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Research Article / Araştırma Makalesi

On the Improved of New Growth Model Namely Korkmaz Model and its Comparison with Some Growth Models

Yeni Büyüme Modeli olan Korkmaz Modelinin Geliştirilmesi ve Bazı Büyüme Modelleri ile Karşılaştırılması

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Abstract

For growth models, in addition to some classical growth models, a new model, Korkmaz model was derived before. In this study, this new Korkmaz model was improved. It is known that growth model was derived by using the following expression: "Growth models has generally sigmoidal shape. In this shape, there is one inflection point. Until this inflection point, the graph is convex. At this inflection point the growth rate reaches maximum value. After this inflection point the graph is concave." Growth models were generally derived by using the last part of this expression. Although Korkmaz model used the entire part up to the last part, it ignored this last part. In this study, Korkmaz model has been improved to take this last part into account as well as using all the other parts. After introducing this improved Korkmaz model, this new model was applied to one data set. In addition to improved Korkmaz model, the growth models such as Logistic, Brody, Gompertz, Von Bertalanffy and Korkmaz model were used. They are compared by using error sum of squares criterion.

Keywords: Growth models, Korkmaz model, Improved Korkmaz model.

Öz

Büyüme modelleri için bazı klasik büyüme modellerine ek olarak daha önce Korkmaz modeli türetilmişti. Bu çalışmada bu yeni Korkmaz modeli iyileştirilmiştir. Büyüme modelinin aşağıdaki ifade kullanılarak türetildiği bilinmektedir: "Büyüme modelleri genellikle sigmoidal bir şekle sahiptir. Bu şeklin bir dönüm noktası vardır. Bu dönüm noktasına kadar grafik konvekstir. Bu dönüm noktasında büyüme hızı maksimum değere ulaşır. Bu dönüm noktasından sonra grafik konkavdır." Büyüme modelleri genellikle bu ifadenin son kısmı kullanılarak türetilmiştir. Korkmaz modeli bütün parçanın tamamını son kısma kadar kullanmış olmasına rağmen son kısmı göz ardı etmiştir. Bu çalışmada Korkmaz modeli bu son kısmı da hesaba katacak ve diğer tüm kısımları da içerecek şekilde iyileştirilmiştir. Geliştirilen Korkmaz modeli tanıtıldıktan sonra, bu yeni model bir veri setine uygulandı. Geliştirilmiş Korkmaz modelinin yanı sıra Lojistic, Brody, Gompertz, Von Bertalanffy ve Korkmaz modeli gibi büyüme modelleri de kullanılmıştır. Hata kareler toplamı kriteri kullanılarak karşılaştırılırlar.

Anahtar Kelimeler: Büyüme modelleri, Korkmaz modeli, Geliştirilen Korkmaz modeli.

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1. Introduction

Growth modeling has been extensively investigated through various mathematical equations, such as those proposed by Brody (1945), Gompertz (1825), Ricker (1979), and von Bertalanffy (1957). The growth curve is characterized by the expanding relationship between an individual's weight and height and time (Fabens, 1965). Growth curves relate the interrelationships between the growth and maturation of an individual's genetic endowment and the environment in which the individual grows (Fitzhugh, 1976). One of the main advantages of using a mathematical equation or model to describe growth is that it combines the information contained in weight or height data into three or four biologically interpretable parameters. These parameters can then be compared across populations and groups used (Goonewardene et al., 1981). Model parameters are estimated by minimizing the differences between observed and predicted values, and these parameters are subsequently used for comparisons across populations. Although they are regarded as biologically interpretable, such parameters do not always have direct biological meaning.

A classic growth curve has sigmoidal shape. This curve can be divided into two phases. One of these phases is the self-acceleration phase, while the other is self-braking phase. The meeting point of these two phases is the infection point.

Some nonlinear models such as monomolecular or Brody model (Brody, 1945) is of the decaying exponential type with no inflection point. For these models, the peak growth rate occurs at birth and decreases continually. The curve of logistic model (Ricker, 1979) is symmetrical around its inflection point. Growth rate increases with age and weight or length until the inflection point, where it reaches maximum value. Then the growth rate decreases until the growth reaches its maximum value. The curve of Gompertz model has an inflection point earlier than a curve of logistic models. In addition, the Gompertz curve is asymmetrical about its inflection point (Bethard, 1997).

The growth models such as monomolecular, logistic, Gompertz, von Bertalanffy models all assume growth is a continuous process resulting in a smooth shaped growth curve. Fitting a growth curve to, for example, the weight of a group of individuals assumes that the shape of the actual growth curve is similar to the fitted curve. With this assumption, the parameters of the growth models used are estimated from field data. Curve fitting consists of estimating parameters in a way that minimizes the differences between observed and predicted values. A situation where the residuals are small would be a “good fit.” Graphical analysis helps in choosing an appropriate growth curve (Bethard, 1997). Graphical analysis has also been emphasized as a useful tool for determining which model provides the best fit for growth data, with the quality of fit assessed by the magnitude of residuals.

In addition to these classical growth models, a new model known as Korkmaz model had previously been derived, and in the present study this model has been improved. The derivation of growth models has typically relied on the following principle: Growth models generally have a sigmoidal shape. In this shape, there is one inflection point. Until this point, the graph is convex. At the infection point, the growth rate reaches its maximum value. After this point, the graph is concave. While most growth models have been derived using only the last part of this principle, the original Korkmaz model was based on all parts of the expression except the final one. In this study, Korkmaz model was improved by incorporating the final part as well. After the improved model was introduced, it was applied to a data set. For comparison, Logistic, Brody, Gompertz, von Bertalanffy, and original Korkmaz models were also fitted, and all models were evaluated using the error sum of squares criterion.

1.1. Some growth models with growth rate

The derivation of Brody model (Brody, 1945) is as follows: This model can be derived from the following differential equation 1.

$$\frac{dy}{dt} = k(A - y) \tag{1}$$

After solving equation 1, equation 2 is found.

$$y = A(1 - be^{-kt}) \tag{2}$$

where y is growth value, A is the maximum growth value, b and k are the growth parameters, t is time

Now the derivation of Logistic model (Ricker, 1979) is as follows: This model can be derived from the following differential equation 3.

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{A}\right) \tag{3}$$

After solving equation 3, equation 4 is found.

$$y = \frac{A}{1+be^{-kt}} \tag{4}$$

Now the derivation of Gompertz model (Gompertz, 1825) is as follows: This model can be derived from the following differential equation 5.

$$\frac{dy}{dt} = kyln\left(\frac{A}{y}\right) \tag{5}$$

After solving equation 5, equation 6 is found.

$$y = Ae^{-e^{b-kt}} \tag{6}$$

Detailed solutions of the differential equations of the three growth models given above and the derivation of the relevant growth models is presented in detail in the article by Gül and Korkmaz (2021).

The monomolecular or Brody model (Brody, 1945) has no inflection point. For this model, the highest growth rate occurs at birth and decreases continually. As an animal gets older, growth value becomes closer to A, but never passes A. As age and growth value increase, growth rate declines linearly. The curve of logistic model is symmetrical around its inflection point. Growth rate increases with age and y until the inflection point, where it decreases until maximum age is reached. Gompertz curve has an inflection point earlier than a logistic curve (Bethard, 1997).

Growth models consist of parameters, most of which have no direct meaning. Since these parameters do not have direct meaning, it is difficult to determine the initial value required for estimation. In this study, it is aimed to use parameters that have direct meaning. Thus, the initial values and final values of the parameters gain meaning. Thus, all parameters of the model mean something. Giving the model with the meaning of all parameters represents a different stance in growth models. Thus, the model can be easier to interpret. The model is presented clearly. For this purpose, the new model, Korkmaz model was first presented in another study (Korkmaz, 2018):

The derivation of this new model (Korkmaz model) is as follows: Since the growth models are generally sigmoidal, the graph is convex until inflection point and then after this point the graph is concave (see Figure 1).

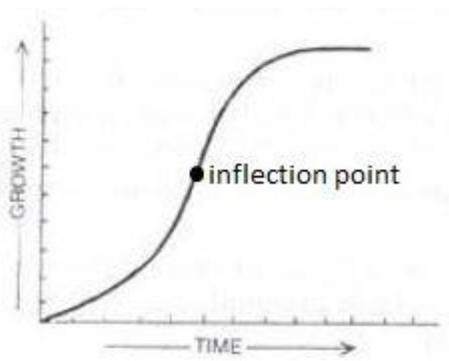


Figure 1. A sigmoidal Shape

For that reason, this new model was derived from the following differential equation 7 with initial conditions:

$$\frac{d^2y}{dt^2} = k(B - y) \quad y(t_i) = B \quad y^{u}(t_i) = 0 \tag{7}$$

where y is the growth value with respect to time, t is time, B is the growth value at inflection point, k is a constant of convexity (k>0), t_i is the time of inflection point.

After solving the differential equation, the general solution of Korkmaz model in equation 8 was found in (Korkmaz, 2018).

$$y = c_1 cos\sqrt{k}t + c_2 sin\sqrt{k}t + B \quad (8)$$

where c_1 and c_2 are the constants of the models.

As can be seen, there are no t_m , the maximum reached time and A, the maximum value reached at t_m values in Korkmaz model. Generally, there is a maximum value reached (A) in growth models. This maximum value was also wanted to be added to Korkmaz model. Because it is thought that Korkmaz model will be stronger by eliminating the deficiency of the maximum value in Korkmaz model.

Now for modifying Korkmaz model firstly, the general solution of Korkmaz model in equation 8 in (Korkmaz, 2018) was used. In this model, there are four parameters, c_1, c_2, k and B.

By using the following initial conditions (9) in equation 8,

$$y(t_m) = A$$
$$y^{\iota}(t_m) = 0 \quad (9)$$

where t_m is the maximum reached time , A is the maximum value reached at t_m
we can get the following equation (10):

$$y^{\iota}(t_m) = \sqrt{k}(c_2 \cos(\sqrt{k}t_m) - c_1 \sin(\sqrt{k}t_m)) \quad (10)$$

By solving $y(t_m) = A$ for c_1, we get

$$c_1 = \frac{-c_2 \sin(\sqrt{k}t_m) + A - B}{\cos(\sqrt{k}t_m)} \quad (11)$$

After substituting equation 11 into equation 10, by solving $y^{\iota}(t_m) = 0$ for c_2 , we get the following equation (12):

$$c_2 = \sin(\sqrt{k}t_m) * (A - B) \quad (12)$$

After putting equation 11 and equation 12 in equation 8, we get the improved Korkmaz model in equation 13.

$$y(t) = \cos\left(\sqrt{k}(t_m - t)\right) * (A - B) + B \quad (13)$$

By using the following initial conditions (14) in equation 8,

$$y(t_i) = B$$
$$y^{\iota\iota}(t_i) = 0 \quad (14)$$

the general solution of Korkmaz model was given in equation 15 in (Korkmaz, 2018).

$$y = c_1[cos\sqrt{k}t - cot\sqrt{k}t_i sin\sqrt{k}t] + B \quad (15)$$

Now for modifying the Korkmaz model secondly, the general solution of Korkmaz model in equation 15 in (Korkmaz, 2018) was used. In this model, there are four parameters, c_1, k, B and t_i.

By using the initial conditions (9) again in equation 15, firstly, we can write the following equations (16 and 17):

$$y(t_m) = c_1[cos\sqrt{k}t_m - cot\sqrt{k}t_i sin\sqrt{k}t_m] + B \quad (16)$$

$$y^{\iota}(t_m) = -c_1\sqrt{k}(\sin(\sqrt{k}t_m) + \cot(\sqrt{k}t_i) * \cos(\sqrt{k}t_m)) \quad (17)$$

By solving $y^{\iota}(t_m) = 0$ for t_m , we get equation 18:

$$t_m = -\frac{\frac{\pi}{2}-\sqrt{k}t_i}{\sqrt{k}} \quad (18)$$

After putting equation 18 in equation 16 for getting $y(t_m)$, we get:

$$y(t_m) \coloneqq c1\left(\sin(\sqrt{k}t_i) + \cot(\sqrt{k}t_i) * \cos(\sqrt{k}t_i)\right) + B$$

By solving $y(t_m) = A$ we can get c1 parameters like below.

$$c1 = (A - B) \sin(\sqrt{k}t_i) \quad (19)$$

By putting equation 19 in equation 15, we get improved Korkmaz model in equation 20.

$$y(t) = \sin\left(\sqrt{k}(t_i - t)\right) * (A - B) + B \quad (20)$$

In [9] after saying C for initial growth value when t=0, $y(0) = c_1 + B = C$

$c_1 = C - B$

Korkmaz model with the meaningful parameters in equation 21 in (Korkmaz, 2018) was found.

$$y = (C - B)[cos\sqrt{k}t - cot\sqrt{k}t_i sin\sqrt{k}t] + B \quad (21)$$

By using the initial conditions (9) again in equation 21, we can write the following equations (22 and 23):

$$y(t_m) = (C - B)[cos\sqrt{k}t_m - cot\sqrt{k}t_i sin\sqrt{k}t_m] + B \quad (22)$$

$$y^{\iota}(t_m) = -(C - B)\sqrt{k}(\sin(\sqrt{k}t_m) + \cot(\sqrt{k}t_i) * \cos(\sqrt{k}t_m)) \quad (23)$$

By solving $y^{\iota}(t_m) = 0$ for t_m , we get the following equation (24):

$$t_m = -\frac{\frac{\pi}{2}-\sqrt{k}t_i}{\sqrt{k}} \quad (24)$$

After putting equation 24 in equation 22 for getting $y(t_m)$, we get the following equation 25:

$$y(t_m) \coloneqq (C - B)\left(\sin(\sqrt{k}t_i) + \cot(\sqrt{k}t_i) * \cos(\sqrt{k}t_i)\right) + B \quad (25)$$

By solving $y(t_m) = A$, we can get t_i parameters like below in equation 26.

$$t_i = \frac{-\arcsin\left(\frac{B-C}{A-B}\right)}{\sqrt{k}} \quad (26)$$

By putting equation 26 in equation 21, we get improved Korkmaz model in equation 27.

$$y(t) = (C - B)\cos(\sqrt{k}t) - \sqrt{(A - C)(A - 2B + C)}sin(\sqrt{k}t) + B \quad (27)$$

2. Materials and Methods

2.1. Material

In this study, one set of data taken from the specific growth rate of *Pseudomonas putida* (NICM 2174) at various time intervals was used (Table 1). The set of data was taken from the study of Annadurai and et all. (2000).

Table 1. Specific growth rate of *Pseudomonas putida* (NICM 2174) at various time intervals

Time (hour)	1	2	3	4	5	6	7	8	9
ODCM	0.216	0.220	0.240	0.250	0.260	0.270	0.280	0.290	0.330
Time (hour)	10	11	12	13	14	15	16	17	18
ODCM	0.360	0.380	0.400	0.460	0.482	0.492	0.522	0.542	0.584
Time (hour)	19	20	21	22	23	24	25	26	27
ODCM	0.594	0.620	0.644	0.644	0.648	0.648	0.648	0.648	0.648

ODCM: Optical density of cell mass

2.2. Methods

In this study firstly, general forms of the improved Korkmaz model besides general forms of Korkmaz model and the other models used in this study were given in Table 2.

Table 2. The general form of the models used in this study

Models	y(t)
Improved Korkmaz	$(A-B)\cos\left(\sqrt{k}\,(t_m-t)\right)+B$ or $(A-B)\sin\left(\sqrt{k}\,(t_i-t)\right)+B$ or $(C-B)\cos(\sqrt{k}t)-\sqrt{(A-C)(A-2B+C)}\sin(\sqrt{k}t)+B$
Korkmaz	$c_1\cos(\sqrt{k}t)+c_2\sin(\sqrt{k}t)+B$ or $c_1[\cos(\sqrt{k}t)-\cot(\sqrt{k}t_i)\sin(\sqrt{k}t)]+B$ or $(C-B)[\cos(\sqrt{k}t)-\cot(\sqrt{k}t_i)\sin(\sqrt{k}t)]+B$
Logistic	$\frac{a}{1+e^{(b-ct)}}$
Brody	$a(1-be^{-ct})$
Gompertz	$ae^{(-e^{(b-ct)})}$
Von Bertalanffy	$a(1-be^{-ct})^3$

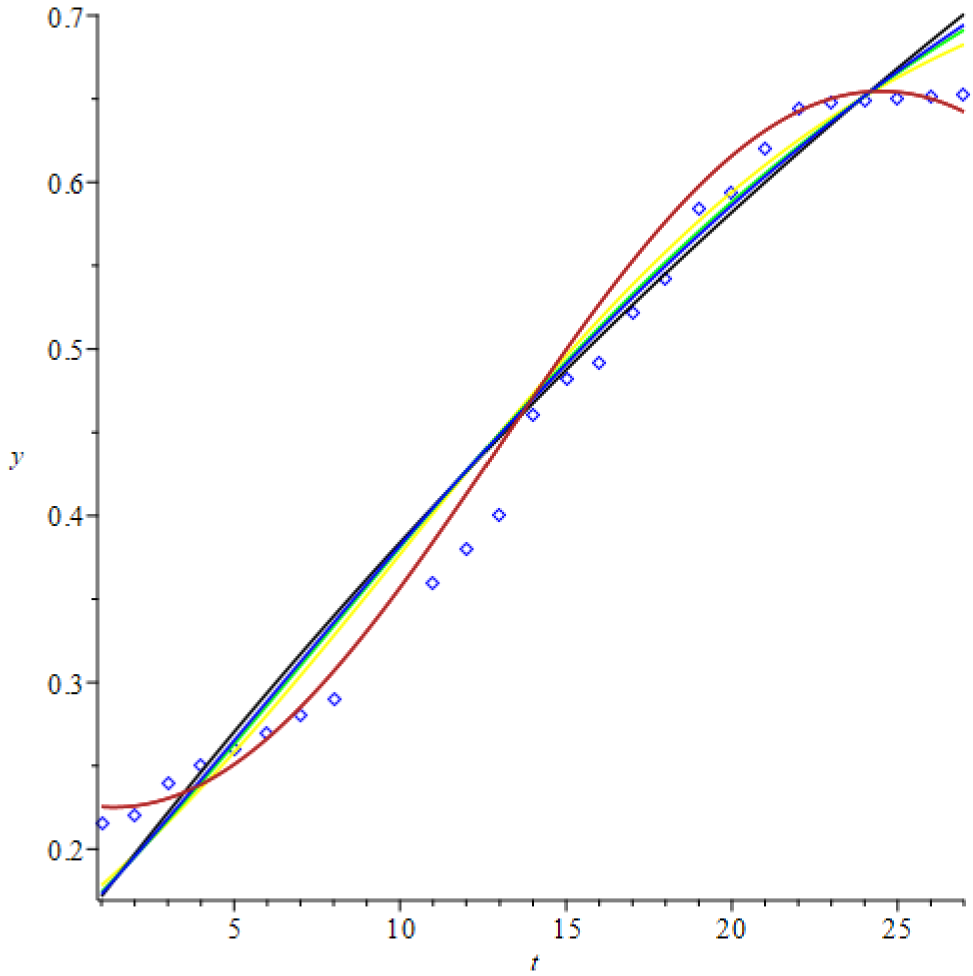
3. Results

The results of Korkmaz model and the other models used in this study were given by using data of Table 1. While Table 2 shows the general form of the models used in this study, Table 3 shows the results of these models by using Table 1. According to errors sum of squares criterion, in Table 3, both Korkmaz model and improved Korkmaz model are the best models with respect to the other models used in this study. The growth curves of improved Korkmaz model, Korkmaz model and the other classical models used in this study are seen in the same graph (Figure 2). As shown in Figure 2, all graphics are close to each other. But especially the graphs of Korkmaz model and Improved Korkmaz model are exactly coincide to each other. In fact, it has been observed that the error sum of squares is exactly the same.

Table 3. The results of the models used in this study according to the data of Table 1

Models	y(t)	SSE
Improved Korkmaz	$(A-B)\cos(\sqrt{k}(tm-t)+B$	0.001956
	$A=0.6546,\quad B=0.4399,\quad k=0.0184,$ $t_m=24.4930$	
	$(A-B)\sin(\sqrt{k}(ti-t)+B$	0.001956
	$A=0.6546,\quad B=0.4399,\quad k=0.0184,$ $t_i=12.9278$	
	$(C-B)[cos\sqrt{k}t-cot\sqrt{k}t_i sin\sqrt{k}t]+B$	0.001956
	$A=0.6546,\quad B=0.4399,\quad k=0.0184,$ $C=0.2288$	
Korkmaz	$(C-B)[cos\sqrt{k}t-cot\sqrt{k}t_i sin\sqrt{k}t]+B$	0.001956
	$C=0.2288,\quad B=0.4399,\quad k=0.0184,\quad t_i=12.9278$	
Logistic	$\frac{a}{1+e^{(b-ct)}}$	0.012249
	$a=0.7592,\quad b=1.3109,\quad c=0.1296$	
Gompertz	$ae^{(-e^{(b-ct)})}$	0.016788
	$a=0.8795,\quad b=0.5551,\quad c=0.0733$	
Von Bertalanffy	$y=a(1-be^{-ct})^3$	0.018551
	$a=0.9691,\quad b=0.4614,\quad c=0.0548$	
Broody	$a(1-be^{-ct})$	0.022361
	$a=1.5713,\quad b=0.9072,\quad c=0.0183$	

SSE: Error Sum of Squares



Color of Logistic, Broody, Korkmaz, Improved Korkmaz, Gompertz, Von Bertalanffy model: Yellow, Black, Red, Brown, Green and Blue, respectively.

Figure 2. The growth curves of all models used in this study according to the data of Table 1.

As known that the parameters of Korkmaz model and improved Korkmaz model are meaningful that's C is initial growth value when t=0, t_i is the time of inflection point, B is the growth value at inflection point, t_m is the maximum reached time, A is the maximum value reached at t_m and k is a constant of convexity (k>0). In Table 4, the original biologically meaningful parameter values of Korkmaz model and improved Korkmaz model and the parameter values obtained from the other models used according to the data of Table 1 were given. These biologically meaningful parameter values in Korkmaz model obtained from other models used, including improved Korkmaz model, were calculated and given in Table 4 for the purpose of comparison. To compare these parameters for the set of data, Table 4 was presented to the readers. In Table 4, the curve of Korkmaz model and improved Korkmaz model has an inflection point later than the curves of the other models used in this study. While in Korkmaz model and improved Korkmaz model, the maximum reached time is 24.4930 and maximum value reached at that time is 0.6546, in the other models used in this study, the maximum reached time is infinity and the different maximum values reached at that time are given in Table 4.

Due to the mathematical equation of Brody model, this model does not have either t_i parameter or B parameter. The classical models used in this study have parameter A only at infinity. It has been observed that all parameter values of Korkmaz model and improved Korkmaz model are exactly the same. Apart from this, all parameter values of Korkmaz model and improved Korkmaz model are very close to each other.

Table 4. The original parameter values of Korkmaz model and the parameter values in Korkmaz model obtained from other models used, including improved Korkmaz model according to the data of Table 1.

Models	C	B	t_i	k or c	A	t_m
Korkmaz and Improved Korkmaz	0.2288	0.4399	12.9278	0.0184	0.6546	24.4930
Logistic	0.1612	0.3796	10.1160	0.1296	0.7592	infinity
Gompertz	0.1540	0.3235	7.5725	0.0733	0.8795	infinity
Brody	0.1459	-	-	0.0183	1.5713	infinity
Von Bertalanffy	0.1514	0.2848	5.9342	0.0548	0.9691	infinity

4. Discussion and Conclusion

In this study, an improved model of the model previously presented as Korkmaz model, called improved Korkmaz model, and other classical models such as Logistic, Broody, Gompertz, Von Bertalanffy were examined in one data set related to growth.

Unlike known classical models, both the Korkmaz model and the improved Korkmaz model are in a cycle because they are trigonometric models. Although there is a cycle, the range of the data set in the study will cause restrictions in this cycle of the models. For this reason, sometimes there may be meaningless in parameters such as parameter B in the model although in this study there is no meaningless parameter in Korkmaz model and improved Korkmaz model. Although there is a turning point value, B in the model, sometimes this value can turn into a meaningless figure in the model because the turning point does not occur in the data range of the model. Therefore, the researcher should take this into consideration when using the model. Although the parameters used in the model are sometimes meaningless, criteria such as error sum of squares, which is one of the fit criteria of the model, are important for the suitability of the model. Criteria such as error sum of squares are important determinants of the fit of the model. In this study, the error sum of squares criterion for comparison the models used. was used. According to this criterion, as seen in Table 3, compared to all models used in this study for the data set, error sum of squares of both Korkmaz model and improved Korkmaz model has the most minimum value. Although in this study the data set taken from the specific growth rate of *Pseudomonas putida* (NICM 2174) at various time intervals were used, improved Korkmaz model used in this study like Korkmaz model could be used for any growth data. In other words, researchers using growth models in their studies could use improved Korkmaz model as well as classical growth models, including Korkmaz model, in their studies on growth data. In fact, although Korkmaz model is not the best model for two sets of data by using error sum of squares criterion in (Korkmaz, 2018), Korkmaz model with improved Korkmaz model is the best model in this study. For that reason, use of improved Korkmaz model in addition to other classical growth models, including Korkmaz model, in their studies on growth data could be suggested to the researchers.

In fact, according to the error sum of squares criterion, both the Korkmaz model and the improved Korkmaz model are the best models in this study, but they may not always be the best model for other data sets. It should be known that in growth models, a model that is best for one data set may not be the best model for another data set. This situation has already been confirmed in Korkmaz's article (Korkmaz, 2018). Therefore, the researcher should choose the best model after completing his/her examination without being biased.

According to the error sum of squares criterion, it was seen that both Korkmaz model and improved Korkmaz model have minimum error sum of squares for the data set. But we cannot say that this will be the case in other data sets. That's while one model is the best model for one data set, that model could not be the best model for the other data set. For that reason, use of improved Korkmaz model in addition to classical growth models with Korkmaz model in their studies on growth data is suggested to the researchers using growth models in their studies.

Even though both Korkmaz model and improved Korkmaz model cannot be the best models for other data sets, Korkmaz model and improved Korkmaz model will still be among the best models. We certainly do not claim that the improved Korkmaz model we present here is "the best model" in the data sets to be used.

As is known, unlike classical models, the Korkmaz model and the improved Korkmaz model have meaningful parameters. Therefore, the initial and final values of the parameters of both Korkmaz model and improved Korkmaz model are within a range that the researcher can predict. Thus, the researcher will have an easier time interpreting the parameters obtained and thus interpreting the model. The researcher will now be able to analyze the model better by seeing the important points in the model more clearly, such as the turning point value, maximum growth value, and initial value. Thus, the researcher will be able to take his/her research on growth further with the parameter data he/she has.

Data Availability

The data set used in this study is from previously reported study and data set which has been cited at relevant place within the text as the reference. The data used to support the findings of this study is available from the author upon request.

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Authors' Contributions

The author has completed all the article independently.

Statement of Conflicts of Interest

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

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