Karadeniz Fen Bilimleri Dergisi, 16(1), 395-414, 2026

Karadeniz Fen Bilimleri Dergisi
The Black Sea Journal of Sciences

DOI: 10.31466/kfbd.1708857
ISSN (Online): 2564-7377
https://dergipark.org.tr/tr/pub/kfbd

Research Article / Araştırma Makalesi

Enhanced Control of Time-Delay Fractional-Order Systems via Smith Predictor

Smith Öngörücüsü ile Zaman Gecikmeli Kesirli Dereceli Sistemlerin Gelişmiş Kontrolü

Nisa SECILMIS[1]*, Tufan DOGRUER[2]

Abstract

The Smith predictor is a widely adopted control strategy for systems with time delays, offering enhanced stability and performance by compensating for the adverse effects of the delay. This study focuses on the design of Smith predictor-based controllers for fractional-order systems characterized by time delays. Fractional-order systems, which have the ability to model real-world processes more accurately than integer-order systems, present challenges in control design due to their non-integer dynamics. The proposed approach combines the capabilities of the Smith predictor with the flexible modeling power of fractional calculus to achieve superior control performance. A hybrid optimization algorithm that merges the strengths of the Firefly Algorithm and Genetic Algorithm is used to determine the controller parameters. Simulation studies on various fractional-order systems with time delays are presented to demonstrate the effectiveness of the proposed method. The results show significant improvements in system stability, response time, and robustness compared to conventional methods, making this approach a promising solution for advanced control applications.

Keywords: Fractional order systems, Smith predictor, Optimization, Parameter tuning.

Öz

Smith öngörücü, zaman gecikmeleri olan sistemler için yaygın olarak benimsenen bir kontrol stratejisidir ve gecikmenin olumsuz etkilerini telafi ederek gelişmiş kararlılık ve performans sağlar. Bu çalışmada, zaman gecikmesiyle karakterize edilen kesirli mertebeden sistemler için Smith öngörücü tabanlı kontrolörlerin tasarımına odaklanılmaktadır. Gerçek dünyada var olan sistemleri tamsayı dereceli sistemlerden daha doğru bir şekilde modelleme yetenekleri olan kesirli dereceli sistemler, tamsayı olmayan dinamikleri nedeniyle kontrol tasarımında zorluklar ortaya koymaktadır. Önerilen yaklaşım, daha üstün bir kontrol performansı elde etmek için Smith öngörücünün yeteneklerini kesirli hesabın esnek modelleme gücüyle birleştirir. Kontrolör parametrelerini belirlemek için Ateş Böceği ve Genetik Algoritmanın üstün taraflarını birleştiren hibrit bir optimizasyon algoritması kullanılarak analiz yürütülmüştür. Zaman gecikmeleri olan çeşitli kesirli dereceden sistemler için benzetim çalışmaları sunularak önerilen yöntemin etkinliği gösterilmiştir. Sonuçlar, geleneksel yöntemlere kıyasla sistem kararlılığında, tepki süresinde ve sağlamlıkta önemli iyileştirmeler ortaya koyarak bu yaklaşımı gelişmiş kontrol uygulamaları için umut verici bir çözüm haline getirir.

Anahtar Kelimeler: Kesir dereceli sistemler, Smith öngörücüsü, Optimizasyon, Parametre ayarı.

[1,2]Tokat Gaziosmanpaşa University, Electrical and Electronics Engineering, Tokat, Turkey

***Corresponding Author / Sorumlu Yazar:** tufan.dogruer@gop.edu.tr

Geliş/Received: 29.05.2025 **Kabul/Accepted:** 25.11.2025 **Yayın/Published:** 08.03.2026

Secilmis, N., Dogruer, T. (2026). Enhanced Control of Time-Delay Fractional-Order Systems via Smith Predictor. *Karadeniz Fen Bilimleri Dergisi*, 16(1), 395-414. https://doi.org/10.31466/kfbd.1708857

1. Introduction

Time delay presents a significant challenge in control systems. When the effect of a change in the input signal appears at the output after a certain delay, the dynamic behavior of the system changes considerably (Normey-Rico & Camacho, 2007). This change makes control operations more complex and hinders the achievement of the desired performance. Delay can reduce system stability, leading to oscillations and potentially instability (Astrom, 1995). Such situations require additional measures to maintain system stability. Time delay in control systems commonly occurs in industrial processes and large-scale systems (Seborg et al., 2016). For example, in chemical processes or power generation plants, it is often observed that the effect of an input signal appears at the output after a certain delay. These delays prolong the system's response time and reduce its sensitivity to instantaneous changes and disturbances. Furthermore, they make it more difficult for operators and controllers to intervene quickly and accurately. To minimize the effects of time delay, various control techniques and methods have been developed. For instance, methods such as the Smith predictor (Smith, 1957), model predictive control (Qin & Badgwell, 2003), and adaptive control (Ioannou & Sun, 1996) offer solutions to compensate for delays and improve system performance. Additionally, optimizing controller parameters and using appropriate mathematical models enable more effective control of systems with delays.

In addition to time delay, the fractional-order nature of a system further complicates control problems. Unlike classical integer-order models, fractional-order systems more accurately represent memory effects and long-term dynamic behaviors of the system (Podlubny, 1998). Such systems provide more realistic models by delivering precise responses over a wider frequency range (Monje et al., 2010). However, the combination of time delay with fractional-order dynamics adversely affects the system's stability and performance (Richard, 2003). The delay effect causes lag in the sensitive dynamic responses of the fractional-order model, thereby complicating controller design even further (Vinagre et al., 2000). The coexistence of time delay and fractional-order systems significantly complicates control operations, and in such cases, conventional PI or PID controllers are generally ineffective (O'dwyer, 2009). Controllers designed for integer-order systems fail to provide the desired stability and control performance when time delays and fractional-order dynamics are present simultaneously. Consequently, the simultaneous presence of time delay and fractional-order dynamics considerably increases system complexity and controllability challenges, necessitating more advanced control methods (Valério & Da Costa, 2012). Advanced control techniques, such as the Smith predictor, offer suitable solutions for effective control of these complex systems (O'dwyer, 2009; Richard, 2003).

The Smith predictor was developed to compensate for the effects of dead time in time-delay systems, thereby enabling more stable and faster control. This model-based approach, proposed by Smith, is particularly employed to address the sensitivity and stability issues encountered by conventional PID controllers in systems with significant dead time. The Smith predictor structure models the delayed portion of the system, allowing the controller to effectively disregard this delay, which results in an accelerated system response. Uma and Rao (Uma & Rao, 2016) designed a two-stage Smith predictor architecture incorporating reference tracking and disturbance rejection control strategies for unstable time-delay systems, demonstrating that the integration of a PID controller with delay filters significantly enhances system performance. Such modifications have extended the applicability of the classical Smith predictor beyond its traditional limitations, enabling its use in more complex systems. Majhi and Atherton (Majhi & Atherton, 1999) proposed a modified Smith predictor incorporating an internal feedback loop to stabilize unstable integrator-containing systems, along with a novel PI-PD controller design. This new predictor architecture particularly improves reference tracking and disturbance rejection capabilities. Notably, the proposed design outperforms classical methods in both transient and steady-state performance.

Recent studies have demonstrated that modifications of the Smith predictor for various types of time-delay systems can achieve enhanced performance. The study by Vrečko et al. (Vrečko et al., 2001) addressed performance comparisons of the Smith predictor and its variants in time-delay systems, revealing that model-based prediction methods provide more stable and effective control compared to classical approaches. This study is particularly significant in highlighting the adaptability of the Smith predictor structure to processes with diverse system dynamics. Raja and Ali (Lloyds Raja & Ali, 2021) developed a novel version of the Smith predictor that improves performance by specifying closed-loop time constants with exact values rather than within a range. This approach eliminates time constant uncertainty, enabling a more distinct and reliable response in the control system. In their study, Safaei and Tavakoli (Safaei & Tavakoli, 2018) designed a fractional-order controller within a Smith predictor structure for integer-order systems with time delay. They emphasized that their study, which bases the design procedure on percentage overshoot and settling time, presents a simple and effective analytical method. They provided two simulation examples to demonstrate the controller's performance. In a different study (İçmez & Can, 2023), the authors aimed to overcome the disadvantages of the traditional Smith Predictor structure for processes with long time delays and unstable dynamics. In the modified Smith Predictor structure developed to address system parameter variations and load disturbances, an I-PD controller was used for set-point tracking and a cascaded PD controller for disturbance rejection. Using the direct synthesis method, the authors emphasized that comparative simulations on unstable second-order process models demonstrated significant improvements in set-point tracking, disturbance rejection, and robustness. In their research, Vu and Lee (Vu & Lee, 2014) proposed a novel fractional-order proportional-integral controller embedded within a Smith predictor structure. They emphasized that the derived analytical tuning rules provide superior closed-loop performance in controlled systems. They validated the method's simplicity, flexibility, and effectiveness through various comparative examples. In another study (Ghorbani et al., 2022), the robust stability of a Smith predictor-based control structure for fractional-order systems with simultaneous uncertainties in gain, time constants, and time delay were investigated. The study proposed a graphical method to analyze the system's stability and auxiliary functions to reduce computational complexity. The authors demonstrated the validity and significance of their method through numerical examples and an experimental study. The authors state in their study that communication dead time in a thermal power system integrated with renewable energy and storage systems leads to instabilities, phase lags, and sluggish corrective actions in frequency and tie-line power deviations (Kumar et al., 2025). To solve this problem, they proposed a Smith predictor-based fractional-order controller. The proposed controller was designed using a modified internal model control method and successfully compensated for the negative effects of communication delay. The feasibility of the method was verified through simulation and hardware-in-the-loop testing. The author focuses on the challenges of controlling time-delayed and inverse-response integrating/unstable processes in their study (Dogruer, 2023). To solve these problems, they propose an I-PD-based Smith predictor structure and optimize the controller parameters using the Equilibrium Optimizer (EO) algorithm. The proposed method improves set-point tracking and disturbance rejection performance, while its superiority is demonstrated through comparative analyses.

In this study, the control of fractional-order systems with time delay is implemented using the Smith predictor structure. Smith predictor-based control is a method rarely encountered in the literature for controlling time-delayed fractional-order systems. This situation highlights the original contribution of the study. To determine the optimal parameters of PID controller within the Smith predictor framework, a hybrid algorithm combining Genetic Algorithm and Firefly Algorithm, referred to as the FAGA algorithm, is employed. Commonly used integral performance criteria in the literature (Tavazoei, 2010), such as the Integral of Time-weighted Absolute Error (ITAE), Integral of Time-weighted Squared Error (ITSE), Integral of Absolute Error (IAE), and Integral of Squared Error (ISE), are utilized as objective functions within the optimization algorithm. Additionally, a multi-objective cost function based on time response performance measures is incorporated into the optimization process to facilitate performance comparisons.

The subsequent sections of the study are organized as follows. The second section, titled Materials and Methods, provides a detailed discussion on the structure of fractional-order systems and PID tuning methods. The third section presents simulation studies conducted to evaluate the effectiveness of the proposed method, examining three different case examples. Finally, the fourth section offers an evaluation of the obtained results, along with a general summary of the study and insights into potential future research directions.

2. Materials and Methods

This section introduces fractional-order systems. The mathematical foundations and approximation models of fractional-order systems are presented. Subsequently, PID controllers and the Smith predictor structure are described in detail. The optimization methods used for tuning the controller parameters are also thoroughly explained.

2.1. Fractional order systems

Fractional-order mathematics is defined as a branch of mathematics in which the orders of derivative and integral operators are considered and analyzed as any real or complex number. Fractional calculus is a concept that has long been recognized in the field of mathematics. Systems described by differential equations in which the orders of differentiation or integration can be expressed as real numbers are referred to as fractional-order systems. Mathematicians such as Liouville-Riemann, Grünwald-Letnikov, and Caputo have made significant contributions to the field of fractional calculus. In particular, the fractional differential equations proposed by Caputo are expressed by Equation (1) (Monje et al., 2010). This formulation represents the Laplace transform of fractional derivatives. The term α denotes the fractional order, and the initial conditions are incorporated into this transformation. Here, α is a positive rational number.

$$L\{D^{\alpha} y(t)\} = s^{\alpha} L\{y(t)\} - \sum_{i=0}^{[\alpha]-1} s^{\alpha-i-1} \frac{d^i y}{dt^i} \tag{1}$$

Equation (2) presents the mathematical expression of the fractional derivative as formulated by Grünwald-Letnikov.

$$ {}_{a}D_{t}^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\frac{t-a}{h}} (-1)^j f\left(t - jh\right) \tag{2}$$

Since the 1960s, researchers have been striving to improve integer-order models that approximate fractional-order systems, aiming to develop finite-dimensional models for inherently infinite-dimensional systems. One of the most fundamental aspects of integer-order approximation methods is the development of fractional-order approximation techniques. These approximation methods differ between the time and frequency domains. In the time domain, two primary operators are commonly used: the Euler and Tustin operators. The Tustin operator is expressed as an infinite impulse response, but it may introduce significant errors at high frequencies. The Euler operator, on the other hand, is directly given in the form of a power series expansion (Vinagre et al., 2001). Additionally, a variety of discrete-time approximation methods for fractional-order operators are found in the literature (Carlson & Halijak, 1960). Compared to the time domain, the frequency domain offers a broader range of operators and is more frequently used. The most notable among these are the Oustaloup, Matsuda, Carlson, and Continued Fraction Expansion (CFE) methods (Krishna, 2011).

2.2. Controller structures and tuning methods

PID (Proportional-Integral-Derivative) controllers are classical control structures widely employed in industrial automation systems. The PID controller adjusts the system output to converge toward a desired reference value by considering the proportional, integral, and derivative components of the error signal. The control signal is generally expressed by the following equation:

$$K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \qquad (3)$$

Here, *e(t)* represents the error between the reference and the system output, K_p is the proportional gain K_i is the integral gain, and K_d is the derivative gain. This structure combines the three terms to ensure that the system responds both stably and rapidly while eliminating steady-state error and maintaining sensitivity to abrupt changes (Åström & Hägglund, 2006).

The effectiveness of PID controllers depends on proper parameter tuning. Therefore, the process of determining controller gains is referred to as tuning. One of the most well-known tuning methods is the Ziegler-Nichols method, which identifies the system's critical gain and period to calculate K_p, K_i, and K_d values using formulas (Ziegler & Nichols, 1942). Alternatively, the Cohen-Coon method analyzes the system's time-delayed response to provide parameters that yield a faster reaction.

In modern control applications, alongside these classical methods, computational intelligence-based optimization techniques such as genetic algorithms (GA), particle swarm optimization (PSO), and differential evolution (DE) are also widely employed (Izci et al., 2023). These techniques enable more optimal determination of PID parameters by utilizing multi-objective criteria.

In PID controller tuning, alongside classical methods, single optimization algorithms have gradually given way to more flexible and robust hybrid optimization approaches. Hybrid algorithms aim to combine multiple optimization techniques by integrating their respective strengths while compensating for individual weaknesses. This approach proves particularly effective for multidimensional, nonlinear, and multi-objective optimization problems such as PID tuning (Mou et al., 2024).

Particularly in PID controller parameter tuning, the FAGA algorithm, a hybridization of the Firefly Algorithm (FA) and Genetic Algorithm (GA), has gained significant attention in recent years. The Firefly Algorithm is a nature-inspired metaheuristic based on the brightness attraction behavior of fireflies (Yang, 2009). FA demonstrates strong capabilities in rapid exploration of the search space and avoiding local minima; however, it may occasionally exhibit instability in converging to the global optimum. Therefore, its global solution capability can be enhanced by incorporating GA's genetic operators (selection, crossover, mutation). The FAGA hybrid structure combines FA's local search power with GA's diversity-enhancing global search ability (Doğruer, 2023). This framework enables optimization of PID parameters through multi-objective performance criteria, yielding superior solutions in terms of both dynamic performance and stability.

Although PID controllers remain one of the most widely used control strategies in industry, they may fail to deliver adequate performance in systems with time delays. In time-delay systems, the corrective action of the PID controller exhibits delayed effectiveness, potentially compromising both system stability and transient performance. This phenomenon can lead to various control issues including excessive overshoot, prolonged settling time, and poor disturbance rejection (Åström & Hägglund, 2006). Furthermore, the conventional PID structure lacks feed-forward capability and model-based predictive features, limiting its ability to provide sufficiently fast and precise responses in complex industrial processes.

To address such limitations of PID controllers, specialized structures known as Smith Predictors are developed. The Smith Predictor represents a model-based approach specifically designed for controlling time-delay systems. By separately handling the system's dynamic model and time delay, it enables the PID controller to act only on the delay-free model (Smith, 1957). This predictive mechanism allows the controller to anticipate the system's future behavior and implement corrective actions in a timelier manner. Figure 1 shows the block diagram of a Smith predictor.

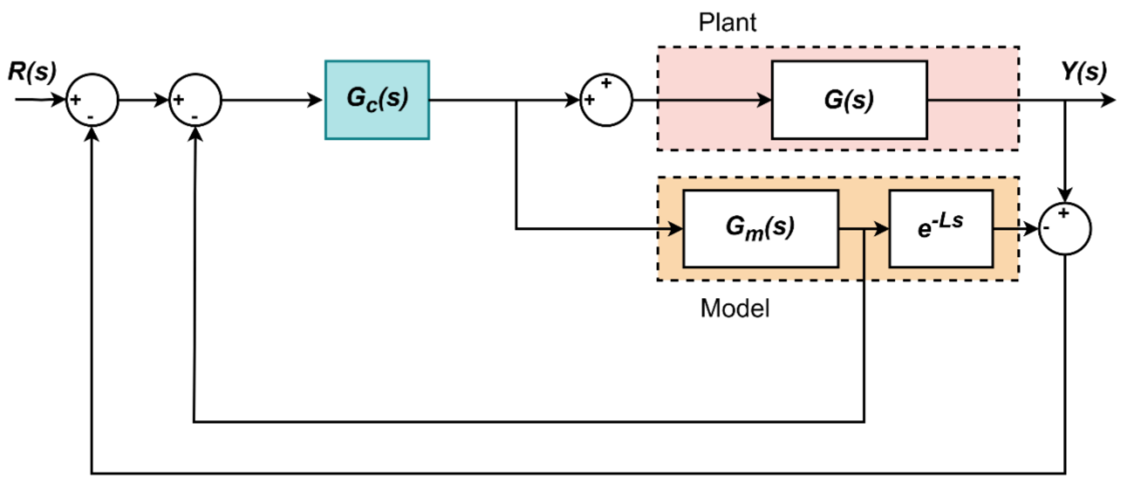

Figure 1. Smith predictor block diagram.

At the core of the structure, the delayed system is modeled as $G(s)=G_m(s)e^{-Ls}$, where $G_m(s)$ represents the delay-free dynamics and e^{-Ls} denotes the time delay term. The Smith predictor uses this model to correct the difference between the actual system output and the model output within the control loop, significantly mitigating the adverse effects of the delay (Chen et al., 2021).

The Smith predictor structure is particularly effective in systems with long time delays, such as chemical processes, heat exchangers, and pipeline systems, where it enhances the performance of PID controllers and strengthens system stability. Today, the Smith predictor can be used in a hybrid manner, both with classical PID controllers and modern intelligent algorithms, enabling more effective control of complex industrial processes.

Integral performance criteria and time-domain performance metrics are commonly used as objective functions in optimization algorithms. Equations (4) - (7) respectively present the IAE, ISE, ITAE, and ITSE criteria (Atherton, 2009). Additionally, Equation (8) presents the multi-objective function incorporating time-domain performance metrics (Bingul & Karahan, 2018). Both the integral performance criteria and time-based objective functions are implemented in the FAGA optimization algorithm for simulation studies.

$$IAE = \int_0^\infty |e(t)|.dt \tag{4}$$

$$ISE = \int_0^\infty e^2(t).dt \tag{5}$$

$$ITAE = \int_0^\infty t.|e(t)|.dt \tag{6}$$

$$ITSE = \int_0^\infty t.e^2(t).dt \quad (7)$$

$$J_{MO} = (1-e^{-\beta})(M_p + e_{ss}) + e^{-\beta}(t_s - t_r) \quad (8)$$

The parameters of the equality given in Equation (8) can be expressed as follows:

J : Total objective function to be minimized for optimization

t_r : Rise time

t_s : Settling time

Mp : Maximum percent overshoot

e_{ss} : Steady-state error

The block diagram of the proposed method is given in Figure 2. In the FAGA algorithm, five different objective functions are defined. The population size of the algorithm is set to 10, and the number of iterations is set to 50. The algorithm starts by defining the lower and upper bounds of the controller parameters. Initially, the algorithm progresses by generating random values for the controller parameters and converges toward the optimal controller parameter values as the number of iterations increases. Once the stopping criterion is met, the controller parameters are determined.

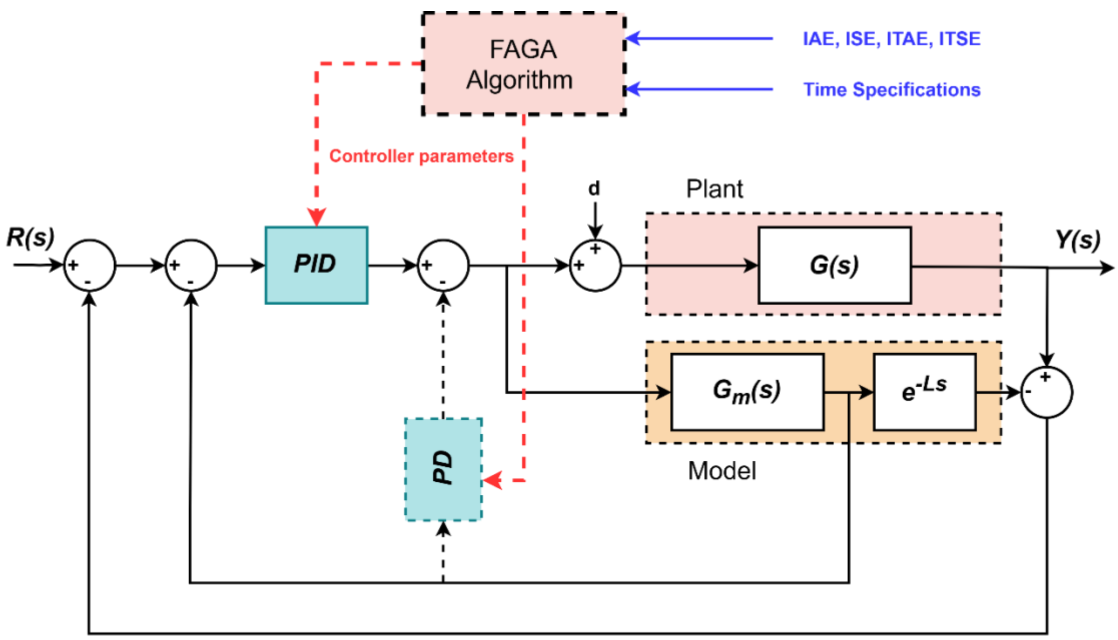

Figure 2. The block diagram of the proposed method.

3. Simulation Study, Findings and Discussion

In this section, a Smith predictor-based controller design is proposed for fractional-order dynamic systems with different time delays. When the control of time-delay systems is addressed using classical methods, significant performance limitations are encountered. In this context, the main objective of the study is to develop an advanced control approach that enables stable and rapid control of systems characterized by both time delay and fractional-order dynamics.

In the controller design process, a hybrid heuristic search method called FAGA formed by combining the Firefly Algorithm with the Genetic Algorithm is employed for parameter optimization. This hybrid structure demonstrates high accuracy and robustness in parametric optimization problems, owing to its ability to maintain global diversity in the search space while also achieving rapid convergence by intensifying the search around local solutions.

The optimization process is carried out based on objective functions, incorporating both integral-based performance indices (*ISE*, *IAE*, *ITAE*, and *ITSE*) and time-domain response characteristics (rise time, settling time, overshoot, peak time) to quantitatively evaluate the system responses. In this way, the aim is to achieve a comprehensive optimization of the system's performance in both the transient and steady-state response.

The conducted simulation studies are applied to various fractional-order system examples, and the transient responses of the Smith predictor-based controller structures designed for each system are thoroughly analyzed. The obtained simulation results are evaluated comparatively, and the control performances are examined. Additionally, various disturbance signals are introduced into the control structures to test the systems' disturbance rejection capabilities. The findings demonstrate that the proposed control strategy offers high stability, fast response, and effective disturbance suppression in time-delay fractional-order systems. In light of these results, the developed method emerges as an effective and reliable solution for the control of delayed fractional-order systems.

Example 1

Consider the fractional order transfer function with a time delay as used in the literature by Özyetkin et al. (Özyetkin et al., 2010). The Smith predictor based PID controller design for this system is presented below.

$$G_p(s) = \frac{-0.5s+1}{(s^{1.2}+1)(2s+1)} e^{-0.6s} \tag{9}$$

The integer-order approximation model of the fractional-order system with time delay given in Equation (9), based on the 5th-order Oustaloup approximation method, is presented in Equation (10). In the Oustaloup approximation, the lower and upper frequency bounds are selected as [10^{-2} and 10^{2}] during the computation of the integer-order model. As observed in Equation (10), the resulting model is a 12th-order system with time delay, and controlling such a high-order system poses a significant control challenge. In particular, the presence of time delay complicates the effectiveness of classical control techniques, thereby necessitating the use of advanced control strategies.

$$G_p(s)=\frac{\begin{array}{l}-0.5s^{11}-55.87s^{10}-1946s^{9}-2.156e04s^{8}-7.677e04s^{7}+1.549e04s^{6}+\\2.963e05s^{5}+3.168e05s^{4}+1.011e05s^{3}+1.219e04s^{2}+493.4s+6.31\end{array}}{\begin{array}{l}5.024s^{12}+487.9s^{11}+1.5e04s^{10}+1.671e05s^{9}+8.089e05s^{8}+1.86e06s^{7}+2.337e06s^{6}\\+1.687e06s^{5}+6.753e05s^{4}+1.399e05s^{3}+1.368e04s^{2}+511.6s+6.31\end{array}}e^{-0.6s} \tag{10}$$

It is well known that the Smith predictor structure provides effective control performance for systems with time delay. Therefore, in this example, a controller is designed using the Smith predictor. In the Smith predictor structure, a PID controller is employed in the forward path. The block diagram of the control system is presented in Figure 3.

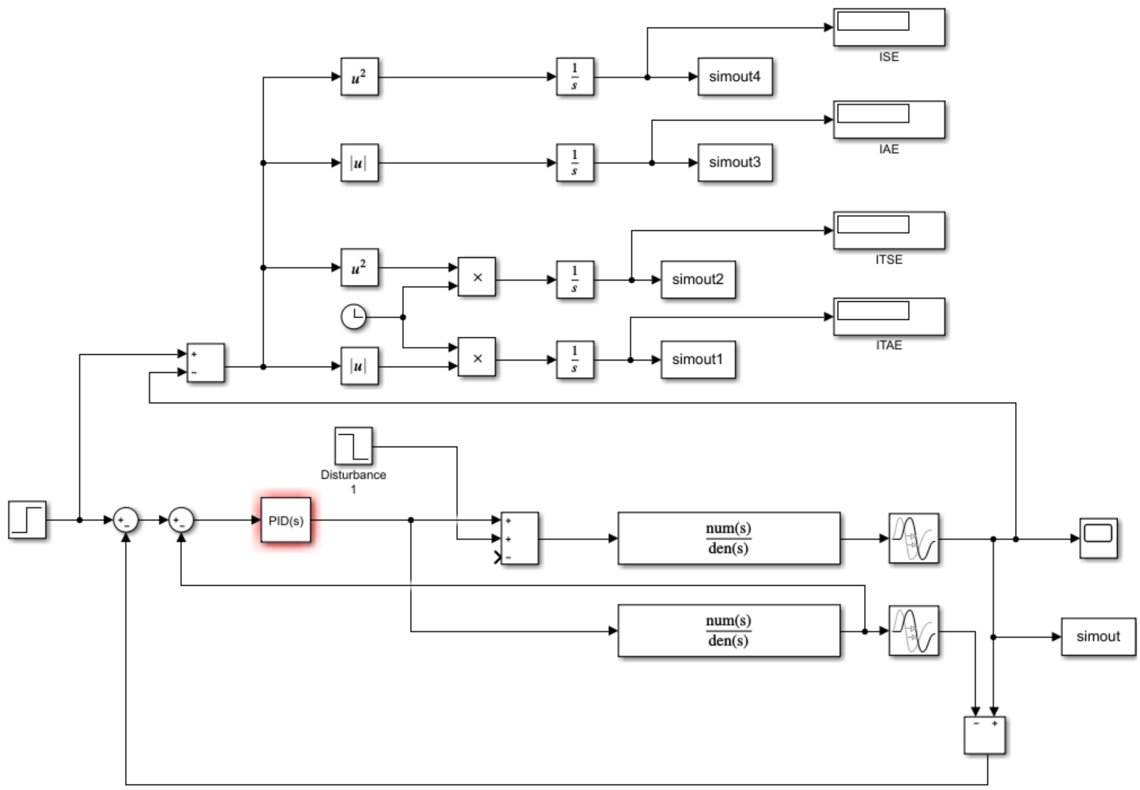

Figure 3. Simulink model for Example 1.

In the optimization algorithm, objective functions are formulated based on integral performance criteria and time response characteristics, aiming to determine the most suitable controller parameters. For this example, the lower and upper bounds of the controller parameters are defined as [0.001 0.001 0.001] and [1.5 1.5 1.5], respectively. Furthermore, the number of iterations and the population size in the FAGA algorithm are set to 50 and 10, respectively, and the algorithm is executed 10 times. The PID controller parameters corresponding to the run that yields the minimum objective function value are selected, and Table 1 is constructed accordingly.

Table 1. PID controller parameters obtained according to different objective functions.

	ISE	IAE	ITSE	ITAE	MO Function
K_p	1.5000	1.5000	1.5000	1.5000	1.4635
K_i	0.7720	0.6218	0.7303	0.6163	0.5549
K_d	1.5000	1.0692	1.5000	1.0343	0.8675

Moreover, the iteration graphs obtained for different objective functions using the FAGA algorithm are presented in Figure 4. As observed from the figure, all objective functions rapidly converge to their minimum values as the number of iterations increases, and stability is achieved in the early iterations. It can be seen that the curves converge around the 5th iteration for all objective functions.

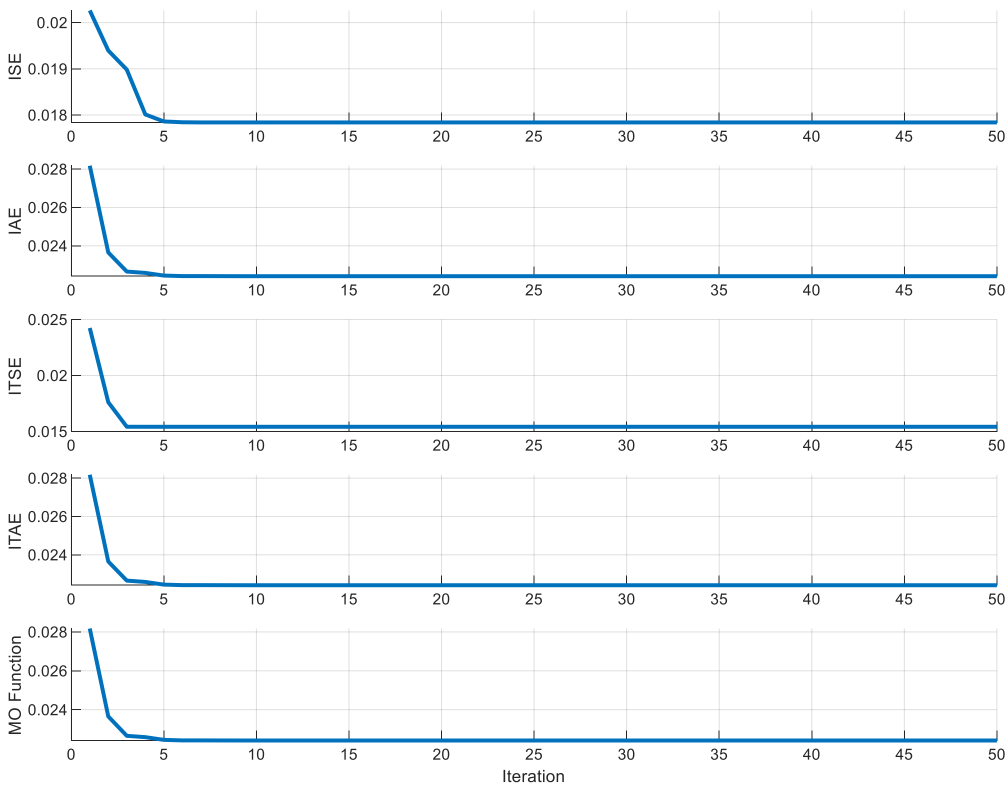

Figure 4. Convergence curves for different objective functions for Example 1.

The unit step responses of the fractional-order system with time delay are obtained by applying the controller parameters listed in Table 1. The step responses are presented in Figure 5.

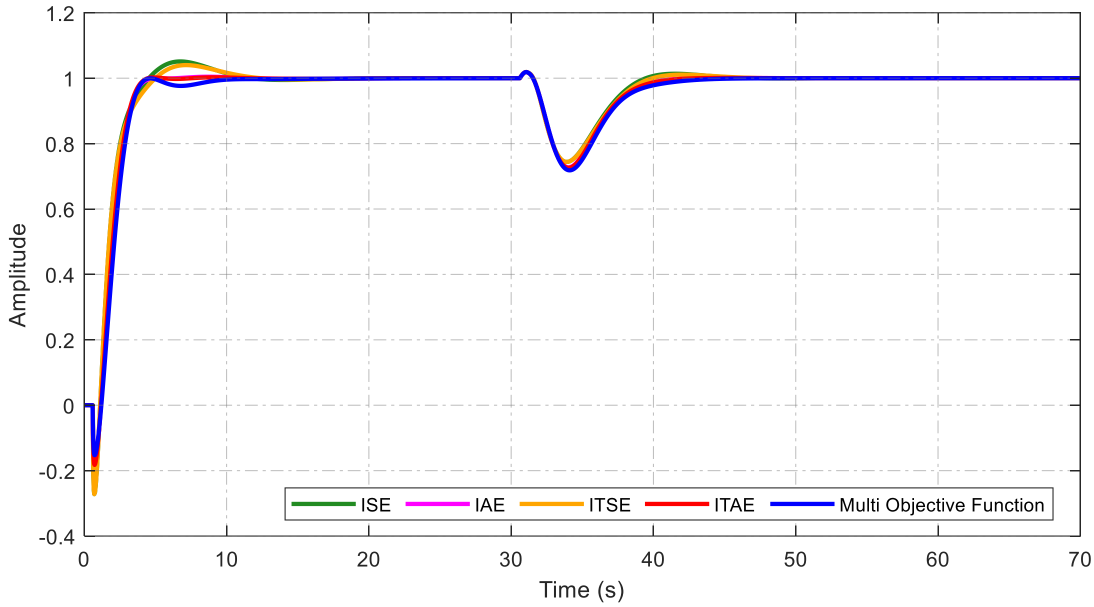

Figure 5. Unit step responses for systems designed according to different objective functions for Example 1.

From the figure, it can be observed that the responses initially move in the negative direction before proceeding positively, indicating that the system is non-minimum phase. Upon examining Figure 5, it is seen that the settling time for all performance criteria is achieved in less than 10 seconds. In terms of maximum overshoot, approximately 5% overshoot is observed with the ISE and ITSE performance criteria, whereas the overshoot remains below 1% for the other criteria. The best performance in terms of overshoot is achieved with the controller designed based on the multi-objective function. It is also evident from the figure that the rise time and peak time characteristics are similar across all performance criteria.

Furthermore, the disturbance rejection performance of the designed controllers is also analyzed. A disturbance with an amplitude of 0.5 is applied to the control system at the 30th second, and the resulting disturbance rejection performances are shown in Figure 5. It can be observed from the figure that the controllers designed for all performance criteria exhibit similar performance in disturbance rejection.

Additionally, the time response characteristics for Example 1 are presented in detail in Table 2.

Table 2. PID controller parameters obtained according to different objective functions.

	ISE	IAE	ITSE	ITAE	MO Function
Rise time (s)	1.9153	1.8524	2.0552	1.8371	1.8980
Settling time (s)	9.2023	3.8908	9.1616	3.8182	7.0482
Peak time (s)	6.7505	8.7632	7.1624	9.1926	69.6478
Overshoot (%)	5.1254	0.4490	4.0047	0.2635	2.2624e-04

An examination of the time responses characteristics reveals that the PID parameters obtained based on all criteria provide effective control of the system. However, it is noteworthy that the PID controller optimized using the multi-objective function offers a more balanced transient response compared to the others. This configuration ensures both a sufficiently fast rise and settling time while maintaining the overshoot at a minimal level.

When comparing the PID parameters, significant deviations are particularly observed in the integral (K_i) and derivative (K_d) gains across different criteria. In the IAE and ITAE criteria, the integral gain is determined to be relatively lower, indicating that the influence of the integral action is limited throughout the error duration. On the other hand, the parameters obtained through the multi-objective function present moderate values that maintain both controller effectiveness and system stability.

Considering the time-domain characteristics, the IAE and ITAE criteria yielded very successful results in terms of settling time. However, this improvement often comes at the cost of increased overshoot. On the other hand, the multi-objective function balances all performance measures, resulting in a well-rounded response with minimal overshoot and reasonable settling time. Regarding the peak time, all configurations exhibit similar performance, with the exception of the multi-objective function approach, where a longer peak time is observed. This may indicate that, although the system reaches its peak more slowly, it experiences less oscillation during the transient response.

In conclusion, using multi-objective functions instead of a single performance criterion more effectively enhances the overall system performance. This approach offers a more suitable solution for engineering applications that aim to optimize both control quality and stability.

Example 2

The fractional-order transfer function with time delay, proposed in the literature by (Narang et al., 2011), has been selected as a case study for analysis in this work. The Smith predictor-based PID controller design for this system is presented below. Additionally, a PD controller is incorporated into the Smith predictor structure to improve the disturbance rejection performance of the system.

$$G_p(s) = \frac{1}{8s^{1.5} + 5s^{0.75} + 1} e^{-4.8s} \tag{11}$$

The integer-order model of the fractional-order system with time delay in Equation (11), obtained using the 5th-order Oustaloup approximation, is given in Equation (12). This equation represents an 11th-order system with time delay.

$$G_p(s)=\frac{\begin{matrix}s^{10}+169.4s^9+9063s^8+1.675e05s^7+1.234e06s^6+3.436e06s^5\\+3.903e06s^4+1.675e06s^3+2.866e05s^2+1.694e04s+316.2\end{matrix}}{\begin{matrix}80s^{11}+1.01e04s^{10}+3.483e05s^9+4.452e06s^8+2.247e07s^7+4.867e07s^6+\\4.713e07s^5+2.173e07s^4+4.857e06s^3+5.194e05s^2+2.34e04s+366.2\end{matrix}}e^{-4.8s} \tag{12}$$

The control system, depicted in Figure 6, uses a PID controller in the forward path of a Smith predictor and a PD controller for disturbance rejection in the Simulink model.

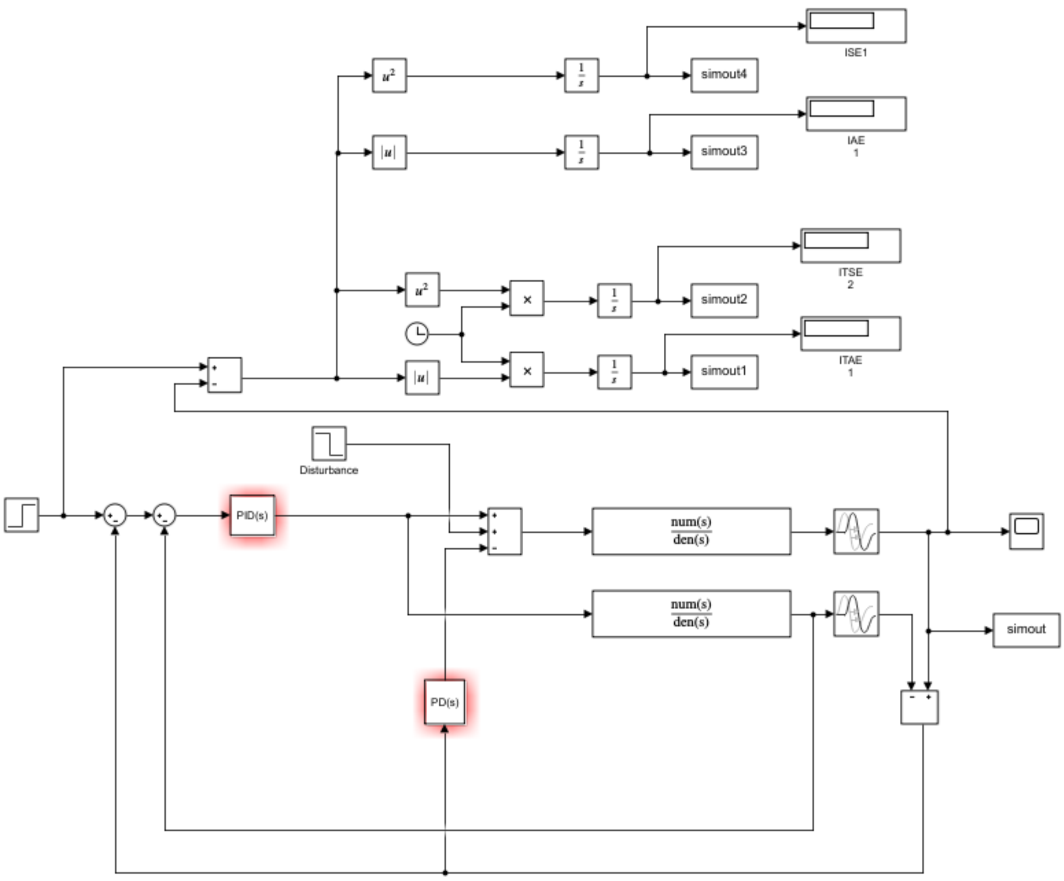

Figure 6. Simulink model for Example 2.

In the optimization algorithm for this example, the lower and upper bounds of the controller parameters are set as [0.001 0.001 0.001 0.001 0.001] and [20 20 20 20 20 20], respectively. The number of iterations and population size for the application of the FAGA algorithm are set to 50 and 10, respectively. To evaluate the reliability and stability of the algorithm, the process is repeated a total of 10 times. The PID controller coefficients obtained from the run that yielded the minimum objective function value are presented in Table 3.

Table 3. PID controller parameters obtained according to different objective functions.

	ISE	IAE	ITSE	ITAE	MO Function
K_p	20	20	16.2416	20	10.6407
K_i	9.96252	10.5867	11.604	7.31198	7.05023
K_d	0.511312	0.923403	9.26426	0.592994	4.52807
K_{p2}	0.846953	0.809045	0.768233	0.793881	0.615535
K_{d2}	1.09414	0.766501	1.45404	0.792867	2.00831

The convergence graphs obtained from the optimization algorithm are presented comparatively in Figure 7. For all objective functions, a rapid decrease in error values is observed within approximately the first 10 iterations, after which the values stabilize and reach their minimum levels.

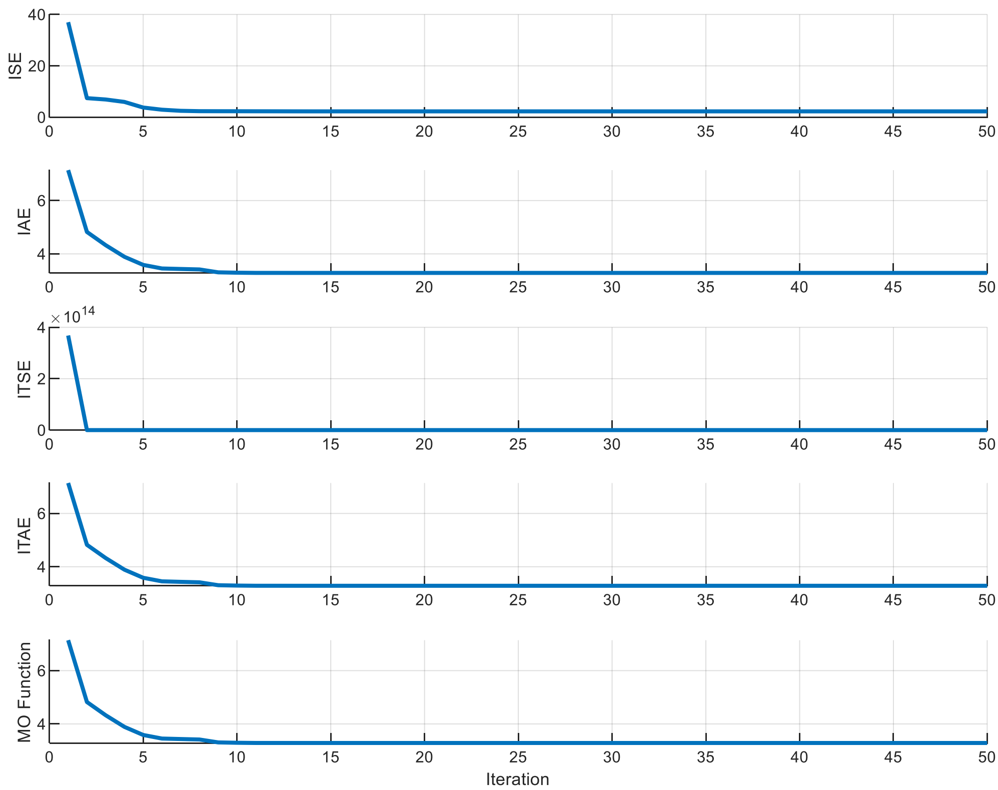

Figure 7. Convergence curves for different objective functions for Example 2.

The unit step responses of the fractional-order system with time delay are obtained by applying the controller parameters given in Table 3. These responses are presented in Figure 8.

Upon examining the figure, it is evident that the system achieves successful reference tracking across all criteria; however, differences exist in terms of maximum overshoot and settling time. Controllers obtained using ITSE and ITAE exhibit lower overshoot and smoother transient behavior. The controller optimized with IAE provides a fast rise time but demonstrates overshoot similar to that of the ISE-based controller. The rise time and peak time characteristics appear to be close across all performance criteria, as seen in the figure.

The disturbance rejection performance of the designed controllers is also analyzed. A disturbance with an amplitude of 0.5 is applied to the control system at the 25th second, and the disturbance rejection responses shown in Figure 8 are observed. It can be seen that the controllers designed according to all performance criteria demonstrate similar disturbance rejection performance.

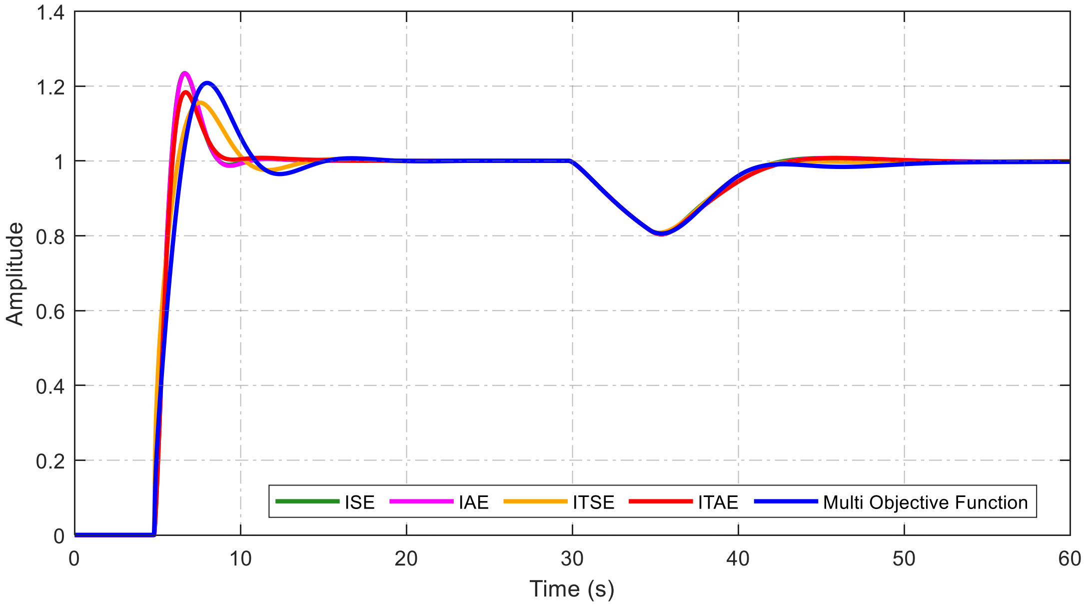

Figure 8. Unit step responses for systems designed according to different objective functions for Example 2.

Furthermore, the detailed time response characteristics for Example 2 are presented in Table 4.

Table 4. PID controller parameters obtained according to different objective functions.

	ISE	IAE	ITSE	ITAE	MO Function
Rise time (s)	0.7608	0.7741	1.0843	0.8081	1.3682
Settling time (s)	8.3576	8.3821	12.3871	8.5805	13.7325
Peak time (s)	6.6160	6.6391	7.5406	6.6706	7.9971
Overshoot (%)	23.4676	23.4175	15.5740	18.3612	20.8386

4. Conclusions and Recommendations

This study proposes a Smith predictor-based control framework for fractional-order systems with time delays. While fractional-order models provide superior modeling accuracy compared to their integer-order counterparts, their control presents significant challenges due to their complex dynamics. To address this issue, the Smith predictor structure is employed to facilitate the control of delay-affected fractional-order systems by effectively compensating for time delays.

Controller parameters were optimized using the FAGA algorithm through objective functions formulated based on multiple error criteria, including ISE, IAE, ITSE, ITAE, and a Multi-Criteria Objective Function. Simulation results demonstrated rapid convergence of the error plots, achieving stability within the first 10 iterations. The performance of the proposed method was validated with concrete data across tested scenarios: In Example 1, a nearly overshoot-free transient response and a settling time of approximately 7 seconds were achieved, while in Example 2, successful performance was observed even for a highly challenging system, with 15% overshoot and a settling time of approximately 8 seconds. In all scenarios, the transient responses of the systems were significantly improved, and a satisfactory level of disturbance rejection capability was attained.

In conclusion, the implemented approach successfully addresses the control challenges of fractional-order systems with time delays, demonstrating effective and robust performance in both set-point tracking and disturbance rejection.

Authors' Contributions

All authors contributed equally to the study

Statement of Conflicts of Interest

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The authors declare that this study complies with Research and Publication Ethics.

References

Astrom, K. J. (1995). PID controllers: theory, design, and tuning. *The international society of measurement and control*.

Åström, K. J., & Hägglund, T. (2006). *Advanced PID control*. ISA-The Instrumentation, Systems and Automation Society.

Atherton, D. (2009). *Control engineering*. Bookboon.

Bingul, Z., & Karahan, O. (2018). A novel performance criterion approach to optimum design of PID controller using cuckoo search algorithm for AVR system. *Journal of the Franklin Institute*, *355*(13), 5534-5559.

Carlson, G. E., & Halijak, C. (1960). *Simulation of the fractional derivative operator and the fractional integral operator* Kansas State University].

Chen, G., Liu, D., Mu, Y., Xu, J., & Cheng, Y. (2021). A Novel Smith Predictive Linear Active Disturbance Rejection Control Strategy for the First-Order Time-Delay Inertial System. *Mathematical Problems in Engineering*, *2021*(1), 5560123.

Dogruer, T. (2023). Design of I-PD controller based modified smith predictor for processes with inverse response and time delay using equilibrium optimizer. *IEEE Access*, *11*, 14636-14646.

Doğruer, T. (2023). Zaman gecikmesi içeren yüksek dereceli salınım sistemler için hibrit ateşböceği-genetik algoritmaya dayalı pıda kontrolör tasarımı.

Ghorbani, M., Tavakoli-Kakhki, M., Tepljakov, A., & Petlenkov, E. (2022). Robust stability analysis of smith predictor based interval fractional-order control systems: A case study in level control process. *IEEE/CAA Journal of Automatica Sinica*, *10*(3), 762-780.

İçmez, Y., & Can, M. S. (2023). Smith predictor controller design using the direct synthesis method for unstable second-order and time-delay systems. *Processes*, *11*(3), 941.

Ioannou, P. A., & Sun, J. (1996). *Robust adaptive control* (Vol. 1). PTR Prentice-Hall Upper Saddle River, NJ.

Izci, D., Ekinci, S., & Hussien, A. G. (2023). Effective PID controller design using a novel hybrid algorithm for high order systems. *PLoS One*, *18*(5), e0286060.

Krishna, B. T. (2011). Studies on fractional order differentiators and integrators: A survey. *Signal processing*, *91*(3), 386-426.

Kumar, D., Raja, G. L., Alkhatib, M., Deep, A. K., & Muduli, U. R. (2025). Smith Predictor Based Fractional LFC Strategy for Storage Supported Thermal Power System Amid Communication Dead Time and Cyber Threats. *IEEE Transactions on Industry Applications*.

Lloyds Raja, G., & Ali, A. (2021). New PI-PD controller design strategy for industrial unstable and integrating processes with dead time and inverse response. *Journal of Control, Automation and Electrical Systems*, *32*, 266-280.

Majhi, S., & Atherton, D. (1999). Modified Smith predictor and controller for processes with time delay. *IEE Proceedings-Control Theory and Applications*, *146*(5), 359-366.

Monje, C. A., Chen, Y., Vinagre, B. M., Xue, D., & Feliu-Batlle, V. (2010). *Fractional-order systems and controls: fundamentals and applications*. Springer Science & Business Media.

Mou, K., Yang, M., Zhang, M., & Wang, D. (2024). Hybrid golden jackal and golden sine optimizer for tuning PID controllers. *Scientific Reports*, *14*(1), 22189.

Narang, A., Shah, S. L., & Chen, T. (2011). Continuous-time model identification of fractional-order models with time delays. *IET Control Theory & Applications*, *5*(7), 900-912.

Normey-Rico, J. E., & Camacho, E. F. (2007). *Model predictive control of dead-time processes*. Springer.

O'dwyer, A. (2009). *Handbook of PI and PID controller tuning rules*. World Scientific.

Özyetkin, M., Yeroglu, C., Tan, N., & Tagluk, M. (2010). Design of PI and PID controllers for fractional order time delay systems. *IFAC Proceedings Volumes*, *43*(2), 355-360.

Podlubny, I. (1998). *Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications* (Vol. 198). elsevier.

Qin, S. J., & Badgwell, T. A. (2003). A survey of industrial model predictive control technology. *Control engineering practice*, *11*(7), 733-764.

Richard, J.-P. (2003). Time-delay systems: an overview of some recent advances and open problems. *automatica*, *39*(10), 1667-1694.

Safaei, M., & Tavakoli, S. (2018). Smith predictor based fractional-order control design for time-delay integer-order systems. *International Journal of Dynamics and control*, *6*(1), 179-187.

Seborg, D. E., Edgar, T. F., Mellichamp, D. A., & Doyle III, F. J. (2016). *Process dynamics and control*. John Wiley & Sons.

Smith, O. J. (1957). Closer control of loops with dead time. *Chemical engineering progress*, *53*, 217-219.

Tavazoei, M. S. (2010). Notes on integral performance indices in fractional-order control systems. *Journal of Process Control*, *20*(3), 285-291.

Uma, S., & Rao, A. S. (2016). Enhanced modified Smith predictor for second-order non-minimum phase unstable processes. *International Journal of Systems Science*, *47*(4), 966-981.

Valério, D., & Da Costa, J. S. (2012). *An introduction to fractional control*. Institution of Engineering and Technology.

Vinagre, B., Podlubny, I., Hernandez, A., & Feliu, V. (2000). Some approximations of fractional order operators used in control theory and applications. *Fractional calculus and applied analysis*, *3*(3), 231-248.

Vinagre, B. M., Petras, I., Merchan, P., & Dorcák, L. (2001). Two digital realizations of fractional controllers: Application to temperature control of a solid. 2001 European Control Conference (ECC),

Vrečko, D., Vrančić, D., Juričić, Đ., & Strmčnik, S. (2001). A new modified Smith predictor: the concept, design and tuning. *ISA transactions*, *40*(2), 111-121.

Vu, T. N. L., & Lee, M. (2014). Smith predictor based fractional-order PI control for time-delay processes. *Korean Journal of Chemical Engineering*, *31*(8), 1321-1329.

Yang, X.-S. (2009). Firefly algorithms for multimodal optimization. International symposium on stochastic algorithms,

Ziegler, J. G., & Nichols, N. B. (1942). Optimum settings for automatic controllers. *Transactions of the American society of mechanical engineers*, *64*(8), 759-765.