Examining Curricula within the Scope of Algebraic Thinking: Examples from Türkiye, Singapore, England and Canada (Ontario)*

RESEARCH ARTICLE

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Abstract

The purpose of this research is to compare the Century of Türkiye Education Model, which has been implemented since the 2024–2025 academic year, and the Türkiye (2018, 2024) secondary school mathematics curriculum, which is still ongoing, with the mathematics curricula of Singapore, England, and Canada (Ontario) in the field of algebraic thinking and algebra learning, revealing similarities and differences. The curricula of Singapore, England, and Ontario have been considered data sources due to their higher average scale scores in internationally recognized assessments such as TIMSS and PISA compared to Turkiye and their geographical diversity across different continents. The algebra learning area at the class levels corresponding to grades 5–8 has been examined through document analysis using three distinct dimensions based on various concepts and processes alongside the components of generalized arithmetic, functional thinking, and modeling languages associated with algebraic thinking. The similarities and differences between the sub-learning areas and achievements related to the algebra learning area have been delineated through horizontal comparisons. As a result of the comparison, it was found that the Ontario and the Century of Türkiye Model curricula present a balanced distribution among generalized arithmetic, functional thinking, and modeling languages. The Singapore curriculum, on the other hand, emphasizes functional thinking and problem solving through patterns and relations, while the England curriculum integrates algebra with reasoning and real-life applications. Overall, the provision of algebraic concepts at each grade level, the support of various processes, the inclusivity of mathematical expressions, and the presence of guiding descriptions and exemplary applications are notable across the curricula.

Keywords: *algebra, algebraic thinking, curriculum, comparative education, the Century of Türkiye Education Model*

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Cebirsel Düşünme Kapsamında Öğretim Programlarının İncelenmesi: Türkiye, Singapur, İngiltere ve Kanada›dan (Ontario) Örnekler

ARAŞTIRMA MAKALESİ

Öz

Bu araştırmanın amacı, 2024-2025 eğitim öğretim yılından itibaren uygulanmaya başlanan Türkiye Yüzyılı Maarif Modeli ile hâlen yürürlükte olan Türkiye (2018, 2024) ortaokul matematik öğretim programını, cebirsel düşünme ve cebir öğrenme alanında Singapur, İngiltere ve Kanada (Ontario) matematik öğretim programlarıyla karşılaştırarak benzerlik ve farklılıkları ortaya koymaktır. Singapur, İngiltere ve Kanada (Ontario) matematik öğretim programları, Türkiye'ye kıyasla TIMSS ve PISA gibi uluslararası geçerliliğe sahip sınavlardaki ortalama ölçek puanlarının daha yüksek olması ve farklı kıtalarda yer almaları nedeniyle veri kaynağı olarak dikkate alınmıştır. 5-8. sınıf düzeylerine karşılık gelen sınıf seviyelerinde cebir öğrenme alanı; cebirsel düşünmeyle ilişkili genelleştirilmiş aritmetik, fonksiyonel düşünme ve modelleme dilleri bileşenlerinin yanı sıra çeşitli kavram ve süreçlere dayalı üç ayrı boyutta doküman analizi yoluyla incelenmiştir. Cebir öğrenme alanına ait alt öğrenme alanları ve kazanımlar arasındaki benzerlik ve farklılıklar yatay karşılaştırmalar yoluyla ortaya konulmuştur. Karşılaştırma sonucunda, Ontario ve Türkiye Yüzyılı Maarif Modeli programlarının genelleştirilmiş aritmetik, fonksiyonel düşünme ve modelleme dili açısından dengeli bir dağılım gösterdiği sonucuna varılmıştır. Buna karşılık, Singapur programı örüntüler ve ilişkiler yoluyla fonksiyonel düşünme ve problem çözmeye; İngiltere programı ise cebiri akıl yürütme ve gerçek yaşam uygulamalarıyla bütünleştirmeye vurgu yapmaktadır. Genel olarak, her sınıf düzeyinde uygun cebirsel kavramların sunulması, çeşitli süreçlerin desteklenmesi, matematiksel ifadelerin kapsayıcılığı, yönlendirici açıklamalar ve örnek uygulamaların bulunması programlar arasında dikkat çeken ortak özelliklerdir.

Keywords: *cebir, cebirsel düşünme, öğretim programı, karşılaştırmalı eğitim, Türkiye Yüzyılı Maarif Modeli*

Introduction

By the late 20th century, education systems' role in nurturing individuals and driving societal progress became critical for nations pursuing economic growth. This necessitated reforms in educational policies and systems, sparking comparative education research. Such studies analyze educational components -curricula, assessment methods, teaching strategies, policies, teacher training, and budgets- across diverse cultures and historical periods to identify similarities and differences (Ergün, 1985). In the 21st century, mathematics education gained prominence as a cornerstone for developing analytical skills vital to global competitiveness (Kuzu et al., 2023; Sadak et al., 2021). Curricula, as reflections of national educational priorities, are central to comparative research. International assessments like Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) reveal performance gaps, prompting scrutiny of high-achieving countries' curricula. Numerous studies critically compare these with domestic programs to identify strengths and disparities (Bacakoğlu, 2022; Bütüner & Güler, 2017; Gürses, 2022; Kuzu et al., 2023).

Algebra, a core content area in the National Council of Teachers of Mathematics Principles and Standards for School Mathematics (NCTM, 2000) has historically shaped curricula through symbolic representations and sequential operations rooted in Al-Khwarizmi's work. However, reducing algebra to symbolic arithmetic or strict rule-following has led to persistent difficulties, such as misinterpretations of the equal sign's relational meaning and oversimplification of variables as mere numerical quantities (Kieran, 2007; Ususkin, 1988; Toluk Uçar, 2018). These issues, compounded by curricula that prioritize quantitative outcomes over operational properties, have hindered conceptual understanding. Post-reform efforts reframed algebra instruction as a process-oriented discipline (NCTM, 2000).

The 1998 NCTM symposium emphasized algebraic thinking's centrality across all grade levels, framing it as a cognitive tool and mathematical language that transcends traditional subject boundaries (Kieran, 1992). Historically, algebraic thinking has enabled real-world modeling and mathematical organization through arithmetic and representations (Lins, 1992). Kaput (2008) categorizes it into generalization (e.g., patterns) and symbolization (e.g., functions), while stressing its role in conceptualizing structures.

Algebraic thinking need not depend on formal expressions; it emerges in arithmetic contexts (Hewitt, 2019; Kieran, 2004; Radford, 2010). Subramaniam and Banerjee (2011) argue that algebra underpins arithmetic, urging students to view arithmetic through "algebraic eyes", a perspective developed long before formal education. Early childhood experiences, such as play-based activities, plant "seeds of algebraic thinking" by linking real-world interactions to reasoning (Blanton & Kaput, 2011; Walkoe & Levin, 2020). Van de Walle et al. (2010) emphasize its lifelong importance, noting that it begins in preschool and extends into daily life.

Research underscores the need to integrate algebra early, combining relational equality (e.g., generalizing operations), symbolic tools, and quantitative representations to bridge abstract gaps (Carraher & Schliemann, 2007; Chimoni et al., 2019; Stephens et al., 2015). Prioritizing inductive/deductive reasoning and introducing topics progressively build foundational proficiency (Kieran, 2011; Cai & Knuth, 2011). Algebraic thinking involves modeling real-world patterns through mathematical language by recognizing relationships between variables, rather than merely calculating outcomes (Kaput & Blanton, 1999). A critical shift from arithmetic to algebra lies in prioritizing structural analysis over numerical results. For instance, while arithmetic validates 3+4=7 as a fixed truth, algebra examines the equivalence structure behind 3+4=4+3 without calculation—a "second-level generalization" where the commutative property transcends specific numbers (Radford, 2000). This structural focus is central to generalized arithmetic: students must reinterpret operations (e.g., "=" as a relational symbol, not a procedural "answer" signal) to grasp algebraic reasoning (Usiskin, 1988).

The absence of explicit "algebraic thinking" headings in curricula does not preclude its support. For example, Fong's (2004) analysis of Singapore's primary curriculum identifies problem-solving, generalization, and functional thinking as key approaches. Cognitive processes like analyzing parts-wholes, generalization/specialization, and construction-reconstruction are emphasized to bolster algebraic thinking. Pattern and shape activities aid in forming arithmetic generalizations and algebraic structures, while functional thinking (using variables to explore how outputs depend on changing quantities) (Kieran, 1997) intersects with generalized arithmetic, blurring rigid distinctions (Bednarz et al., 1996; Fong, 2004).

Algebraic thinking is regarded as a fundamental cognitive process that directly influences students' mathematical achievement and higher-order problem-solving abilities (Kaput, 2008; Kieran, 2004). Developing this form of thinking from an early age helps students overcome the difficulties they face in transitioning from arithmetic to school algebra and enables them to make mathematical generalizations (Blanton & Kaput, 2005; Stephens et al., 2017). In the literature, algebraic thinking is conceptualized through three main components: generalized arithmetic, functional thinking, and modeling language (Chimoni et al., 2018). Within this framework, it is emphasized that students should not only perform symbolic manipulations, but also understand algebraic structures through various representations, relationships, and generalizations (Cai & Hwang, 2022; Carpenter & Levi, 2000).

"Patterns and relationships," which serve as a bridge for algebra and play a significant role in the development of functional thinking, have been incorporated as a sub-learning domain in the constructivist approach within the 2005 curriculum and included in subsequent curricula. The algebra achievements in the curricula analyzed by Toluk Uçar (2018) were based on Kieran's (1996) classification of algebra activities and the criteria for evaluating algebra content developed by the National Research Council (NRC, 1998). It has been concluded that the algebra content in the curricula developed under the constructivist approach in 2005, 2013, and 2017 encompasses functional, structural, representational, and modeling approaches. The analysis conducted within the framework of Kieran's (1996) classification of algebra activities, which conceptualizes algebra as an activity, demonstrates that these curricula each include all three categories: generative, transformational, and meta-level (Toluk Uçar, 2018).

International research has consistently shown a strong relationship between algebraic thinking and student achievement. Specifically, findings from TIMSS and PISA indicate that countries with strong performance in sub-dimensions such as functional thinking and generalization (e.g., Singapore, Canada-Ontario, England) also exhibit higher average mathematics scores (Mullis et al., 2020). In contrast, Türkiye's below-average performance in both assessments suggests that algebraic thinking is not sufficiently developed among students (Ministry of National Educatiom [MoNE], 2023). Although there are studies in the Turkish context focusing on algebra instruction and students' conceptual difficulties (Akkan & Çakıroğlu, 2012), few have undertaken an in-depth comparison of curricula in terms of algebraic thinking. Furthermore, similarities and differences among different countries' curricula have not been systematically examined through the lens of the components of algebraic thinking.

Chimoni et al. (2018) have investigated various approaches to studying algebra, consistently focusing on identifying and analyzing fundamental content sequences, key concepts, processes, and forms of reasoning in algebraic thinking. Their examination spans four essential dimensions: (1) Content Spiral, which examines the structure and relationships within three fundamental sequences generalized arithmetic, functional thinking, and modeling; (2) Concepts, addressing core algebraic ideas such as equality, variables, and operations; (3) Processes, encompassing noticing, hypothesizing, representing, generalizing, justifying, and verifying (Blanton et al., 2011); and (4) Forms of reasoning, which involve inductive, deductive, and abductive reasoning (Jeannotte & Kieran, 2017).

While the 2018 Mathematics Curriculum is still being implemented in Türkiye, the Century of Türkiye Education Model was introduced in the 2024–2025 academic year. This change necessitates a re-evaluation of national curricula from the perspective of algebraic thinking. However, the existing literature does not sufficiently discuss how these two Turkish curricula address algebraic thinking or how they differ from their international counterparts. Comparing the curricula of high-performing countries in TIMSS and PISA, such as Singapore, England, and Ontario, provides an important opportunity to reveal Türkiye's strengths and weaknesses in fostering algebraic thinking. Comparative education researchs provides countries with the opportunity to analyze the similarities and differences in their education systems, learn from others, and improve their own systems in line with contemporary needs and existing shortcomings (Kuzu et al., 2023; Hemmi et al., 2021; Serçe &Acar, 2021).

The Century of Türkiye Education Model (Ministiry of National Education [MoNE], 2024) introduced explicit emphasis on algebraic thinking as a core theme for the first time and incorporated algorithmic thinking as a contemporary extension of algebraic reasoning (Wing, 2006, 2017; Radford, 2010). By positioning these ideas within both process and content dimensions, the new model offers a more systematic and coherent framework for teaching algebra (MoNE, 2024). Therefore, it was included in this study to highlight how Türkiye's renewed curriculum vision reflects structural changes in the development of algebraic thinking.

Moreover, the literature highlights that curriculum analyses serve as a critical starting point for understanding students' learning opportunities (Schmidt & Prawat, 2006; Valverde et al., 2002). Examining written curricula, particularly in cross-national comparisons, systematically reveals how the components of algebraic thinking are represented and emphasized within different education systems. In this context, the present study not only introduces the intended goals of the current curricula but also sheds light on potential structural shortcomings that may be related to Türkiye's relatively low performance in TIMSS and PISA. Furthermore, clarifying the differences between Türkiye's 2018 Mathematics Curriculum and the 2024 the Century of Türkiye Education Model in terms of algebraic thinking will provide a valuable foundation for future implementation-based research and policy development efforts.

Therefore, the purpose of this study is to compare the Century of Türkiye Education Model and the 2018 Mathematics Curriculum with those of Singapore, England, and Ontario. The comparison focuses on algebraic thinking competencies across equivalent grade levels (5th, 6th, 7th, and 8th grades) regarding the curricula implemented during the 2023-2024 academic year. The study is conducted within the framework of the three dimensions proposed by Chimoni et al. (2018) -fundamental components of algebraic thinking, concepts, and processes- and aims to provide a more comprehensive analysis than previous limited or descriptive comparisons. As a result, the study aims to contribute to the improvement of algebra teaching in Türkiye and to provide evidence-based recommendations for policymakers. To achieve this goal, the study seeks to answer the following research questions:

1. What are the differences and similarities in the algebra learning domain present in the mathematics curricula of Türkiye, Singapore, the United Kingdom, and Canada (Ontario) regarding the development and stages of algebraic thinking, the addressed content, and the learning outcomes?

   1.1. How do the algebra-related learning outcomes in the mathematics curricula of Türkiye, Singapore, the United Kingdom, and Canada (Ontario) compare in terms of algebraic thinking at the 5th, 6th, 7th, and 8th grade levels?

   1.2. What are the similarities and differences between the sub-learning areas related to algebra in the curricula of these countries at the 5th, 6th, 7th, and 8th grade levels?

Context of the Study

Türkiye

Türkiye's education system operates under a centralized structure, with all educational policies governed by the Ministry of National Education (MoNE). Following reforms in 2012, compulsory education covers twelve years, organized as 4 years of primary, 4 years of lower secondary, and 4 years of upper secondary education. In the 2024-2025 academic year, Türkiye introduced a renewed curriculum through «The Century of Türkiye Education Model" which adopts a process-oriented approach emphasizing conceptual skills and holistic development. In this study, the curriculum implemented in 2018 is referred to as the Türkiye Mathematics Curriculum (TMC) and the curriculum introduced in 2024 is referred to as The Century of Türkiye Education Model.

Singapore

Singapore's education system is governed by the Ministry of Education, with English as the main language of instruction. The mathematics curriculum progresses through structured stages, offering standard and foundation tracks at the primary level to cater to different learning needs. The secondary level continues this spiral structure, aligning with students' developing proficiency (Singapore Ministry of Education [SMOE], 2020a, 2020b). This curriculum is referred to as the Singapore Mathematics Curriculum (SMC) throughout this study.

England

England's education system is organized into key stages (KS), with mathematics emphasizing conceptual fluency and reasoning. KS1 and KS2 cover ages 5-11, while KS3 and KS4 cover secondary education. The national curriculum allows for localized flexibility, aiming to foster critical thinking and mathematical literacy (Department for Education [DfE], 2013). The National Curriculum framework analyzed in this research is referred to as the England Mathematics Curriculum (EMC) (DfE, 2013).

Canada (Ontario)

Canada has a decentralized education system where each province governs its own policy. Ontario's mathematics curriculum is organized spirally from grades 1 to 8 and encompasses six domains, including social-emotional learning (SEL) and coding. It emphasizes the application of mathematics in interdisciplinary contexts and the development of transferable skills (Ontario Ministry of Education [OMOE], 2020). This specific document is referred to as the Ontario Mathematics Curriculum (OMC) (OMOE, 2020).

Methodology

This qualitative study compares mathematics curriculas from Türkiye (2018, 2024), Singapore, the UK, and Canada (Ontario) through the lens of algebraic thinking. Employing document analysis, a core qualitative method for examining written/printed materials (Wach, 2013; Kıral, 2020), the research integrates descriptive, content, and discourse analysis techniques (Yıldırım & Şimşek, 2016). Curricula, as publicly accessible official documents, are analyzed independently of their original purpose, with the researcher's interpretive focus adding qualitative depth (Kıral, 2020). The data were categorized into sub-themes and codes under the central framework of algebraic thinking to facilitate cross-national comparisons.

Data Sources

The researchers obtain documents for analysis from appropriate sources, selected depending on the topic under investigation (Kıral, 2020). The documents to be examined in this study include the Türkiye Mathematics Curriculums (Ministry of National Education [MoNE], 2018, 2024), the Singapore Mathematics Curriculum (SMOE, 2020a; 2020b), the United Kingdom Mathematics Curriculum (DOE, 2013), and the Ontario Mathematics Curriculum from Canada (OMOE, 2020), with a focus on addressing problems related to the area of algebra learning and the associated learning outcomes. These documents focus on addressing issues in algebra learning and related learning outcomes. The selection of source documents was influenced by their above-average performance in international assessments and the fact that the original sources are in English and Turkish.

The documents analyzed in this study were selected based on the criteria that the source countries perform above average in international assessments and that the documents are available in English or Turkish (Kıral, 2020). The primary data sources include:

- **Türkiye:** The 2018 Mathematics Curriculum (MoNE, 2018) and the Century of Türkiye Education Model Mathematics Curriculum (MoNE, 2024).
- **Singapore:** The Primary and Secondary Mathematics Syllabuses (SMOE, 2020a; 2020b).

- **England:** The National Curriculum in England: Mathematics Programmes of Study (DfE, 2013).
- **Canada (Ontario):** The Ontario Mathematics Curriculum Grades 1-8 (OMOE, 2020).

These documents were specifically chosen to address learning outcomes related to algebraic thinking. To ensure comparability across different educational systems, the grade levels were matched as follows: The 5th and 6th grades in Türkiye correspond to P5-P6 in Singapore, Y5-Y6 in England, and Grades 5-6 in Ontario. Similarly, the 7th and 8th grades in Türkiye are equivalent to S1-S2 in Singapore, Y7-Y8 in England, and Grades 7-8 in Ontario.

Data Analysis

The curricula that constitute the data sources form the written documents of the research. The process of reviewing the documents progressed as follows:

1. The mathematics curricula of Türkiye, Singapore, the United Kingdom, and Canada (Ontario Province) were accessed through the official websites of the relevant institutions.
2. Previous studies were reviewed to examine and analyze the documents within the framework of algebraic thinking. The details are included in the section that forms the theoretical foundation of this research, establishing a specific review framework (Table 1). The curricula were analyzed based on the first three fundamental dimensions of algebraic thinking compiled by Chimoni et al. (2018). The fourth dimension, which pertains to classroom practices and types of reasoning, was excluded from the analysis. This decision was made because the current study is strictly a document analysis of official curricula (the intended curriculum). Since classroom practices represent the 'enacted curriculum' and cannot be directly observed through curriculum documents alone, including this component would have compromised the methodological validity of the comparison.
3. The researchers translated documents written in English into Turkish and subsequently had these translations validated for accuracy by a mathematics education expert and an English teacher.

4. The researchers classified the competencies and learning objectives in the curricula based on their country of origin, grade level, and sequence (e.g., "Tr 5-1", see Table 2). Unidentified dimensions were categorized as "-" in the analysis framework presented in Table 2. The data presented in Table 2 demonstrate that a competency statement can include various concepts or processes.
5. The data obtained from the documents were examined comparatively under the main theme of algebraic thinking. The expressions of the learning outcomes were presented, as shown in Table 2, regarding which fundamental dimension they belong to, which concepts they encompass, and which processes they target. The data were supported by graphics.

Table 1

Central Themes, Sub-Themes, and Codes to Be Used in The Data Analysis Process (Chimoni et al., 2018)

Algebraic Thinking (Main Theme)			
	Fundamental Components of Algebraic Thinking (Sub-theme)		
1st dimension	Generalized arithmetic Topics covered include properties of numbers, properties of operations, symbols, algebraic expressions, missing numbers, and representing unknown quantities with letters.	Functional Thinking (Codes: Patterns, designs, relationships between quantities, relationships between variables, dependent and independent variables, covariance, one-to-one mapping)	Modeling Languages (Codes: Creating a mathematical model and utilizing various representation forms such as graphs and tables for problem-solving
2 nd dimension	Concepts (sub-theme) Codes: Equal symbol, equality, equivalence, inequality, properties of numbers, properties of operations, patterns, variables, one-to-one correspondence, covariance, unknown quantity.		
3rd dimension	Processes (sub-theme) Codes: Noticing, representation, assumption, justification, generalization, and validation.		

Table 2

An Applied Example of the Analytical Framework Used in Document Review.

Learning Outcome Code	Learning Outcome Statement	Algebraic Thinking Component	Concept	Process
Tr 5-1	It generates the desired steps based on the specified number and shape patterns.	Functional thinking	Pattern (design)	Noticing
TM 5-2	To be able to make deductions about the conservation of equality and operation properties	Generalized Arithmetic	Equality, properties of operations	Justification
Sin 5-1	Patterns in Number Sequences	Functional thinking	Pattern (design)	Noticing
Sin 8-13	The graph of a two-variable linear equation.	Functional thinking	Variable, covariance	Representing
Sin 6-1	Using a letter to represent an unknown number.	Generalized Arithmetic	Unknown quantity	Generalizing
Eng 6-4	Find pairs of numbers that satisfy an equation with two unknowns	Functional thinking	Covariance	Noticing, verification
Eng 7-3	Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms, and factors	Generalized Arithmetic	Variable, equality, inequality	Representing, generalizing
Ca 7-11	To apply the process of mathematical modeling in order to represent, analyze, predict, and provide insights into real-life situations.	Modeling languages	Covariance	Noticing, representing, making assumptions, justification, generalization, verifying

Note. *Tr: Türkiye (2018 Mathematics Curriculum); TM: Century of Türkiye Education Model (CTEM); Sin: Singapore; Eng: England; Ca: Canada (Ontario). While other curricula are structured around "sub-learning areas," the CTEM is organized around "themes." These themes are treated as equivalent to sub-learning areas for this comparison.*

Validity and Reliability of the Research

The documents used in this study were obtained from the official websites of the units responsible for education in the respective countries. The stability, reproducibility, and accuracy of the findings obtained from the document analysis are vital for reliability (Özkan, 2023). Therefore, the researchers regularly repeated the coding process to maintain continuity and consistency in data analysis. In addition, two experts in the field re-examined the analyses of themes, sub-themes, and codes, and any discrepancies identified during this process were resolved through consensus. The coding process was conducted based on the criteria outlined in Chimoni et al.'s (2018) framework (Table 2). Coding was carried out by three experts: two academics with doctoral degrees in mathematics education and one mathematics teacher with more than ten years of professional experience. For each country, the learning outcomes in the mathematics curriculum were listed, and the corresponding algebraic thinking components were recorded in the coding form. The experts worked independently during the initial phase and later discussed and reconciled any differences to reach a common agreement. Inter-rater reliability was calculated using Cohen's Kappa coefficient, which was found to be 0.90, indicating a high level of agreement among coders. For example, the learning outcome "represent linear relationships using tables and graphs" was categorized under the functional thinking dimension.

These procedures ensured the methodological rigor and trustworthiness of the document analysis.

Findings

Findings Related to the First Sub-Problem

Grade levels present the findings related to the first sub-problem.

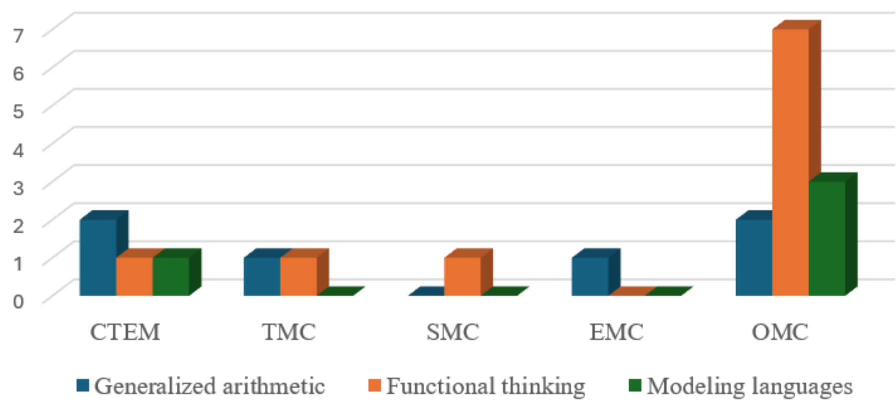
Findings for the 5th grade level concerning dimensions of algebraic thinking

While Türkiye (2018) and the England lack explicit 5th-grade algebra content, their number/operation domains include elements supporting algebraic thinking such as pattern-based bridging from arithmetic to algebra and functional thinking. The Century of Türkiye Education Model, on the other hand, has created individual themes at each grade level that center on algebraic thinking. Singapore integrates algebra under a combined "Numbers and Algebra" unit at Grade 5, whereas Ontario (Canada) structures algebra as a distinct learning area with sub-domains. Pattern generalization tasks (e.g., identifying term values and symbolic relationships) are pivotal in fostering algebraic thinking.

Notably, Ontario's explicit algebra framework contrasts with other nations' embedded pre-algebraic components. Comparative analysis (Figure 1) highlights similarities/differences in generalized arithmetic, functional thinking, and modeling across countries' 5th-grade curricula, focusing on alignment with algebraic proficiency.

Figure 1

Components of Algebraic Thinking in 5th Grade Curricula

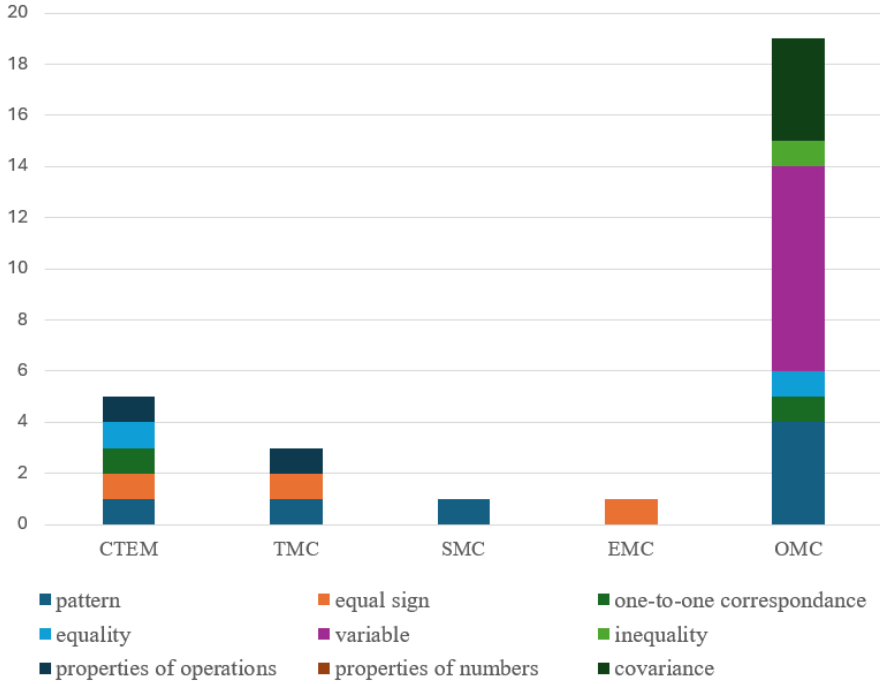


While statements related to all components of algebraic thinking are included in Canada (Ontario) and The Century of Türkiye Education Model, the concepts presented in Figure 2 differ among the statements in other countries.

*In the graphs, the Century of Türkiye Education Model is referred to as CTEM.

Figure 2

Concepts Covered in the 5th-Grade Learning Outcomes and the Frequency of Their Usage



In the 5th grade level, all components of algebraic thinking have been identified in Canada (Ontario), highlighting a remarkable diversity of concepts related to algebraic reasoning. While process expressions have been identified in Canada and Türkiye (See Figure 3 and Figure 4), no process components have been identified in Singapore and the England.

Figure 3

An Example of an Application Aimed at Supporting Functional Thinking Skills in the 5th-Grade TMC (MoNE, 2018, p. 51)

For example, find the 6th step of the number pattern that starts with 7 and adds three at a time. If Büsra starts the collection with 7 marbles in the first week and then adds 3 marbles, how many marbles will there be in the collection after 5 weeks?

For example, in the figure pattern below, there are studies on the number pattern of square and triangular numbers, or how many squares or triangles can grow from the changing squares as desired.

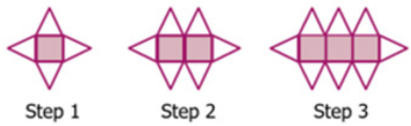
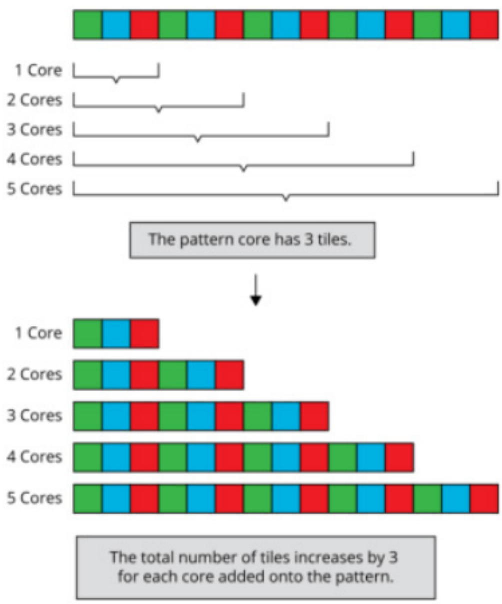


Figure 4

An Example of an Application Aimed at Supporting Functional Thinking in 5th Grade OMC (OME, 2020, p. 295)



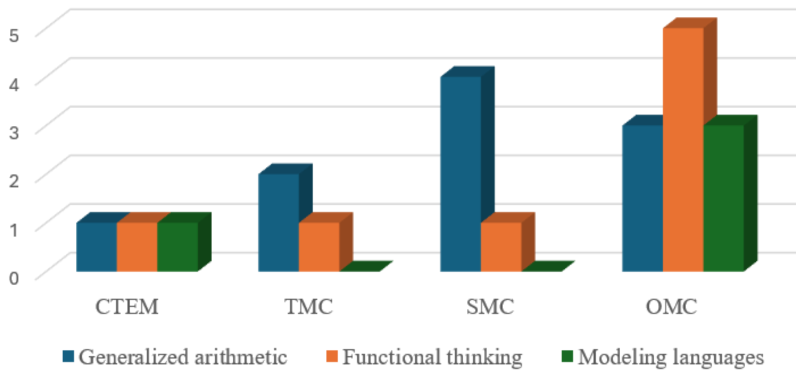
Türkiye's Tr 5-1 outcome fosters functional thinking by identifying/ extending numerical patterns, integrating cultural elements to bridge math with real-life contexts. Tr 5-2 emphasizes inverse multiplication-division relationships, reinforcing generalized arithmetic (Tr 5-1; Tr 5-2). This is also addressed in the Century of Türkiye Education Model, which emphasizes the generalization of pattern rules. This expectation differentiates the Century of Türkiye Education Model and others. In the process components, explanations are given for various skills, and process skills such as making assumptions, generalizing, testing, and transferring to similar tasks are employed to develop algebraic thinking. Singapore's Sin 5-1 focuses on number sequence patterns but omits shape-based pattern tasks in its P5 standards, despite revisiting foundational concepts (SMC, 2020a). The EMC/SMC frameworks lack explicit pattern-related outcomes critical for pre-algebra transitions, though they stress the equal sign's conceptual role in algebraic thinking (EMC, 2013).

Findings on the dimensions of algebraic thinking at the 6th-grade level

The Century of Türkiye Education Model, TMC, EMC, and OMC curricular frameworks include the concept of "variable," while the SMC does not address this concept. All of the curricula include unknown quantities, but the achievement statements regarding algebraic expressions only refer to the letter or symbol used in the TMC as being designated as a variable. It has been observed that the OMC has more comprehensive achievement statements than the other programs, encompassing many process skills. The achievements related to algebra learning at the 6th-grade level in the Century of Türkiye Education Model, TMC, SMC, EMC, and OMC have been analyzed, and the distribution according to the first dimensions of algebraic thinking is presented in Figure 5.

Figure 5

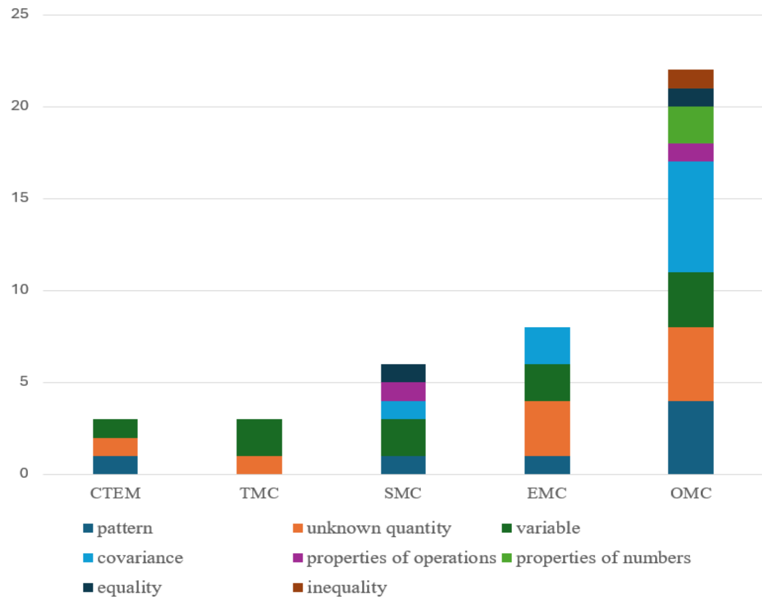
The Distribution of the Achievements in the Algebra Learning Area of the 6th-Grade Curriculum is Based on the Components of Algebraic Thinking



While TMC limits algebraic expression evaluation to natural numbers, EMC supports functional thinking through number sequence generalization, two-variable equations, and relational emphasis (e.g., geometric formulas as algebraic expressions). The Century of Türkiye Education Model, on the other hand, emphasized relational situations in its process components and included the generalization of "the sum of the measures of the interior angles of polygons" and "finding an exterior angle of polygons". In this respect, it differs from TMC and other curricula. The unknown meaning of the variable was used predominantly. At the same time, attention was drawn to the algorithmic structure of algebraic expressions as in OMC. OMC uniquely combines modeling, equation solving and inequality analysis in a single output (Figure 6). Patterns and covariation (critical for functional thinking) are absent in TMC/SMC despite their role in linking quantity relations. In particular, linear expressions for covariation are not addressed in these curricula.

Figure 6

Concepts Included in the 6th-Grade Algebra Attainments and Their Frequency of Use



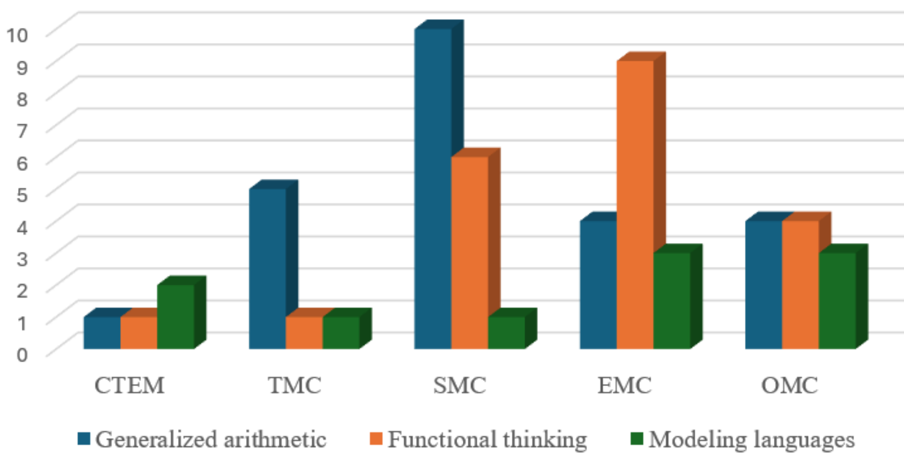
When the processes identified for the outcomes are examined, it is seen that processes such as representation and generalization, conjecture and verification for the construction or interpretation of algebraic expressions lead to the recognition of written representation, verbal representation and awareness of variables as processes in algebra learning outcomes. Algebra learning outcomes include more than one concept as well as various processes in most of the outcome statements. While TMC focused more on noticing, the Century of Türkiye Education Model included processes such as reasoning, noticing, interpreting, and inferring as domain skills in its learning outcomes. OMC gave skills related to reasoning in general terms and did not distribute them in learning outcomes.

Findings on the dimensions of algebraic thinking at the 7th-grade level

The achievements related to the 7th-grade level of algebra learning, including the Century of Türkiye Education Model, TMC, SMC, EMC, and OMC, were analyzed sequentially (Figure 7). The achievements belonging to Key Stage 3 of EMC were presented as a whole without being separated into 7th, 8th, and 9th grade levels. This study examined EMC achievements without separating them according to grade levels.

Figure 7

Distribution of the 7th-Grade Curriculum's Algebra Learning Outcomes According to the Components of Algebraic Thinking.

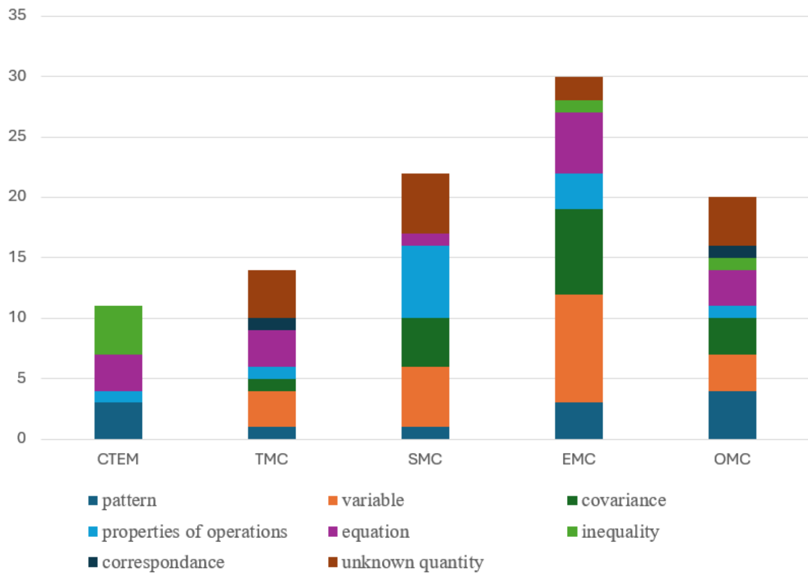


In the algebra learning domains analyzed at the seventh grade level, the number of objectives related to functional thinking increased significantly in both SMC and EMC compared to the previous grade level. However, while SMC has more content related to generalized arithmetic, EMC contains more statements related to functional thinking. It was observed that the components were represented in similar amounts in SMC.

OMC, which stands out with its pattern-based algebra teaching approach, had the most content supporting patterns at the seventh grade level, while covariances, which explore relationships between variables and support the development of functional thinking, were least prominent in TMC and most prominent in SMC and EMC (Figure 8). the Century of Türkiye Education Model, on the other hand, did not include any content related to covariance, but drew attention to the structure of algebra consisting of sequential operations and emphasized the algorithms. In all programs except the Century of Türkiye Education Model, while the concept of variable was included, covariance was also emphasized, the Century of Türkiye Education Model only gave the meaning of unknown and there was no expression related to covariance (Figure 8).

Figure 8

Algebraic Concepts of Algebraic Thinking in Grade 7 Achievements and Their Frequency of Use.



An analysis of the processes emphasized in the outcome statements revealed that in the outcomes classified as generalized arithmetic, functional thinking, and modeling languages in the basic components of algebraic thinking, the representation process helped to classify generalized arithmetic; in the statements drawing attention to simultaneous change, the processes of noticing and justification helped to classify functional thinking; and processes such as assuming, verifying, and justifying helped to classify modeling languages. It was determined that an outcome or target learning statement may specify one or more processes, or it may not specify any process at all. When the processes identified for the outcomes were examined, it was seen that there were processes such as representation and generalization, conjecture and verification for the construction or interpretation of algebraic expressions, whereas in the TMC, there were processes for written representation, verbal representation and variable recognition. OMC included more than one concept as well as more than one process in most of the learning outcome statements. The Century of Türkiye Education Model elaborated the learning outcomes with process components but did not include a skill such as noticing, instead emphasizing linguistic skills such as inference, mathematical verification or proof. Process-oriented skills are not defined in other programs.

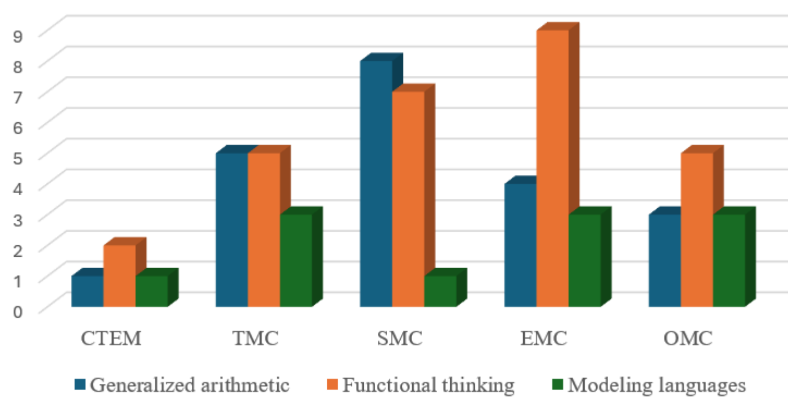
Findings related to algebraic thinking dimensions at the 8th-grade level

The gains related to the 8th-grade level of algebra learning in the context of the Century of Türkiye Education Model, TMC, SMC, EMC, and OMC have been analyzed sequentially. The gains associated with EMC Key Stage 3 have been presented as a whole without being separated into 7th, 8th, and 9th grade levels. In this research, the EMC gains have been examined without separation according to grade levels.

All gains listed in EMC Key Stage 3 have been provided in a single list, leaving the disaggregation of gains by grade levels to the school and the teacher. The analysis of all gains at the 8th-grade level has richer functional thinking content compared to other programs, and this can be explained in this way.

Figure 9

Distribution of algebra learning outcomes at the 8th-grade level according to the components of algebraic thinking.

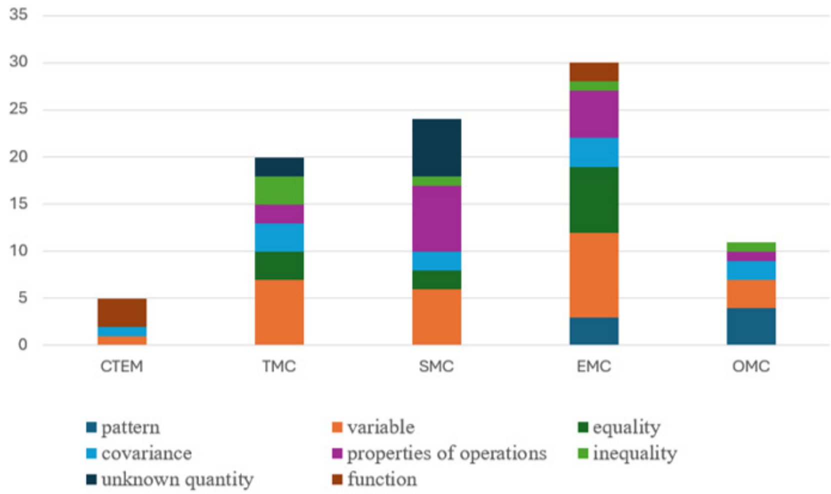


The TMC and the Century of Türkiye Education Model 8th-grade curriculum shows balanced emphasis across algebraic thinking components (Figure 9), with functional thinking supported through covariance and linear relationships; yielding the highest quantitative focus (Figure 10). While

SMC uniquely integrates functions to advance functional thinking, OMC sustains patterns/covariance across all grades, ensuring holistic progression. All curricula exhibit increased achievements in algebra, encompassing all algebraic thinking dimensions, though TMC and SMC diverge in conceptual prioritization (e.g., TMC's quantitative dominance vs. OMC's process continuity).

Figure 10

Algebraic Concepts of Algebraic Thinking Used in 8th-Grade Learning Outcomes and Their Frequency of Use.



At the 8th-grade level, the prior curriculum reduced concept diversity. EMC and SMC exhibit broader conceptual range via multi-concept integration within single statements, while TMC achieves similar diversity through comparable methods. Process analysis reveals parallels in generalized arithmetic and functional thinking across curricula. However, modeling language expressions diverge, particularly in OMC, which emphasizes distinct processes (e.g., hypothesis validation).

The diversity of concepts in TMC decreased in the Century of Türkiye Education Model, and the concept of function was added at this grade level. The concept of variable was not included in the learning outcomes, but the variable status of ordered pairs in the coordinate system was indirectly included. It is noteworthy that the diversity of concepts decreased with the decrease in the number of learning outcomes. While conceptual skills are included holistically in OMC, the Century of Türkiye Education Model imposes skills such as inferring and constructing on process components and learning outcomes.

Findings Related to the Second Sub-Problem

The findings address the similarities and differences among the sub-learning areas within the algebra learning domain as presented in the curricula of various countries. Table 3 illustrates the sub-learning areas categorized by class levels within the algebra learning domain.

Table 3

Sub-Learning Areas and Themes Related to Algebra Across Countries and the Number of Learning Outcomes

Cirruculum	Grade 5	Grade 6	Grade 7	Grade 8
The Century of Türkiye Education Model 2024	Algebraic thinking with operations (n=4)	Algebraic thinking and variations with operations (n=3)	Algebraic thinking and variations with operations (n=4)	Algebraic thinking and variations (n=4)
Türkiye 2018	-	Algebraic expressions (n=3)	Algebraic Expressions (n=3) Equality and Equation (n=4)	Algebraic expressions and identities (n=4) Linear equations (n=7) Inequalities (n=2)
Singapore	-	Algebra (n=5)	Algebraic expressions and formulas (n=8) Functions and Graphs (n=5) Equations and Inequalities (n=4)	Algebraic expressions and formulas (n=8) Functions and graphs (n=2) Equations and inequalities (n=6)

Cirruculum	Grade 5	Grade 6	Grade 7	Grade 8
England	-	Algebra (n=5)	Algebra (n=16)	Algebra (n=16)
Ontario	Patterns and Relationships (n=4) Equations and inequalities (n=4) Coding (n=3) Mathematical modeling (n=1)	Patterns and Relationships (n=4) Equations and inequalities (n=4) Coding (n=2) Mathematical modeling (n=1)	Patterns and Relationships (n=4) Equations and inequalities (n=4) Coding (n=2) Mathematical modeling (n=1)	Patterns and Relationships (n=4) Equations and inequalities (n=4) Coding (n=2) Mathematical modeling (n=1)

Note. *While the 2018 Mathematics Curriculum and other international curricula are structured around "sub-learning areas," the Century of Türkiye Education Model (CTEM) is organized around "themes." For the purpose of this comparative analysis, these themes are treated as equivalent to sub-learning areas.*

SMC integrates algebra with numbers but introduces distinct algebra sub-domains (e.g., "Functions and Graphs" coded N6) from grade 6, with sequential coding (e.g., N6.1-N6.5 in Grade 7; N6.7-N6.8 in Grade 8) fostering a spiral structure. EMC lacks sub-domain divisions, offering only unstructured outcome lists for grades 7-9. OMC uniquely structures algebra into four sub-domains: patterns/relationships, equations/inequalities, coding, and mathematical modeling. Its focus on coding and modeling under algebra distinguishes it from other programs. TMC emphasizes numerical patterns (linear relationships) and algebraic expression generalization but excludes two-variable equations. The Century of Türkiye Education Model, on the other hand, has organized the learning outcomes under the theme of Algebraic Thinking and does not use learning domain/sublearning domain statements. The statements are not written as outcomes in TMC, but as learning outcomes by emphasizing skills and detailed step by step with process components. After the theme, learning and teaching experiences and explanations to guide teachers are included. The guiding explanations are similar to the OMC. In addition, enrichment and support activities are included under the title "Differentiation". In this regard, it differs from all other programs. While SMC and OMC prioritize systematic progression, TMC and EMC remain less granular. OMC's inclusion of coding and TMC's omission of multivariable concepts highlight curricular contrasts.

In the section provided as a guide for both the attainment statement and content in the algebra learning area for the 6th-grade level, emphasis is placed on the formulas learned in mathematics and science classes (Department for Education, 2013). Similarly, the interdisciplinary learning framework OMC expands students' learning experiences in algebra through examples that reference their prior learning in geometry or by connecting formulas learned in different disciplines, such as the sciences (Figure 11). It has also been identified that guiding content is associated with the sub-learning area of coding within the algebra learning area, which exists at each grade level. In the Century of Türkiye Education Model, the algorithmic structure of algebraic expressions is also included and attention is drawn to this structure at each grade level. With the renewal of TMC, the Century of Türkiye Education Model program content named the section where the interdisciplinary approach is included as "Building Bridges" and related the learning outcomes under the theme of algebraic thinking at each grade level to the history of mathematics or daily life problems.

Figure 11

Guide note for the algebra learning area of the 6th grade OMC, pertaining to the sub-learning area of equality and inequality (p. 355)

Note

- When students are working with formulas, they are evaluating expressions.
- Replacing the variables with numerical values often requires the use of brackets. For example, the expression 4.5*m* becomes 4.5(*m*) and then 4.5(7) when *m* = 7. The operation between 4.5 and (7) is understood to be multiplication.
- Many coding applications involve algebraic expressions being evaluated. This may be carried out in several steps. For example, the instruction: "input 'the side of a square', sideA" is instructing the computer to define the variable "sideA" and store whatever the user inputs into the temporary location called "sideA". The instruction: "calculate sideA*sideA, areaA" instructs the computer to take the value that is stored in "sideA" and multiply it by itself, and then store that result in the temporary location, which is another variable called "areaA".

At the 6th-grade, while TMC focuses solely on introducing algebraic expressions, SMC and OMC include solving simple linear equations. EMC uniquely targets two-variable equation pairs. At the 7th-grade SMC, EMC, and OMC address linear equations and relationships, whereas TMC limits coverage to basic equations/equalities. EMC and OMC further emphasize inequalities and their graphical representation. At the 8th grade level, while all programs have a more intensive algebra content, the TMC only includes the factorization of perfect square expressions for algebraic expressions and does not include grouping for factorization. At the same time, there is no inclusion of the solution or graphical representation of quadratic equations. Both the SMC and EMC incorporate all methods used in the factorization of algebraic expressions, including quadratic equations and their graphical representations. In the OMC, linear equations and inequalities are included, while the graphical representations of quadratic equations are not. With the Century of Türkiye Education Model, the concept of functions began to be included in the program along with linear equations at the 8th grade level. In SMC and EMC, the concept of function started to be included from the 7th grade onwards. Like TMC, the Century of Türkiye Education Model also focused on linear relationships and did not include quadratic equations and their graphs. Notably, a distinct divergence from the TMC is observed in algebraic manipulation: while the TMC included specific factorization topics, the analysis reveals that algebraic identities and perfect square expansions have been excluded from the Century of Türkiye Education Model curriculum.

Discussion and Conclusion

This study comparatively analyzes 2023–2024 mathematics curricula in Türkiye, Singapore, the UK, and Canada (Ontario), focusing on algebraic thinking components. Learning objectives, pivotal to curricular frameworks, shape equitable access by standardizing content for teachers and students. The main reasons for this comparative study's focus on "algebraic thinking" are based on both algebra's central role in mathematics and students' common difficulties in this area. Besides differences in international student achievement make comparing curriculum approaches particularly interesting.

Algebraic thinking, encompassing generalized arithmetic, functional thinking, and modeling languages, relies on a spiral, reasoning-driven structure. Swafford and Langrall (2000) define it as operating with unknowns, highlighting its roots in arithmetic. Booth (1988) argues students' algebraic struggles stem from unresolved arithmetic gaps, underscoring the need to integrate algebraic reasoning with arithmetic foundations (e.g., number properties). Singapore's curriculum (Fong, 2004) exemplifies this through problem-solving and generalization, prioritizing cognitive processes like pattern analysis.

The findings highlight several critical implications for curriculum design and pedagogical practice. First, comprehensive curricula, such as Canada (Ontario)'s integration of coding and modeling within algebra, promote equity by providing structured guidance that ensures all students access standardized content and examples. Second, addressing unresolved gaps in arithmetic-to-algebra transitions -evident in Turkiye's limited coverage of quadratic equations-is essential to prevent foundational misunderstandings that hinder algebraic proficiency. With the new curriculum design, Turkiye has significant differences and continuities in both content and approach to algebraic topics and algebraic thinking. The Century of Türkiye Education Model is presented in a more structured and process-oriented way while retaining the basic concepts. It directly incorporates algebraic thinking skills such as generalization, abstraction, pattern recognition and representation into the learning outcomes and has a structure that defines skills and strongly emphasizes conceptual understanding. While it is similar to Ontario in this respect, it facilitates comparative analysis by clearly defining skills and including a skill in each learning outcome. As detailed in the results, topics such as identity and perfect square expansions have been removed from the Century of Türkiye Education Model. This exclusion indicates a pedagogical shift from symbolic manipulation and procedural fluency toward a more concept-based approach focused on functional thinking and linear relationships (Blanton and Kaput, 2011; Kieran, 2004).

Finally, prioritizing reasoning skills over procedural fluency, as seen in Singapore's spiral curriculum design, strengthens conceptual depth by embedding algebraic thinking within arithmetic properties and real-world problem-solving, thereby fostering lifelong analytical competencies. Verschaffel et al. (2007) emphasize that generalized arithmetic bridges arithmetic and algebra by enabling students to model complex problems through relational generalization. In this regard, the Century of Türkiye Education Model differs from the 2018 Turkish curriculum in that reasoning skills are prioritized over procedural fluency, and the objective language has been updated with statements that focus on the level of analysis and synthesis rather than action-oriented statements. The objectives include actions such as "interpret, relate, generalize, infer" that encourage higher-order thinking. Pattern-based activities in early education, paired with opportunities to articulate rules and generalizations mathematically, strengthen algebraic thinking by fostering problem-solving reflection (Ayber and Tanışlı, 2017; Palabıyık and Akkuş İspir, 2011).

Canada's Ontario curriculum prioritizes patterns/relationships across grades but relegates numeric patterns to "Numbers and Operations" at Grade 5, neglecting explicit relational quantity analysis. This omission risks hindering arithmetic-to-algebra transitions and functional thinking development (Özyıldırım Gümüş, 2022). In contrast, OMC introduces variables and inequalities at Grade 5, aligning with evidence that pre-algebra students can grasp covariation and functional relationships through structured learning (Blanton et al., 2017; Akın, 2020; Pittalis et al., 2020). The Century of Türkiye Education Model curriculum includes relational situations in algebraic thinking from Year 5 onwards, emphasises functional thinking and reflects a contemporary algebraic approach. Both curricula emphasise the algorithmic structure of problem solving by creating models with algebraic structures, but the Century of Türkiye Education Model differs significantly in that it addresses this as a learning outcome. Early exposure to variable dynamics mitigates future challenges (Blanton et al., 2017; Akın, 2020).

At Grade 6, curricula often focus on generalized arithmetic (e.g., writing expressions, solving equations) but overlook covariance. Strengthening functional thinking requires explicit emphasis on variable interpretation, equation pairs, and inequality analysis. NCTM (2000) advocates introducing functional relationships earlier to enhance algebraic reasoning and conceptual depth. To strengthen algebraic thinking across curricula, three strategic priorities emerge. First, integrating pattern-based relational tasks (such as exploring covariation between quantities) across grade levels can bridge arithmetic and algebraic reasoning. Second, embedding variables and inequalities in pre-algebra stages scaffolds functional thinking by familiarizing students with dynamic relationships early, as demonstrated by Canada (Ontario)'s OMC framework. Third, aligning curricula with spiral models, which iteratively reinforce core concepts (e.g., symbolic representation, equation-solving), ensures progressive skill development. These steps address current gaps, such as Ontario's underemphasis on relational analysis in early grades, while promoting equitable access to advanced mathematical competencies through structured, conceptually rich learning pathways.

OMC's algebra curriculum uniquely integrates modeling languages through coding, tables, and graphics, emphasizing functional reasoning and interdisciplinary problem-solving. In contrast, TMC, EMC, and SMC lack explicit process articulation (e.g., prediction-validation) (Gadanidis et al., 2021). While

EMC's Key Stage 3 provides only a learning list (delegating planning to schools), its Key Stage 2 aligns with TMC/SMC but shows stronger functional thinking support at Grade 6 (Gökçe and Aydoğan Yenmez, 2022). SMC emphasizes generalized arithmetic across grades, with functional thinking increasing at higher levels due to linear equation exploration (Blanton and Kaput, 2004; Steinweg et al., 2023).

Generalized arithmetic establishes foundational algebraic reasoning but fails to fully articulate quantity relationships. Introducing covariance and variable dynamics in pre-algebra (e.g., OMC's Grade 5 inequalities) mitigates cognitive gaps during concrete-to-abstract transitions (Blanton et al., 2017; Akın, 2020). However, overemphasizing algebra as a problem-solving tool risks neglecting relational reasoning. Integrating modeling activities and multiple representations (e.g., coding, graphical analysis) fosters deeper conceptual understanding (Türkmen and Tanışlı, 2019).

To address gaps in algebraic thinking development, strategic reforms should prioritize three areas. First, early integration of algebraic reasoning -beginning in primary education- ensures arithmetic and algebra evolve synergistically, as advocated by Stephens et al. (2017) and Blanton et al. (2018). This approach prevents the misconception that algebraic thinking starts only with formal algebra instruction. Second, interdisciplinary methods, such as Canada (Ontario)'s coding and modeling integration, contextualize algebra through real-world applications, fostering engagement and conceptual depth (Gadanidis et al., 2021). Third, curricula should unify arithmetic and algebra, avoiding artificial segregation. Singapore's "Numbers and Algebra" framework and Canada's spiral design exemplify this integration, ensuring continuity and reducing cognitive dissonance during transitions (Toluk Uçar, 2018; Stephens et al., 2017).

Comparative analysis of curricula is hindered by inconsistent granularity in competency definitions—some programs overly simplify outcomes, while others lack specificity. This variability complicates cross-national evaluations of algebraic proficiency. Additionally, uneven teacher guidance frameworks, such as England's reliance on optional NCTM guides (Gökçe and Aydoğan Yenmez, 2022), risk perpetuating inequities. Without standardized support, educators may struggle to implement complex concepts like covariation or modeling languages effectively, underscoring the need for structured, equity-focused curricular frameworks. To sum up, in the Century of Türkiye Education Model, in which the number of learning outcomes has decreased compared to the past, algebra is no longer an area where only operation skills are taught, but it is handled within a structure that enables students to generalize, make transitions between representations, and develop functional thinking skills by interpreting relationships and relations. In this respect, the Maarif Model is similar to the contemporary mathematics curricula of countries such as Ontario and Singapore.

Recommendations

- The algebraic thinking components of countries with evaluation scores above the international average can be examined and compared within their overall learning areas.
- Written instructional materials for the second primary education level can be examined to identify tasks that promote algebraic thinking.
- Written instructional materials for the second level of primary education can be analyzed by examining the components of functional thinking and modeling languages related to algebraic thinking.
- Curricula or written instructional materials from countries with similar cultural structures to Turkiye can be studied with a focus on algebraic thinking.
- Investigations may be conducted within a different analytical framework for algebraic thinking.

Genişletilmiş Özet

Giriş

21. yüzyılda, ileri matematiksel muhakeme ve analitik becerilerle donatılmış bireylere olan talebin küresel olarak artması, matematik müfredatında cebirsel düşünmeye yeniden odaklanılmasına neden olmuştur. Genelleştirilmiş aritmetik, işlevsel muhakeme ve modellemeyi kapsayan cebirsel düşünme, öğrencilerin kavramsal anlayış ve problem çözme yeteneklerini geliştirmeleri için temeldir. Ancak, ülkeler bu becerilerin müfredatlarında nasıl ve ne zaman tanıtıldığı ve vurgulandığı konusunda önemli farklılıklar göstermektedir. Bu çalışma, farklı ulusal müfredatların cebirsel düşünmenin gelişimini nasıl desteklediğine dair karşılaştırmalı bir analiz yaparak bu farklılıkları anlama ihtiyacından doğmuştur. Bu çalışmanın amacı, Türkiye (2018 versiyonu ve 2024 Türkiye Yüzyılı Maarif Modeli), Singapur, İngiltere ve Kanada (Ontario) ortaokul matematik müfredatlarını karşılaştırarak 5. sınıftan 8. sınıfa kadar cebirsel

düşünmenin bileşenlerini (genelleştirilmiş aritmetik, fonksiyonel düşünme ve modelleme dilleri) nasıl içerdiklerini araştırmak ve ortaya çıkarmaktır.

Yöntem

Nitel bir doküman analizi yaklaşımı kullanılarak, dört ülkenin ulusal matematik müfredatları gözden geçirilmiş ve Chimoni ve diğerleri (2018) tarafından geliştirilen analitik bir çerçeve kullanılarak kodlanmıştır. Bu çerçeve, müfredat içeriğini üç ana boyutta sınıflandırmaktadır: (1) cebirsel düşünmenin temel bileşenleri, (2) anahtar kavramlar (ör. eşitlik, değişkenler, kovaryans) ve (3) bilişsel süreçler (ör. temsil etme, genelleme, gerekçelendirme). Öğrenme çıktıları her sınıf düzeyinde (5-8) karşılaştırmalı olarak incelenmiş ve verilerin yorumlanmasını desteklemek için tablolar ve şekiller gibi görsel araçlar kullanılmıştır. İncelenen dokümanlar ilgili bakanlıkların veya resmi kurumların herkes tarafından erişilebilir resmi internet sitelerinden ulaşılmıştır.

Bulgular

Analiz, Kanada’nın (Ontario) cebirsel düşüncenin en bütüncül adaptasyonu sunduğunu, açıkça ifade edilmiş sonuçlar ve kodlama ve finansal okuryazarlık da dâhil olmak üzere disiplinler arası bağlantılarla desteklendiğini ortaya koymuştur. Singapur, erken sınıflardan itibaren cebiri “Sayı ve Cebir” içine yerleştiren, işlemsel akıcılığı ve problem çözmeyi vurgulayan yapılandırılmış ve sarmal bir müfredat sunmaktadır. İngiltere’nin müfredatı, cebir içeriğini Key Stage 3’te (7-9. sınıflar) içermekte ve kazanımlar sınıf düzeylerine göre ayrılmadığı için ayrıntıdan yoksundur ve bu durum karşılaştırmalı eğitim araştırmacıları için bir zorluktur. Türkiye’nin 2018 öğretim programı daha işlemsel bir odak gösterirken, 2024 Türkiye Yüzyılı Maarif Modeli (TYMM) soyutlama, gerekçelendirme ve genelleme gibi becerileri vurgulayan süreç odaklı, kavramsal açıdan zengin öğrenme çıktılarına doğru önemli bir yenilik göstermektedir. Tüm ülkelerde değişken kavramı 6. sınıfta tutarlı bir şekilde ortaya çıkarken, işlevsel düşünme ve modelleme Ontario ve TYMM öğretim programlarında daha kapsamlı bir şekilde temsil edilmektedir.

Tartışma, Sonuç ve Öneriler

Karşılaştırmalı sonuçlar, cebirsel düşünmenin en iyi şekilde yapılandırılmış, sarmal bir yaklaşım sunan ve süreç becerilerine açıkça atıfta bulunan müfredatlarda desteklendiğini göstermektedir. Ontario'nun müfredatı, işlevsel akıl yürütme ve modellemeyi öğrenme çıktılarına ve disiplinler arası bağlamlara yerleştirmesiyle öne çıkmaktadır. TYMM, küresel standartlarla uyumlu, süreç ve beceri temelli bir yapı sunarak Türkiye için bir ilerlemeye işaret etmektedir. Bununla birlikte, bazı müfredatlarda kovaryans veya çok değişkenli fonksiyonlar gibi temel kavramların kapsamındaki tutarsızlıklar, cebirsel muhakemenin desteklenmesinde potansiyel eksikliklere işaret etmektedir. Cebirsel düşünmeye (özellikle örüntü tanıma, sembolik manipülasyon ve gerçek dünya modellemesi) erken yaşta maruz kalmak, uzun vadeli matematiksel yeterlilik için gerekli olduğunu kanıtlamaktadır. Sınıf seviyeleri boyunca cebirsel düşünmeyi geliştirmek şunları gerektirir:

- İlişkisel akıl yürütme, örüntüler ve sembolik genellemenin erken ve tutarlı entegrasyonu;
- Soyut ve gerçek dünyadaki matematiksel bağlamlar arasında köprü kurmak için modelleme ve fonksiyonel düşünmeye vurgu yapılması;
- Yalnızca işlemsel akıcılıktan ziyade muhakeme ve süreç becerilerini açıkça ifade eden öğrenme çıktıları.
- TYMM ve Kanada (Ontario) müfredatları, diğer eğitim sistemlerinde gelecekte yapılacak reformlar için referans teşkil edebilecek çağdaş bir cebirsel düşünme yaklaşımını örneklemektedir. Müfredatın bu ilkelerle uyumlu hale getirilmesi, öğrencilerin daha derin matematiksel anlayış kazanma fırsatlarını artırır ve onları modern dünyada karmaşık problem çözmeye hazırlar.

Tüm bunlar göz önünde bulundurularak gelecekteki araştırmacılara şu öneriler verilebilir:

- Uluslararası ortalamanın üzerinde değerlendirme puanlarına sahip ülkelerin cebirsel düşünme bileşenleri, genel öğrenme alanları içinde incelenebilir ve karşılaştırılabilir.

- Ortaokul sınıf düzeyleri için yazılı öğretim materyalleri, cebirsel düşünmeyi teşvik eden görevleri belirlemek için incelenebilir.
- İlköğretimin ikinci seviyesi için yazılı öğretim materyalleri, cebirsel düşünme ile ilgili işlevsel düşünme ve modelleme dillerinin bileşenleri incelenerek analiz edilebilir.
- Türkiye'ye benzer kültürel yapıya sahip ülkelerden müfredatlar veya yazılı öğretim materyalleri cebirsel düşünme odağında çalışılabilir.
- Araştırmalar, cebirsel düşünme için farklı bir teorik çerçeve içinde yürütülebilir.

Teşekkür: Araştırma sürecimde desteğini esirgemeyen kıymetli danışmanıma teşekkür ederim.

Yazar Katkıları: Makalenin her bölümü yazarlar tarafından eşit düzeyde katkı sunularak hazırlanmıştır.

Çıkar Çatışması: Yazarlar, bu çalışmayla ilgili herhangi bir çıkar çatışmasının olmadığını beyan etmektedir.

Etik Beyanı: Tüm yazarlar çalışmanın tarafsız ve etik araştırma ilkelerine uygun olarak yürütüldüğünü teyit etmekte ve bu beyanın doğruluğunu teyit etmektedirler. Veri toplama, analiz ve sonuçların yorumlanması dâhil olmak üzere araştırma süreci boyunca yazarlar hiçbir finansal, akademik veya kişisel çıkardan etkilenmemiş ve tüm etik ihlallerde tüm sorumluluğun makale yazarlarına ait olduğunu beyan etmektedirler.

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Veri Kullanılabilirliği Beyanı: Bu çalışma sırasında oluşturulan veya analiz edilen veriler, talep üzerine yazarlardan temin edilebilir.

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