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Solutions for the Drinfeld-Sokolov Equation Using an IBSEFM Method

Faruk Dusunceli[1]
[1]Faculty of Architecture and Engineering, Mardin Artuklu University, Mardin, Turkey
✉: farukdusunceli@artuklu.edu.tr

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ABSTRACT

In this study, the Drinfeld-Sokolov system is solved by the application of the improved Bernoulli sub-equation function method (IBSEFM). We have found new solutions different from the others articles in the literature. In addition, we carried all the computations out and the graphics plot in this article by software Wolfram Mathematica 9.

Keywords: Improved Bernoulli sub-equation function method, Drinfeld-Sokolov equations, New solutions

Drinfeld-Sokolov Denkleminin IBSEFM Yöntemiyle Yeni Çözümleri

ÖZ

Bu çalışmada, geliştirilmiş Bernoulli fonksiyon yönteminin Drinfeld-Sokolov sistemine uygulanması sunulmuştur. Literatürdeki diğer makalelerden faklı yeni çözümler bulduk. Ek olarak, bu makaledeki tüm hesaplamalar ve grafik çizimleri Wolfram Mathematica 9 programı yardımıyla yapılmıştır..

Anahtar Kelimeler: Geliştirilmiş Bernoulli Denklem Metodu, Drinfeld-Sokolov denklemi, Yeni çözümler

INTRODUCTION

Nonlinear partial differential equations (NPDEs) have modeled Nonlinear complex phenomena in various scientific fields such as plasma physics, fluid mechanics, optical fibers, nonlinear optics, solid state physics and so on. The investigation of exact solutions of NPDEs will help to be better understanding the complex phenomena. In this paper, the Drinfeld-Sokolov (DS) system of equations [1-3] is investigated by using the improved Bernoulli sub-equation function method(IBSEFM) [4]. DS equation system is an example of a system of nonlinear equations possessing Lax pairs of a special form [1] and it was introduced by Drinfeld and Sokolov.

The Drinfeld-Sokolov (DS) equation is given by

$$u_t + (v^2)_x = 0$$
$$v_t - av_{xxx} + 3dvu_x + 3kv_xu = 0 \tag{1}$$

where $u = u(x,y,t),\ v = v(x,y,t)$ and a,d,k are constants.
Various analytical approaches have been used in obtaining the exact solutions to the Drinfeld-Sokolov (DS) system of equations. Wazwaz [5] used the sine-cosine and tanh methods to DS, El Wakil and Abdou [6] used the modified extend tanh-function method for finding exact solutions for five model of nonlinear differential equations, one of them is the DS system. And Zangh et al. [7] used the complex system for complex DS system.

The IBSEFM

Improved Bernoulli sub-equation function method (IBSEFM) formed by modifying the Bernoulli sub-equation function [8-10] method will be given in this part.

Step 1. Let's consider the following fractional differential equation;

$$P\left(u, u_x, u_t, u_{xt}, \cdots\right) = 0, \tag{2}$$

and take the wave transformation;

$$u(x,t) = U(\gamma),\ \gamma = x - ct \tag{3}$$

where c is constant and, it will be determined later.
Substituting Eq.(3) into Eq.(2), we obtain the following nonlinear ordinary differential equation;

$$N\left(U, U', U'', U''', \cdots\right) = 0. \tag{4}$$

Faruk DUSUNCELI orcid.org/0000-0002-2368-7983

Step 2. Considering trial equation of solution in Eq.(4), it can be written as following;

$$U(\eta)=\frac{\sum_{i=0}^{n}a_iF^i(\eta)}{\sum_{j=0}^{m}b_jF^j(\eta)}=\frac{a_0+a_1F(\eta)+a_2F^2(\eta)+\cdots+a_nF^n(\eta)}{b_0+b_1F(\eta)+b_2F^2(\eta)+\cdots+b_mF^m(\eta)}. \quad (5)$$

According to the Bernoulli theory, we can consider the general form of Bernoulli differential equation for F' as following;

$$F'=wF+dF^M,\ w\neq 0,\ d\neq 0,\ M\in\mathbb{R}-\{0,1,2\}, \quad (6)$$

where $F=F(\eta)$ is Bernoulli differential polynomial. Substituting above relations in Eq.(4), it yields equations of polynomial $\Omega(F)$ of F as following;

$$\Omega(F)=\rho_sF^s+\cdots+\rho_1F+\rho_0=0. \quad (7)$$

According to the balance principle, we can determine the relationship between n,m and M.

Step 3. The coefficients of $\Omega(F)$ all will be zero yield us an algebraic system of equations;

$$\rho_i=0,\ i=0,\cdots,s. \quad (8)$$

Solving this system, we will specify the values of $a_0,\cdots,a_n$ and $b_0,\cdots,b_m$.

Step 4. When we solve nonlinear Bernoulli differential equation Eq.(6), we obtain the two following situations according to b and d;

$$F(\eta)=\left[\frac{-d}{w}+\frac{E}{e^{w(M-1)\eta}}\right]^{\frac{1}{1-M}},\quad w\neq d, \quad (9)$$

$$F(\eta)=\left[\frac{(E-1)+(E+1)\tanh\left(w(1-M)\eta/2\right)}{1-\tanh\left(w(1-M)\eta/2\right)}\right]^{\frac{1}{1-M}},\quad w=d,\ E\in\mathbb{R}. \quad (10)$$

Application

In this section, the Drinfeld-Sokolov (DS) equation is solved by using the wave transformation on Eq. (1) in the application of the improved Bernoulli sub-equation function method.

$$u(x,y,t)=U(\gamma),\ \gamma=x-ct,$$
$$v(x,y,t)=V(\gamma),\ \gamma=x-ct, \quad (11)$$

we get the following system of nonlinear ordinary differential equations:

$$-cU'+(V^2)'=0,$$
$$cV'+aV'''-3dVU'-3kUV'=0 \quad (12)$$

Integrating the first equation in the system (12), we get

$$cU=V^2 \quad (13)$$

Inserting Eq.(13) into the second equation of Eq. (12), we get the following single nonlinear ordinary differential equation:

$$c^2V'+acV'''-6dV^2V'-3kV^2V'=0 \quad (14)$$

Finally, integration Eq (14), we have

$$c^2V+acV''-(2d+k)V^3=0 \quad (15)$$

Balancing Eq. (15) by considering the highest derivative and power, we obtain

$$m+M=n+1.$$

Choosing $M=3,\ m=1,$ gives $n=3.$ Thus, the trial solution to Eq. (1) takes the following form:

$$U(\gamma)=\frac{a_0+a_1F(\gamma)+a_2F^2(\gamma)+a_3F^3(\gamma)}{b_0+b_1F(\gamma)} \quad (16)$$

where $F'=wF+dF^3, w\neq 0, d\neq 0.$ Substituting Eq. (16), its second derivative along with $F'=wF+dF^3, w\neq 0, d\neq 0$ into Eq. (15), yields a polynomial in F. Solving the system of the algebraic equations, yields the values of the parameter involved. Substituting the obtained values of the parameters into Eq. (16), yields the solutions to Eq. (1).

For $w\neq d$, we can find following coefficients:

Case 1.

$$a_1=\frac{a_0b_1}{b_0}, a_2=\frac{2wa_0}{\sigma}, a_3=\frac{2wa_0b_1}{\sigma b_0}, k=-2w+\frac{4\alpha^2\sigma^4b_0^2}{a_0^2}, c=2\alpha\sigma^2, d=w \quad (17)$$

Case 2.

$$a_1=\frac{a_0b_1}{b_0},a_2=-\frac{ka_0}{\sigma}+\frac{4\alpha^2\sigma^3b_0^2}{a_0},a_3=-\frac{ka_0b_1}{\sigma b_0}+\frac{4\alpha^2\sigma^3b_0b_1}{a_0},c=2\alpha\sigma^2,d=-\frac{k}{2}+\frac{2\alpha^2\sigma^4b_0^2}{a_0^2},w=-\frac{k}{2}+\frac{2\alpha^2\sigma^4b_0^2}{a_0^2}; \quad (18)$$

Case 3.

$$a_1=\frac{a_0b_1}{b_0},a_2=\frac{2\sqrt{2}d\sqrt{\alpha}a_0}{\sqrt{c}},a_3=\frac{2\sqrt{2}d\sqrt{\alpha}a_0b_1}{\sqrt{c}b_0},k=-2d+\frac{c^2b_0^2}{a_0^2} \quad (19)$$

Case 4.

$$a_1=\frac{a_0b_1}{b_0},a_2=\frac{2wa_0}{\sigma},a_3=\frac{2wa_0b_1}{\sigma b_0},\alpha=\frac{\sqrt{2d+k}a_0}{2\sigma^2b_0},c=\frac{\sqrt{2d+k}a_0}{b_0},w=d \quad (20$$

Substituting Eq. (17) into Eq. (16), gives

$$u_1(x,y,t)=\frac{\left(a_0+\frac{2wa_0}{\left(e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}\right)\sigma}+\frac{a_0b_1}{\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}}b_0}+\frac{2wa_0b_1}{\left(e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}\right)^{3/2}\sigma b_0}\right)^2}{2\alpha\sigma^2\left(b_0+\frac{b_1}{\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}}}\right)^2}$$

$$v_1(x,y,t)=\frac{a_0+\frac{2wa_0}{\left(e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}\right)\sigma}+\frac{a_0b_1}{\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}}b_0}+\frac{2wa_0b_1}{\left(e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}\right)^{3/2}\sigma b_0}}{b_0+\frac{b_1}{\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}}}} \quad (21)$$

Substituting Eq. (18) into Eq. (16), gives

$$u_2(x,y,t)=\frac{\left(a_0+\frac{-\frac{ka_0}{\sigma}+\frac{4\alpha^2\sigma^3b_0^2}{a_0}}{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{-\frac{k}{2}+\frac{2\alpha^2\sigma^4b_0^2}{a_0^2}}{\sigma}}+\frac{a_0b_1}{b_0\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{-\frac{k}{2}+\frac{2\alpha^2\sigma^4b_0^2}{a_0^2}}{\sigma}}}+\frac{-\frac{ka_0b_1}{\sigma b_0}+\frac{4\alpha^2\sigma^3b_0b_1}{a_0}}{\left(e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{-\frac{k}{2}+\frac{2\alpha^2\sigma^4b_0^2}{a_0^2}}{\sigma}\right)^{3/2}}\right)^2}{2\alpha\sigma^2\left(b_0+\frac{b_1}{\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{-\frac{k}{2}+\frac{2\alpha^2\sigma^4b_0^2}{a_0^2}}{\sigma}}}\right)^2}$$

$$v_2(x,y,t)=\frac{a_0+\frac{2wa_0}{\left(e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}\right)\sigma}+\frac{a_0b_1}{\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}}b_0}+\frac{2wa_0b_1}{\left(e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}\right)^{3/2}\sigma b_0}}{b_0+\frac{b_1}{\sqrt{e^{-2\sigma(x-2t\alpha\sigma^2)EE}-\frac{w}{\sigma}}}} \quad (22)$$

Substituting Eq. (19) into Eq. (16), gives

$$u_3(x,y,t) = \frac{\left(a_0 + \frac{2\sqrt{2w}\sqrt{\alpha}a_0}{\sqrt{c}\left(e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}\right)} + \frac{a_0 b_1}{\sqrt{e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}}\, b_0} + \frac{2\sqrt{2w}\sqrt{\alpha}a_0 b_1}{\sqrt{c}\left(e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}\right)^{3/2} b_0}\right)^2}{c\left(b_0 + \frac{b_1}{\sqrt{e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}}}\right)^2}$$

$$v_3(x,y,t) = \frac{a_0 + \frac{2\sqrt{2w}\sqrt{\alpha}a_0}{\sqrt{c}\left(e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}\right)} + \frac{a_0 b_1}{\sqrt{e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}}\, b_0} + \frac{2\sqrt{2w}\sqrt{\alpha}a_0 b_1}{\sqrt{c}\left(e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}\right)^{3/2} b_0}}{b_0 + \frac{b_1}{\sqrt{e^{-\frac{\sqrt{2}\sqrt{c}(-ct+x)}{\sqrt{\alpha}}}EE - \frac{\sqrt{2w}\sqrt{\alpha}}{\sqrt{c}}}}}$$

Substituting Eq. (20) into Eq. (16), gives

(24)

$$u_4(x,y,t) = \frac{b_0\left(a_0 + \frac{2da_0}{\left(e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}\right)\sigma} + \frac{a_0 b_1}{\sqrt{e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}}\, b_0} + \frac{2da_0 b_1}{\left(e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}\right)^{3/2}\sigma b_0}\right)^2}{\sqrt{2d+ka_0}\left(b_0 + \frac{b_1}{\sqrt{e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}}}\right)^2}$$

$$v_4(x,y,t) = \frac{a_0 + \frac{2da_0}{\left(e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}\right)\sigma} + \frac{a_0 b_1}{\sqrt{e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}}\, b_0} + \frac{2da_0 b_1}{\left(e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}\right)^{3/2}\sigma b_0}}{b_0 + \frac{b_1}{\sqrt{e^{-2\sigma\left(x-\frac{\sqrt{2d+kta_0}}{b_0}\right)}EE - \frac{d}{\sigma}}}}$$

Choosing the suitable values of parameters, we performed the numerical simulations of the obtained solutions for (21) equation by plotting their 2D and 3D.

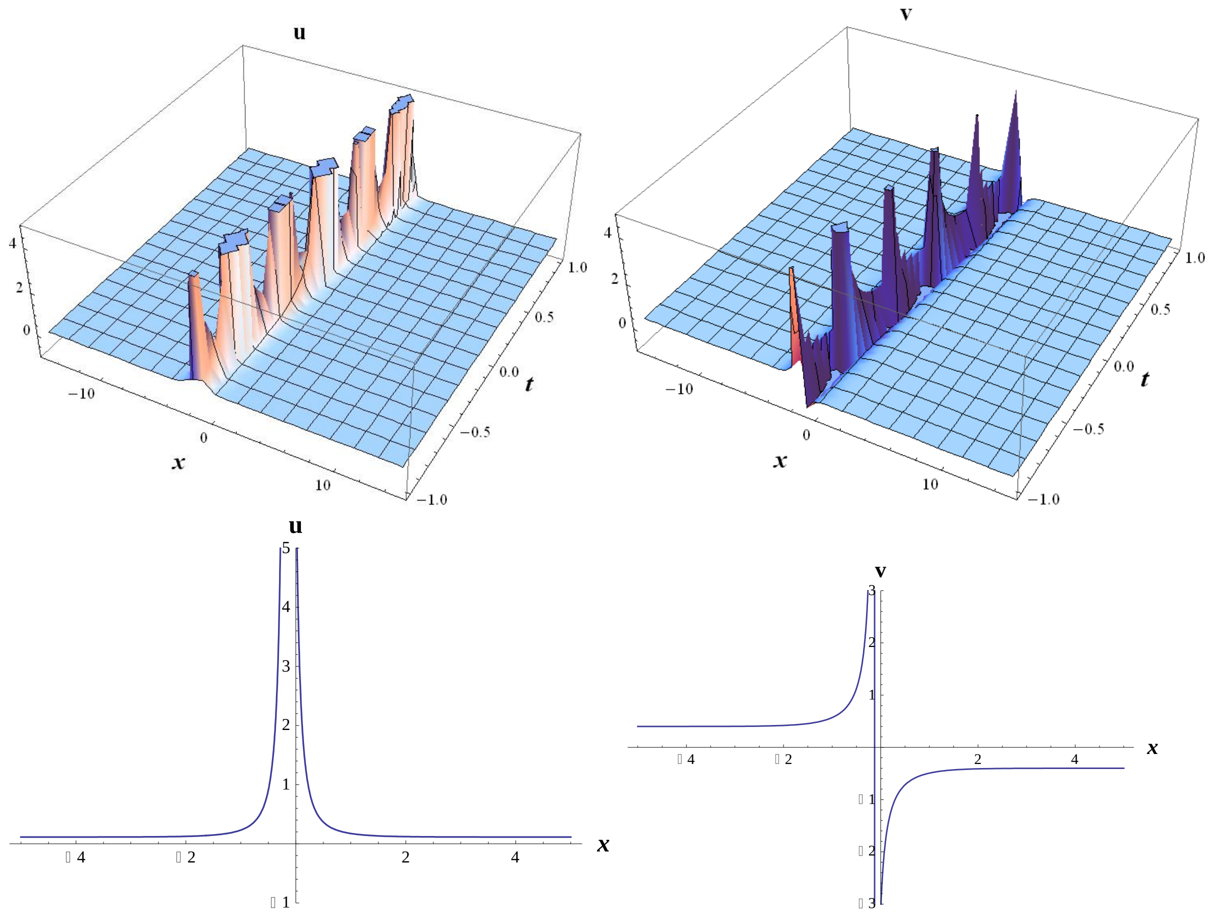

Figure-1 The 3D and 2Dsurfaces of the solution Eq.(21) for suitable values

CONCLUSIONS

In this article, new solutions are obtained for the Drinfeld-sokolov equation by using the IBSEFM method. We have seen that the results we obtained are new solutions when we compare them with previous ones. Our results might be useful in explaining the physical meaning of various nonlinear models arising in the field of nonlinear sciences. IBSEFM is powerful and efficient mathematical tool that can be used to handle various nonlinear mathematical models.

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