MODIFIED ADAPTIVE WEIGHT RAO-3 ALGORITHM FOR CONSTRUCTION TIME-COST TRADE-OFF OPTIMIZATION PROBLEMS

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Keywords

Multi-objective optimization, Rao-3 algorithm, Modified adaptive weight approach, Time-cost optimization.

Abstract

The Modified Adaptive Weight Approach (MAWA) represents a straightforward method commonly applied to solve time–cost optimization problems. MAWA is a flexible and effective approach that can be applied to various multi-objective optimization problems, including time–cost optimization in construction projects. These algorithms operate on a population of potential solutions that are randomly initialized within the boundaries of the solution space. MAWA assigns uniform weight factors to all individuals in the population without accounting for their specific characteristics. However, each solution possesses unique fitness attributes relative to its position in the solution space. In this study, a multi-objective optimization model combining the Rao-3 algorithm with the MAWA is introduced to generate a set of Pareto-optimal solutions. Two construction project case studies, drawn from existing technical literature and comprising 81 to 146 activities, are analyzed to evaluate the effectiveness of the proposed MAWA-Rao-3 method. The results are benchmarked against those from previously established models that produced approximate Pareto fronts or near-optimal solutions. The findings demonstrate that the MAWA-Rao-3 algorithm performs efficiently in addressing time-cost trade-off problems within the field of construction engineering and management.

İNŞAAT ZAMAN-MALİYET ÖDÜNLEŞİM OPTİMİZASYONU PROBLEMİ İÇİN DEĞİŞTİRİLMİŞ UYARLANABİLİR AĞIRLIKLI RAO-3 ALGORİTMASI

Anahtar Kelimeler

Çok amaçlı optimizasyon, Rao-3 algoritması, Değiştirilmiş uyarlanabilir ağırlık yaklaşımı, Zaman-maliyet optimizasyonu.

Öz

Değiştirilmiş Uyarlanabilir Ağırlık Yöntemi (MAWA), zaman–maliyet optimizasyon problemlerini çözmek için yaygın olarak kullanılan basit bir yöntemdir. MAWA, zaman-maliyet optimizasyonu da dahil olmak üzere çeşitli çok amaçlı optimizasyon problemlerine uygulanabilen esnek ve etkili bir yaklaşımdır. Bu tür problemler genellikle çok amaçlı optimizasyon problemleri olarak ele alınmakta olup, bu problemlerin çözümünde çoğunlukla meta-sezgisel algoritmalar kullanılmaktadır. Bu algoritmalar, çözüm uzayı sınırları içerisinde rastgele başlatılan bir çözüm popülasyonu üzerinde çalışır. MAWA, popülasyondaki tüm bireylere özgül niteliklerini dikkate almadan eşit ağırlık faktörleri atar. Ancak her çözüm, çözüm uzayındaki konumuna bağlı olarak kendine özgü uygunluk özelliklerine sahiptir. Bu çalışmada, Pareto-optimal çözümler elde etmek amacıyla Rao-3 algoritması ile MAWA'nın birleştirildiği çok amaçlı bir optimizasyon modeli sunulmuştur. Teknik literatürden alınan ve 81 ile 146 faaliyet içeren iki inşaat projesi örneği, önerilen MAWA-Rao-3 yönteminin etkinliğini değerlendirmek amacıyla incelenmiştir. Elde edilen sonuçlar, yaklaşık Pareto cepheleri veya neredeyse optimal çözümler üreten önceki modellerin sonuçlarıyla karşılaştırılmıştır. Bulgular, MAWA-Rao-3 algoritmasının inşaat mühendisliği ve yönetimi alanındaki zaman-maliyet dengeleme problemlerini çözmede etkili bir şekilde çalıştığını göstermektedir.

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1. Introduction

Multi-objective optimization problems require balancing conflicting objectives, making it challenging to find a single optimal solution. In construction projects, optimizing time, cost, and quality simultaneously is particularly difficult due to the inherent contradictions among these goals. As a result, construction managers must conduct trade-off analyses during the planning and scheduling stages to identify feasible alternatives that best satisfy project priorities Zhang and Xing (2010).

The earliest attempt to apply optimization techniques to time-cost trade-off problems (TCTPs) was made by Meyer et al. (1965), who utilized mathematical models, including linear programming, integer programming, and dynamic programming. Later, Feng et al. (1997) pointed out that these models often assume continuous relationships among decision variables. However, real-world construction activities rely on discrete operational choices involving time, cost, and quality, which make continuous mathematical models less practical. Moreover, integer and dynamic programming approaches can become computationally intensive for complex networks.

Vanhoucke et al. (2007) explored the use of heuristic algorithms for solving TCTPs, emphasizing their simplicity and efficiency for large-scale problems. Unlike mathematical models, heuristic methods apply straightforward decision rules, making them accessible for practical use. However, the reliability of the solutions, especially regarding their global optimality, remains a concern. To overcome these drawbacks, researchers have increasingly turned to metaheuristic algorithms, which are better suited for solving complex multi-objective or many-objective optimization problems by exploring the solution space more thoroughly.

Kumar et al. (2024) developed a TCTP optimization framework tailored specifically to Indian highway projects, utilizing the NSGA-II algorithm. Their model simultaneously minimized time and cost while respecting constraints such as resource availability and activity precedence. The findings showed that NSGA-II outperformed traditional techniques, offering a broad range of Pareto-optimal solutions. To further refine the selection process, the weighted sum method was used, confirming the algorithm's robustness and practicality for complex construction scenarios in India.

The literature clearly indicates that trade-off optimization in construction has been a long-standing challenge. Researchers have adopted three main categories of approaches to tackle TCTPs:

1. Mathematical models such as linear and non-linear programming, aim for exact solutions but are often unsuitable for large or discrete problems due to high computational demands (Liu et al. 1995).
2. Heuristic methods offer faster results using rule-based approaches but do not guarantee optimality.
3. Metaheuristic algorithms, on the other hand, provide flexible and scalable techniques for identifying a set of optimal trade-off solutions, making them more applicable to real-world construction planning (Eshtehardian et al., 2008).

Albayrak (2020) presented a novel hybrid algorithm (NHA), combining Particle Swarm Optimization (PSO) with Genetic Algorithm (GA) to address the TCT problem. Results indicated that the NHA achieved shorter and more cost-effective project durations than standard PSO.

More recently, Agarwal et al. (2024) suggested a Multi-Objective Particle Swarm Optimization (MOPSO) model for solving TCTPs. Drawing inspiration from swarm intelligence, their model was tested on a real-world project and proved effective in generating a diverse set of Pareto-optimal solutions, outperforming several existing optimization techniques.

In recent years, there has been increasing attention in applying metaheuristic algorithms (MHAs) to solve TCTPs more effectively. These algorithms are particularly suited for navigating complex and high-dimensional search spaces. Various MHAs have demonstrated promising results in solving optimization problems, including GA (Deb et al., 2002), TLBO (Rao et al., 2011), Slime Mould Algorithm (SMA) (Li et al., 2020), Arithmetic Optimization Algorithm (AOA) (Abualigah et al., 2021a), Aquila Optimizer (AO) (Abualigah et al., 2021b), and the Crayfish Optimization Algorithm (COA) (Jia et al., 2023).

More recently, Rao (2020) introduced a new class of metaphor-free and parameter-less optimization algorithms, known as the Rao algorithms. These algorithms are similar to TLBO and Jaya and use a single, simple update mechanism to guide the search process. Despite their conceptual simplicity, the Rao algorithms have shown strong performance across a wide range of constrained and unconstrained optimization problems (Rao, 2020; Rao and Keesari, 2020).

In the context of TCTPs, two widely adopted solution strategies in the literature are the Non-Dominated Sorting (NDS) approach and the Modified Adaptive Weight Approach (MAWA) (Toğan and Eirgash, 2019; Eirgash et al. 2019). While NDS is effective for identifying diverse Pareto-optimal solutions, it can become computationally intensive for large-scale or high-dimensional problems. In contrast, the MAWA technique is more computationally efficient and helps direct the search toward promising regions of the trade-off surface.

To the best of our knowledge, the integration of the MAWA technique with the Rao-3 algorithm has not yet been investigated for solving TCTPs. In this study, we propose a multi-objective optimization procedure that combines the simplicity of the Rao-3 algorithm with the guided focus of MAWA. A set of benchmark TCTPs is used to evaluate the proposed MAWA-Rao-3 framework, and the results are analyzed in terms of its ability to generate high-quality Pareto front solutions. The findings indicate that the MAWA-Rao-3 algorithm demonstrates robust and competitive performance, making it a promising approach for solving multi-objective TCTP problems.

- A novel multi-objective optimization model integrating Rao-3 algorithm with the MAWA is proposed for solving TCTP problems.
- The Critical Path Method (CPM) is employed as a project scheduling technique to derive time and cost objectives for optimization.
- The proposed MAWA-Rao-3 algorithm demonstrates strong capability in generating Pareto-optimal solutions for complex construction planning scenarios.
- Two real-life case studies involving construction projects with 81 and 146 activities are investigated to assess the practical applicability and performance of the proposed model.
- Hypervolume (HV) and Number of Function Evaluations (NFEs) are used as performance indicators to validate the effectiveness of the MAWA-Rao-3 algorithm.

2. Literature Review

TCTP in construction management have been extensively studied, with various optimization approaches proposed over the past decades. Traditional methods like the Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) provided foundational frameworks for schedule optimization (Kelly, 1961). However, these deterministic approaches often failed to capture the complex, multi-objective nature of real-world construction projects. Zheng et al. (2005) addressed these limitations by applying the GA-MAWA method to an 18-activity project, adjusting mutation and selection parameters to avoid local optima. Their approach demonstrated improved performance compared to earlier heuristic and mathematical techniques. Similarly, Parveen and Saha (2012) tested MAWA-GA on projects with 7 and 18 activities. They confirmed that the method effectively generated Pareto-optimal solutions and outperformed Zheng et al. (2004) by increasing the number of iterations in the optimization process.

The MAWA represents a significant advancement in multi-objective optimization, addressing limitations of traditional weighted sum methods (Toğan et al. 2020). The development of metaheuristic algorithms has revolutionized TCTP solutions, with genetic algorithms (GAs) demonstrating particular promise (Feng et al., 1997). Eirgash and Dede (2018) employed the TLBO algorithm, which incorporates the concept of assigning various teachers to different student groups along with an adaptive teaching factor. In their study, this algorithm was integrated with the MAWA. The proposed model was tested on construction projects consisting of 18 and 63 activities and demonstrated its capability to produce effective Pareto front solutions. The outcomes of the study outperformed those of similar works in the literature. Notably, the model successfully reached the global optimum for the 18-activity project and delivered optimal solutions for the 63-activity project as well. In a different study, Dede (2018) examined TLBO-MAWA and TLBO-NDS methods in the project with 7 activities. According to the results, TLBO method performed successfully in both the methods. Also, Toğan and Eirgash (2019) proposed the TLBO-MAWA model, applying it to projects with 7, 18, and 63 activities. Their findings showed that the TLBO method produced high-quality solutions and stood out for its algorithmic simplicity. Although numerous innovative algorithms and approaches have been developed for the TCTP, to the best of the authors' knowledge, the work by (Kim and de Week 2005; Ya-ping, and Ying, 2006; Toğan and Eirgash 2019) represents the only notable effort that specifically utilizes MAWA within the context of TCTP. Therefore, the proposed MAWA-Rao-3 method aims to address a significant gap in the existing literature.

A brief summary of related literature is provided in Table 1. The highlights of this study are summarized as follows:

Year	Author (s)	Approach	Problem size	Title Topics
2005	Elbeltagi, Hegazy, and Grierson	GA, MA, PSO, ACO, SFL	18	Five metaheuristic algorithms like GA, MA, PSO, ACO, and SFL are evaluated in comparison, highlighting the weak performance of GA and the strong robustness demonstrated by the PSO approach.
2005	Zheng et al.	MAWA-GA	7	Small scale 7 activity problem is solved using the modified adaptive weighted algorithm.
2008	Eshtehardian, Afshar and Abbasnia	GA	18	Fuzzy set theory allows the genetic algorithm (GA) to effectively address stochastic TCTPs.
2012	Zhang and Ng	ACO	18	Minimizing both of construction Time and Cost via ACO algorithm
2009	Afshar, Ziaraty, Kaveh and Sharifi	ACO	18	The Multi-Colony Non-Dominated Archiving Ant Colony Optimization (NA-ACO) approach assigns distinct ant colonies to individually focus on each objective.
2016	Bettemir and Birgönül	Siemens Algorithm	63	The study demonstrates that the minimum cost slope method yields the optimal solution for the continuous TCTP problem.
2016	Aminbakhsh and Sönmez	PSO	18, 63 and 630	Optimization of Time and Cost with discrete PSO algorithm for the large-scale DTCTPs.
2012	Sönmez and Bettemir	Hybrid-GA	18 ,29, 63 and 630	A hybrid GA algroithm is developed by integrating the strengths of simulated annealing (SA) with quantum simulated annealing, enhancing its optimization capabilities
2019	Toğan and Eirgash	NDSII-TLBO	18, 63 and 630	This study employs TLBO algorithm to optimize the TCTP problems in construction projects.
2020	Toğan et al.	New MAWA GA, TLBO, and Jaya	18, 63, and 630	Novel adaptive weight formulations have been proposed to enhance the efficiency of time–cost optimization processes.
2023	Bettemir and Birgönül	HMHH, DE	81, 146, and 291	Solving the discrete TCTP using an adaptive search domain approach enhances the optimization process by dynamically adjusting the solution space.
2024	Sulub et al.	OAOA, NDSII-AOA	63, 81 and 146	An Arithmetic Optimization Algorithm incorporating an Opposition Jumping Rate mechanism has been developed to effectively solve TCTP optimization problems.
2025	Eirgash	OBJA	9, 19	The impact of jumping rate on the O-Jaya algorithm for solving discrete TCTP problems.

The paper is structured as follows: First, we present the theoretical foundations of time-cost optimization, including key problem formulations and their significance in construction management. Next, we introduce the MAWA and detail its integration with the Rao-3 optimization algorithm for solving various discrete TCTP problems in construction projects. The subsequent section evaluates the MAWA-Rao-3 algorithm's performance through comprehensive benchmarking against established optimization problems. The paper concludes with a detailed discussion of findings and their practical implications for construction project management.

3. Methodology

Figure 1 illustrates the research methodology employed in this study, which addresses multi-objective optimization in project management with an emphasis on balancing time and cost. The approach begins with the development of a comprehensive model integrating these three interrelated objectives. Subsequently, several optimization algorithms are examined, including the proposed MAWA-Rao-3 and the baseline Rao-3, alongside benchmark algorithms such as NDSII-AOA, NDSII-TLBO, and NDSII-AO. These algorithms are applied to case studies of varying complexity, specifically projects comprising 81 and 146 activities. The performance of the algorithms is assessed considering the number of objective functions (NFEs) and the Hypervolume (HV) indicator. Additionally, the traditional Critical Path Method (CPM) is utilized to determine the overall project duration. the modified adaptive weight approach (MAWA) is employed as a multi-objective optimization approach to enhance the solution quality.

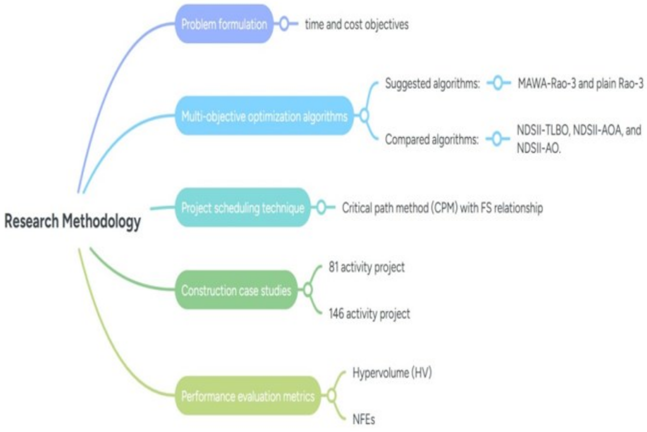


Figure 1. Methodology map of this study

3.1. Problem formulation of Trade-offs Problems

The trade-off optimization process aims to reduce both project duration and cost at the same time, identifying optimal solutions that can be applied across all activities in construction projects.

Objective 1: Minimizing of project time (PT): The minimization of project duration is as follows:

$$Min\ PT = \max\ (ES_i + d_i),\ \forall i = 1,2,\ldots,m \qquad (1)$$

where:

- PT = total project duration
- ES_i = earliest start time of activity i
- d_i = duration of activity i
- m = total number of activities in the project
- ES_i is the earliest start time of activity i.

Objective 2: Minimizing of project cost (PC): The minimization of project cost is formulated as follows:

$$Min\ PC = \sum_{i=1}^{m} cos\ t_i \qquad (2)$$

where:

- PC = total project cost
- $cost_i$ = cost of activity iii for the chosen execution mode

3.2. Modified Adaptive Weight Approach (MAWA) in Multi-Objective Optimization

The modified adaptive weight approach (MAWA) is a computationally efficient and conceptually simple method widely used in addressing multi-objective optimization problems, particularly TCTP problems commonly encountered in the construction engineering field (Toğan et al. 2020). These problems are characterized by conflicting objectives, making metaheuristic algorithms essential for exploring trade-off solutions and generating Pareto-optimal approximations. Within the MAWA framework, a population of potential solutions is randomly initialized within the defined bounds of the solution space and subsequently evaluated based on multiple objective functions. To guide the search and balance trade-offs, MAWA employs a weighting strategy based on solution quality. According to Zheng et al. (2004), the adaptive weighting mechanism in MAWA follows four key conditions, which are formulated as follows:

1. $S_c^{max} \neq S_c^{min}$ ve $S_t^{max} \neq S_t^{min}$

$$V_c = S_c \big/ (S_c^{max} - S_c^{min}) \qquad Wc = V_c/(V_c+V_t)$$
$$V_t = S_t \big/ (S_t^{max} - S_t^{min}) \qquad Wt = V_t/\ (V_c+V_t) \qquad (3)$$

2. $S_c^{max} = S_c^{min}$ and $S_t^{max} = S_t^{min}$

$$W_c = 0{,}5 \qquad W_t = 0{,}5 \qquad (4)$$

3. $S_c^{max} \neq S_c^{min}$ and $S_t^{max} = S_t^{min}$

$$W_c = 0{,}1 \qquad W_t = 0{,}9 \qquad (5)$$

4. $S_c^{max} = S_c^{min}$ and $S_t^{max} \neq S_t^{min}$

$$W_c = 0{,}9 \qquad W_t = 0{,}1 \qquad (6)$$

Let S_t^{max} and S_t^{min} denote the maximum and minimum values, respectively, of the project duration objective function in the current iteration. Similarly, let S_c^{max} and S_c^{min} represent the maximum and minimum values of the total direct cost objective function in the same iteration. The parameters v_t and v_c are expressed as the ratios between the minimum values and the ranges (i.e., differences between maximum and minimum values) for project duration and total cost, respectively, and are defined as:

$$F(x) = W_t{*}\left(\frac{S_t - S_t^{min} + m)}{S_t^{max} - S_t^{min} + m)}\right) + W_c{*}\left(\frac{S_c - S_c^{min} + m}{S_c^{max} - S_c^{min} + m}\right) \quad (6)$$
$$m = 0{,}001$$

Here, x denotes a candidate solution in the current generation, and $F(x)$ refers to its corresponding fitness value. The terms S_c and S_t represent the total cost and project duration of the x^{th} solution, respectively. The parameter m is a small positive random value within the range (0, 1), while w_c and w_t are the adaptive weights assigned to cost and time, respectively. To prevent situations where $S_c^{max} = S_c^{min}$ or $S_t^{max} = S_t^{min}$, the random value m is incorporated into Equation (6), as suggested by Zheng et al. (2004).
This value is sufficiently small to have a negligible impact on the normalization process while ensuring stable fitness evaluation and avoiding division-by-zero errors during optimization.

3.3. Rao-3 optimization algorithm

The Rao-3 optimization algorithm, introduced by Rao (2020) as the third variant in a series that includes Rao-1 and Rao-2, is a parameter-free metaheuristic method. Its design eliminates the need for algorithm-specific hyperparameter tuning, thereby enhancing user accessibility and simplifying its implementation.

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i}(X_{j,best,i} - |X_{j,worst,i}|) + r_{2,j,i}(|X_{j,k,i} \text{ or } X_{j,l,i}| - (X_{j,l,i} \text{ or } X_{j,k,i})) \quad (7)$$

Where,

- X_i : Current solution
- X_{best} : Best solution in the current population
- X_{worst} : Worst solution in the current population
- X_j, X_k : Randomly selected different solutions from the population
- r_1, r_2: Random numbers in the range [0, 1]

In the term $|X_{j,k,i}$ or $X_{j,l,i}|$, candidate solution k is compared with another candidate l randomly chosen from the existing candidates in the population. The term $|X_{j,k,i}|$ is chosen if k is more suitable than l. Else if, $|X_{j,l,i}|$ is selected. The same rule applies to the second term (i.e. $X_{j,l,i}$ or $X_{j,k,i}$). The flowchart of the process can be tracked in Figure 2.

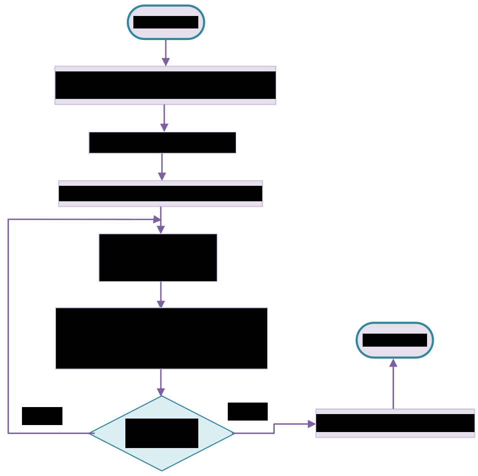


Figure 2. Flowchart of the MAWA-Rao-3 algorithm for TCTP

4. Numerical Examples

The effectiveness of the MAWA-Rao-3 model in solving TCTP problems was assessed using two medium-scale benchmark cases with 81 and 146 activities, derived from Bettemir and Birgonul (2023). The algorithm was implemented in MATLAB and executed on a system with an Intel® Core™ i3-3110M CPU (2.40 GHz) and 6 GB RAM. As TCTPs are multi-objective in nature, conventional techniques such as non-dominated sorting (NDS) and the MAWA (Deb et al., 2002; Zheng et al. 2004) are commonly used. In this study, the MAWA-Rao-3 algorithm was applied to evaluate its performance on the selected problems.

4.1. Numerical example of 81-activity project

A complex construction project consisting of 81 activities, originally presented by Bettemir and Birgonul (2023), is resolved to evaluate the performance of the proposed algorithm. Each activity in the project features six discrete execution modes, adding to the scheduling complexity. An indirect cost rate of $2000 per day is assumed for the duration-based cost analysis. Due to the combinatorial nature of the problem, the project presents **a** solution space of approximately 1.072×10^{63} possible scheduling scenarios. To assess the effectiveness of the proposed MAWA-Rao-3 algorithm, its results are compared with those obtained using the original Rao-3 algorithm and the benchmark solution reported by Bettemir and Birgonul (2023), which was generated using a hybrid heuristic–metaheuristic (HHMH) approach. A summary of the comparative trade-off solutions is provided in Table 2. The results indicate that the MAWA-Rao-3 algorithm consistently achieves competitive or improved outcomes in terms of both time and cost trade-offs. For instance, at a project duration of 345 days, the MAWA-Rao-3 produces a total cost of

$3,491,150, which is lower than the $3,498,200 obtained by the plain Rao-3 under the same duration. Similar patterns are observed for other time points (e.g., 346, 347, 348 days), where MAWA-Rao-3 frequently outperforms the plain Rao-3 in cost efficiency, and competes closely with the benchmark HHMH results. In terms of computational performance, the proposed MAWA-Rao-3 model demonstrates a significant advantage. While both MAWA-Rao-3 and plain Rao-3 used the same number of populations (NOP = 200), MAWA-Rao-3 achieved these with only 30,000 function evaluations (NFE), compared to 1,250,000 NFEs required by the HHMH method. This corresponds to just 2.5% of the computational cost, highlighting the remarkable efficiency of the MAWA-Rao-3 in solving complex, medium-scale TCTPs.

Consequently, the MAWA-Rao-3 algorithm proves to be an effective and computationally effective approach for addressing TCTP problems in construction project planning. It offers a compelling alternative to more computationally intensive metaheuristic methods without compromising on the quality of the approximate Pareto-optimal solutions.

Table 2. Trade-off solutions of 81 activity TCTP problem

Sr. No	Bettemir and Birgonul (2023) HHMH		This paper			
			Plain Rao-3		MAWA-Rao-3	
	PCT	PCC	PCT	PCC	PCT	PCC
1	389	3321550	345	3498200	345	3491150
2			346	3499000	346	3496950
3			347	3550000	347	3519600
4			359	3507700	359	3492500
5			349	3527950	349	3512200
6			348	3523050	348	3517700
7			360	3517500	360	3505050
8			358	3518700	358	3517950
9			362	3513700	362	3509000
10			342	3545700	342	3533100
NOP			200		200	
NOI			180		150	
NFE	1250000		36000		30000	

Note: HHMH: Hybrid heuristic meta-heuristic

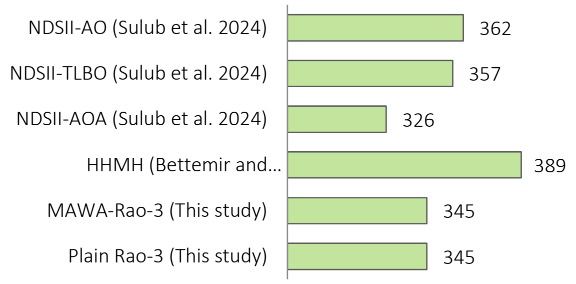
Table 3 presents a comparative evaluation of the proposed MAWA-Rao-3 algorithm against three other state-of-the-art multi-objective optimization models previously reported by Sulub et al. (2024): NDSII-TLBO, NDSII-AO, and NDSII-AOA. All methods are applied to the same 81-activity time–cost trade-off project to identify efficient Pareto front solutions. A direct comparison with Bettemir and Birgonul (2023) is not feasible in terms of schedule efficiency, as the authors provided only selected trade-off solutions rather than a detailed schedule or comprehensive Pareto front. Therefore, for fairness and rigor, the performance of the MAWA-Rao-3 is assessed against the NDSII-based algorithms. The results clearly show that MAWA-Rao-3 obtained satisfactory solutions, making it an attractive and efficient option for solving TCTP problems in the construction domain.

Table 3. Pareto front solutions obtained by the comparison algorithms for the 81-activity TCTPs

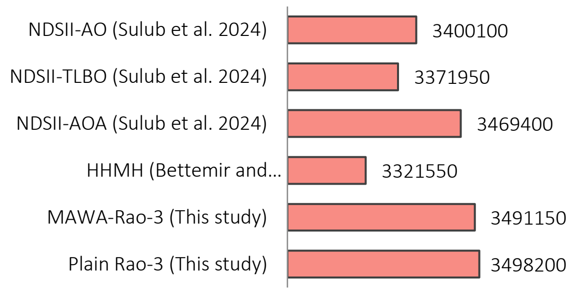
Sr. No	Sulub et al. 2024						(This study)	
	NDSII-TLBO		NDSII-AO		NDSII-AOA		MAWA-Rao-3	
	PCT	PCC	PCT	PCC	PCT	PCC	PCT	PCC
1	357	3371950	362	3400100	285	3601500	345	3491150
2	370	3358100	346	3414150	326	3469400	346	3496950
3	348	3383750	355	3402700	320	3486050	347	3519600
4	311	3451350	296	3526750	287	3563850	359	3492500
5	333	3406350	353	3340050	298	3538600	349	3512200
6	319	3448950	290	3521400	312	3487400	348	3517700
7	341	3396600	322	3422000	304	3522350	360	3505050
8	310	3477450	386	3537950	307	3500850	358	3517950
9	299	3492400	381	3350950	286	3593250	362	3509000
10	297	3503950	284	3575350	291	3549950	342	3533100
NOP	100		200		100		200	
NOI	90		180		180		150	
NFE	36000		36000		36000		30000	

Note: Teaching learning-based optimization (TLBO), Aquila optimizor (AO)

The comparative analysis evaluates six optimization algorithms based on project time (days) and cost ($). Among the tested methods, NDSII-AOA (Sulub et al., 2024) demonstrates superior time efficiency, completing projects in just 326 days - significantly faster than other NDSII variants (NDSII-TLBO: 357 days; NDSII-AO: 362 days) and Rao-3-based approaches (345 days). HHMH (Bettemir and Birgonul, 2023) emerges as the most cost-effective solution at $3,321,550, though it requires the longest project duration (389 days). NDSII-TLBO presents a balanced alternative, offering moderate time efficiency (357 days) with competitive costs ($3,371,950). The Rao-3 variants (MAWA-Rao-3 and plain Rao-3) show room for improvement, performing moderately in time (345 days) but lagging in cost efficiency ($3,491,150-$3,498,200). Hence, Figure 3, visualizes the results from Table 3, demonstrating the better performance of MAWA-Rao-3 in finding the shorter project duration. In simpler terms, when minimizing project duration is a higher priority for decision-makers compared to cost, the approximate Pareto front solutions obtained using the MAWA-Rao-3 is as good as comparison algorithms.



(a) Project time (days)



(b) Project cost ($)

Figure 3. Desired approximate Pareto front solutions of TCTP for 81-activity project

4.1. Case study of 146-activity project

A medium-sized construction project consisting of 146 activities, originally analyzed by Bettemir and Birgonul (2023), is utilized as a test case to further validate the effectiveness of the proposed models. An indirect cost rate of $4000 per day is applied in this study. The scheduling complexity of the problem is immense, with approximately 1.12×10^{102} potential combinations. While Bettemir and Birgonul (2023) presented only a single preferred solution, the MAWA-Rao-3 algorithm provides a comprehensive set of Pareto front solutions for this project. These solutions, illustrated through both tabular and graphical formats, offer project managers detailed insights into the trade-offs between cost and duration, enabling more informed decision-making.

Table 4 presents the trade-off solutions obtained through the plain Rao-3 algorithm, highlighting the relationship between project cost and duration through Pareto optimal solutions. Notably, the MAWA-Rao-3 algorithm searches a narrower region of the solution space compared to the standard Rao-3 yet succeeds in achieving superior outcomes on the Pareto front. This suggests that MAWA-Rao-3 is more efficient in locating high-quality solutions, offering a broader range of viable options tailored to specific project requirements. As previously discussed, although the case study originates from the work of Bettemir and Birgonul (2023), their research provides only a single desirable outcome rather than a complete Pareto front. Therefore, for a more meaningful comparison, the performance of the proposed MAWA-Rao-3 algorithm is also evaluated against other contemporary multi-objective algorithms, including NDSII-TLBO, NDSII-AOA, and NDSII-AO.

Table 4. Pareto front solutions obtained by the comparison algorithms for the 146-activity TCTP

Sr. No	Bettemir and Birgonul (2023) HHMH		This paper			
			Plain Rao-3		MAWA-Rao-3	
	PCT	PCC	PCT	PCC	PCT	PCC
1	560	6238500	552	6490000	530	6432750
2			541	6542750	545	6372750
3			544	6537500	548	6375750
4			541	6570500	556	6374000
5			554	6538500	591	6379000
6			594	6591250	552	6475000
7			656	6564000	571	6424250
8			569	6464250	561	6469750
9			555	6487000	548	6525750
10			570	6456250	579	6399250
NOP			200		200	
NOI			200		200	
NFE	6000000		40000		40000	

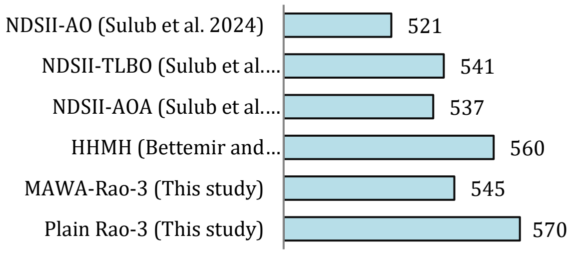
The results presented in Table 5 offer a comprehensive comparative analysis of the proposed MAWA-Rao-3 algorithm with three state-of-the-art multi-objective optimization methods, NDSII-TLBO, NDSII-AO, and NDSII-AOA, for solving a 146-activity TCTP. From the perspective of solution quality, the MAWA-Rao-3 algorithm demonstrates remarkable balance between project completion time (PCT) and project completion cost (PCC). Its solutions range from 530 to 579 days for PCT and from \$6,237,250 to \$6,482,750 for PCC. While NDSII-AOA provides faster solutions with PCTs as low as 487 days, they come at a significantly higher cost, reaching up to \$6,890,000. Conversely, MAWA-Rao-3 maintains a better equilibrium, offering competitive PCTs without excessive financial burden, making it highly suitable for real-world applications where cost control is critical. While NDSII-AOA offers faster schedules and NDSII-AO delivers marginal improvements in completion time, both fall short in terms of cost efficiency and computational demand. Moreover, the normalized function evaluations (NFE) correspond to roughly 7%, comparing 40,000 to 6,000,000. That means, MAWA-Rao-3 requires significantly fewer function evaluations compared to the HHMH algorithm.

Table 5. Pareto front solutions of comparison algorithms for 146 activity TCTP problem

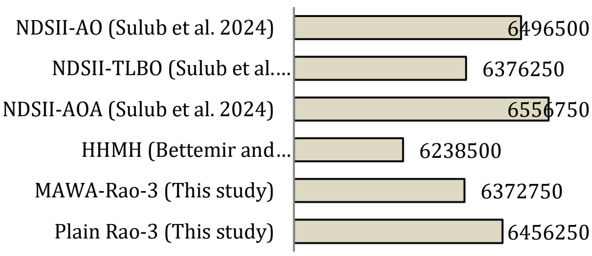
Sr. No	Sulub et al. 2024				(This study)			
	NDSII-TLBO		NDSII-AO		NDSII-AOA		MAWA-Rao-3	
	PCT	PCC	PCT	PCC	PCT	PCC	PCT	PCC
1	523	6458000	521	6496500	537	6556750	530	6432750
2	535	6394250	541	6542750	480	6835000	545	6372750
3	537	6412500	526	6613250	523	6566000	548	6375750
4	538	6467850	523	6655250	510	6595750	556	6374000
5	539	6435250	554	6538500	509	6639000	591	6379000
6	537	6593570	537	6562000	504	6642250	552	6475000
7	541	6376250	520	6722000	487	6755750	571	6424250
8	530	6544520	520	6664000	515	6582000	561	6469750
9	539	6537550	524	6657000	490	6737750	548	6525750
10	528	6697650	520	6497000	499	6689000	579	6399250
NOP	100		100		100		200	
NOI	100		200		200		200	
NFE	40000		40000		40000		40000	

Note: Teaching learning-based optimization (TLBO), Aquila optimizor (AO)

A direct comparison of scheduling efficiency with the previously proposed model by Bettemir and Birgonul (2023) is not feasible, as they only presented the desired solution without a detailed schedule in table format. Therefore, to ensure a fair comparison, the MAWA-Rao-3 algorithm was evaluated against the NDSII-TLBO, NDSII-AOA, and NDSII-AO algorithms. Figure 4, which visualizes data from Tables 4 and 5, highlights the suitabilility of MAWA-Rao-3 in achieving shorter project durations. Simply put, when minimizing project duration is prioritized over cost, the approximate Pareto front solutions produced by MAWA-Rao-3 are comparable to those obtained by the other algorithms.



(a) Project time (days)



(b) Project cost (\$)

Figure 4. Desired approximate Pareto front solutions of TCTP for 146-activity project

5. Performance Metrics for TCTP Problems

The literature does not provide a standardized performance metric for evaluating multi-objective optimization algorithms, given their inherent complexity compared to single-objective problems. Various quality indicators, such as convergence and diversity, have been introduced (Basseur and Zitzler, 2006; Hamdy et al. 2016). This paper used the Hypervolume (HV) metric for evaluation, each representing different performance aspects. This concept is illustrated in Figure 5 and is supported by findings in the literature (Senouci and Mubarak, 2016).

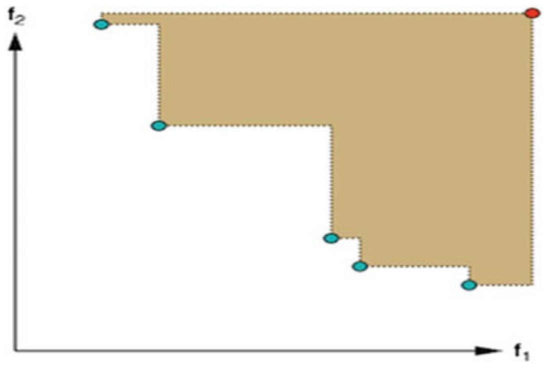


Figure 5. HV in case of bi-objective minimization problems

$$HV = \int_R \prod_{i=1}^{m} \left(f_i^{ref} - f_i(x)\right) dx \quad (8)$$

Where:

R: the dominant region defined by the pareto front.

m: the number of objectives

f_i^{ref}: the reference point for the *i-th* objective, usually set slightly worse than the worst solution.

$f_i(x)$: the value of *i-th* objective for solution *x.*

Figure 6 presents a comparative analysis of algorithmsapplied to projects with 81 and 146 activities. The comparison includes NDSII-TLBO, NDSII-AOA, NDSII-AO, and the standard Rao-3 algorithms. Based on the Hypervolume (HV) metric, MAWA-Rao-3 exhibits comparable performance, underscoring its effectiveness in producing a well-distributed and comprehensive approximation of the Pareto front. This demonstrates the suitability of MAWA-Rao-3 for solving time–cost trade-off problems (TCTP).

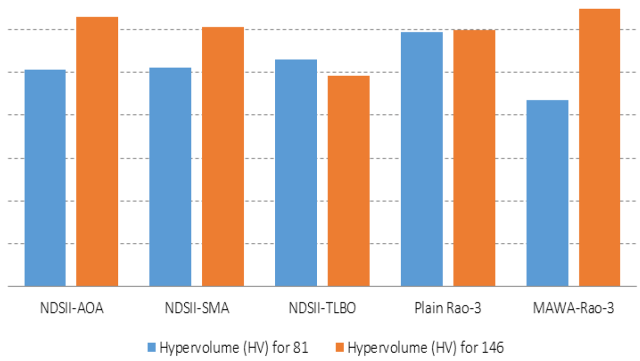


Figure 6. Performance metrics comparison of proposed algorithms.

6. Conclusion

This study introduced a novel multi-objective optimization model, MAWA-Rao-3, which integrates the Modified Adaptive Weight Approach (MAWA) with the parameter-less Rao-3 metaheuristic algorithm, to address time–cost trade-off problems (TCTPs) in construction project management. Unlike conventional weight assignment strategies, MAWA adaptively adjusts weight factors based on the evolving fitness characteristics of each solution, enhancing the algorithm's ability to balance exploration and exploitation during the optimization process.

The performance of the proposed model was validated through two benchmark construction project scenarios comprising 81 and 146 activities, respectively, sourced from the existing technical literature. These test cases represent medium-scale, real-world projects and encompass varying levels of complexity. The obtained results were benchmarked against those generated by other established models that employ metaheuristic strategies for TCTP optimization, including methods that approximate the Pareto front.

The empirical findings demonstrated that the MAWA-Rao-3 algorithm effectively generates high-quality Pareto-optimal solutions, offering a more balance between project time and cost objectives. Notably, the model yielded a broader and more diverse set of non-dominated solutions compared to conventional Rao-3 and other existing approaches. This indicates that the MAWA component significantly contributes to the applicability and adaptability of the search process.

Furthermore, due to its parameter-less nature, the Rao-3 framework facilitates easy implementation without the need for extensive hyperparameter tuning. Overall, the integration of MAWA with Rao-3 provides a computationally efficient and reliable approach for solving complex TCTPs.

While this uniform weighting strategy simplifies the implementation and maintains computational efficiency, it overlooks the inherent diversity among solutions. Specifically, candidate solutions may exhibit distinct fitness characteristics and varying levels of proximity to the Pareto front, which are not captured when a homogeneous weighting scheme is applied. Consequently, although MAWA serves as a practical tool in multi-objective optimization, its effectiveness may be constrained by its limited capacity to adaptively respond to the dynamic nature of the solution landscape.

Future work may extend this framework by incorporating additional objectives such as quality, environmental impact, or risk, and evaluating the model's performance on larger-scale projects or under uncertainty. Additionally, hybridization with a non-dominated sorting or enhancement with learning mechanisms could further improve convergence behavior and solution diversity.

The case study problems can be obtained from the following link:

https://drive.google.com/drive/folders/1R5w6SUZPkn6obihwgvgZ9MBpMqKRzKCS?usp=sharing

Author Contribution

Mohammad Azim Eirgash: Conducted the analysis and interpretation, edited the manuscript, developed and implemented the modeling process, performed the literature review, and led the writing of the article.

Yusuf Baltaci: Supervised the research, provided critical revisions and suggestions, contributed to the methodological framework, and supported the finalization and approval of the manuscript.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Data Availability Statement

The data, models, and codes that support the findings of this study are available from the corresponding author upon reasonable request.

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