Using Conformable Fractional Derivative to Model the Capacitance Variation of XLPE During a Prolonged Cable Water Absorption Test

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Highlights

➢ Introduces the Conformable fractional derivative method to model insulator capacitance in power cables, demonstrating that equivalent capacitance evolves as a power function of time and effectively captures the dynamic effects of moisture intrusion.

➢ Demonstrates that the power law model outperforms the exponential model, with curve-fitted functions closely following experimental data across all measured voltages, thereby confirming the model's accuracy and offering a new perspective for reliable cable engineering applications.

Article Info	Abstract
	The study of water diffusion in cables has garnered significant attention due to its impact on the reliability and longevity of cable materials in diverse engineering applications. Moisture intrusion in a cable can result in changes in mechanical and electrical properties, making it essential to understand the underlying diffusion mechanisms. Traditional models often oversimplify this process, assuming homogeneity in time and space, and therefore fail to account for the nuanced behaviours observed in practice. This study addresses the Conformable fractional derivative approach to model the insulator capacitance of the power cable wrestling from the diffusion behaviour of water within cables during the water absorption test. The capacitance of the cable insulator due to water diffusion is evaluated as a process dependent on time. Equivalent capacitance is expressed as a power function of time. The accuracy, flexibility, and advantages of the conformable fractional derivative in modelling are implied. The accuracy of the model has been tested with experimental data. The fractional derivative offers a new perspective for better understanding and modelling the effect of the water absorption process on the insulator capacitance in power cable engineering applications.

Uzun Süreli Bir Kablo Su Emme Testi Sırasında XLPE'nin Kapasitans Değişimini Modellemek İçin Uyumlu Kesirli Türev Kullanımı

Makale Bilgileri	Öz
	Kablolardaki su difüzyonu çalışmaları, çeşitli mühendislik uygulamalarında kablo malzemelerinin güvenilirliği ve ömrü üzerindeki etkisi nedeniyle önemli ölçüde ilgi görmektedir. Bir kabloya nem girişi, mekanik ve elektriksel özelliklerinde değişimlere neden olabilir; bu da altta yatan difüzyon mekanizmalarının anlaşılmasını hayati kılar. Geleneksel modeller bu süreci genellikle zaman ve uzayda homojenlik varsayarak basitleştirir ve dolayısıyla uygulamada gözlemlenen karmaşık davranışları hesaba katmakta yetersiz kalır. Bu çalışma, kablolarda su emme testi sırasında meydana gelen difüzyon davranışından kaynaklanan yalıtkan kapasitansını modellemek için Uyumlu kesirli türev yaklaşımını ele almaktadır. Su difüzyonuna bağlı olarak kablo yalıtkanının kapasitansı zamana bağlı bir süreç olarak değerlendirilmiştir. Eşdeğer kapasitans zamanın bir üs fonksiyonu olarak ifade edilmiştir. Uyumlu kesirli türevin modellemedeki doğruluğu, esnekliği ve avantajları vurgulanmaktadır. Modelin doğruluğu, deneysel verilerle test edilmiştir. Kesirli türev yaklaşımı, güç kablosu mühendisliği uygulamalarında su emme sürecinin yalıtkan kapasitansına etkisini daha iyi anlamak ve modellemek için yeni bir bakış açısı sunmaktadır.

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1. Introduction

The cable Cables are essential components of electrical systems and must be tested in accordance with relevant standards (Moore, 1997; Thue, 2017). Marine cables are an important part of the cable market (Beyer, 2021; Worzyk, 2009). Marine cables are susceptible to aging, particularly due to the formation of water trees within the insulation material (Al-Arainy et al., 2008; "IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021; Karhan, Çakır, & Uğur, 2021; Priya & Mubashira Anjum, 2014). To ensure reliable operation, marine cables are tested according to standards such as IEEE 1580-2021, NEMA WC 53, and NEMA WC 57("IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021; "WC 53 standard," 2020; "WC 57 standard," 2021). The Accelerated Water Absorption test is conducted to evaluate how materials, particularly composite materials and cable insulation, absorb moisture over time ("IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021). This test is crucial because moisture absorption can degrade the strength and stiffness of materials, affecting their long-term performance ("WC 53 standard," 2020; "WC 57 standard," 2021). By accelerating the absorption process, researchers or users can predict how materials will behave in real-world conditions without waiting for extended periods (Beyer, 2021; Moore, 1997; Thue, 2017; "WC 53 standard," 2020; "WC 57 standard," 2021; Worzyk, 2009). For marine cables, this test ensures that insulation remains effective even in high-humidity environments, preventing electrical failures and maintaining safety standards ("IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021; "WC 53 standard," 2020; "WC 57 standard," 2021). Water absorption is commonly studied to understand the phenomenon better (Karhan, Cakir, & Ugur, 2020; Karhan, Uzunoğlu, ISSI, & UĞUR, 2017; Karhan, Yılmaz, & Uğur, 2017) and to mitigate it (Furuheim, Nilsson, Hvidsten, & Hellesø, 2013; Hellesø, Hvidsten, Balog, & Furuheim, 2011; Resner & Paszkiewicz, 2021). Water diffusion in marine cables occurs when moisture gradually penetrates the insulation material, potentially leading to electrical degradation over time. This process is influenced by factors such as temperature, pressure, and the permeability of the insulation. To mitigate water ingress, marine cables often incorporate radial water barriers or specialized polymer coatings that slow down diffusion and extend the cable's operational lifespan (Furuheim et al., 2013; Hellesø et al., 2011; Resner & Paszkiewicz, 2021). The Accelerated Water Absorption test is specifically defined in the IEEE 1580-2021 standard("IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021). According to Table 13 of the same standard, after removing the cable sheath, electrical and mechanical tests are conducted on the insulation material. The results from these tests are used to calculate the dielectric constant, the increase in capacitance, the stability factor, and the relative permeability of the insulator (Badmera & Patel, 2017; "IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021). These calculated values are then compared with the standard values provided in Table 13 to determine the test's success ("IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021). The note below Table 13 refers to Section 6.14 of the NEMA WC 57 standard, which outlines the test procedure ("WC 57 standard," 2021). This section also specifies that two alternative methods—electrical and gravimetric—may be used to perform the test. Among these, the electrical method is detailed in the IEEE 1580-2021 standard. Furthermore, NEMA WC 57 refers to additional calculations found in Section 2.6 of the NEMA WC 53 standard, thereby

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linking all three documents for comprehensive guidance("WC 53 standard," 2020).

The water diffusion process in marine cables can be modeled using the diffusion equation, specifically Fick's laws of diffusion (Helleso, Henoen, & Hvidsten, 2008; Nordas, Helleso, & Hvidsten, 2008). This equation describes how water molecules move through the insulation material due to concentration gradients (Helleso et al., 2008; Nordas et al., 2008). Solutions to the diffusion equations of different types can reveal how deeply a diffusant or a pollutant infiltrates a medium (Blazek, 2015). The water diffusion in a medium can also be modeled using a fractional diffusion equation (Atangana & Bildik, 2013; Baleanu, Agheli, & Al Qurashi, 2016; Bildik & Deniz, 2019; Bohaienko & Bulavatsky, 2018; Gülkaç, 2016; Mirza, Akram, Shah, Akhtar, & Muneer, 2021; Murio, 2008; Yao, Ren, & Gong, 2020; K. Zhang, 2019; Zhuang & Liu, 2007), which extends classical diffusion models by incorporating fractional-order derivatives (Baleanu et al., 2016; Murio, 2008; K. Zhang, 2019; Zhuang & Liu, 2007). Fractional Diffusion Equations considers Memory Effects, Nonlocality, and Anomalous Transport (Atangana & Bildik, 2013; Bildik & Deniz, 2019; Bohaienko & Bulavatsky, 2018; Mirza et al., 2021; Yao et al., 2020). These equations are particularly useful for describing anomalous diffusion, where the transport process deviates from classical Fickian behavior (Atangana & Bildik, 2013; Bildik & Deniz, 2019; Mirza et al., 2021; Yao et al., 2020).

The amount of diffused water within the cable insulation influences its capacitance (Badmera & Patel, 2017; "IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021). Capacitance in an insulated cable is primarily determined by the dielectric properties of the insulation material. When water diffuses into the insulation, it alters these properties in several ways:

1. Increase in Dielectric Constant: Water has a high dielectric constant (~80), much higher than typical insulation materials. As water permeates the insulation, the overall dielectric constant of the material increases, leading to higher capacitance.

2. Loss of Insulation Integrity: The presence of water can degrade the insulation, increase leakage currents, and affect the cable's electrical performance.

3. Frequency Dependence: The impact of water diffusion on capacitance can vary with frequency, as moisture-related changes in dielectric properties may introduce additional losses.

4. Water Treeing Effects: In high-voltage cables, prolonged exposure to moisture can lead to water treeing, which further alters capacitance and can eventually cause insulation failure.

To mitigate the effects of water diffusion in marine cables, engineers employ several strategies to maintain insulation integrity and electrical performance:

1. Water-Blocking Insulation: Specialized polymer coatings and cross-linked polyethylene (XLPE) are used to reduce water permeability and slow down diffusion.

2. Radial Water Barriers: Metallic layers, such as aluminum or lead sheaths, act as barriers to prevent moisture ingress.

3. Hydrophobic Materials: Insulation materials with hydrophobic properties repel water, minimizing absorption.

4. Gel-Filled Cables: Some cables incorporate water-blocking gels that prevent moisture from spreading within the insulation.

5. Periodic Testing and Monitoring: Accelerated water absorption tests and capacitance measurements help detect early signs of moisture-related degradation.

Fractional-order derivatives (FDs) have gained widespread use in engineering for modeling and control applications (Gutierrez, Rosário, & Tenreiro Machado, 2010; Kothari, Mehta, & Prasad, 2019). In recent years, fractional-order circuit components have emerged as a significant research focus (Elwakil, 2010; B. Zhang & Shu, 2022), particularly for their effectiveness in modeling specific supercapacitor behaviors(Freeborn, Elwakil, & Allagui, 2018; Freeborn, Maundy, & Elwakil, 2013, 2015; Hidalgo-Reyes, Gómez-Aguilar, Escobar-Jiménez, Alvarado-Martínez, & López-López, 2019; Kopka, 2017; Lewandowski & Orzyłowski, 2017; Ortigueira, Martynyuk, Kosenkov, & Batista, 2023). Studies on circuits incorporating fractional elements often utilize the Grünwald-Letnikov, the Caputo, and the Riemann–Liouville derivatives, as these formulations provide valuable insights into circuit theory and practical applications (Martínez, Rosales, Carreño, & Lozano, 2018; Piotrowska, 2018; Sikora, 2017). From a circuit theory standpoint, investigating oscillator circuits with novel fractional components such as capacitors and inductors is crucial. Different types of fractional derivatives exist, with the Riemann-Liouville and Caputo approaches being commonly employed to solve differential equations (Podlubny, 1998). In(Khalil, Al Horani, Yousef, & Sababheh, 2014), a new and simplified FD known as the conformable fractional derivative (CFD) was proposed. It is essentially an ordinary derivative multiplied by $t^{1-\alpha}$ where the order of the conformable fractional derivative is α, where $0 < \alpha \leq 1$ (Khalil et al., 2014). The CFD adopts the conventional limit definition of a derivative, making it more intuitive compared to traditional FD formulations. However, when compared to other fractional derivatives, CFD is not strictly fractional—it is essentially a first derivative scaled by a fractional power of the independent variable. This concept was further refined in (Abdeljawad, 2015) and it is more physically interpretable than other fractional derivatives (Zhao & Luo, 2017). Analytical studies on

electrical circuits incorporating capacitors modeled with CFDs have been conducted (Palaz & Mutlu, 2021a, 2021b, 2022; Piotrowska, 2018). Since CFD explicitly depends on time, exploring whether it can effectively represent capacitance variations due to water absorption may be a worthwhile investigation. Its DC and AC behavior are presented in (Palaz & Mutlu, 2021a). The energy of a capacitor modeled with the CFD is examined in (Palaz & Mutlu, 2022). A two-capacitor problem with a CFD and an LTI capacitor is inspected in (Palaz & Mutlu, 2021b).

In the literature, the conformable fractional derivative-based diffusion equations are also used to model water or pollutant diffusion within the cement, soil, etc. (Bayrak, Demir, & Ozbilge, 2023; Kim, Mert Coskun, Ordu, & Mutlu, 2024; Tajadodi, Khan, Gómez-Aguilar, Khan, & Khan, 2021; Zhou, Yang, & Zhang, 2018). The CFD has an explicit time dependency. Both the diffusion equation and the cable insulator capacitance variation may have the same time dependency, or the term $t^{(1-\alpha)}$ may link the diffused water and the capacitance to each other since the amount of water also defines the cable capacitance. It might be worth examining whether the capacitance variation due to water absorption can be modeled with CFD or not. To the best of our knowledge, the capacitance of a cable insulator has not been modeled using a CFD capacitor and an LTI capacitor in the literature yet. In this study, using the water chamber designed in (Ulutay, Yurtsever, Öztaş, Akyol, & Mutlu, 2024), the water absorption experiment for XLPE material has been made, and then, a CFD capacitor connected in series with an LTI capacitor has been used to model the electrical circuit of the insulator section of the cable, the equivalent capacitance of the cable insulator is calculated as a function of the time, and the numerical analysis is performed with Matlab™ to find the equivalent parameters. It has been examined whether the CFD capacitor-based equivalent model during the water absorption is applicable or not.

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This paper is as follows. The insulator examined, the water absorption chamber test system used in the experiments, the conformable fractional derivative, the CFD capacitor, and the equivalent electrical circuit of the insulator, the parameters of the circuit are presented in the second section. In the third section, the experimental results are given, and the parameters of the CFD capacitor have been calculated. The paper is finalized with the conclusions section.

2. Material and Method

2.1. The Conformal Fractional Derivative and the CFD-based Capacitor Constitutional Law

The conformable fractional derivative (CFD) is defined as

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}} = f'(t)t^{1-\alpha} = \frac{df(t)}{dt}t^{1-\alpha} \tag{1}$$

If a capacitor is modelled using CFD, its constitutive law is expressed as

$$i_{C_{\alpha}}(t) = C_{\alpha}\frac{d^{\alpha}v_{C_{\alpha}}(t)}{dt^{\alpha}} = C_{\alpha}\frac{dv_{C_{\alpha}}(t)}{dt}t^{1-\alpha} \tag{2}$$

where, $i_{C_{\alpha}}(t)$, $v_{C_{\alpha}}(t)$, and C_{α} are the CFD capacitor current, the CFD capacitor voltage, and the CFD capacitor coefficient.

The integer order (the traditional) diffusion equation which is a first order differential equation with respect to time is given as

$$D\Delta u = D\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{\partial u}{\partial t} \tag{3}$$

where u is the mass of water absorbed per unit volume of the polymer and D is the diffusion constant.

Considering radial symmetry of the cable, Eq. (3) is reduced to

$$D\Delta u = \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\partial u}{\partial t} \tag{4}$$

Its solution is given as the following infinite series:

$$u(r,t) = E_{\infty} + \sum_{1 \leq n \leq \infty} E_n J_0(\alpha_n r)e^{-D(\alpha_n)^2 t} \tag{5}$$

where α_n are eigenvalues determined by boundary conditions (e.g., zero at boundary radius), E_n are coefficients from initial conditions, and $J_0(\alpha_n r)$ is the Bessel function of the first kind.

In the paper (Gao & Zhou, 2019), instead of E, the symbol C is used. In this paper, not be confused with the capacitance, instead of C, E is used throughout the paper. The solution of radial water diffusion in polymers given in Eq. (5) is an infinite series and its terms can be found using fixed concentration at the surface of the insulator immersed under water. Each one of the terms has an exponential function depending on and decreasing with respect to time. The infinite sum of the terms expressed as $e^{-D(\alpha_n)^2 t}$ can be regarded as a measure of the water diffused within the cable and, therefore, the capacitance of the insulator depends on the sum. The coefficients of the sum are hard to find using small experimental data due to its infinite series nature. A new method which can be used with small data would be quite useful for evaluation of the experimental results in water immersion tests. Its solution in (Gao & Zhou, 2019) has been approximated as

$$u(r,t) = E_{\infty} - (E_{\infty} - E_0)e^{-\beta t} \tag{6}$$

where E_0 is the initial concentration of the volume within the polymer, and E_{∞} is the final concentration of the volume within the polymer, during the saturation stage.

Similarly, to Eq. (6), the capacitance increase of the cable is expressed as

$$C(t) = C_0 + \Delta C(1 - e^{-t/\tau}) = C_{\infty} + \Delta C e^{-t/\tau} \tag{7}$$

where $C(t)$ is the capacitance at time t, C_0 is the initial capacitance (in dry or air conditions), ΔC is the maximum possible increase in capacitance due to full water saturation, C_{∞} is the final capacitance, τ is the time constant (depends on insulation permeability, temperature, and water conductivity), and t It is time since the immersion.

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As done for pollutant transport within the soil in (Kim et al., 2024), due to its cylindrical topology, the conformable fractional diffusion equation governing water diffusion in the cable within the test chamber can be expressed as:

$$D_\alpha \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} \right) = \frac{\partial^\alpha u}{\partial t^\alpha} \tag{8}$$

where $\frac{\partial^\alpha u}{\partial t^\alpha}$ is the αth fractional derivative of the mass of water absorbed, and D_α is the fractional diffusion constant.

Using conformable fractional derivative, Eq. (8) turns into

$$D_\alpha \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} \right) = t^{1-\alpha} \frac{\partial u}{\partial t} \tag{9}$$

The term $t^{1-\alpha}$ can be regarded as a measure of the water diffused within the cable. The water has a higher electrical permittivity than the cable insulator and the water treeing channels have a lower volume than the rest of the insulator volume (Furuheim et al., 2013; Hellesø et al., 2011; Karhan, Uzunoğlu, et al., 2017; Priya & Mubashira Anjum, 2014). Therefore, the amount of water can be regarded as the defining parameter behind the permittivity change and, therefore, the capacitance increments in a cable. It may be possible that this time-dependent term $t^{1-\alpha}$ may appear within the measured capacitance.

Using the fractional diffusion, the capacitance model of the cable's insulator is shown in Figure 1. The power loss within the insulator of the cable is ignored in this model. The capacitance of the cable before water absorption is modeled with the constant capacitor C_0.
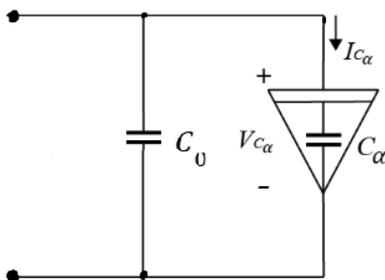

Figure 1. The cable insulator model.

The capacitance of the cable after the water immersion is given as

$$C(t) = C_0 + C_\alpha t^{1-\alpha} \tag{10}$$

where, C_0, C_α, and α are parameters to be found using the water-absorption experimental data.

C_0 is known due to its measurability before the test. At $t=0$, $C(0) = C_0$. The increase of capacitance due to the water absorption is expressed multiplying the fractional capacitance coefficient C_α with the term $t^{1-\alpha}$. The capacitance of the cable can also be given as

$$C(t) = C_0 + C_\alpha t^\beta \tag{11}$$

It is an important advantage of the CFD-based capacitor modeling that the power parameter α provides the accuracy and flexibility needed to obtain a power function. Let $C_d(t) = C(t) - C_0$ the differential capacitance which can be found by subtracting the capacitance before the absorption test from the capacitance during the absorption test.

$$C_d(t) = C(t) - C_0 = C_\alpha t^\beta \tag{12}$$

2.2 Overview of XLPE Material Used

In this study, cross-linked polyethylene (XLPE) was used for the evaluation of water absorption properties. XLPE is a highly durable material, particularly suited for electrical insulation in medium and high-voltage cables due to its excellent electrical, mechanical, and thermal properties. XLPE is commonly used in power cables, including medium and high-voltage cables, submarine cables, and underground cables, thanks to its excellent insulating properties and resistance to environmental degradation. Its high thermal stability and low water absorption make it particularly suitable for high-performance cable systems, where long-term reliability is critical. The cross-linking process, a key feature of XLPE, is achieved using peroxide-based catalysts, which initiate the formation of covalent bonds between polyethylene chains, enhancing the material's rigidity and heat resistance. This process

significantly improves the material's mechanical strength, chemical resistance, and dielectric properties, while also limiting water absorption, making it highly resistant to moisture and ensuring long-term durability under wet conditions. In comparison to EPR and XLPO, XLPE stands out for its superior thermal stability, lower water absorption, and long-term durability under both dry and wet conditions, making it the preferred choice for high-voltage power cables, submarine cables, and cables in humid environments. While XLPO offers good moisture resistance and flexibility, XLPE provides the most reliable performance in demanding environments, especially where high-temperature stability and long-term insulation integrity are required.

2.3 On the Water Absorption Test System

The procedures outlined in the Accelerated Water Absorption section of IEEE 1580-2021, as referenced in Table 13 of the NEMA WC 53 standard, are described in detail, including the relevant calculation methods. A low voltage XLPE cable is subjected to this test to evaluate its performance under accelerated water absorption conditions ("IEEE Recommended Practice for Marine Cable for Use on Shipboard and Fixed or Floating Facilities," 2021; "WC 53 standard," 2020; "WC 57 standard," 2021).

According to the IEEE 1580-2021 standard, a water chamber is required for the testing of marine cables. This chamber must be capable of maintaining the water temperature at 75 °C (Badmera & Patel, 2017; Kim et al., 2024).

The water absorption test for a marine power cable is conducted to determine the amount of water absorbed by the insulation upon exposure C (Badmera & Patel, 2017). To utilize the device in such a test, its performance must first be assessed. This study involves experiments to evaluate the functionality of the designed water absorption chamber. Before testing, the device is filled with tap water, and the chamber is positioned within the test room to ensure protection during high-voltage experiments. Consequently, no active or passive cooling occurs during the test. The test involves placing a 4.5-meter-long, 0.6/1 kV-rated marine cable—after the removal of its outer halogen-free thermoplastic sheath and armoring inside the chamber.

3. Experimental Results

XLPE material of a power cable is tested using the water absorption chamber shown in Figure 2. It has been found that it passes the tests described in IEEE 1580-2021, NEMA WC 53, and NEMA WC 57 standards considering the experimental data of two weeks. In the test, a 4.5-meter long 0.6/1 kV-rated marine cable after removing the outer halogen-free thermoplastic sheath and armoring is placed within the chamber as shown in Figure 2 and the device is turned on.
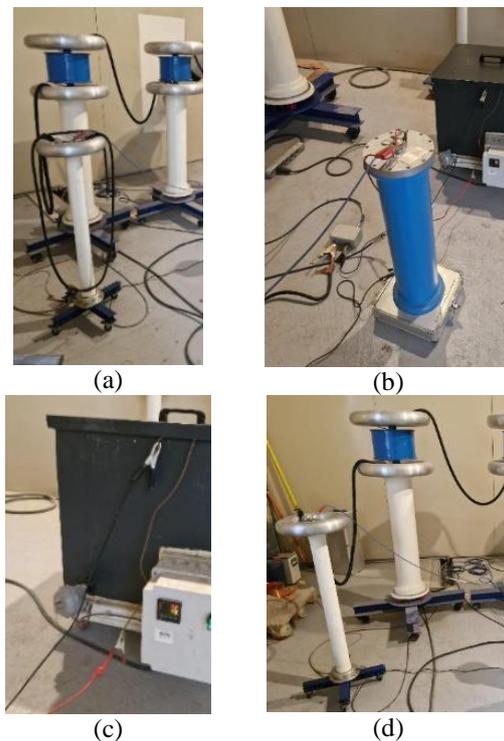
    (a)   (b)

    (c)   (d)

Figure 2. Photographs of the water absorption test system before and during the tests.

The water temperature of the chamber reaches 75 0C in almost 3 hours after being turned on and keeps its

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temperature constant at 75 °C (Ulutay et al., 2024), An impedance measurement system called HTM-30 C, L& tanδ Measuring Bridge is used to measure the cable's impedance, dissipation factor, capacitance, and relative permittivity by applying 1600 V, 3200 V, and 4800 V respectively to the insulator for 21 days consequently to see the water diffusion dynamics.

The measured capacitance values of XLPE material are presented in Table 1 and plotted in Figure 3. The capacitance increase is calculated using Eq. (7) and shown in Figure 4. Using the curve-fitting method, C_∞ and α parameters can be found for the data presented in Table 2.

Table 1. The Measured Capacitance of XLPE During 75 °C Water absorption test

| t (Day)	$C(t)$ [pF] measured with 1600 Volt	$C(t)$ [pF] measured with 3200 Volt	$C(t)$ [pF] measured with 4800 Volt
1	477,863	477,863	478,281
2	479,231	478,361	480,850
3	479,852	478,393	482,835
4	480,847	479,852	482,920
5	482,835	481,046	483,830
6	482,920	482,835	484,824
7	483,100	483,830	485,321
8	483,202	485,602	487,310
9	484,824	487,807	487,807
10	487,807	487,808	488,225
11	487,808	490,971	492,234
12	487,835	491,931	493,190
13	488,802	492,779	495,620
14	489,202	494,879	497,751
15	489,796	497,553	498,249
16	497,752	502,028	502,723
17	498,250	502,723	517,639
18	498,303	503,022	517,782
19	542,499	532,555	538,025
20	564,376	537,527	542,499
21	597,015	576,806	567,359
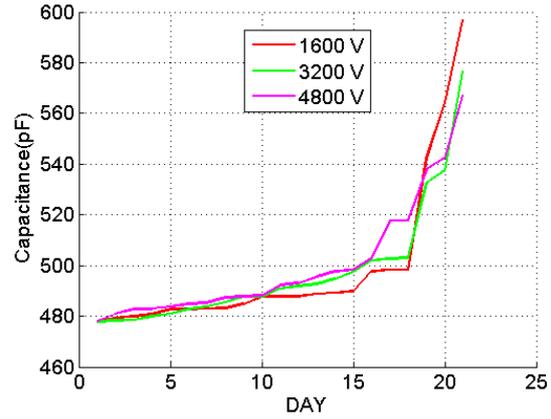
Figure 3. Variation of the Capacitance of XLPE versus time measured with 1600 V, 3200 V, and 4800 V.
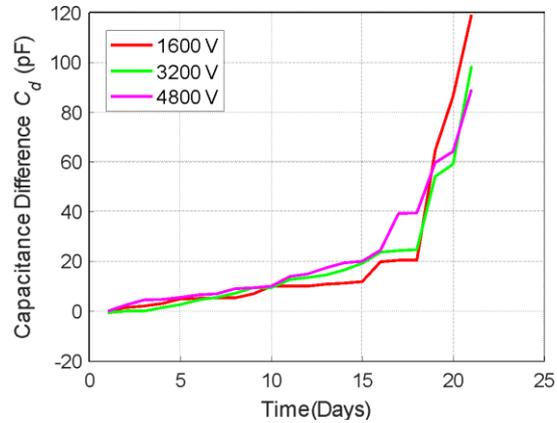

Figure 4. The capacitance increase of XLPE versus time measured with 1600 V, 3200 V, and 4800 V.

4. Results and Discussion

The capacitance increases slowly at the beginning, and it passes the absorption test. Then, a sudden increase in capacitance occurs after two weeks and passing the test criteria. If the experiment were just done for two weeks, the dynamics would be missed. Since the dataset represents capacitance measurements over 21 days, the analysis shifts from absorption behavior to electrical property variation in XLPE material. Capacitance behavior over time can be broken down to three phases as follows.

1. Initial Phase (Days 1–5): Capacitance starts at 477.863 pF and gradually increases to 482.835 pF. This suggests early-stage polarization effects, where absorbed water slightly alters the dielectric constant.

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2. Intermediate Phase (Days 6–15): Capacitance fluctuates between 483.100 pF and 489.796 pF, showing a gradual increase. The relatively small variation indicates that charge storage capacity is stabilizing, likely due to limited ionic movement in the polymer matrix.

3. Sharp Increase (Days 16–21): Capacitance jumps from 497.752 pF to 597.015 pF, indicating significant dielectric modifications. This could be caused by water diffusion into microstructural voids, increasing overall permittivity. The surge after day 16 might reflect ionic migration or conductive path formation, leading to electrical property degradation.

Key Insights can be articulated as follows:

- XLPE initially exhibits stable dielectric behavior, followed by progressive enhancement due to water ingress.

- The sudden increase beyond day 16 suggests possible breakdown mechanisms, such as increased leakage currents or permanent structural polarization.

- Further characterization like impedance spectroscopy or tan δ analysis can help determine if the polymer's dielectric integrity is compromised.

Using the least squares method, the parameters of the power function model $C_d(t) = C_\propto t^\beta$ are obtained for the three measurement voltages 1600 V, 3200 V, and 4800 V, respectively. The power functions found are given as follows.

$$C_{d1600V}(t) = 1.659x10^{-8}\ t^{7.457}\quad \text{for 1600 V}\qquad(13)$$

$$C_{d3200V}(t) = 1.642x10^{-5}\ t^{5.087}\quad \text{for 3200 V}\qquad(14)$$

$$C_{d4800V}(t) = 16.94x10^{-4}\ t^{3.541}\quad \text{for 4800 V}\qquad(15)$$

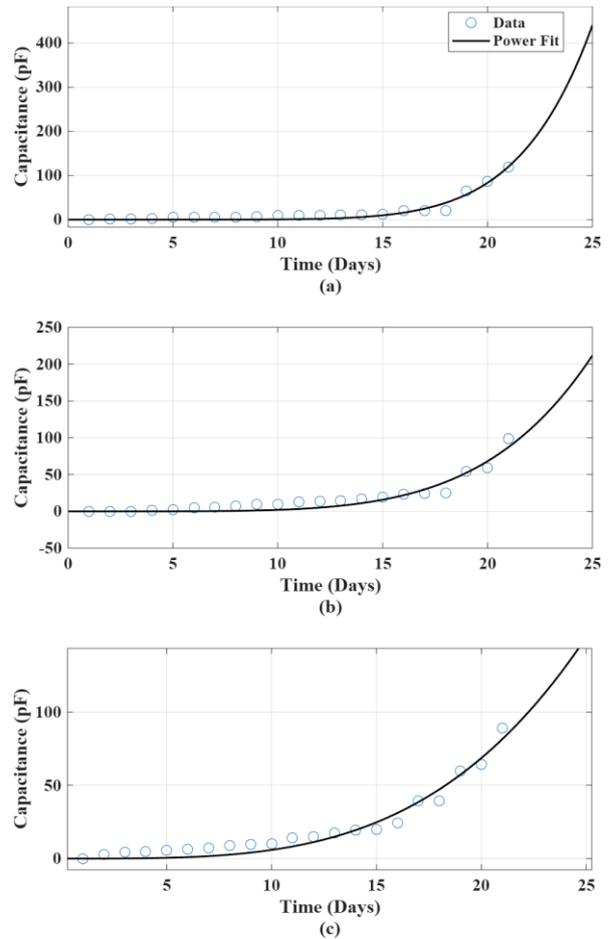The curves obtained using these functions are shown in Figure 5.

Figure 5. The curve-fitted capacitance increase using a power function for (a) 1600 V, (b) 3200 V, and (c) 4800 V.

The curve-fitting functions are evaluated by computing the sum of squares error (SSE), the coefficient of determination (R^2), and the root mean square error (RMSE). The coefficient of determination ranges from 0 to 1. An R^2 of 0 means the model explains none of the variability of the response data around its mean. An R^2 of 1 means the model explains all the variability—i.e., a perfect fit. In practical terms: $R^2 > 0.9$ is considered excellent, especially in physics or engineering fields.[10²]

Table 2. The performance of the curve-fitted power law and exponential functions

Model	The Measurement Voltage	SSE	R^2	RMSE
The power function	1600 V	909.45	0.952	6.918
The power function	3200 V	939.82	0.919	7.033
The power function	4800 V	480.49	0.956	5.028
The exponential function	1600 V	1.088e4	0.432	23.93
The exponential function	3200 V	5227	0.554	16.59
The exponential function	4800 V	3824	0.656	14.19

The parameters of the exponential model given by E. (7), $C_d(t) = \square(1 - e^{-bt})$ are obtained using the least squares method, for the three measurement voltages 1600 V, 3200 V, and 4800 V, respectively. The exponential functions found are given as follows.

$$C_{d1600V}(t) = 9437(1 - e^{-0.000241\,\square}) \text{ for 1600 V}$$

$$(16)$$

$$C_{d3200V}(t) = 5079\,(1 - e^{-0.0004081\,\square}) \text{ for 3200 V}$$

$$(17)$$

$$C_{d4800V}(t) = 1.112\text{x}10^4\,(1 - e^{-0.0002068\,\square})$$

for 4800 V

$$(18)$$

To compare the performance of both of these models, the functions given by Eq.s (13)-(18) are plotted and shown in Figure 6.

The comparison with the traditional model is also presented both in tables and in graphs. The provided graphs given in Figure 6 and the data in Tables 2-4 show the power law model is more accurate than the exponential model and the curve-fitted function follows the experimental data better for all the measured voltages.
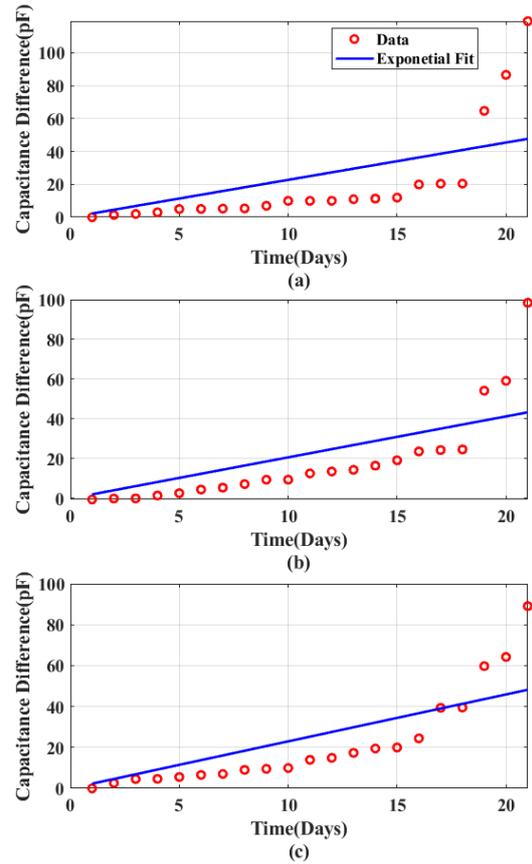
Figure 6. The curve-fitted capacitance increase using an exponential function for (a) 1600 V, (b) 3200 V, and (c) 4800 V.

Coefficients of the curve-fitted functions with 95% confidence intervals of the power and the exponential functions are given in Tables 3 and 4, respectively. The low variation of the interval of the coefficients of the power law function given in Table 3 and the high variation of the interval of the coefficients of the exponential function given in Table 4 show that the power function performs better than the exponential function as also the graphics confirms.

The statistical metrics (SSE, R^2, RMSE) are also used for the exponential model, and the values shown in Table 2 shows that the power function model performs better since its SSE and RMSE is lower and its R^2 is higher than that of the exponential function.

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Table 3. The parameters and confidence intervals of the power law functions curve-fitted.

Model	The Measure-ment Voltage	$C_\propto$	$\propto$
The power function	1600 V	1.659e-8 (-5.485e-8, 8.803e-8)	7.457 (6.023, 8.891)
	3200 V	1.642e-5 (-4.35e-5, 7.634e-5)	5.087 (3.861, 6.312)
	4800 V	0.001694 (-0.001267, 0.004655)	3.541 (2.948, 4.135)

Table 4. The parameters and confidence intervals of the exponential functions curve-fitted.

Model	The Measure-ment Voltage	a	b
The expo-nential function	1600 V	9437 (-7.21e+6, 7.229e+6)	0.000241 (-0.1845, 0.1849)
The expo-nential function	3200 V	5079 (-1.741e+6, 1.751e+6)	0.0004081 (-0.1403, 0.1412)
The expo-nential function	4800 V	1.112e+04 (-5.798e+6, 5.821e+6)	0.0002068 (-0.1081, 0.1085)

The proposed CFD-based model offers a significant advancement in accurately predicting the capacitance behavior of power cables under water immersion conditions. This enhanced predictive capability directly translates into more reliable estimations of cable lifespan, particularly in moisture-prone environments. By capturing subtle dielectric variations over time, the model enables early detection of insulation degradation and latent faults—well before conventional testing methods signal failure. Furthermore, it may serve as a robust supplementary tool to standard IEC and IEEE testing protocols, offering a non-invasive, model-based diagnostic layer that can reduce maintenance costs and improve operational safety. Its integration into cable monitoring systems could thus provide utilities and manufacturers with a proactive asset management strategy, extending service life and minimizing unexpected downtimes.

5. Conclusion

In this study, the water absorption of XLPE material is examined considering its capacitance variation. The data has been measured for 21 days to see the absorption dynamics. A capacitive network with a time-dependent capacitor is used to model the measured cable capacitance profile. We have used a capacitor model based on the conformable fractional derivative to model the effect of the water absorption process. In our opinion, this indicates that it is possible to model water diffusion within the power cables with fractional derivatives using CFD or other types of fractional derivatives and we suggest it as future work. The system is nonlinear since the measured capacitance is voltage-dependent. The paper is also a first in the literature presenting a physically existing CFD-modelled capacitor in the water immersion test. It has been found that the CFD-based model can predict the measured capacitance with a very high R² value higher than 0.9 considering all the experimental values, i.e., The accuracy of the model has been tested with experimental data. Also, the comparison made with the traditional model shows that the CFD-based model is more accurate considering statistical metrics such as SSE, R², and RMSE.

Capacitance per unit length of a cable can also be measured using a TDR device. Since the inductance per unit length of a cable stays constant and does not depend on the water diffused into the cable, by measuring the speed of propagation within the cable, the amount of the capacitance change would give the amount of the diffusion and the CFD-based model can be used to predict when the amount of the water is going to be in the unacceptable level. In our opinion, at least three measurements are needed to find the parameters considering the CFD-based model and, using the measurements, the failure time or perhaps the life of the cable can be predicted under heavy operation mediums.

In (Karakulak & Mutlu, 2023), an LTspice model of the capacitor modeled with the CFD is given. Such a model can be modified for the simulation of the dynamics of the water absorption phenomenon as future work.

The parameters of the circuit are calculated using the least-squares method. Usage of an Excel Spreadsheet has been suggested for the preparation of a Thermal Endurance Test Report (Tan, Önder, Yerişenoğlu, & Mutlu, 2023). A similar Excel Spreadsheet can be prepared to find the parameters of the CFD-capacitor-based model using the least-squares method. since Excel is not an expensive program.

Water diffusion within cables plays a critical role in determining their long-term performance and durability, particularly in applications where exposure to moisture is inevitable. Traditional diffusion models often assume homogeneity in both time and space, failing to capture the complex, heterogeneous nature of water transport processes observed in real-world scenarios. To address this limitation, the concept of conformable fractional derivatives emerges as a powerful and flexible mathematical tool, enabling a more accurate representation of diffusion behavior in our opinion. We also suggest modeling water diffusion or water concentration in polymers within the cables using CFD or other types of fractional derivatives as future work. Unlike traditional models, water diffusion may also be evaluated as a process with heterogeneous properties in both time and space. The nonlinear effect of the test voltage on the diffusion mechanism and the measured capacitance can also be examined from the first principles in the future. If the fractional diffusion behavior is examined in more detail, this approach may not only enhance the accuracy of diffusion models but also open up new possibilities for optimizing cable design and improving engineering practices.

Declaration of Competing Interest

The authors declared no conflicts of interest concerning the research, authorship, and/or publication of this article.

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