Fen, Matematik, Girişimcilik ve Teknoloji Eğitimi Dergisi
Journal of Science, Mathematics, Entrepreneurship and Technology Education
https://dergipark.org.tr/tr/pub/fmgted
© ISSN: 2667-5323

Strategy Flexibility Exhibited by Gifted Middle Schoolers While Solving Non-Routine Problems

Sümeyye Balikçi[1], Yeliz Yazgan[2], Gizem Yapar Sögüt[3]

[1] *Doktora, Yıldırım Bilim Sanat Merkezi, Bursa, smlybalk_matematik@hotmail.com, ORCID ID: 0000-0003-1217-8431*
[2] *Prof. Dr., Bursa Uludağ Üniversitesi, yazgany@uludag.edu.tr, ORCID ID: 0000-0002-8417-1100*
[3] *Eşrefoğlu Rumi İmam Hatip Ortaokulu, Bursa, gzmypr@hotmail.com, ORCID ID: 0000-0001-8157-402x*

ABSTRACT
The aim of this study is to examine the strategic flexibility demonstrated by gifted middle school students while solving non-routine problems. The study involved seven sixth-grade, six seventh-grade, and five eighth-grade students who were identified as gifted and were attending middle schools in the Yıldırım district of Bursa, Turkey. Individual interviews were conducted with each student, during which they were presented with seven non-routine problems. The students' problem-solving processes and approaches were analyzed. Their performances were evaluated based on three main categories: selecting the most appropriate strategy, inter-task flexibility, and intra-task flexibility. Each category was scored on a scale from 1 to 4. The findings reveal that students were generally successful in selecting suitable strategies and in shifting strategies across different tasks. However, despite their overall high flexibility scores, it was observed that they struggled to change strategies or use multiple strategies simultaneously within a single problem. These findings suggest that the components of strategic flexibility may develop independently from one another.

ARTICLE TYPE
Research

ARTICLE INFORMATION
Received: 29.05.2025
Accepted: 17.07.2025

KEYWORDS:
Mathematics Education, Middle School Students, Non-Routine Problem, Non-Routine Problem Solving, Strategic Flexibility

Üstün Yetenekli Ortaokul Öğrencilerinin Rutin Olmayan Problemleri Çözerken Gösterdikleri Strateji Esnekliği

ÖZET
Bu çalışmanın amacı, üstün yetenekli ortaokul öğrencilerinin rutin dışı problemleri çözerken sergiledikleri stratejik esnekliği incelemektir. Araştırmaya, Bursa ili Yıldırım ilçesindeki ortaokullarda öğrenim gören ve üstün yetenekli olarak tanılanmış yedi altıncı sınıf, altı yedinci sınıf ve beş sekizinci sınıf öğrencisi yer almıştır. Her biriyle ayrı ayrı görüşmeler yapılmış ve öğrencilere yedi adet rutin olmayan problem yöneltilmiş, öğrencilerin çözüm süreçleri ve biçimleri incelenmiştir. Öğrencilerin problem çözme süreçleri üç temel kategori üzerinden değerlendirilmiştir: en uygun stratejiyi kullanma, görevler arası esneklik (inter-task flexibility) ve görev içi esneklik (intra-task flexibility). Bu kategorilerin her biri, 1 ile 4 arasında puanlanarak analiz edilmiştir. Araştırma bulguları, öğrencilerin uygun strateji seçimi ve farklı problemler karşısında strateji değiştirme konularında genel olarak başarılı olduklarını ortaya koymaktadır. Bununla birlikte, genel esneklik puanlarının yüksek olmasına rağmen, öğrencilerin tek bir problem üzerinde çalışırken strateji değiştirme veya birden fazla stratejiyi bir arada kullanma konusunda zorlandıkları görülmektedir. Elde edilen bulgular, stratejik esnekliğin boyutlarının birbirinden bağımsız gelişebileceğine işaret etmektedir.

MAKALE TÜRÜ
Araştırma

MAKALE BİLGİLERİ
Gönderilme Tarihi: 29.05.2025
Kabul Edilme Tarihi: 17.07.2025

ANAHTAR KELİMELER:
Matematik Eğitimi, Ortaokul Öğrencileri, Rutin Olmayan Problem, Rutin Olmayan Problem Çözme, Stratejik Esneklik.

Introduction

The focus of education processes are the advancements in mental processes today, where science and technology highly develop. It is essential to raise individuals who can keep up with this development (OECD, 2023; World Economic Forum, 2024). From this point of view, in mathematics education, which is very important for the development of mental skills, it becomes essential how the courses in the curriculum can be taught to provide students with the ability to apply learned procedures in a meaningful, flexible and creative way (Hatano, 2003; Boaler, 2022**).** This need in mathematics education has drawn attention to problem solving skills, so many studies have been carried out on problem solving (Schoenfeld, 2021**).** While each study provides benefits to reach the targeted level in mathematics education, new research continues to be carried out to adapt to changing and developing needs. Aiming to understand the complex structure of problem solving processes and to improve this structure in a positive way, many researchers have focused on solution strategies, one of the first components of the problem solving process that comes to mind, and as a result, strategic flexibility has gained attention (Leikin, 2020). When all this information is evaluated, it can be said that the flexibility skills demonstrated by students in the problem-solving process are positively correlated with their success in mathematics (Er ve Diğerleri, 2024; Kılıç ve Uçar, 2023).

Non-routine problems are problems in which a person cannot reach a solution by using the rules and algorithms that he/she already knows, requiring a different perspective (Woodward et al., 2012). Unlike routine problems, non-routine problems require higher-order thinking skills such as reasoning, organizing data, and finding patterns (Polya, 1945). Non-routine problems improve problem solving skills in general. Indeed, recent studies have demonstrated that engaging with such problems markedly improves students' capacities in mathematical creativity, critical thinking, and problem-solving skills **(**Boesen et al., 2023; Star & Rittle-Johnson, 2022). Still, they are not widely included in mathematics teaching programs, environments, and textbooks (Kolovou, van den Heuvel-Panhuizen, & Bakker, 2009), so students are unfamiliar with such problems and need help solving them (Yeo, 2009). As evidenced by recent research, this issue persists in current practice, as elementary and middle school textbooks predominantly focus on routine problems, while non-routine problems receive limited attention. Vural's (2019) analysis of elementary school curricula further supports this observation by revealing that non-routine problems are largely neglected. Among the strategies used to solve such problems, the most common ones in the literature are: "look for a pattern", "guess and check", "simplify the problem", "make a drawing", "make a table", "work backward", "act it out", "write an equation", "eliminate" and "use logical reasoning" (Herr & Johnson, 2002; Posamentier & Krulik, 2008). These strategies are flexible paths that guarantee a solution. However, they broaden the point of view and can be easily adapted to different problems in different subject areas (Tiong, Hedberg, & Lioe, 2005). Recent curriculum developments emphasize the importance of students actively employing these strategies and provide resources to guide teachers in this regard (NCTM, 2023; OECD, 2023**).** Studies have shown that students at various grade levels can inherently use non-routine problem solving strategies, and interventions focused on these strategies increase students' success in solving such problems (Lee, Yeo, & Hong, 2014). These findings indicate that flexible thinking in mathematics can be developed from an early age and contributes positively to mathematical achievement (Kılıç & Uçar, 2023; Schoenfeld, 2021).

The concept of flexibility is intertwined with problem solving and strategy use. Within the scope of problem solving, which is one of the process standards of NCTM (2000), the expression "…so that they can apply and adapt the strategies they develop to other problems and in other contexts." is included. Although the word "flexibility" is not used here, the expression of adapting strategies to different contexts directly indicates "flexibility". In another document, NCTM (2010) recommends that students be encouraged to use more than one solution strategy for a problem. Verschaffel, Luwel, Torbeyns, and Van Dooren, (2009) defined strategic flexibility as "conscious or unconscious selection and use of the most appropriate solution strategy on a given mathematical item or problem, for a given individual, in a given context" (p. 343). According to the definition on which this study is based, an individual with strategic flexibility is supposed to have knowledge of multiple strategies, choose the

most appropriate strategy (or strategies) for the problem to be solved, and switch to a different strategy when the chosen strategy does not work (Krems, 2014). Algebraic equations (e.g., Star & Rittle Johnson, 2008), addition and subtraction (e.g., Selter, 2001), mental calculation and estimation (e.g., Threlfall, 2009) are areas which studies have been made on strategic flexibility. According to these studies, students instinctively have the ability to choose appropriate and various strategies without any intervention, and training in this direction can further develop this instinct. In addition, the components of convenience, accuracy, and fluency are essential in selecting and developing these strategies. Recent studies support that integrating these components into the educational process leads to significant improvements in students' achievement levels and self-confidence (Torbeyns et al., 2021).

There is a vast of studies on gifted students in mathematics. Although it is not possible to include each of these studies here due to space limitations, some points can be mentioned in terms of problem solving. In most of the definitions made in the literature, mathematical giftedness is almost identified with the ability to solve problems (especially non-routine problems are meant here). Maker (2003) states that the distinguishing features of the mathematically gifted individual are their problem-solving power and flexible and effective approach to the problem-solving process. Indeed, recent studies have revealed that gifted students demonstrate higher performance than their peers in strategic flexibility, in-depth analysis, and creative problem-solving during the problem-solving process(Leikin & Levav-Waynberg, 2020; Assmus et al., 2022). More specifically, as indicators of mathematical giftedness, being able to define and formulate problems and problem solving steps correctly (Greenes, 1981), being able to to make decisions during problem solving (Schoenfeld, 1985), being able to solve a problem in different ways (Leikin, Koichu, & Berman, 2009), being able to use creative strategies in solving problems (Sheffield, 2003), being able to solve complex problems (Niederer, Irwin, Irwin, & Reilly, 2003), and being determined and persistent in problem solving (Sriraman, 2003) are accepted. However, this does not mean that a gifted child in mathematics should have all of these characteristics (Bulgar, 2008). From this point of view, children's strategy choice suggests that there may be individual and contextual characteristics in addition to superior abilities. The quality of the learning environment, teacher guidance, and students' metacognitive awareness can influence strategy selection (Zhou et al., 2021). Understanding the role of ability, individual differences and contextual characteristics in flexibility selection and revealing the unknown aspects of this issue supports the fact that focusing on studies with gifted students is the right step (Vershaffel, 2023).

Related Studies

Only a few studies examine strategic flexibility in terms of non-routine problem solving. The first of these studies was carried out by Elia, van den Heuvel-Panhuizen, and Kolovou (2009). In this study, the researchers aimed to examine the strategy use and flexibility of a large group of fourth grade students who are high-achievers while they are solving non-routine problems. To this end, they asked the participants three problems and identified two types of strategic flexibility, which form the basis of further studies: "inter-task flexibility" (changing strategies across problems) and "intra-task flexibility" (changing strategies within problems). Arslan and Yazgan (2015) conducted another follow-on study similar to Elia et al. (2019) with a smaller group of high-achieving students in the sixth, seventh and eighth grades. However, the twelve students in their study worked in pairs while solving the four non-routine problems. Bräuning (2016), on the other hand, gave a broader perspective on the subject of flexibility and associated the concept of strategic flexibility of teachers with pedagogical content knowledge and content knowledge. In this study, conducted with 57 pre-service teachers at a university in Germany, the participants were given a single non-routine problem and asked to solve it with two different strategies. The researcher claims that teachers with relatively better inner-task flexibility give better feedback to student solutions because they can easily understand students' thoughts and change their perspectives. On the other hand, Keleş and Yazgan (2021) carried out a study focusing on strategic flexibility with a gifted group mainly consisting of high school students (eight, nine, ten, and eleventh grade students). Each of the 50 participants individually solved seven non-routine problems in their

study. Unlike other studies, Gavaz et al. (2021) examined whether an intervention could improve strategic flexibility of regular fifth graders' in non-routine problems. In their study involving an experimental group of 65 students, the researchers used eight non-routine problems in pre-and post-test.

The findings of the studies mentioned in the previous paragraph can be summarized as follows: a) except those in the fourth grade, gifted and high-achieving students are above average in terms of success in non-routine problem solving and strategic flexibility. b) Intra-task strategic flexibility levels of students are weaker than their inter-task strategic flexibility levels. c) There is a high correlation between strategic flexibility and success in solving non-routine problems. d) Problem-solving success and strategic flexibility of regular students are below average, but intervention provided to them has led to positive progress in both.

Importance and Aim of the Study

This study has similarities and differences with the related studies mentioned in the previous section. First, the types of strategic flexibility identified by Elia et al. (2009) are used in this study. In addition, the grade levels are the same as in the study of Arslan and Yazgan (2015). However, this study differs from Elia et al. (2009) and Arslan and Yazgan (2015) in terms of including gifted students, working with students individually, and asking for more problems for students. In these respects, the study is similar to the study by Keleş and Yazgan (2021), although the grade levels are different. Most importantly, as Star (2018) points out, the study of strategic flexibility on similar subjects in mathematics education has been a limitation. Therefore, this study aims to contribute to the literature by focusing on non-routine problems, an area where limited research on flexibility has been conducted.

In this context, the specific research question can be addressed: To what extent do gifted sixth, seventh, and eighth grade students have strategic flexibility in non-routine problem solving?

Methodology

Research model

The design of this study was determined as a case study for the following reasons: Case studies examine one or more events, environments, programs, social groups, or other interconnected systems in their reality, depending on a specific place and time. Among the case study designs, the holistic single case design was used. The holistic case design is used in the study of extreme, contradictory, and idiosyncratic situations that do not meet the general standards (Yıldırım & Şimşek, 1999). The holistic single case study type was preferred in the present study because gifted students are individuals outside the standards. Following these characteristics, the current study elaborates on strategic flexibility with a group of gifted students in their context without any intervention (McMillian, 2004).

Participants

The study was carried out with seven sixth-graders, six seventh-graders, and five eighth-graders. Participants were aged between 12 and 14 years. All participants were also receiving education in the field of general talent at two different Science and Art Centers (SACs) located in the province of Bursa. SACs are schools in Turkey that provide extracurricular education to gifted students during out-of-school hours. Students are selected by aptitude tests to these schools and can apply from the fields of general talent, music, and visual arts. There are titles such as "Numbers and Operations", "Problem-solving Methods", and "Patterns" in the mathematics courses at SAC, and the selection of appropriate strategies for solving problems and reasoning-oriented acquisitions are included (Ministry of National

Education, 2019). As seen in Table 1, most participants (about 67%) were male. The socioeconomic levels of all participants are in the interval of intermediate.

Table 1

Distributions On Grade Levels And Genders Of Participants

Grade	Male	Female	Total
6th grade	4	1	7
7th grade	4	2	6
8th grade	4	3	5
Total	12	6	18

Sampling

We used criterion sampling as the sampling method of the current study. Criterion sampling is the sample creation from people who meet the predetermined qualifications related to the problem (Patton, 1987). In this respect, criterion sampling, which includes rich situations, provides an opportunity for in-depth study (Patton, 2014). In this context, we selected our participants according to the requirements of giftedness, attending SAC, and taking mathematics courses.

Instrument

Seven non-routine problems were used in this study (See Appendix). Problems related to "guess and check", "make a systematic list", "work backward", "look for a pattern", "simplify the problem", "make a drawing", "reasoning", and "write an equation" strategies were chosen from different sources (Altun, 2013; Posamentier & Krulik, 2008, Van de Walle, 1990; Verhage & de Lange, 1997). Since the chosen problems were used in different former studies (e. g. Altun, Bintaş, Yazgan, & Arslan, 2004), no validation studies were needed. Nevertheless, to enhance the content validity and ensure the suitability of the problems for the target age group, expert reviews were obtained from three scholars specializing in mathematics education. In Table 2, possible strategies that could be used were presented for each problem.

Table 2

Possible Strategies For Each Problem.

Problem number	Make a systematic list	Guess and check	Make a drawing	Work backward	Write an equation	Reasoning	Simplify the problem	Look for a pattern
1	X							
2	X		X					
3							X	X
4			X				X	X
5	X	X						
6			X			X		
7				X	X			

Procedure and Scoring

Each student was interviewed individually by the second and third authors. Seven non-routine problems were asked each student. Each question was presented to the students in turn on separate papers. The researchers only intervened if the participants demanded it or wasted too much time on the wrong strategy. When the participant got stuck on a strategy or finished working on one problem, the interviewer asked some questions to probe her/his thinking and discover whether s/he had any flexibility. Some of them are "Are you sure that this answer is correct?", "How can you be sure that your answer is correct?", "Is there another solution or strategy for this problem?", "Could this problem be solved easier?", "Would you like to think more about the problem?". Each interview lasted approximately 30-90 minutes. After taking the consent of the participants, all interviews were video or audio-taped by the researchers. The participants were asked to think aloud during the problem solving process and to write down each step in the solution process.

Three criteria were used to determine the strategic flexibility of the participants: Choosing and using of the most appropriate strategy (C_1), switching to a different strategy when a strategy does not work in solving a problem or using multiple strategies together to solve a problem (intra-task flexibility-C_2), and changing the solution strategy when moving from one problem to another (inter-task flexibility-C_3). C_2 and C_3 were determined based on the work of Elia et al. (2009). However, C_1 was added as a separate criterion in the current study. While scoring the criteria, the strategies used in all questions were taken into consideration as a whole. In addition, the strategies used were examined and scored separately as 1,2,3, or 4 points for each criterion (see Table 3 for details) by the first and second authors independently. In this way, each participant had a flexibility score ranging from 4 to 12. For inter-coder agreement, the Cohens' kappa coefficient calculated based on the coding of the two authors was found to be .951, .965, and .957 for C_1, C_2, and C_3, respectively. These values indicate almost perfect concordance between the two coders.

Table 3

Scoring System For Each Criterion

Score	C_1	C_2	C_3
1	Use of appropriate strategy for only 1 problem	No intra-task flexibility	Use of 2 or less different strategies across problems
2	Use of appropriate strategies for 2-3 problems	Intra-task flexibility in one problem	Use of 3 different strategies across problems
3	Use of appropriate strategies for 4-5 problems	Intra-task flexibility in two problems	Use of 4 different strategies across problems
4	Use of appropriate strategies for 6-7 problems	Intra-task flexibility in three or more problems	Use of 5 or more different strategies across problems

Findings

The means calculated for C1, C2, C3, and the total scores are 3.78, 2.83, 3.89, and 10.5, respectively. These means, along with the data presented in Table 4, point to three important findings. First, the students participating in the study were successful in selecting and using the most appropriate strategy

during problem-solving and in changing strategies when transitioning between problems. This indicates that the students had developed strategic thinking and flexibility skills. Second, the students' intra-task flexibility levels were the lowest among the categories. In other words, students had difficulty using more than one strategy simultaneously or switching strategies during the process of solving a single problem. This suggests some limitations in their ability to apply flexible thinking instantly. Third, considering the highest possible score, the students' overall flexibility levels were above average. Although three students received relatively low scores of 8 and 9, the total flexibility scores of the remaining students were considerably high. When examined by grade level, sixth-grade students had the highest average score with 10.86, followed by seventh graders with 10.83 and eighth graders with an average of 9.6. Another noteworthy finding is that all students who received full points in the C2 category also received full points in the C1 and C3 categories. This indicates that some students demonstrated consistent success across all types of flexibility. Although it was not the main focus of the research, it is also worth noting that 117 out of 126 solutions provided by the students were correct, which points to a generally high level of problem-solving performance.

Table 4

The Criterion And Total Scores Of Each Student.

Student number	Grade	C_1	C_2	C_3	Total
1	8	3	3	3	9
2	8	3	3	4	10
3	8	4	3	4	11
4	8	4	2	4	10
5	8	3	2	3	8
6	7	4	4	4	12
7	7	4	2	4	10
8	7	4	3	4	11
9	7	3	3	4	10
10	7	4	4	4	12
11	7	4	2	4	10
12	6	4	4	4	12
13	6	4	4	4	12
14	6	4	1	4	9
15	6	4	4	4	12
16	6	4	3	4	11
17	6	4	2	4	10
18	6	4	2	4	10

At this point, examples of student answers that meet the C_1, C_2, and C_3 criteria will be presented to support the quantitative findings qualitatively.

Figure 1

Answer Given To Sixth Problem By Student 6

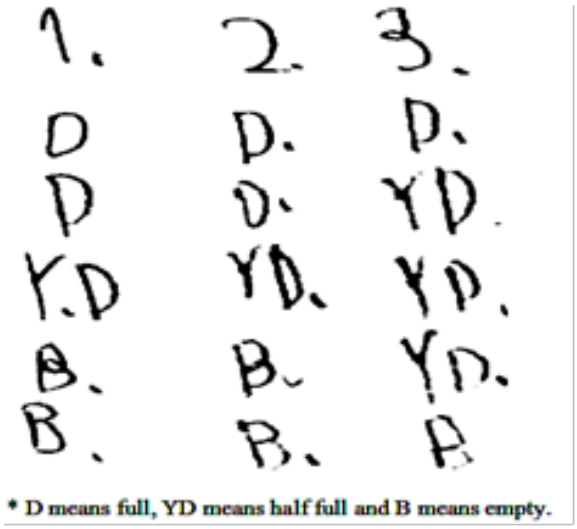
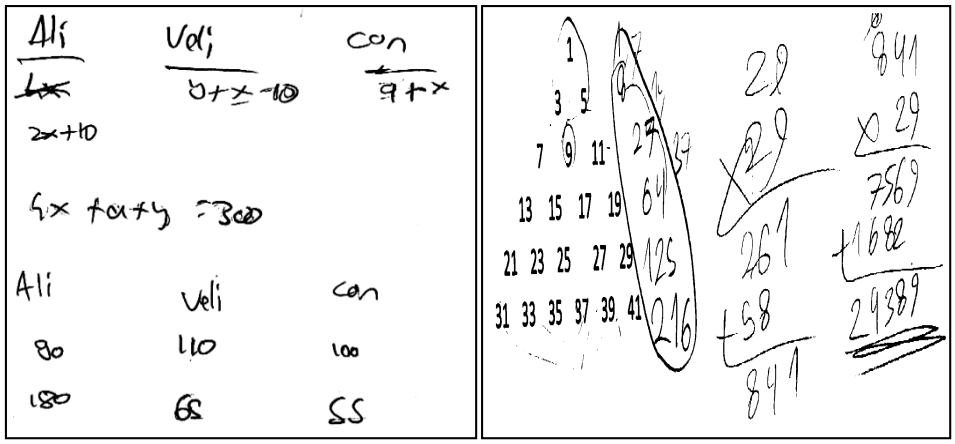


Figure 1 shows the answer given by student S6 to the sixth problem. This student selected the most appropriate and effective strategy to solve the problem and successfully reached the correct solution. In other words, the student exhibited behavior in line with the C1 criterion. Moreover, it is observed that the student applied the chosen strategy with determination and managed the problem-solving process carefully.

Figure 2

Answers Given To The Seventh And Third Problems By S2, Respectively



(a) (b)

C_2 had two components: switching to another strategy when the chosen strategy did not work or using multiple strategies while solving a problem. For the first component, S2's answer to the seventh problem can be a good example. As can be seen from the Figure 2a, S2 first tried to solve this problem by "write an equation", but when he realized that it was difficult; he resorted to the "work backward" strategy. S13 used a combination of "simplify the problem" and "look for a pattern" strategies to solve the third problem (Figure 2b). So, she has provided a good example of the second component of C_2.

Figure 3

Answers Given By S15 To The Second And Fifth Problems

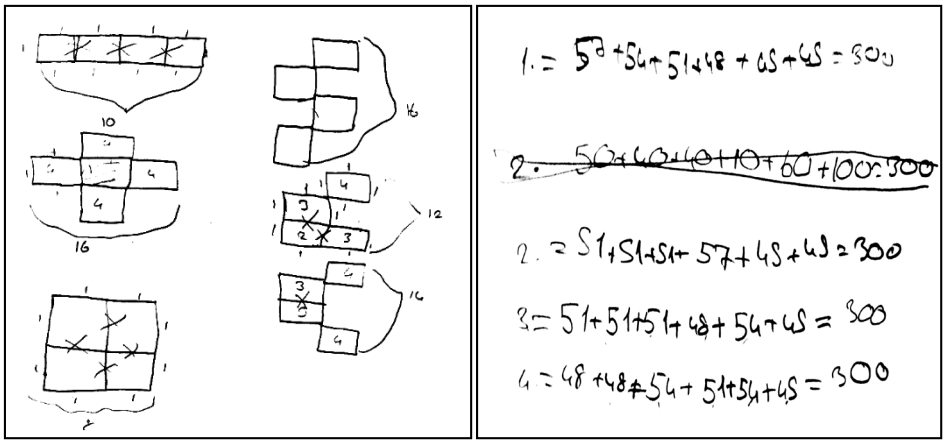


Figure 3 shows S15's answers to the second and fifth problems. These students used the "make a drawing" strategy in her first solution and "guess and check" strategy in the other. Considering the answers she gave to all the problems together, it was observed that she benefited from seven different strategies in different questions. This situation shows that she fully meets the C_3 criterion.

Discussion

Problem solving requires creative ability to explore new ways to solve problems, as well as a flexible perspective and openness to different ideas (Kitchener, 2011). Non-routine problem solving may be an excellent way to develop this attitude. In this context, the main focus of this study is strategic flexibility exhibited by gifted middle-schoolers during non-routine problem solving.

Given that almost one-third of our participants achieved maximum score, the most conspicuous finding of the present study is the high strategic flexibility exhibited by gifted students. This finding is consistent with the results found by Arslan and Yazgan (2015) and Keleş and Yazgan (2021). It seems that students have experience with non-routine problems in SAC. The participants were particularly successful in the C_1 and C_3 criteria, as their strategy repertoire is rich. Additionally, the number, structures, and contexts of problems used in this study were quite sufficient to reveal strategic flexibility of the students. However, in accordance with the findings of Elia et al. (2009), and Arslan and Yazgan (2015), the participants of this study did not tend to try a different strategy because they were very sure of the strategy they used when solving the problems. This situation may be an explanation for the lowness of the intra-task flexibility (C_2). For example, only one student got 1 point for intra-task flexibility, and he was the only one who didn't hesitate over his strategy and came up with a solution for all questions in his first trial. It is also interesting that the students who are successful at intra-task flexibility are also successful at choosing a proper strategy for different problems. The conclusion drawn by Yazgan and Keleş (2022), indicating that the ability of students to select the most appropriate strategy on their first attempt serves as a strong indicator of strategic flexibility, aligns closely with the findings of the present study.

Another remarkable finding is that as the grade level increases, the level of strategic flexibility decreases. Keleş and Yazgan (2021) also reached similar findings. The reason for this may be that as the grade level rises, students' ability to think flexibly changes inversely with the time they spend in formal education. For example, especially towards the end of middle school, more emphasis is placed on algebra and thus on establishing equations. For this reason, even gifted students tend to use the write an equation strategy more than others. Besides, after completing the eighth grade, students in Turkey take a national high school entrance exam. In order to be successful in this exam, students need to memorize ready-made formulas and algorithms. We think that this situation also reduces their strategic

flexibility. Recent studies have also highlighted points similar to the findings obtained in this study. For instance, Kramarski and Michalsky (2015) stated that exam-focused educational environments impose constraints on students' problem-solving strategies, which in turn foster a negative attitude toward learning. Additionally, Mok et al. (2020) indicated that standardized national exams adversely affect students' problem-solving processes, leading them to rely excessively on certain fixed strategies.

The findings of this study may have different reflections on the mathematics education of the gifted. Although the students participating in this study showed good levels of strategic flexibility, they were weaker in terms of intra-task flexibility. Even if a problem has been solved, opening up to class discussion about whether it can be solved in a different way can make students more aware of different strategies. In this way, their repertoire of strategies grows, and so do they tend to use more than one strategy together or change their chosen strategy when it doesn't work. Teachers should create such learning environments and encourage students to change their perspectives when stuck on a problem. To enable teachers and even prospective teachers to achieve such awareness, strategic flexibility needs to be emphasized in both in-service and pre-service training. The notion that teachers ought to possess strategic flexibility and effectively communicate this competence to their students is consistent with the teacher attributes emphasized by Karabulut (2019).

Of course, there are limitations to this study, and in connection with this, there are some suggestions that can be made for the future studies. We carried out this study only with gifted middle school students. Similar studies with students of different grades and abilities may yield more firm and generalizable results. In the current study, students were not exposed to an instruction focused on strategic flexibility. In future studies, such an education can be given, and its effect on students' strategic flexibility can be examined more deeply. We did not have the chance to follow the same students for several years. Through a longitudinal study, a more detailed understanding of the development of strategic flexibility can be obtained. In addition, the number of our participants was limited to 18 students. Another study involving more students could show how verifiable the results obtained in this study were. This study did not focus on the accuracy of participants' solutions. However, about 93 percent of the answers were correct. Considering the high level of strategic flexibility of the participants, whether there is a relationship between the level of strategic flexibility and the accuracy of solutions can be an exciting research topic.

Conclusion

In conclusion, this research aimed to identify the strategy flexibilities of gifted middle school students in non-routine problems. Based on the analyses presented above, it can be concluded that gifted students can choose and apply the right strategy in solving non-routine problems. However, it cannot be said that they successfully changed strategies or used more than one strategy together in a question. This may be because they think they do not need to change or use a new strategy because they use it correctly. Actually, according to Threlfall and Hargreaves (2008), gifted students use different strategies and have better flexible strategic thinking during problem solving. Despite this, the fact that even gifted individuals are not very successful in flexibility is a point that needs to be emphasized. This suggests that more progress is needed on non-routine problems, solution strategies and flexibility in classroom settings. In addition, the decrease in strategic flexibility with the increase in grade levels suggests that flexibility ability is negatively affected by the duration of exposure to formal education. This issue requires educators, administrators, and academicians, to conduct in-depth studies of education systems and processes by making self-criticism. On the other hand, including non-routine problems in textbooks, introducing different strategies to students, and encouraging their use can also increase strategic flexibility. As Hong et al. (2023) point out, strategic flexibility is not limited to students' strategy knowledge, but can also be influenced by children's cognitive capacities, motivation and contextual characteristics. Without neglecting the idea that strategic flexibility can be influenced by

many factors in learning environments, it may be a good idea for future research to create and examine environments that contribute to the development of flexibility.

Ethics Declaration

This research was conducted in accordance with the articles in the Higher Education Institutions Scientific Research and Publication Ethics Directive. We declare that no ethical violations have been made.

Conflict of Interest Statement

We declare that this study was published without any academic or financial conflict of interest.

References

Assmus, J., Ertküçük, A., & Grigutsch, S. (2022). Gifted students' approaches to solving non-routine mathematical problems: Insights from a process-oriented perspective. *International Journal of Science and Mathematics Education, 20*(4), 789–812. https://doi.org/10.1007/s10763-021-10189-9

Altun M. (2016). *Ortaokullarda (5, 6, 7 ve 8. sınıflarda) matematik öğretimi*. Bursa: Aktüel Yayıncılık.

Altun, M., Bintas, J., Yazgan, Y. & Arslan, C. (2004). *İlköğretim çağındaki çocuklarda problem çözme gelişiminin incelenmesi.* Uludağ Üniversitesi Bilimsel Araştırma Projeleri Birimi. https://www.researchgate.net/profile/Yeliz-Yazgan-2/publication/305769065_ILKOGRETIM_CAGINDAKI_COCUKLARDA_PROBLEM_COZME_GELISIMININ_INCELENMESI/links/57a0602a08aec29aed23755e/ILKOGRETIM-CAGINDAKI-COCUKLARDA-PROBLEM-COeZME-GELISIMININ-INCELENMESI.pdf

Arslan, C., & Yazgan, Y. (2015). Common and flexible use of mathematical non routine problem solving strategies. *American Journal of Educational Research, 3(*12), 1519-1523. doi:10.12691/education-3-12-6.

Bräuning, K. (2016). *Investigating multiple strategy use and strategy flexibility in problem solving by pre-service teachers.* In T. Fritzlar, D. Aßmus, K. Bräuning, A. Kuzle, & B. Rott (Eds.), Problem solving in mathematics education. Proceedings of the 2015 joint conference of ProMath and the GDM working group on problem solving (pp. 41-55). Münster: WTM.

Boesen, J., Lithner, J., & Palm, T. (2023). Students' creative reasoning and mathematical problem solving: New insights from classroom settings. *Educational Studies in Mathematics, 112*(1), 45–67. https://doi.org/10.1007/s10649-022-10154-4

Boaler, J. (2022). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. Jossey-Bass.

Bulgar, S. (2008). Enabling more students to achieve mathematical success; a case study of Sarah. In B. Sriraman (Ed.), *Creativity, giftedness, and talent development in mathematics* (133-154). Charlotte, NC: Age Publishing Inc.

Er, F. S. K., Selek, H. K. G., & Yazgan, Y. (2024). Beşinci Sınıf Öğrencilerinin Problem Çözmede Stratejik Esnekliklerinin Matematiğin Doğasına İlişkin Görüşleri Çerçevesinde İncelenmesi. *Dokuz Eylül Üniversitesi Buca Eğitim Fakültesi Dergisi*, (59), 98-119.

Elia, I., Van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in nonroutine problem solving by primary school high achievers in mathematics. *Zentralblatt Didaktik für Mathematik (ZDM), 41*(5), 605-618. doi:10.1007/s11858-009-0184-6

Gavaz, H. O., Yazgan, Y., & Arslan, Y. (2021). Non-routine problem solving and strategy flexibility: A quasi-experimental study. *Journal of Pedagogical Research, 5*(3), 40-54. doi:10.33902/JPR.2021370581.

Greenes, C. (1981). Identifying the gifted student in mathematics. *Arithmetic Teacher, 28*(6), 14–17. doi:10.5951/AT.28.6.0014

Hatano, G. (2003). Foreword. In A.J. Baroody & A. Dowker (Eds), *The development of arithmetic concepts and skills* (pp. xi-xiii). New Jersey, NJ: Lawrence Erlbaum Associates.

Herr, T., & Johnson, K. (2002). *Problem-solving strategies: Crossing the river with dogs*. Emeryville: Key Curriculum Press.

Hong, W., Star, J. R., Liu, R., Jiang, R. & Fu, X. (2023). A Systematic Review of Mathematical Flexibility: Concepts, Measurements, and Related Research. *Educational Psychology Review.* 35:104 https://doi.org/10.1007/s10648-023-09825-2.

Karabulut, T. (2019). *Altıncı sınıf öğrencilerinin matematiksel problem çözmedeki stratejik esneklikleri ve bu konuyla ilgili öğretmen görüşleri.*(Yüksek Lisans Tezi). Bursa Uludağ Üniversitesi.

Keleş, T., & Yazgan, Y. (2021). Gifted eighth, ninth, tenth and eleventh graders' strategic flexibility in non-routine problem solving. *The Journal of Educational Research*, 1-14. doi:10.1080/00220671.2021.1937913

Kılıç, H., & Uçar, Z. (2023). Secondary school students' flexibility in mathematical problem solving: An analysis of strategies. *International Journal of Educational Research*, 118, 102106. https://doi.org/10.1016/j.ijer.2023.102106

Kitchener, R.F (2011). Personal epistemology and philosophical epistemology: The view of a philosopher. In J. Elen, E. Stahl, R. Bromme, & G. Clarebout, (Eds.), *Links between beliefs and cognitive flexibility: Lessons learned* (pp. 79– 103). Rotterdam: Springer

Kolovou, A., van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks – a needle in a haystack. *Mediterranean Journal for Research in Mathematics Education 8*(2), 31-68.

Kramarski, B., & Michalsky, T. (2015). Enhancing mathematical reasoning and metacognitive awareness through computerized collaborative learning. *Learning and Instruction*, 39, 1-11. https://doi.org/10.1016/j.learninstruc.2015.05.002

Krems, J. F. (2014). Cognitive flexibility and complex problem solving. In P.A. Frensch and J. Funke (ed.), *Complex problem solving the European perspective* (pp. 206-223). New York, NY: Psychology Press.

Lee, N. H., Yeo, J. S. D., & Hong, S. E. (2014). A metacognitive based instruction for primary four students to approach non-routine mathematical word problems. *ZDM, 46* (3), 465-480. doi:10.1007/s11858-014-0599-6

Leikin, R., Koichu, B., & Berman, A. (2009). Mathematical giftedness as a quality of problem-solving acts. In R. Leikin, A. Berman and B. Koichu, (Eds.*), Creativity in mathematics and the education of gifted students* (115-127). Rotterdam: Sense Publishers.

Leikin, R. (2020). Advanced mathematical thinking and creativity: A look at high achieving learners. *ZDM–Mathematics Education*, 52(1), 3–13. https://doi.org/10.1007/s11858-019-01088-5

Leikin, R., & Levav-Waynberg, A. (2020). Characteristics of gifted students' mathematical creativity: Connections to flexibility and multiple solution strategies. *ZDM–Mathematics Education, 52*(1), 25–37. https://doi.org/10.1007/s11858-019-01107-5

Maker, J. (2003). New directions in enrichment and acceleration. In N. Colangelo & G. Davis (Eds.). *Handbook of gifted education* (pp. 163-173). Boston, MA: Allyn and Bacon.

McMillian, J. H. (2004). *Educational research: Fundamentals for the consumer*. New York, NY: HarperCollins College Publishers.

Ministry of National Education (2019). *Bilim ve sanat merkezi matematik dersi öğretim programı etkinlik kitabı.* Ankara: Milli Eğitim Bakanlığı. Özel Eğitim ve Rehberlik Hizmetleri Genel Müdürlüğü.

Mok, I. A. C., & Lam, S. F., Cheng, M. M. H., & Ho, S. K. W. (2020). Standardized testing and creativity: The impact of test preparation on students' problem-solving skills. *Journal of Educational Psychology*, 112(7), 1349–1360. https://doi.org/10.1037/edu0000421

NCTM (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

NCTM(2010). Why is teaching with problem solving important to student learning? Reston, VA: NCTM

NCTM. (2023). *Catalyzing change in early childhood and elementary mathematics: Initiating critical conversations*. National Council of Teachers of Mathematics.

Niederer, K., Irwin, R. J., Irwin, K. C., & Reilly, I. L. (2003). Identification of mathematically gifted children in New Zealand, *High Ability Studies, 14*(1), 71-84. doi:10.1080/13598130304088

Patton, M. Q. (1987). *How to use qualitative methods in evaluation*. Newsbury Park, London, New Dehli: Sage Publications.

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice (4. ed.),* Thousand Oaks, CA: Sage Publications.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Posamentier, A. S. & Krulik, S. (2008*). Problem solving strategies for efficient and elegant solutions, grades 6-12: A resource for the mathematics teacher*. USA: Corwin Press.

Schoenfeld, A. (1985). *Mathematical problem solving*. USA: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (2021). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. *Educational Psychologist*, 56(4), 261–278. https://doi.org/10.1080/00461520.2021.1902311

Selter, C. (2001). Addition and subtraction of three-digit numbers: German elementary children's success, methods and strategies. *Educational Studies in Mathematics, 47*(2), 145-173. doi:10.1023/a:1014521221809

Sheffield, L. J. (2003). *Extending the challenge in mathematics: Developing mathematical promise in K-8 students*. USA: Corwin Press.

Sriraman, B. (2003). Mathematical giftedness, problem solving, and the ability to formulate generalizations: The problem-solving experiences of four gifted students. *Journal of Secondary Gifted Education, 14*(3), 151-165. doi:10.4219/jsge-2003-425.

Star, J. (2018). Flexibility in mathematical problem solving: The state of the field. In F. J. Hsieh, (Ed.) *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*: Vol. 1 (pp. 15-25). EARCOME8-National University of Taiwan.

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction, 18*(6), 565-579. doi:10.1016/j.learninstruc.2007.09.018

Star, J. R., & Rittle-Johnson, B. (2022). Promoting flexibility in mathematics: A review of instructional approaches and practices. *Journal for Research in Mathematics Education*, 53(2), 123–148. https://doi.org/10.5951/jresematheduc-2022-0010

Threlfall, J., (2009). Strategies and flexibility in mental calculation. *ZDM, 41*(5), 541-555. doi:10.1007/s11858-009-0195-3

Threlfall, J., & Hargreaves, M. (2008). The problem-solving methods of mathematicallygifted and older average-attaining students. High Ability Studies, 19,83–98. doi:10.1080/13598130801990967

Tiong, J. Y. S., Hedberg, J. G., & Lioe, L. T. (2005). A metacognitive approach to support heuristic solution of mathematical problems. *Proceedings of the Redesigning Pedagogy: Research, Policy, Practice Conference*. Singapore National Institute of Education. https://repository.nie.edu.sg/bitstream/10497/141/1/2005a15.pdf

Torbeyns, J., Schneider, M., Xin, Z., & Siegler, R. S. (2021). Bridging the gap between numerical understanding and strategic flexibility: Recent insights. *Cognitive Development, 59*, 101063. https://doi.org/10.1016/j.cogdev.2021.101063

OECD. (2023). *OECD Future of Education and Skills 2030: Curriculum analysis*. https://www.oecd.org/education/2030-project/

OECD. (2023). *Mathematics curriculum redesign: Toward 21st century learning*. OECD Publishing. https://www.oecd.org/education

Van de Walle, J. A. (1990). *Elementary school mathematics, teaching developmentally*. Newyork, NY: Addison-Wesley/Longman.

Verhage, H., & de Lange, J. (1997). Mathematics education and assessment. *Pythagoras, 42*, 14-20.

Verschaffel, L. (2023). Strategy flexibility in mathematics. *ZDM – Mathematics Education* https://doi.org/10.1007/s11858-023-01491-6.

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education, 24*(3), 335–359. doi:10.1007/bf03174765.

Vural, E. (2019). *İlkokul 4. sınıf düzeyinde doğal sayılarla ilgili rutin ve rutin olmayan problemlerin öğrenim ve öğretim durumları* (Master's thesis, Lisansüstü Eğitim Enstitüsü).

World Economic Forum. (2024). *Future of Jobs Report 2024*. https://www.weforum.org/reports/future-of-jobs-report-2024

Woodward J, Beckmann S., Driscoll M., Franke M., Herzig P., Jitendra A., Koedinger, K.R., & Ogbuehi P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide*. National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/MPS_PG_043012.pdf

Yeo, K. K. J. (2009). Secondary 2 students' difficulties in solving non-routine problems. *International Journal for Mathematics Teaching and Learning, 10*, 1–30.

Zhou, Z., Sun, Y., & Wang, S. (2021). Strategy use and metacognitive monitoring in mathematical problem solving among high-achieving students. *Journal of Educational Psychology, 113*(8), 1471–1487. https://doi.org/10.1037/edu0000674

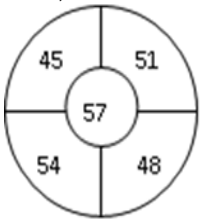
Appendix

1) "A rectangle has an area of 120 sq. cm. Its length and width are whole numbers. What are the possibilities for the two numbers? Which possibility gives the smallest perimeter?"
2) "Four squares are arranged so that each square touches at least one other square. Any two squares touch each other according to these rules: they can touch on the corners, or they can touch entirely across one side but not partially across aside. What are the possible perimeters?"
3) "What is the sum of the numbers in the 29th row of the following array?"

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
31 33 35 37 39 41

4) "In the picture, you see a house of cards. 18 cards were needed to make this house. How many cards are needed for a ten storey house of cards?"

5) "a) At least how many shots are needed on this dartboard to score 300 points?
b) Find four different combinations where the sum is 300."



6) "Three men stranded in the center of the desert have 15 canteens, all the same size. Five are full of water, five are exactly half full, and five are empty. Each man plans to take a different route out of the desert. How can they share the water and the canteens equally so that if they come to an oasis, they will have an equal ability to take on more water?"
7) "Ali, Veli, and Can gain 300 Turkish liras in total after working at a job together. They do not have the same amount of money, so they decide to share it fairly. First, Ali gives half of his money to Veli and Can equally. And then Veli gives 10 Turkish liras to Ali. Now each of them has the same amount of money. Find the amount of money each boy has at the beginning."