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Constructing the Unity of the Cosmic Body in the Timaeus and Archytas's Solution to the Doubling the Cube Problem[1]

Abstract: This paper examines the mathematical model of the Cosmos presented in Plato's *Timaeus*, focusing on how geometrical proportions construct the unity of the Cosmos's body. It argues that Plato's proportions are fundamentally based on magnitudes rather than numbers, a significant point of contention in scholarly interpretations. Engaging with influential scholars such as Heath, Cornford, and Taylor, who contend that Plato's ratios involve numbers due to his Pythagorean influences, this paper challenges these views. It emphasizes Plato's deployment of geometrical concepts, particularly βάθος (depth) in a specific way, which strongly indicates a concern with continuous magnitudes rather than discrete numbers. Crucially, in constructing the body of the Cosmos, Plato appears to allude to Archytas's solution to the Delian problem. Archytas's famed solution geometrically determines two mean proportionals between two lines through the intersection of three-dimensional solids. This paper contends that Plato, in his account of the proportional unity of the Cosmos's solid body, draws inspiration from this landmark achievement in stereometry.
Keywords: Archytas, Timaeus, Doubling the cube, Cosmos, geometry,

Evrenin Bedeninin Birliğinin İnşası ve 'Küpü İki Katına Çıkarma Sorunu'na Arkhutas'ın Çözümü

Öz: Bu makale, Platon'un *Timaios* diyaloğunda sunulan Kozmos'un matematiksel modelini, geometrik oranların Kozmos'un bedeninin birliğini nasıl inşa ettiğine odaklanarak incelemektedir. Makale, Platon'un Cosmos'un birliğini inşa ederken kullandığı oranın sayılardan ziyade temelde büyüklüklere dayandığını savunmaktadır. Platon'un oranlarının Pythagorasçı etkiler nedeniyle sayıları içerdiğini öne süren Heath, Cornford ve Taylor gibi etkili akademisyenlerle tartışmaya

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girerek, bu makale söz konusu görüşlere meydan okumaktadır. Platon'un geometrik kavramları, özellikle de ayrık sayılardan ziyade sürekli büyüklüklerle ilgili bir kaygıya güçlü bir şekilde işaret eden βάθος (derinlik) kavramını kullanımını vurgulamaktadır. Daha da önemlisi, Kozmos'un bedenini inşa ederken Platon'un, Archytas'ın Delos problemine getirdiği çözüme atıfta bulunduğu görülmektedir. Archytas'ın meşhur çözümü, üç boyutlu katı cisimlerin kesişimi yoluyla iki doğru arasında iki orta orantılıyı geometrik olarak belirler. Bu makale, Platon'un Kozmos'un katı bedeninin orantısal birliğine dair anlatımında, stereometrideki (katı cisimler geometrisi) bu dönüm noktası niteliğindeki başarıdan ilham aldığını öne sürmektedir.
Anahtar kelimeler: Arkhutas, Kozmos, Timaios, Delos Sorunu, Geometri.

Introduction

In the *Timaeus*, Plato employs geometric proportions to articulate the construction of the Body of the Universe. Given that this body is conceived as three-dimensional, its unity is ensured through a continuous geometrical proportion involving two mean proportionals between four elements (That means, two *means* between two lines, such as a:b::b:c::c:d. In this arithmetical construction, b and are means between a and d). These means geometrically bind the four fundamental elements—fire, earth, air, and water. However, Plato's silence on whether these ratios pertain to discrete quantities (numbers), volumes, or weights has led to scholarly skepticism. Crombie, for instance, suggests that the passage should not be interpreted with excessive literalness (Crombie, 2013: 201). Nevertheless, a meticulous examination of the history of ancient Greek mathematics offers valuable clues for discerning the precise nature of Plato's mathematical framework. Two primary candidates emerge for consideration: proportions of numbers (arithmetical) and proportions of magnitudes (geometrical).

The prevailing tradition of interpretation posits that the proportional relationships Plato establishes are numerical. This paper challenges this orthodox view, arguing instead that Plato refers to proportions between continuous magnitudes, a conception deeply intertwined with Archytas of Tarentum's solution to the Delian problem—the doubling of the cube. The pertinent passage in the *Timaeus* (31b-32c) describes the binding of the four kinds (water, earth, air and fire) to constitute the unified body of the Cosmos. Plato achieves this binding through a continuous proportion, justifying its necessity by the solid, three-dimensional nature of the cosmic body. This explicit emphasis on solidity and the requirement of two mean proportionals directly echoes the core problematic and achievement of Archytas's solution, which provides a method for finding two mean proportionals between any two given lines (magnitudes). Thus, Plato's utilization of such a proportion constitutes an indirect yet profound reference to Archytas's mathematical innovation.

1. The Solution of Archytas

The Delian problem, concerns the task of doubling the volume of a given cube. This requires finding the length of the side of the new cube, or more fundamentally, the factor by which the original side must be increased to achieve this doubling, and critically, a method to construct or determine this factor. While historical accounts (Theon of Smyrna, 1978: 1-2), (Plutarch, 1878a: 385 – 386), (Plutarch, 1878b: 482), (Netz, 2004: 294) of the problem's origins vary, they consistently highlight the central challenge: constructing a cube with twice the volume of a given one. The solution necessitates finding a line x such that if a is the side of the original cube, then $x^3=2a^3$. This implies $x=\sqrt[3]{2}$. Such a length is incommensurable with a (and a^2), a fact that likely contributed to the problem's difficulty and its legendary framing as a divine injunction from the Oracle at Delos. According to same sources, the Delians, unable to solve the problem, consulted Plato, who re-framed the challenge, emphasizing its mathematical significance rather than offering a direct solution. The historicity of this particular narrative is debated (Kouremenos, 2011: 347). For example, Knorr suggests that it is invented in Plato's Academy. (Knorr, 1993: 22).

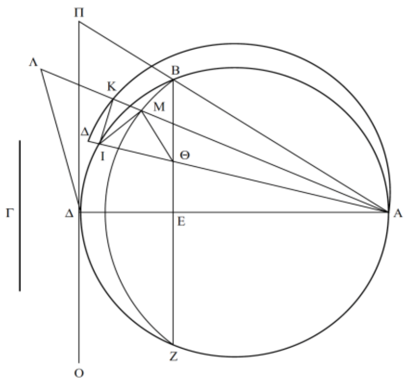
A crucial breakthrough occurred when Hippocrates of Chios reduced the problem of doubling the cube to the problem of finding two mean proportionals in continuous proportion between two straight lines (Netz, 2004: 294). That is, given two lines a and b (where for the Delian problem, b=2a), one must find x and y such that a:x::x:y::y:b. From this proportion, If b=2a, then $x^3=2a^3$. Archytas of Tarentum, a contemporary of Plato, provided an ingenious solution to this problem of finding two mean proportionals by employing the intersection of three-dimensional geometric solids: a section of a cone, a cylinder, and a torus (generated by rotating a circle). Our knowledge of Archytas's solution primarily derives from Eutocius's commentary on Archimedes' On the Sphere and Cylinder. As Netz presents it in his edition, the construction is as:

Archytas's solution is not based on a simple 2D drawing but on the intersection of three complex surfaces in three-dimensional space:

A Semicylinder: A half-cylinder is erected perpendicularly on a semicircle that has the longer line AΔ as its diameter. A Cone: A large triangle (AΠΔ) in the base plane is rotated around the axis AΔ, generating the surface of a cone. A Rotated Semicircle (Torus section): A second semicircle, also standing perpendicular to the base plane on the diameter AΔ, is rotated around a point A. This rotation cuts a curve into the surface of the semicylinder. The entire construction is designed to find the single point in space (K) where these three surfaces—the cone, the semicylinder, and the curve from the rotated semicircle—all intersect. Once the intersection point K is located, the solution is found by projecting it back onto the original 2D plane. A perpendicular line is dropped from K down to the base circle, hitting it at a point I. The geometry of this setup, particularly the relationships created by right angles in the various semicircles, results in a series of similar triangles (△ΔKA, △AKI, and △IAM). The properties of these similar triangles mathematically guarantee the continuous proportion:

AΔ:AK=AK:AI=AI:AM

Since the construction sets the line AM to be equal to the second given line Γ, the proportion is solved (Netz, 2004: 290-293).



[2]

Regarded as one of the earliest significant works in stereometry, Archytas's solution was more than just an ingenious answer to the Delian problem. It demonstrated a profound, and for its time, novel understanding of how complex three-dimensional curves and surfaces could be manipulated to determine specific continuous linear magnitudes. This conceptual advance becomes relevant when considering Plato's efforts to articulate the structure of a three-dimensional cosmos through proportional means, as it highlights the understanding of magnitudes within spatial contexts that Plato could draw upon.

2. the *Timaeus*

In the *Timaeus* Plato presents a mathematical model of the universe. This endeavor is framed by his broader metaphysical distinctions. While mathematics and its objects belong to the intelligible sphere with its second place to the Forms, physical being belongs to the perceptible sphere, as argued in the *Republic's* Divided Line (Plato, 1969: 509d–511e)[3]. This fundamental distinction between the intelligible and the perceptible is paramount. Plato, at the beginning of his natural

[2] This image comes from the ancient manuscripts and is most probably the Archytas' own construction. The image from Netz, R., *The Works of Archimedes* v. 1, s. 294.

[3] İn the Divided Line analogy, Plato divides the mental states due to their objects. First he divides perception and thinking. Then, divide them again. So a fourfold structure emerges: ideas, mathematical objects, physical objects and images like shadows and reflections. This is a hierarchical structure and the hierarchy is determined by the exactness of the objects or the clearness of their knowledge.

philosophy in the Timaeus, reminds this distinction (Plato, 1925: 27d-28a), though he doesn't touch upon the mathematics' intelligible character. After saying that Becoming is the copy (εἰκών) of Being which is the model (παράδειγμα), he states that explanations also must resemble their objects, therefore the explanation of the Cosmos, meaning physics can only be a probable story (εἰκός μῦθος) (Plato, 1925: 29d).

This world is in the constant state of Becoming and subject to change. Naturally, some commentators such as Kahn (Kahn, 2013:177), Taylor (Taylor, 1928:59) and Brisson (Brisson, 2018:216) state that natural philosophy is an εἰκός μῦθος because its subject is a world of change. These interpretations are reasonable but incomplete. The significance of the εἰκός μῦθος lays not solely in the constant change of the phenomenal world. It suggests that Plato is constructing a relationship between two distinct spheres of being, and the discourse about this construction does not possess the necessity and exactness of the propositions of διάνοια, nor the explanatory power and generality of the propositions of νόησις.[4]

Consequently, the physics in the *Timaeus* cannot adhere the logical forms of dialectics and mathematics. Therefore, the application of these mathematical operations cannot be based on any logically necessary connections. Instead, the relation between mathematics and the cosmos is established through *metaphors*. It means that Plato uses mathematics as a source of inspiration, and he is searching for *the most suitable candidates* to construct the Cosmos in a context different from the ones that are expressed in the logical forms.

[4] This does not lead to the conclusion that Plato's narrative in the Timaeus is an unwarranted effort, or that Plato understood only a children's tale from εἰκός μῦθος. Furthermore, later in the dialogue, Plato discusses those who talk about the local Gods, saying that they have neither εἰκός nor necessary proof (40e). For Plato, then, the mathematical explanation of nature involves not a pure story, but a proof that does not have the necessity and certainty, making nature known in a kind of intermediate state of mind that transcends the πίστις but does not reach to διάνοια.

3. The allusion to Archytas

With these epistemological and methodological considerations in mind, we turn to the pivotal passage in the *Timaeus*: (31b-32c):

> Now that which has come into existence must be of bodily form, visible and tangible; yet without fire nothing could ever become visible, nor tangible without some solidity nor solid without earth. Hence, in beginning to construct the body of the All, God was making it of fire and earth. But it is not possible that two things alone should be conjoined without a third; for there must be some intermediary bond to connect the two. And the fairest of bonds is that which most perfectly unites into one both itself and the things which it binds together; and to effect this in the fairest manner is the natural property of proportion. For whenever any three numbers, cubic or square (ἀριθμῶν τριῶν εἴτε ὄγκων εἴτε δυνάμεων), is such that as the first term is to it, so is it to the last term, and again, conversely, as the last term is to the middle, so is the middle to the first,—then the middle term becomes in turn the first and the last, while the first and last become in turn middle terms, and the necessary consequence will be that all the terms are interchangeable, and being interchangeable they all form a unity. Now if the body of the All had had to come into existence as a plane surface, having no depth (βάθος), one middle term would have sufficed to bind together both itself and its fellow-terms; but now it is otherwise: for it behaved it to be solid of shape (στερεοειδῆ) , and what brings solids (**τὰ στερεὰ**) into unison is never one middle term alone but always two. Thus, it was that in the midst between fire and earth God set water and air, and having bestowed upon them so far as possible a like ratio one towards another—air being to water as fire to air, and water being to earth as air to water, —he joined together and constructed a Heaven visible and tangible. (Plato, 1925: 31b – 32c)

This passage directly answers the implicit question of why there are four primary elements. Plato's explanation hinges on the crucial roles of *proportion* and the *solidity* (three-dimensionality) of the Cosmos's body. These two aspects, particularly the necessity of two means for binding solids, strongly allude to Archytas's solution.

Even before this explicit statement, subtle allusions to mathematical concepts appear. Plato initiates his construction with fire and earth, positing the necessity of a "third thing" or bond. While some commentators[5] interpret this primarily through the lens of mediating the opposition between these elemental extremes,

[5]For example Salamone says that ''Fire and Earth had been commonly regarded on the two extreme and contrary elements (or end elements), since Fire belongs to the Sky in opposition to the earth'' and equalize the proportion put forward by Plato with the Golden ratio, which is formulated as (A+B):A=A:B by her. (Salamone, 2019: 2)

one can also discern an allusion to a fundamental mathematical principle. Plato's assertion that "there must needs be some intermediary bond to connect the two" resonates with the definition found in later mathematical texts, such as Euclid's Elements, V, Def. 8:

"A proportion in three terms is the least possible."[6]

This definition means that, for a proportion at least three terms are needed. The "bond" in physics is analogous to the "mean" in mathematics. This *mean proportional* requires at least two extreme terms for its establishment, creating a minimum of three terms. In mathematical practice, means are the results of operations or constructions. Within the narrative framework of the *Timaeus*, the mathematical concept of a mean is metaphorically transformed into the physical "bond" between elements; a mathematical definition becomes a cosmological principle. The necessity of the middle term, therefore, stems not only from physical contrariety but also from the mathematical requirement that a proportion involves at least three terms, one being the mean proportional.

The second allusion to mathematics is the reason why he chooses the proportion for binding the four elements. In proportions, terms are changeable. In Book V, corollary of Prop 7, we see this statement: If any magnitudes are proportional, then they are also proportional *inversely* (my emphasis). If we express a proportion like this: a:b::b:c, its inversion would be this: b:a::c:b, which is the same inversion Plato talked about when he says that "the middle term becomes in turn the first and the last, while the first and last become in turn middle terms, and the necessary consequence will be that all the terms are interchangeable, and being interchangeable they all form a unity" (Plato, 1925: 32a). But again, this statement turns into some principle that makes the body of the cosmos unified.

[6] All translations of Euclid are from: Euclid, *Euclid's Elements*. Translated by Sir Thomas Little Heath, Dover, New York, 1956.

The most significant allusion to Archytas emerges with Plato's distinction between plane and solid constructions:

> Now if the body of the All had had to come into existence as a plane surface, having no depth (βάθος), one middle term would have sufficed to bind together both itself and its fellow-terms; but now it is otherwise: for it behaved it to be solid of shape (στερεοειδῆ) , and what brings solids (τὰ στερεὰ) into unison is never one middle term alone but always two. (Plato, 1925: 31bc– 321)

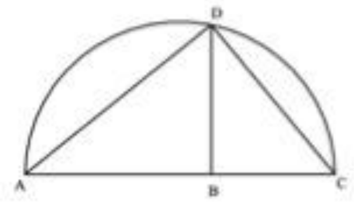
Plato is explicitly referencing mathematical principles here. The critical question is whether he refers to proportions of numbers or magnitudes. There are mainly two candidates. The first one is the proportion of numbers and the second one is the proportion of magnitudes. If Plato alludes to Archytas' solution, then this proportion must be about magnitudes. Most scholars say that this passage is about the proportion of numbers (there is also exceptions). This issue will be disputed later. But for now, it can be said that because Plato's language is heavily geometrical here, he is talking about the magnitudes. But if it is an allusion to Archytas it is also an allusion to a *construction* but not some kind of a definition or a general statement of mathematics.

Archytas's solution is precisely such a construction, reliant on prior definitions and propositions, but itself a dynamic generation of lines through intersecting solids. Having established general principles through earlier allusions, Plato now seemingly has the Demiurge construct the universe[7] by invoking contemporary geometrical constructions. The allusion is thus to the method of finding mean proportionals.

Therefore, the allusions are the construction of the finding the means. The first part of the passage is a reference to method of finding one mean proportional. Plato says that one term would be enough to bind itself and the other terms if the universe was a plane surface. In Euclid's Elements book VI, Prop. 13, we find the construction of the finding one mean.

[7] 'He bonded and *constructed* the Heaven' (συνέδησεν καὶ συνεστήσατο οὐρανὸν) (32b)

In Book VI prop 13, Euclid show how to find a mean proportional to two given straight lines. The solution is based on a geometrical construction:



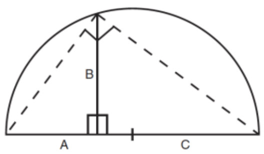
Euclid shows how to deal with this kind of problem in Ancient Greek, which is some geometrical construction to reach interrelated similar triangles satisfy the a:b::b:c relation.

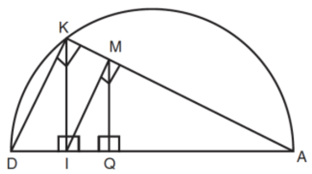
Now let's remember the *Timaeus'* narrative's story-like context. As said before, the mathematical means turn into bonds in cosmology and what is a mathematical conclusion in the practice itself turns into a cosmological principle in the *Timaeus*. In mathematical or *dianoiatic* thinking, DB is the line we reach by some constructions by using *two dimensional mathematical objects.* We need to construct a right triangle for applying to similarity of triangles and for doing so we need to construct a semi-circle; two dimensional objects. But in the story-like Platonic perspective, the line DB turns into a *bond* in this two-dimensional schema. When we look at the final representation of the construction (below), the lines AB, BC and BD are in a connection, and the two-dimensional objects are in a unity thanks to the *proportion* and the mean BC. Therefore, when Plato talks about being plane, he is not talking about plane numbers but *planes themselves.* And he gets his inspiration from the two-dimensionality which is involved in the method itself. Therefore, terms themselves are lines. They come together in a two-dimensional space, and they are making those two-dimensional objects being a unity.

The allusion to the Archytas' solution is now obvious. The second part of the passage makes same connections but this time for *three-dimensionality.* Let's turn to Archytas' solution after some considerations.

Huffman points out that Archytas' solution is the earliest example of stereometry that we have (Huffman, 2005: 355). But why does this solution is an example of solid geometry? It is not because of the mentioned problem is about the cubes, but because it solves a different and more general problem with stereometry which is the finding the two mean proportionals. Therefore, the same relation between proportion and similarity of triangles is working here but this time in the third dimension. Archytas' solution has already this statement: 'as ΔA to AK, that is KA to AI, (15) so IA to AM, (16) through the similarity of the triangles.'

Therefore, the function of three-dimensional objects formed by moving two-dimensional objects in the three-dimensional space is to reach the similar right triangles that are interrelated. This construction is like an *expansion* of the finding one mean into the *3d space* as Netz states (Netz, 2022: 70).

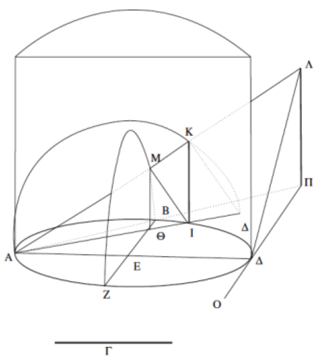




[8]

The figure on the left is the one used for the finding one mean between two lines. The figure on the right is a part of the Archytas' construction. From this point of view that Archytas' solution is an expansion of the first method is more obvious. But there is an important difference between them. The first one represents the final construction of the solution of finding one mean. But the second one, is only a part of the final construction, although it is the schema to which Archytas tries to reach:

[8] Images are from Netz, ibid., 65 and 68.



[9]

The final construction is in the three-dimensional space, and it includes three-dimensional objects. Therefore, for finding one mean we need two-dimensional objects. But to find two means, we need three-dimensional objects.

Now let's remember the Plato's narrative's context and it's meaning again: means turn into bonds and what is a conclusion in the practice itself turns into a principle in the hands of Plato. In dianoiatic thinking, KA and AI in the configuration of Archytas are the lines we reach by some constructions by using *three-dimensional mathematical objects.* We need to construct right triangles for applying to similarity of triangles and for doing so we need to construct three-dimensional objects. But in the story-like Platonic perspective, the lines KA and AI turn into bonds in this three-dimensional schema. When we look at the final representation of the construction, the lines DA, AK, AI and AM are in a bond and the three-dimensional objects are in a *unity* thanks to the *proportion* and the *means* KA and AI as a *metaphor*. Therefore, when Plato talks about being solid, he is not talking about the solid numbers but the *solids themselves.* The importance of this difference lies in that: the mean proportionals between numbers should be integers but the means between magnitudes doesn't have that limitation and can be irrational also. And he gets his inspiration from the three-dimensionality which is involved in the method itself. Therefore, terms themselves are not three-

[9] A modern construction by Huffman: Huffman, Carl A., *Archytas of Tarentum*, p 356

dimensional, they are lines, but they are coming together in the three-dimensional space, and they are making those three-dimensional objects being a unity in a metaphorical sense.

From this perspective, the point K, where the three solid objects connected to form interrelated similar triangles, is the point of the *union,* again in a metaphoric sense; means turn into principles, so it is like that the place of the point K depends on the proportion. Because of the continuous proportion of four lines, those solids become a union in the point K.

This is the inspiration that Plato's adheres. There are also some reasons why Plato would indicate the Archytas' solution.

The first one is about the 'strictness. Mathematical objects have strict and definite relationships within themselves. On the other hand, pre-nature state of the world has completely random and contingent relationships (Plato, 1925: 47e – 53c). Mathematically constructed nature, on the other hand, has "probable" relationships. When the random is associated with the strictness, probable or likely relations emerge (Plato, 1925: 53d). What part of mathematics is Plato referring to when he ascribes likeliness to nature? More precisely, what aspect of the practice of mathematics does he refer to when he moves nature from randomness to a realm of knowledge that is at least probable? Since he speaks as if there is a necessary relation between solids and continuous proportions, this strictness cannot come from the solid in the empirical world. So, we need to look at a field where there is a necessary relationship between solids and continuous proportion. In other words, we need to look at 3-dimensionality, which is not found in the physical world, but in the ideal world; that is, to the ideal mathematical objects in 3D.

The place where ideal 3D objects and continuous proportions with two mean proportionals *necessarily* come together is the Archytas' solution. So, when Plato says that "... for it behaved to be solid of shape, and what brings solids into unison is never one middle term alone but always two.'' (Plato, 1925: 32b). He indicates this: it is the total body of the universe in *stereoedies*, that it is in the three-dimensional form, while the second stereo, which is analogized with it, corresponds to the three-dimensionality in Archytas' solution. Just as in Archytas' solution, two mean proportionals are reached by the intersection of 3-dimensional objects the universe gains its material unity thanks to this proportion.

The second reason is *irrationality.* Archytas' solution can construct *irrational* magnitudes. When Burkert says that "Archytas could determine $\sqrt[3]{2}$ (used in doubling the cube) geometrically, but not arithmetically" (Burkert, 1972: 221) is not wrong.[10] Therefore, there emerges the possibility that Plato allows *irrational magnitudes in his rational explanation.* Because there is still something irrational in the Cosmos, even after the Demiurges put it in an order.

The second part of the dialogue, Plato starts a new explanation of cosmos based on ἀνάγκη[11], necessity[12]. Necessity is the errant cause, indicates the pre-order state of cosmos, meaning chaos. Cosmos emerged not because Reason overrules the Necessity but because *persuades* (πείθειν*)* her (Plato, 1925: 48a). Therefore, the Cosmos is something like the ordered Necessity. The most important part of this Necessity is the *receptacle* (ὑποδοχή) which is the third thing besides Being and Becoming and is composing the real substance of four Kinds. Plato, while reconsidering four Kinds second time in this part of the dialogue, makes an important claim. There is still constant flux, meaning water becomes fire, fire becomes air and so on in an unbroken circle (Plato, 1925: 49b-d). Because of that;

[10]He is not wrong if we take the formulation $\sqrt[3]{2}$ not as a number but as expressing the incommensurable line in length and square also, but commensurable in cube that emerges as the first of the mean proportionals between two lines, one of them is the double of the other.

[11] ἀνάγκη has a complicated role in the cosmology of the Timaues. But it is not the purpose of this work to get into the detail of this concept.

[12] It is not the necessity that a logical or mathematical proposition has. It is Cosmos' necessity, a second kind of cause besides *aitia* (teleological cause), which is chaotic randomness.

> Whatsoever object we perceive to be constantly changing from one state to another, like fire, that object, be it fire, we must never describe as “this” (τοῦτο) but as “suchlike,” (τοιοῦτον) nor should we ever call water “this” but “suchlike” nor should we describe any other element, as though it possessed stability. (Plato, 1925: 49d).

This means that, because of constant becoming, even in an ordered universe, there is still something that can’t be *said* in these Four Kinds. The terms ‘this’ and ‘that’ indicate something stable, but there is nothing stable in these kinds, so they have a part that can’t be *spoken* properly. The only thing that can be really labeled about four kinds is *χώρα* (Plato, 1925: 52a), which is home of four Kinds, something like space and matter at the same time. But it is not an idea nor an object of thought, it can be grasped only with an apprehension like dreaming (Plato, 1925: 52b). Because its stability does not come from having some kind of abiding quality, but from that it doesn’t have any shape nor quality in-itself, it may receive the opposite forms (Plato, 1925: 50c-51b). Therefore, it is not a coincidence that Plato talks about Becoming associated with the four kinds as *irrational* (ἄλογον) (Plato, 1925: 28a, 42d, 43b, 43e). But they are put in a *rational* order so the Cosmos could emerge. Transition from chaos to cosmos is not same as an irrationality’s becoming rationality, but it is putting irrational things in some rational order; by placing them in a mathematical proportion. Therefore, just as Archytas' solution can put *irrational* lines in a certain proportional (ἀνάλογος) order, Demiurges constructs the cosmos by establishing the *irrational* kinds in a certain proportional (ἀνάλογος) order. Moreover, Plato uses this term long before he explains its indication of *instability,* exactly in the place where he constructs the body of the cosmos with the proportion: ‘from these such-likes (ἔκ τε δὴ τούτων τοιούτων)…’ (Plato, 1925: 32b).

There have also been those who have claimed that Plato used irrationals in the philosophy of nature. Popper points out that Platonic solids are constructed of triangles containing the values surds and says:

> The motive of this construction is the attempt to solve the crisis of atomism by incorporating the irrationals into the last elements of which the world is built. Once this has been done, the difficulty of the existence of irrational distances is overcome" (Popper, 1952: 150)

Novak, on the other hand, is saying. "Plato might have felt that the irrationals best find something exemplifying them in the world of change, the sensible world of flux" (Novak, 1982:23).[13] My interpretation is completely different from the attitude of these two authors. Popper refers to irrationals contained in the Platonic solids to overcome paradoxes about the paths that need to be traveled, while Novak mentions that the irrational may have found their place in the world of flux, but he does not indicate anything about proportion. But there is no indication of Archytas nor Plato's construction of the body of the Cosmos in their works.

Ancient Greek mathematics recognized distinct, though related, objects: numbers (ἀριθμοί, the objects of arithmetic, understood as discrete pluralities of units), magnitudes (μεγέθη, the objects of geometry, such as lines, surfaces, solids, understood as continuous quantities), and ratios (λόγοι, applicable to both numbers and magnitudes). Plato explicitly states that the Demiurge shaped the primary bodies "with the help of forms (shapes) and numbers" (εἴδεσί τε καὶ ἀριθμοῖς) (Plato, 1925:53b). Furthermore, in the construction of the World Soul (Plato, 1925:35b-36b), he unequivocally employs ratios of numbers. If Plato were to use only ratios of numbers for the construction of the cosmic body, he would seemingly neglect ratios of magnitudes, a cornerstone of the geometrical tradition with which he was intimately familiar. Such an omission would render his mathematical cosmology curiously incomplete, especially given the body's explicitly spatial, and therefore magnitudinal, nature.

[13] Novak's two-article does not include the application of irrationals to nature. In fact, he only says the passage to which we are referring in this regard. His approach to it is more of an essay that explores the impact of developments concerning the irrational on Plato's philosophy and dialectic. In particular, in the latter, he mentions the effects of the antufairesis method for that interpretation see: Novak, Joseph A. ''Plato and Irrationals Part 2'', *Aperion, Vol 17 iss 1*, 1983.

4. Refuting Numerical Interpretations: The Case for Magnitudes

Heath, Cornford, Taylor and Knorr argue that the proportion discussed in Plato's Timaeus pertains not to magnitudes, but to numbers. Knorr doesn't give any reason for his evolution though he is consistent (Knorr, 1975: 66, 168, 172, 203, 223, 224). But the others have specific reasons. There are mainly two reasons. One of them is about Plato's Pythagoreanism and the other one is about two terms used by Plato: δύναμις and ὄγκος in the phrase "ἀριθμῶν τριῶν εἴτε ὄγκων εἴτε δυνάμεων" (Plato, 1925: 32a).

Heath claims that the mathematics of Plato's *Timaeus* is a kind of Pythagorean. And in this passage, he is putagorizei (Heath, 1908: 294). Taylor similarly emphasizes the Pythagoreanism of the *Timaeus* and makes it clear that there can be no irrationals in these ratios (Taylor, 1928: 97), as a Pythagorean text would allow only proportions of numbers, not magnitudes because the proportion of magnitudes has ability to be constructed with the irrational magnitudes. Taylor has a highly speculative argument about Plato. According to Taylor, irrationals are numbers for Plato (Taylor, 1926). His reference to Pythagoreanism of Timaeus is not because of philosophical consistency but because of dramatic consistency. This attitude is not very plausible for Plato because mostly his philosophical intentions is much stronger than his dramatic aims in his dialogues. Also, the tradition that says discovery and the public discussion of irrationality may not be valid and probably it is a legend (Szabo, 1978: 88). After all, the solution of Archytas, which belongs to a concrete source of Pythagoreanism, defies this tradition by itself because irrational values can be handled through it.

While Plato undoubtedly "pythagorizes" in the *Timaeus*, and his philosophy incorporates numerous Pythagorean elements (e.g., recollection, mimesis, limit/unlimited), the extent and nature of this Pythagoreanism must be carefully delineated. From the very beginning of his cosmology, Plato establishes a clear ontological and epistemological distinction between objects of thought (intelligibles) and objects of perception (sensibles). This foundational Platonic dichotomy limits the interpretation of the Timaeus as a purely or uncritically Pythagorean text. If the empirical harmonics that Plato criticizes in the *Republic* (Plato, 1969: 530d-531c) bear resemblance to Archytas's approach, then Plato's sharp distinction between the intelligible and perceptible domains might carry an implicit critique of any tendency to ground harmonics or mathematics solely in the sensible world—an attitude Plato deprecates. Thus, Plato has Timaeus, a Pythagorean, articulate views that, in crucial respects (such as the primacy of intelligible mathematicals), diverge from or refine certain contemporary Pythagorean practices. Plato's unwavering emphasis on the intelligible/perceptible distinction should thus preclude an interpretation that treats the Timaeus as a straightforward Pythagorean manifesto.

While Plato certainly "pythagorizes" in the Timaeus, and his overall philosophy includes many Pythagorean elements—such as recollection, mimesis, and the concepts of limit and the unlimited in the dialogues such as Meno, Phaidon, Philebus and Phaedrus—it's essential to define the boundaries of his Pythagoreanism. From the beginning of his natural philosophy, Plato explicitly distinguishes between objects of thought and objects of perception. This separation sets limits on interpreting the *Timaeus* as a purely Pythagorean text.

If the Pythagorean harmonics that is criticized by Plato in the *Republic* (Plato, 1969: 531c) resemblances the harmonics of Archytas (Huffman, 2005: 41) the distinction between intelligible and perceptible includes a hidden philosophical criticism towards Archytas. Because Archytas directs harmonics towards the sensible world (Huffman, 2005: 63) which is the criticized attitude by Plato in the *Republic*. Therefore, Plato makes a Pythagorean speak in an anti-Pythagorean perspective in some respects. So, the categorical distinction he makes between perceptible and intelligible, and the fact that he places this distinction at the beginning of the natural philosophy, should, prevent us from treating this text in our interpretation as if it were a purely Pythagorean text.[14]

The second main reason they have is about two terms used by Plato, δύναμις and όγκος: ''…any three numbers, (whether) cubic or square (ἀριθμῶν τριῶν εἴτε ὄγκων εἴτε δυνάμεων)…'' Heath argues that Plato means by planes (ἐπίπεδος) and solids (**τὰ στερεὰ**) square and cube numbers (Heath, 1921: 89). And he gives a reason about the mentioned terms: ''Plato speaks first of δύναμις and ὄγκος and then of "planes" and "solids" in such a way as to suggest that δύναμις correspond to ἐπίπεδος and ὄγκος to στερεὰ.'' (Heath, 1908: 294). But Heath knows the weak point of his own interpretation. If we consider the expressions δύναμις and ὄγκος as squares and cubes, Plato states that he is talking about a ratio between three squares and three cube numbers, but there is no such ratio. He doesn't answer to his own criticism, but Cornford tries to overcome this situation by some grammatical valuation.[15]

A more compelling reading, advanced by Pritchard, suggests that the phrase 'ἀριθμῶν τριῶν εἴτε ὄγκων εἴτε δυνάμεων' should be understood more distributively as "three arithmoi (numbers), or three onkoi (bulks/volumes), or three dynameis (powers/squares/areas)". This interpretation allows Plato to articulate a general principle of proportion applicable across different mathematical domains—numbers, geometrical solids (volumes), and plane figures (areas/squares)—rather than restricting it to specific types of numbers (Pritchard, 1992: 192).

Also, these comments altogether ignore Plato's mentioning about 'βάθος. Bathos, meaning *depth*, is strictly a geometrical concept. Interestingly, we see this

[14] Some has noticed that also, for example, (Prior, 2013: 87). 7

[15]Cornford says that this obstacle ''can be obviated by construing the genitives eite onkon eite dunameon ontinonoun not (as is commonly done) as in apposition to aritmon, but as depending on the meson.'' (Cornford, 1997: 47).

term in Euclid's Elements only one time[16]; in the definition of solid as a geometrical shape: Book XI, Def. 1: ''A solid is that which has length, breadth, and depth (βάθος)''. Plato, on the hand, use this term in context of geometry, especially stereometry regularly. We can see this in the *Republic*, when he organizes the sciences (Plato, 1969: 528b, 528d). This fact, even more obvious when we consider the *Laws*. In the *Laws* Plato again mentions βάθος in context of stereometry (Plato, 1968: 747a, 817e, 819d, 819e, 820a).

I think, these commentaries could be emerged naturally because the concept of ratio is applicable both numbers and magnitudes and therefore there arises some kind of perplexity. Euclid treats ratio and proportion two times, one of them for magnitudes in book V and VI, and one of them In Book VII, def 20, the proportion of numbers described as this: ''Numbers are proportional (ἀνάλογος) when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.'' But in Book V, def 6 says that ''let magnitudes which have the same ratio[17] be called proportional.''

In this manner, Grattan-Guinness mentions ratio in Euclid's Elements a different kind of quantity (Grattan-Guiness, 1996: 362). The most striking different is that ratios and proportions about magnitudes that they include *incommensurables* contrary to ones about numbers. In Book X prop 5, 7 and 11 Euclid's terminology and statements allows indirectly incommensurable magnitudes in proportion. For example, the definition 11 says: ''If four magnitudes are proportional and the first is commensurable with the second, then the third also is commensurable with the fourth; but, if the first is incommensurable with the second, then the third also incommensurable with the fourth''. This definition important that it is directly allows incommensurables in proportion. And notice

[16] Through the search with Perseus and TLG.
[17] In Book V def. 4 Euclid states that ''magnitudes are said to have a ratio to one another which can, when multiplied, exceed one another.'' Remember that this multiplication is not same as multiplication of numbers so incommensurable magnitudes too can be multiplied as their size increased.

that Euclid doesn't mention about incommensurable in length or in square, so it includes all kind of incommensurables, such as incommensurable in length and square which is the *irrational* in Euclid's language. But of course, if incommensurable magnitudes are in proportion, this proportion cannot be the one that is of numbers, which is a condition stated in prop. 7.

The second issue is that in Euclid geometry, geometrical concepts are used for numbers and geometrical objects also. In Book VII, there are *plane* numbers, *solid* numbers, *square* numbers, *cubic* numbers etc. So, when Plato is mentioning planes and solids without specifying *which* planes and solids it is natural to show a tendency to argue that he is talking about proportion about numbers with the help of Pythagorean elements of the dialogue. But again, there is another hint besides his chois of term βάθος. When we look Elements again, we see that when Euclid talks about numbers, he uses στερεός in *masculine* (In Book VII def 17: ὁ στερεός) gender but when talks about geometrical shapes he uses same concept in *neuter* gender (In Book XI def 1: στερεόν. And all of Book XI). It is because of, those terms are adjectives while as a name ἀριθμός is a masculine word and again as a name μέγεθος and σχῆμα is a neuter word. When we look at Plato itself, we see that he uses *neuter* form also when he talks about the need for two means: τὰ στερεὰ.

In addition, the proportions in which the soul is established do not include irrationals and are established by the proportion of integers. Cornford mentions the possibility that Plato had in mind numerical values that he uses to construct the soul while establishing the proportion of matters.[18] Although the ratios in which the souls are established can be formulated as a:b::b:c::c:d, Wagner and Netz directly state that the two ratios are different from each other. ".. organizing solids ('body') through the structure of four lines in continuous proportion, the other is the idea of organizing *more* abstract structures ('soul') through the numerical ratios of musical harmony'' (Wagner and Netz, 2023: 77).

Conclusion

This paper has argued that Plato's account of the construction of the Cosmos's body in the Timaeus (31b-32c) relies fundamentally on proportions of continuous magnitudes rather than discrete numbers, and that in doing so, Plato makes a significant, albeit indirect, allusion to Archytas of Tarentum's celebrated solution to the Delian problem. By closely examining Plato's terminology—particularly his use of βάθος (depth) to distinguish plane from solid constructions and the neuter plural τὰ στερεά (solids) when stipulating the necessity of two mean proportionals—we have contended that the textual evidence points decisively towards a geometrical, rather than a purely arithmetical, understanding of these cosmic bonds.

The traditional interpretation, often emphasizing Plato's Pythagorean influences and a numerical reading of terms like ὄγκοι and δυνάμεις, encounters significant difficulties, not least the mathematical inelegance of forcing a continuous proportion with two means into a framework of only square or cube numbers. Conversely, recognizing the allusion to Archytas's stereometric achievement provides a compelling explanatory framework. Archytas's solution, a landmark in early solid geometry, demonstrated precisely how two mean proportionals between two given lines (magnitudes) could be constructed through the intersection of three-dimensional solids. This method not only addressed the problem of doubling the cube but also provided a paradigm for generating and relating magnitudes, including those representing irrationals, within a rigorous geometrical system. Such an achievement would have been of profound interest to Plato, offering a concrete mathematical analogy for the Demiurge's task of unifying a three-dimensional, solid Cosmos. The distinction Plato draws between the single mean sufficient for a plane surface and the two means necessitated by a solid body directly mirrors the progression from two-dimensional to three-dimensional geometrical constructions for finding one and two mean proportionals, respectively.

The significance of this interpretation extends beyond a mere technical point in the history of mathematics. It underscores the depth and sophistication of Plato's engagement with the cutting-edge mathematical research of his contemporaries. It suggests that when Plato turned to mathematics to articulate the structure of the sensible world, he drew not only upon established arithmetical and harmonic principles, as evident in the construction of the World Soul, but also upon the most advanced developments in geometry, particularly the nascent field of stereometry. This reading positions the Timaeus not only as a profound work of cosmology and metaphysics but also as a testament to Plato’s conviction that the deepest understanding of the physical world is inextricably linked to the highest achievements of mathematical science.

Ultimately, Plato's mathematically structured Cosmos, bound by proportions that find their paradigm in advanced geometry, is presented as an εἰκών of the intelligible Forms—an ordered and beautiful manifestation of divine goodness. The choice of a proportion requiring two means for the solid body of the universe, a principle rooted in the challenging domain of stereometric construction and the theory of magnitudes, reflects Plato's commitment to finding the "fairest of bonds." This interpretation, by highlighting the role of magnitudes and the allusion to Archytas, contributes to a richer appreciation of the mathematical architecture of Plato's likely story, revealing a cosmology deeply informed by the geometrical insights that were transforming the intellectual landscape of the fourth century BCE. The Timaeus, in this light, continues to invite scholarly inquiry into the intricate dialogue between Platonic philosophy and the mathematical sciences of its age.

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