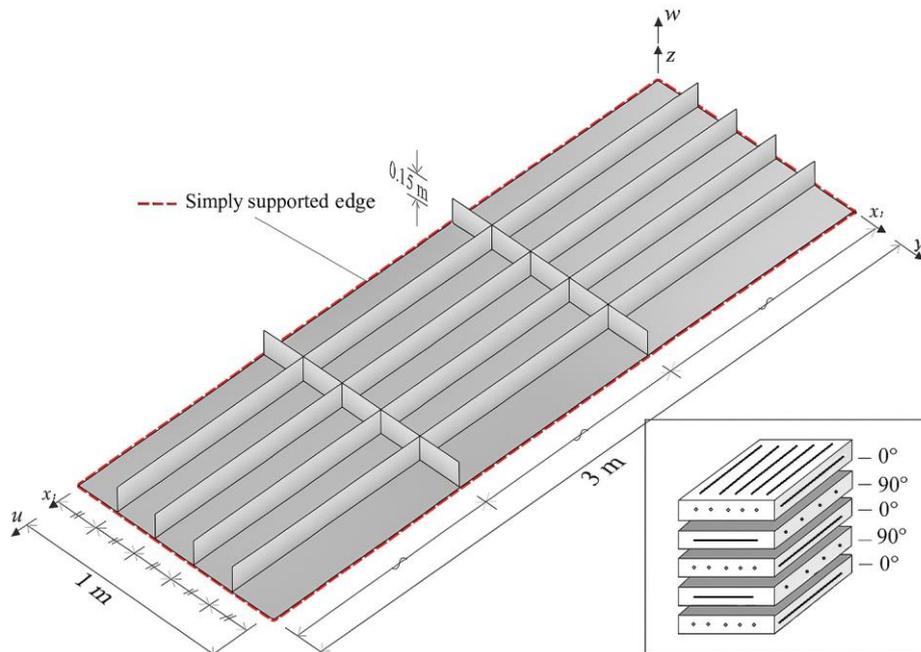


Figure 2. Boundary condition of the displacement, dimensions and stacking sequence of the laminates.

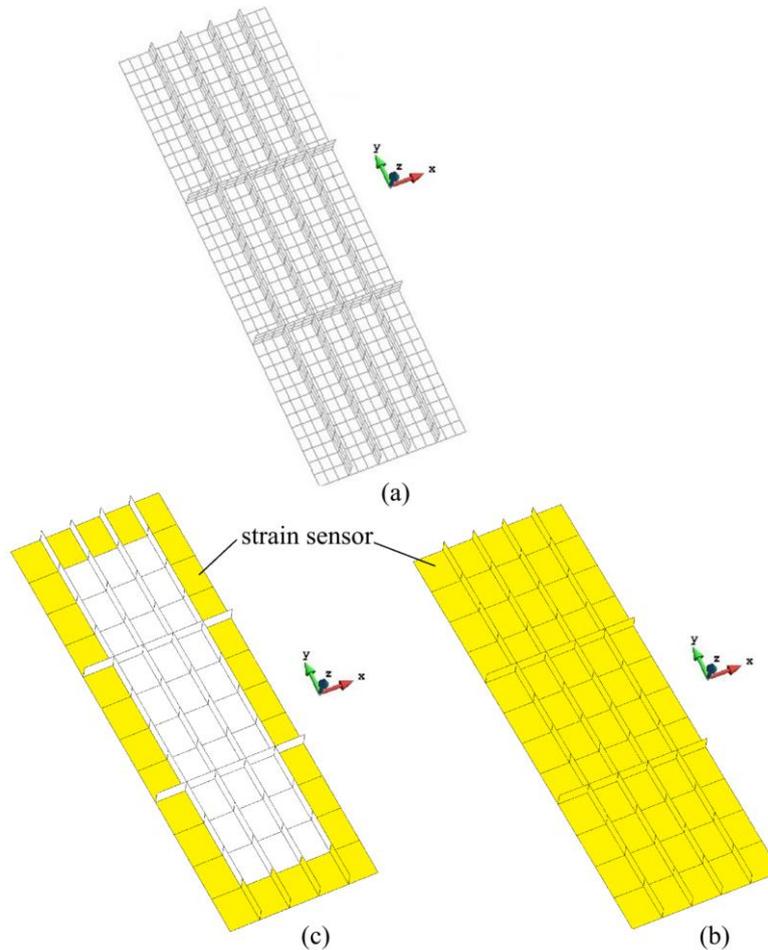


Figure 3. a) Mesh resolution of FEM analysis, b) Mesh resolution and sensor arrangement of iFEM-Full analysis, c) Mesh resolution and sensor arrangement of iFEM-Reduced analysis.

3. Results and Discussions

The main criterion for evaluating the accuracy of the displacement reconstruction of the full/reduced sensor arrangement cases is based on the percentage differences between the maximum (critical) values of total displacements and total rotations between the iFEM and FEM analyses. Figure 4-5 compares the contours of the total displacement and the total rotation of the two iFEM models with those of the FEM analysis. Based on these results, the percentage difference in the total

displacement between the iFEM and FEM analyses is approximately 6.1% for the "Full" sensor distribution model. The percentage difference increases slightly to 7.8% when using the "reduced" sensor distribution, which is still within the range of reliability despite the drastic reduction in the number of sensors to almost a quarter. The results of the total rotation show that iFEM can predict the maximum values of the rotations with 15% and 20% errors when using the "Full" and "Reduced" number of sensors, respectively.

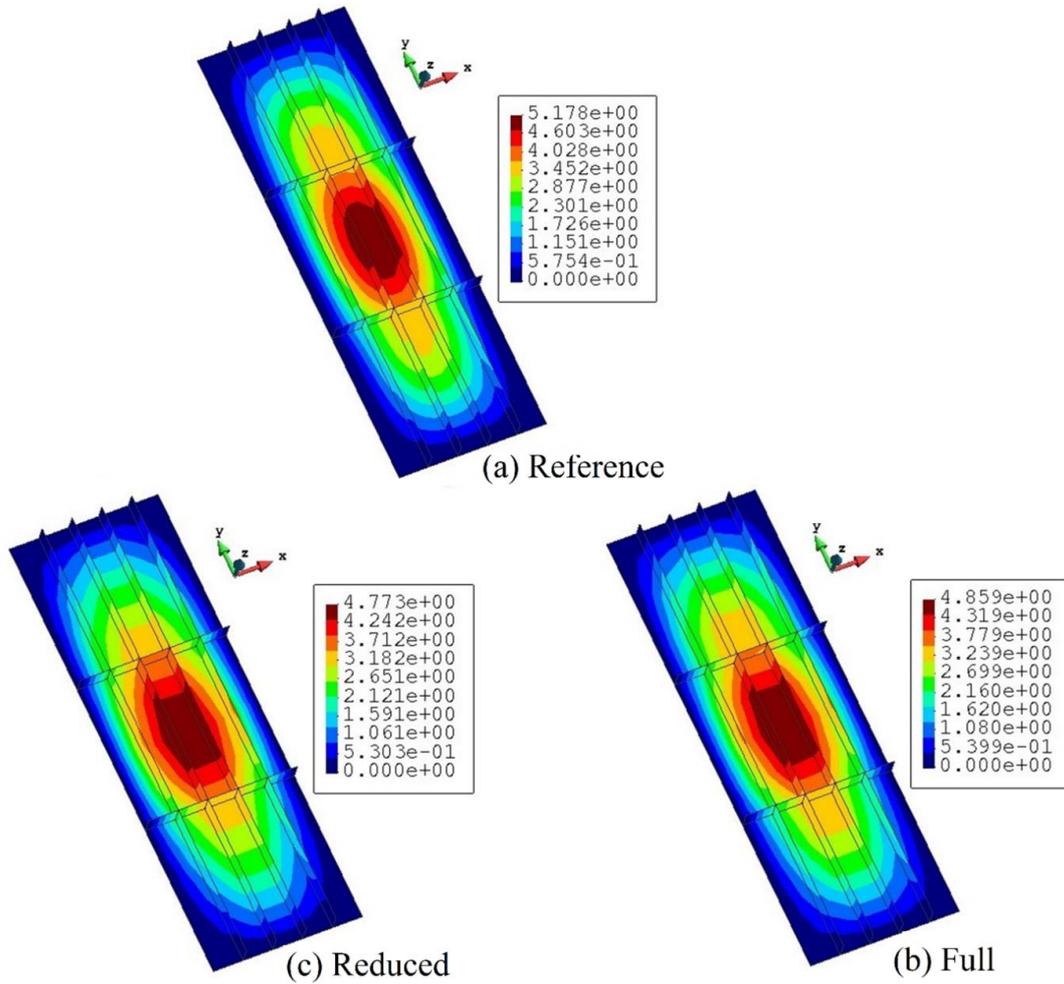


Figure 4. Total displacement contours [mm] obtained from a) FEM analysis, b) iFEM analysis with full number of sensors, c) iFEM analysis with reduced number of sensors.

In iFEM analysis of stiffened composite plates, lower errors in total displacements are usually observed compared to higher errors in rotational displacements. This discrepancy is because the displacements are determined by integrating strain fields, which smooths out noise and local inaccuracies and makes the displacements less sensitive to sparse strain measurements. In contrast, rotational displacements are based on curvature or strain gradients, making them more prone to errors, especially when strain measurements are sparse or not optimally positioned. In addition, the presence of stiffeners leads to strong local

variations in bending strains, which are difficult to capture accurately without dense sensor coverage near these regions. Lower order element formulations and the use of shear deformable plate theories can contribute to these errors by limiting the accuracy of curvature reconstruction. To improve rotational accuracy, more strain sensors should be placed near high-slope zones, and higher-order finite element formulations or mixed formulas may be required. In addition, the reconstructed contour plots predict the distribution of displacement and rotation with high consistency by reference FEM solution. These qualitative and quantitative results show that iFEM-iQS4 can predict the deformation fields even with a

limited number of sensors. Therefore, the iQS4 element has high potential and practical implementation for performing iFEM-based shape recognition of TSCPs. Although iFEM has a high potential to reconstruct the displacement and stress fields, it can face some challenges, especially when a reduced number of sensors is used. One of the main limitations of the reduced iFEM model is its sensitivity to sensor density and placement, especially under complex or non-uniform loading conditions. An insufficient number of sensors can lead to poor resolution of the strain field, which directly affects the accuracy of the reconstruction of displacements and stresses — especially in areas with strong curvature or near structural discontinuities such as stiffeners or cut-

outs. From an industrial perspective, the implementation of such a system on real structures such as an airplane wing poses a practical challenge. These include the high cost of sensor hardware, the difficulty of integrating sensors into composite structures without compromising integrity, and the need for robust data acquisition systems that can function in harsh environments. Future work should address these issues by investigating dynamic loading scenarios where the time-dependent behavior and modal responses need to be accurately captured. Furthermore, extending the framework to more complex composite architectures, such as angle-ply or quasi-isotropic laminates, would improve its applicability to modern aerospace structures.

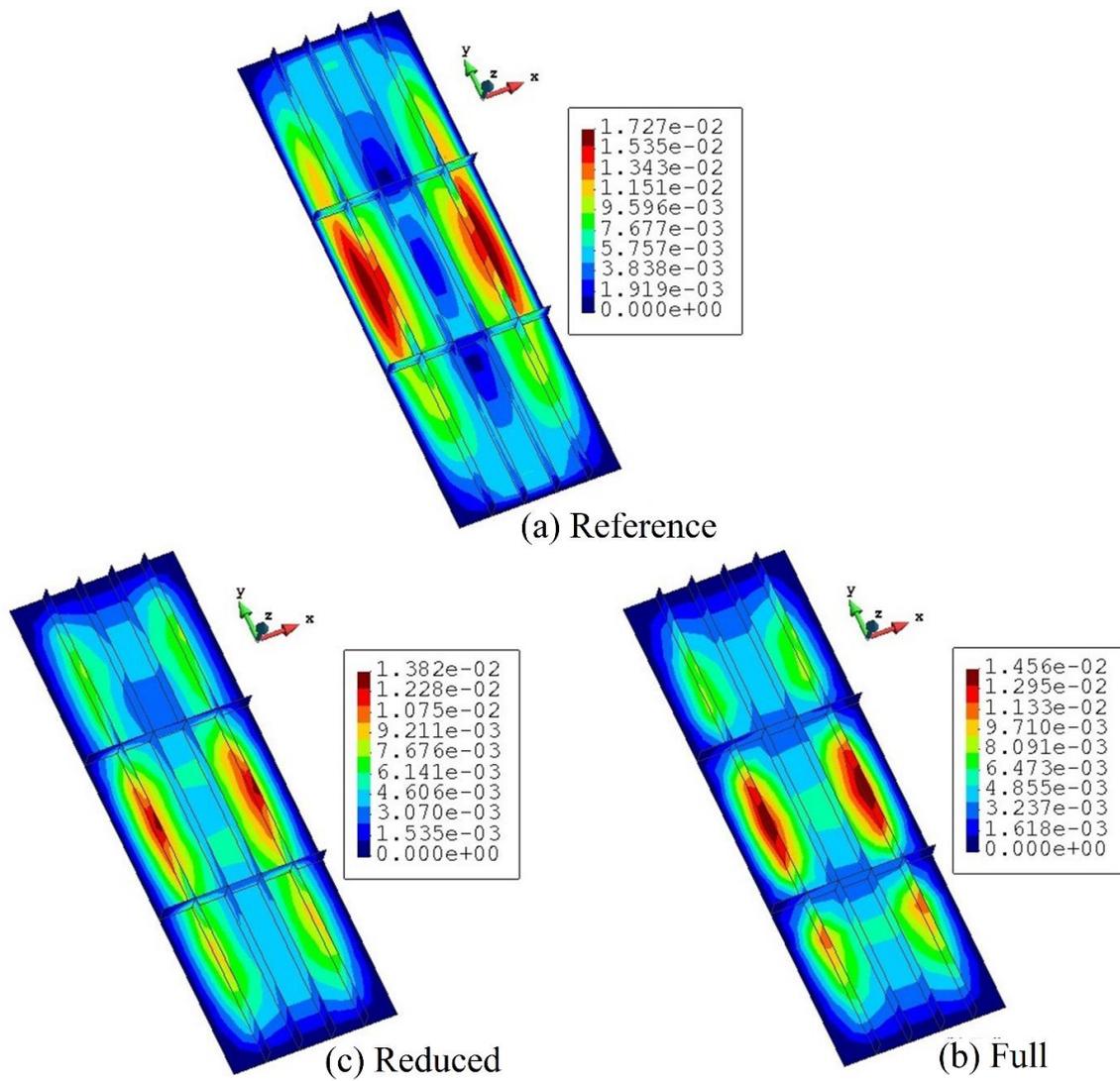


Figure 5. Total rotation contours [rad] obtained from a) FEM analysis, b) iFEM analysis with full number of sensors, c) iFEM analysis with reduced number of sensors.

In this paper, in addition to shape sensing, stress sensing analysis is also evaluated using equivalent (von Mises) stress analysis. To this end, iFEM analyses are performed to reconstruct the von Mises stress contours over the entire domain of the TSCP using a discrete number of

simulated strain data. The stress contours and results are shown in Figure 6. They obviously confirm the great potential of iFEM technology to predict the stress distribution over the entire structure. The percentage error between the iFEM and FEM analyses is less than 2% in this case.

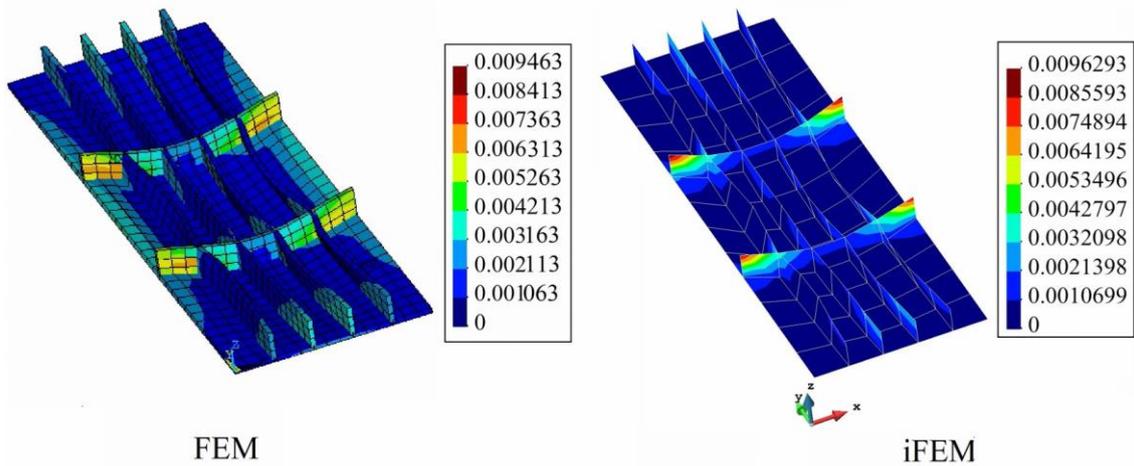


Figure 6. Von Mises stress contours [MPa] obtained from iFEM and FEM analyses.

4. Conclusions

Thin stiffened composite panels (TSCPs) are frequently used in aerospace engineering. These components are exposed to extreme environmental conditions and can fail significantly. Therefore, a proper structural health monitoring (SHM) approach is important to ensure that these components are still in service. The inverse finite element method (iFEM) is a relatively new method for shape identification based on an error minimization procedure between experimental/simulated and analytical strain values obtained on specific parts of the structure. The mathematical framework of iFEM is based on the first shear deformation theory (FSDT). Therefore, it is more suitable for thin and moderately thick isotropic materials. However, for orthotropic materials such as multilayer and sandwich composites, complementary methods are required to obtain a reliable solution, which means additional time and cost. In this work, an attempt is made to evaluate the efficiency of the iFEM method for shape and stress sensing of TSCPs for the first time in literature. For this purpose, a rectangular stiffened composite panel is modelled and iFEM analyses are performed. The results obtained are compared with FEM results, which show a good agreement with the reference solutions. The percentage difference between the iFEM and FEM analyses for the total displacement and stress fields is about 6% and 2%, respectively, demonstrating the accuracy and efficiency of the iFEM approach for shape and stress sensing of TSCPs, which can be used for other types of composite structures in the future.

Declaration of Ethical Standards

The author declares that he complies with all ethical standards.

Declaration of Competing Interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

All data generated or analyzed during this study are included in this published article.

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