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Soft Intersection-plus Product of Groups

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Abstract

Soft set theory constitutes a mathematically rigorous and algebraically expansive framework for representing and analyzing systems permeated by epistemic uncertainty, vagueness, and parameter-contingent variability—hallmarks of foundational problems in decision sciences, engineering, economics, and information theory. Within this algebraic landscape, we formally introduce and investigate a novel binary operation—termed the soft intersection–plus product—defined on soft sets whose parameter sets are structured as groups. The operation is rigorously developed within an axiomatic framework that guarantees compatibility with generalized notions of soft subsethood and soft equality, thereby maintaining the algebraic integrity of the induced structure. A meticulous algebraic analysis is carried out to establish key structural attributes of the operation, including closure, associativity, commutativity, idempotency, and distributivity over other soft set operations, as well as its behavior with respect to identity and absorbing elements, and its interaction with the null and absolute soft sets. Our findings reveal that the soft intersection–plus product not only conforms to the algebraic constraints imposed by group-parameterized domains but also induces a well-behaved and internally coherent algebraic system over the soft set space. Two principal contributions emerge from this investigation: (i) the integration of the proposed product enhances the internal algebraic cohesion of soft set theory by embedding it within a formally consistent, axiom-preserving framework; and (ii) it serves as a foundational component for the development of a generalized soft group theory. Beyond its abstract significance, the proposed operation offers a robust mathematical basis for the design of soft computational models governed by algebraic principles, with prospective applications in multi-criteria decision-making, algebraic classification frameworks, and uncertainty-aware data analysis across group-parameterized semantic environments. As such, the formal apparatus developed herein not only expands the theoretical frontier of soft algebra but also affirms its relevance in both pure mathematics and applied analytical disciplines.

Keywords Soft sets; Soft subsets; Soft equalities; Soft intersection–plus product.

1. Introduction

A wide spectrum of mathematically sophisticated frameworks has been developed to model and analyze phenomena governed by uncertainty, vagueness, and indeterminacy—conditions frequently encountered across domains such as engineering, economics, social sciences, and medical diagnostics. Despite their conceptual richness, classical paradigms such as fuzzy set theory and probabilistic models exhibit fundamental epistemological and algebraic limitations. Specifically, fuzzy set theory, as formulated by Zadeh (1965), relies on the subjective assignment of membership functions, while probabilistic frameworks presuppose the existence of repeatable experiments and well-defined distributional structures—assumptions that are often untenable in real-world environments marked by epistemic ambiguity or non-replicability.

In response to these constraints, Molodtsov (1999) introduced soft set theory as an axiomatically minimal yet structurally flexible formalism, wherein uncertainty is represented via parameter dependence rather than probabilistic likelihoods or fuzzy grades. Since its inception, the algebraic structure of soft set theory has been significantly enriched. Foundational operations such as union, intersection, and AND/OR-products were first introduced by Maji et al. (2003) and subsequently reconceptualized by Pei and Miao (2005) from an information-theoretic standpoint to facilitate applications in relational and multivalued settings. Ali et al. (2009) advanced this operational schema by defining restricted and extended variants of core operations, thereby increasing the granularity and expressive precision of soft algebraic systems. A broad corpus of subsequent investigations—including Yang (2008), Feng et al. (2010), Jiang et al. (2010), Ali et al. (2011), Neog and Sut (2011), Fu (2011), Ge and Yang (2011), Singh and Onyeozili (2012a,2012b,2012c, 2012d), Zhu and Wen (2013), Onyeozili and Gwary (2014), and Sen (2014) have further clarified semantic ambiguities, introduced generalized equality relations, and defined novel binary soft products, thereby deepening the formal algebraic infrastructure of soft set theory. More recently, significant progress has been achieved through the introduction of a wide array of novel operations rigorously analyzed within formal algebraic frameworks. Notable among these are contributions by Eren and Çalışıcı (2019), Stojanović (2021), Sezgin et al. (2023a,2023b), Sezgin and Aybek (2023), Sezgin and Dağtoros (2023), Sezgin and Demirci (2023), Sezgin and Çalışıcı (2024), Sezgin and Yavuz (2023a, 2023b; 2024), Sezgin and Çağman (2024, 2025), Sezgin and Sarıalioğlu (2024a, 2024b), and Sezgin and Şenyiğit (2025) whose work have established a robust, extensible, and internally consistent algebraic landscape for soft set theory.

One pivotal axis of this development has been the formalization and generalization of soft subsethood and soft equality. The foundational notion of soft subsets introduced by Maji et al. (2003) was extended by Pei and Miao (2005) and Feng et al. (2010), while Qin and Hong (2010) introduced soft congruences that embedded equivalence relations into the soft set universe. Jun and Yang (2011) refined the theoretical landscape with the introduction of J-soft equalities and associated distributive identities, and Liu et al. (2012) developed the notions of L-soft subsets and L-equalities, thereby revealing the breakdown of classical distributive laws within generalized soft contexts. Feng and Li (2013) provided a systematic typology of soft subsets under L-equality and demonstrated associativity, commutativity, and distributivity properties within certain quotient structures that yield commutative semigroup behavior. Broader generalizations—such as g-soft, gf-soft, and T-soft equalities—have been explored by Abbas et al. (2014, 2017), Al-shami (2019), and Al-shami and El-Shafei (2020), introducing congruence-theoretic and lattice-enriched interpretations of soft algebraic systems.

A significant redefinition of the theoretical foundation was undertaken by Çağman and Enginoğlu (2010), who eliminated inconsistencies in the original formulation and established a coherent operational calculus. Parallel research streams have investigated soft binary products over algebraic structures. For example, the soft intersection–union product has been extended to rings (Sezer, 2012), semigroups (Sezgin, 2016), and groups (Muştuoğlu et al., 2016), leading to the construction of algebraic entities such as soft union rings, semigroups, and groups. Conversely, the soft union–intersection product has been systematically explored within group-theoretic (Kaygısız, 2012), semigroup-theoretic (Sezer et al., 2015), and ring-theoretic (Sezgin et al., 2017) frameworks, with the algebraic properties of the resulting structures shown to depend critically on the presence of identity and inverse elements in the parameter domain.

Building upon this foundational corpus, the present study introduces a novel binary operation—the soft intersection–plus product—defined over soft sets indexed by parameter sets possessing group-theoretic structure. The operation is rigorously axiomatized and subjected to exhaustive algebraic analysis. We formally establish its closure, associativity, commutativity, idempotency, and distributivity, and examine its interactions with identity and absorbing elements. Furthermore, we verify its compatibility with generalized soft subsethood and soft equality, ensuring that it integrates seamlessly into the broader algebraic apparatus of soft set theory. A comparative evaluation is also undertaken with respect to previously defined soft binary operations, with particular attention to expressive capacity and algebraic coherence across soft subset classifications. In addition, the behavior of the product in relation to the null and absolute soft sets is explicitly characterized. Our theoretical findings demonstrate that the soft intersection–plus product satisfies key axiomatic properties and enables the coherent aggregation of soft information across group-structured parameter domains. This construction provides a principled generalization of classical group-theoretic ideas to the soft set context and lays the algebraic groundwork for a generalized soft group theory founded on rigorously defined binary operations. The remainder of the paper is organized as follows: Section 2 presents essential preliminaries and foundational algebraic definitions. Section 3 introduces the soft intersection–plus product and develops its algebraic theory in detail. Section 4 consolidates the principal results and outlines avenues for future research aimed at deepening the algebraic formalism of soft sets and extending their applications in abstract algebra and uncertainty quantification.

2. Preliminaries

This section presents a rigorous and systematic re-articulation of the foundational definitions and algebraic postulates that serve as the formal substratum for the theoretical framework elaborated in the subsequent sections. Soft set theory, originally introduced by Molodtsov (1999), was conceived as a parameterized formalism for modeling systems characterized by epistemic uncertainty and contextual indeterminacy. However, the initial formulation lacked the algebraic precision required for robust theoretical development. In response, the axiomatic refinement proposed by Çağman and Enginoğlu (2010) marked a pivotal advancement, rectifying inherent structural inconsistencies and introducing a logically coherent and algebraically tractable framework. Their reformulation not only endowed the theory with enhanced formal integrity but also significantly extended its applicability to a broad spectrum of mathematical and computational domains, including algebraic systems, decision theory, and soft computation. The present study adopts this refined axiomatic model as its foundational basis. Consequently, all ensuing algebraic constructions, operational definitions, and theoretical generalizations are rigorously developed within the logical boundaries established by this enhanced formulation. This adherence ensures internal consistency, structural fidelity, and full alignment with contemporary standards in the algebraic theory of soft systems. Throughout the remainder of this manuscript, all references to soft sets, soft operations, and their associated algebraic behaviors are to be interpreted in the context of this revised formalism unless explicitly stated otherwise.

Definition 2.1. (Çağman and Enginoğlu, 2010) Let E be a parameter set, U be a universal set, $P(U)$ be the power set of U, and $\mathcal{H} \subseteq E$. Then, the soft set $f_{\mathcal{H}}$ over U is a function such that $f_{\mathcal{H}}: E \to P(U)$, where for all $w \notin \mathcal{H}$, $f_{\mathcal{H}}(w) = \emptyset$. That is,

$$f_{\mathcal{H}} = \{(w, f_{\mathcal{H}}(w)) : w \in E\}$$

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2.2. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. If $f_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $f_{\mathcal{H}}$ is called a null SS and indicated by $\emptyset_E$, and if $f_{\mathcal{H}}(w) = U$, for all $w \in E$, then $f_{\mathcal{H}}$ is called an absolute SS and indicated by U_E.

Definition 2.3. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. If $f_{\mathcal{H}}(w) \subseteq g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is a soft subset of $g_{\aleph}$ and indicated by $f_{\mathcal{H}} \tilde{\subseteq} g_{\aleph}$. If $f_{\mathcal{H}}(w) = g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is called soft equal to $g_{\aleph}$, and denoted by $f_{\mathcal{H}} = g_{\aleph}$.

Definition 2.4. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, the intersection of $f_{\mathcal{H}}$ and $g_{\aleph}$ is the $\mathcal{SS}$ $f_{\mathcal{H}} \tilde{\cap} g_{\aleph}$, where $(f_{\mathcal{H}} \tilde{\cap} g_{\aleph})(w) = f_{\mathcal{H}}(w) \cap g_{\aleph}(w)$, for all $w \in E$.

Definition 2.5. (Çağman and Enginoğlu, 2010) Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. Then, the complement of $f_{\mathcal{H}}$ denoted by ${f_{\mathcal{H}}}^{c}$, is defined by the soft set ${f_{\mathcal{H}}}^{c}: E \to P(U)$ such that ${f_{\mathcal{H}}}^{c}(e) = U \setminus f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 2.6. (Sezgin et al., 2025b) Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-subset of $g_{\aleph}$, denoted by $f_K \tilde{\subseteq}_S g_{\aleph}$, if for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two SSs f_K and $g_{\aleph}$ are said to be soft S-equal, denoted by $f_K =_S g_{\aleph}$, if $f_K \tilde{\subseteq}_S g_{\aleph}$ and $g_{\aleph} \tilde{\subseteq}_S f_K$.

It is obvious that if $\mathscr{f}_K =_S \mathscr{g}_\aleph$, then $\mathscr{f}_K$ and $\mathscr{g}_\aleph$ are the same constant functions, that is, for all $w \in E$, $\mathscr{f}_K(w) = \mathscr{g}_\aleph(w) = \mathcal{M}$, where $\mathcal{M}$ is a fixed set .

Definition 2.7. (Sezgin et al., 2025b) Let $\mathscr{f}_K$ and $\mathscr{g}_\aleph$ be two $\mathcal{SS}$s. Then, $\mathscr{f}_K$ is called a soft A-subset of $\mathscr{g}_\aleph$, denoted by $\mathscr{f}_K \tilde{\subseteq}_A \mathscr{g}_\aleph$, if, for each $a, b \in E$, $\mathscr{f}_K(a) \subseteq \mathscr{g}_\aleph(b)$.

Definition 2.8. (Sezgin et al., 2025b) Let $\mathscr{f}_K$ and $\mathscr{g}_\aleph$ be two $\mathcal{SS}$s. Then, $\mathscr{f}_K$ is called a soft S-complement of $\mathscr{g}_\aleph$, denoted by $\mathscr{f}_K =_S (\mathscr{g}_\aleph)^c$, if, for all $w \in E$, $\mathscr{f}_K(w) = \mathcal{M}$ and $\mathscr{g}_\aleph(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = U \setminus \mathcal{D}$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all $\mathcal{SS}$s over U, whose parameter sets are G; that is, each element of $S_G(U)$ is an $\mathcal{SS}$ parameterized by G.

Definition 2.9. (Sezgin and Durak, 2025) Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Then, the soft union-difference product $\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G$ is defined by

$$(\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G)(x) = \bigcup_{x=yz} \left(\mathscr{f}_G(y) \setminus \mathscr{g}_G(z)\right), \qquad y, z \in G$$

for all $x \in G$.

For additional information on SSs, we refer to Aktas and Çağman (2007), Alcantud et al. (2024), Ali et al. (2015), Ali et al. (2022), Atagün et al. (2019), Atagün and Sezgin (2015), Atagün and Sezer (2015), Atagün and Sezgin (2017), Atagün and Sezgin (2018), Atagün and Sezgin (2022), Feng et al. (2008), Gulistan and Shahzad (2014), Gulistan et al. (2018), Jana et al. (2019), Karaaslan (2019), Khan et al. (2017), Mahmood et al. (2015), Mahmood et al. (2018), Manikantan et al. (2023), Memiş (2022), Özlü and Sezgin (2020), Riaz et al. (2023), Sezer and Atagün (2016), Sezer et al. (2017), Sezer et al. (2013), Sezer et al. (2014), Sezgin et al. (2019a, 2019b), Sezgin and İlgin (2024a, 2024b), Sezgin et al. (2022), Sezgin and Onur (2024), Sezgin et al. (2024a,2024b), Sezgin and Orbay (2022), Sezgin et al. (2025a), Sun et al. (2008), Tunçay and Sezgin (2016), Ullah et al. (2018).

3. Soft Intersection-Plus Product of Groups

In this section, we introduce and formally define a novel binary operation on soft sets—termed the soft intersection–plus product—which is constructed over parameter domains endowed with an intrinsic group-theoretic structure. The operation is subjected to a comprehensive algebraic investigation aimed at delineating its fundamental structural properties, including closure, associativity, commutativity, idempotency, and its adherence to generalized soft equality and soft subsethood relations. Particular emphasis is placed on examining the product's behavior within established soft inclusion hierarchies and its structural alignment with the broader algebraic taxonomy of soft operations. The analysis is further enriched by a comparative study of the product's interactions with preexisting soft binary operations, thereby illuminating its integrability and algebraic coherence within the existing operational lattice. To substantiate the theoretical exposition, a series of illustrative examples is constructed, each tailored to expose nontrivial operational dynamics and reveal nuanced algebraic phenomena inherent in the proposed formulation. Collectively, these findings affirm that the soft intersection–plus product is structurally sound, algebraically expressive, and capable of serving as a core constituent in the algebraic generalization and deepening of soft set theory.

Definition 3.1. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s over U. Then, the soft intersection-plus product $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G$ is defined by

$$(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G(y) + \mathscr{g}_G(z)\right) = \bigcap_{x=yz} \left(\left(\mathscr{f}_G(y)\right)' \cup \mathscr{g}_G(z)\right), \qquad y, z \in G$$

for all $x \in G$.

Note here that since G is a group, there always exist $y, z \in G$ such that $= yz$, for all $x \in G$. Let the order of the group G be n, that is, $|G| = n$. Then, it is obvious that there exist n distinct representations combinations for each $x \in G$ such that $x = yz$, where $y, z \in G$. Besides, for more on plus $(+)$ operation of sets, we refer to Sezgin et al. (2023c).

Note 3.2. The soft intersection-plus product is well-defined in $S_G(U)$. In fact, let f_G, g_G, σ_G, $k_G \in S_G(U)$ such that $(f_G, g_G) = (\sigma_G, k_G)$. Then, $f_G = \sigma_G$ and $g_G = k_G$, implying that $f_G(x) = \sigma_G(x)$ and $g_G(x) = k_G(x)$ for all $x \in G$. Thereby, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{i/p} g_G)(x) &= \bigcap_{x=yz} \left(f_G{}^c(y) \cup g_G(z)\right)\\ &= \bigcap_{x=yz} \left(\sigma_G{}^c(y) \cup k_G(z)\right)\\ &= (\sigma_G \otimes_{i/p} k_G)(x)\end{aligned}$$

Hence, $f_G \otimes_{i/p} g_G = \sigma_G \otimes_{i/p} k_G$.

Example 3.3. Consider the group $\mathrm{G} = \{\mathfrak{q}, \mathfrak{b}\}$ with the following operation**:**

$\cdot$	$\mathfrak{q}$	$\mathfrak{b}$
$\mathfrak{q}$	$\mathfrak{q}$	$\mathfrak{b}$
$\mathfrak{b}$	$\mathfrak{b}$	$\mathfrak{q}$

Let f_G and g_G be two SSs over $U = D_2 = \{< x, y >: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\mathfrak{q}, \{e, y\}), (\mathfrak{b}, \{yx\})\} \text{ and } g_G = \{(\mathfrak{q}, \{x, y\}), (\mathfrak{b}, \{e, x\})\}$$

Since $\mathfrak{q} = \mathfrak{q}\mathfrak{q} = \mathfrak{b}\mathfrak{b}$, $(f_G \otimes_{i/p} g_G)(\mathfrak{q}) = \left(f_G{}^c(\mathfrak{q}) \cup g_G(\mathfrak{q})\right) \cap \left(f_G{}^c(\mathfrak{b}) \cup g_G(\mathfrak{b})\right) = \{x, y\}$ and since $\mathfrak{b} = \mathfrak{q}\mathfrak{b} = \mathfrak{b}\mathfrak{q}$, $(f_G \otimes_{i/p} g_G)(\mathfrak{b}) = \left(f_G{}^c(\mathfrak{q}) \cup g_G(\mathfrak{b})\right) \cap \left(f_G{}^c(\mathfrak{b}) \cup g_G(\mathfrak{q})\right) = \{e, x\}$ is obtained. Hence,

$$f_G \otimes_{i/p} g_G = \{(\mathfrak{q}, \{x, y\}), (\mathfrak{b}, \{e, x\})\}$$

Proposition 3.4. The set $S_G(U)$ is closed under the soft intersection-plus product. That is, if f_G and g_G are two SSs, then so is $f_G \otimes_{i/p} g_G$.

PROOF. It is obvious that the soft intersection-plus product is a binary operation in $S_G(U)$**.** Thereby, $S_G(U)$ is closed under the soft intersection-plus product.

Proposition 3.5. The soft intersection-plus product is not associative in $S_G(U)$

PROOF. Let Consider the SSs f_G and g_G over $U = \{e, x, y, yx\}$ in Example 3.3. Let $\hbar_G = \{(\mathfrak{q}, \{x\}), (\mathfrak{b}, \{y, yx\})\}$ be an SS over U. Since $f_G \otimes_{i/p} g_G = \{(\mathfrak{q}, \{x, y\}), (\mathfrak{b}, \{e, x\})\}$, then

$$(f_G \otimes_{i/p} g_G) \otimes_{i/p} \hbar_G = \{(\mathfrak{q}, \{yx\}), (\mathfrak{b}, \{y, yx\})\}$$

Moreover, since $g_G \otimes_{i/p} \hbar_G = \{(\mathfrak{q}, \{yx\}), (\mathfrak{b}, \{y, yx\})\}$, then

$$f_G \otimes_{i/p} (g_G \otimes_{i/p} \hbar_G) = \{(\mathfrak{q}, \{x, yx\}), (\mathfrak{b}, \{x, y, yx\})\}$$

Thereby, $(\mathfrak{f}_G \otimes_{i/p} \mathfrak{g}_G) \otimes_{i/p} \hbar_G \neq \mathfrak{f}_G \otimes_{i/p} (\mathfrak{g}_G \otimes_{i/p} \hbar_G)$.

Proposition 3.6. The soft intersection-plus product is not commutative in $S_G(U)$.

PROOF. Consider the SSs $\mathfrak{f}_G$ and $\mathfrak{g}_G$ over $U = \{e, x, y, yx\}$ in Example 3.3. Then,

$$\mathfrak{f}_G \otimes_{i/p} \mathfrak{g}_G = \{(Ձ, \{x, y\}), (б, \{e, x\})\}, \text{ and } \mathfrak{g}_G \otimes_{i/p} \mathfrak{f}_G = \{(Ձ, \{y, yx\}), (б, \{e, yx\})\}$$

implying that $\mathfrak{f}_G \otimes_{i/p} \mathfrak{g}_G \neq \mathfrak{g}_G \otimes_{i/p} \mathfrak{f}_G$.

Proposition 3.7. U_G is the right absorbing element of the soft intersection-plus product in $S_G(U)$.

PROOF. Let $x \in G$. Then,

$$\begin{aligned}(\mathfrak{f}_G \otimes_{i/p} U_G)(x) &= \bigcap_{x=yz} \left(\mathfrak{f}_G{}^c(y) \cup U_G(z)\right) \\ &= \bigcap_{x=yz} (\mathfrak{f}_G{}^c(y) \cup U) \\ &= U_G(x)\end{aligned}$$

for all $x \in G$. Thus, $\mathfrak{f}_G \otimes_{i/p} U_G = U_G$.□

Proposition 3.8. U_G is not the left absorbing element of the soft intersection-plus product in $S_G(U)$.

PROOF. Consider the SS $\mathfrak{f}_G$ in Example 3.3. Then,

$$U_G \otimes_{i/p} \mathfrak{f}_G = \{(Ձ, \emptyset), (б, \emptyset)\}$$

implying that $U_G \otimes_{i/p} \mathfrak{f}_G \neq U_G$.□

Remark 3.9. U_G is not the absorbing element of the soft intersection-plus product in $S_G(U)$.

Proposition 3.10. The soft intersection-plus product is not idempotent in $S_G(U)$.

PROOF. Consider the SS $\mathfrak{f}_G$ in Example 3.3. Then, for all $x \in G$,

$$\mathfrak{f}_G \otimes_{i/p} \mathfrak{f}_G = \{(Ձ, U), (б, \{x\})\}$$

implying that $\mathfrak{f}_G \otimes_{i/p} \mathfrak{f}_G \neq \mathfrak{f}_G$. □

Proposition 3.11. Let $\mathfrak{f}_G$ be a constant SS. Then, $\mathfrak{f}_G \otimes_{i/p} \mathfrak{f}_G = U_G$.

PROOF. Let $\mathfrak{f}_G$ be a constant SS such that, for all $x \in G$, $\mathfrak{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(\mathfrak{f}_G \otimes_{i/p} \mathfrak{f}_G)(x) &= \bigcap_{x=yz} \left(\mathfrak{f}_G{}^c(y) \cup \mathfrak{f}_G(z)\right) \\ &= U_G(x)\end{aligned}$$

Thereby, $\mathfrak{f}_G \otimes_{i/p} \mathfrak{f}_G = U_G$.□

Proposition 3.12. Let $\mathfrak{f}_G$ be a constant SS. Then, $U_G \otimes_{i/p} \mathfrak{f}_G = \mathfrak{f}_G$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(U_G \otimes_{i/p} \mathscr{f}_G)(x) &= \bigcap_{x=yz} \left(U_G{}^c(y) \cup \mathscr{f}_G(z)\right)\\ &= \bigcap_{x=yz} \left(\emptyset \cup \mathscr{f}_G(z)\right)\\ &= \mathscr{f}_G(x)\end{aligned}$$

Thereby, $U_G \otimes_{i/p} \mathscr{f}_G = \mathscr{f}_G$.□

Proposition 3.13. Let $\mathscr{f}_G$ be an $\mathcal{SS}$. Then, $\emptyset_G \otimes_{i/p} \mathscr{f}_G = U_G$.

PROOF. Let $\mathscr{f}_G$ be an $\mathcal{SS}$. Then, for all $x \in G$,

$$\begin{aligned}(\emptyset_G \otimes_{i/p} \mathscr{f}_G)(x) &= \bigcap_{x=yz} \left(\emptyset_G{}^c(y) \cup \mathscr{f}_G(z)\right)\\ &= \bigcap_{x=yz} \left(U \cup \mathscr{f}_G(z)\right)\\ &= U_G(x)\end{aligned}$$

Thereby, $\emptyset_G \otimes_{i/p} \mathscr{f}_G = U_G$.□

Proposition 3.14. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G \otimes_{i/p} \emptyset_G = \mathscr{f}_G{}^c$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(\mathscr{f}_G \otimes_{i/p} \emptyset_G)(x) &= \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \emptyset_G(z)\right)\\ &= \bigcap_{x=yz} (\mathscr{f}_G{}^c(y) \cup \emptyset_G)\\ &= \mathscr{f}_G{}^c(x)\end{aligned}$$

Thereby, $\mathscr{f}_G \otimes_{i/p} \emptyset_G = \mathscr{f}_G{}^c$.

Proposition 3.15. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G{}^c \otimes_{i/p} \mathscr{f}_G = \mathscr{f}_G$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(\mathscr{f}_G{}^c \otimes_{i/p} \mathscr{f}_G)(x) &= \bigcap_{x=yz} \left((\mathscr{f}_G{}^c)^c(y) \cup \mathscr{f}_G(z)\right)\\ &= \bigcap_{x=yz} \left(\mathscr{f}_G(y) \cup \mathscr{f}_G(z)\right)\\ &= \mathscr{f}_G(x)\end{aligned}$$

Thereby, $\mathscr{f}_G{}^c \otimes_{i/p} \mathscr{f}_G = \mathscr{f}_G$.□

Proposition 3.16. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G \otimes_{i/p} \mathscr{f}_G{}^c = \mathscr{f}_G{}^c$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(\mathscr{f}_G \otimes_{i/p} \mathscr{f}_G{}^c)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \mathscr{f}_G{}^c(z)\right) = \mathscr{f}_G{}^c(x)$$

Thereby, $\mathscr{f}_G \otimes_{i/p} \mathscr{f}_G{}^c = \mathscr{f}_G{}^c$.□

Theorem 3.17. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two SSs. Then, $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G = U_G$ if and only if $\mathscr{f}_G \,\tilde{\subseteq}_A\, \mathscr{g}_G$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two SSs. Suppose that $\mathscr{f}_G \,\tilde{\subseteq}_A\, \mathscr{g}_G$. Then, $\mathscr{f}_G(y) \subseteq \mathscr{g}_G(z)$, for each $y, z \in G$. Thus, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z)\right) = U_G(x) = U$$

Thereby, $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G = U_G$.□

Conversely, suppose that $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G = U_G$. Then, $\left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)(x) = U_G(x) = U$ for all $x \in G$. Thus, for all $x \in G$,

$$U_G(x) = U = \left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z)\right)$$

This implies that $\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z) = U$, for all $y, z \in G$. Thus, $\mathscr{f}_G(y) \subseteq \mathscr{g}_G(z)$, for each $y, z \in G$. Thereby, $\mathscr{f}_G \,\tilde{\subseteq}_A\, \mathscr{g}_G$. Note here that, $\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z) = \left(\mathscr{f}_G(y) \backslash \mathscr{g}_G(z)\right)'$, for all $y, z \in G$.

Proposition 3.18. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two SSs. If $\mathscr{f}_G = U_G$ and $\mathscr{g}_G = \emptyset_G$, then $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G = \emptyset_G$.

PROOF. Let $\mathscr{f}_G = U_G$ and $\mathscr{g}_G = \emptyset_G$. Then, for all $x \in G$, $\mathscr{f}_G(x) = U_G(x) = U$ and $\mathscr{g}_G(x) = \emptyset_G(x) = \emptyset$. Thus, for all $x \in G$,

$$\begin{aligned}\left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)(x) &= \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z)\right)\\ &= \bigcap_{x=yz} \left(U_G{}^c(y) \cup \emptyset_G(z)\right)\\ &= \bigcap_{x=yz} (\emptyset \cup \emptyset)\\ &= \emptyset_G(x)\end{aligned}$$

Thereby, $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G = \emptyset_G$.□

Proposition 3.19. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two SSs. If $\mathscr{g}_G \,\tilde{\subseteq}_S\, (\mathscr{f}_G)^c$, then $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G = \mathscr{f}_G{}^c$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two SSs and $\mathscr{g}_G \,\tilde{\subseteq}_S\, (\mathscr{f}_G)^c$. Hence, for all $x \in G$, $\mathscr{f}_G(x) = A$ and $\mathscr{g}_G(x) = B$, where A and B are two fixed sets and $B \subseteq A'$. Thus, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z)\right) = \mathscr{f}_G{}^c(x)$$

Thereby, $\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G = \mathscr{f}_G{}^c$. □

Proposition 3.20. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two SSs. Then, $\left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)^c = \mathscr{f}_G \otimes_{u/d} \mathscr{g}_G$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two SSs. Then, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)^c(x) = \left(\bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z)\right)\right)'$$

$$= \bigcup_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \mathscr{g}_G(z) \right)'$$
$$= \bigcup_{x=yz} \left(\mathscr{f}_G(y) \cap \mathscr{g}_G{}^c(z) \right)$$
$$= \bigcup_{x=yz} \left(\mathscr{f}_G(y) \setminus \mathscr{g}_G(z) \right)$$
$$= (\mathscr{f}_G \otimes_{u/d} \mathscr{g}_G)(x)$$

Thereby, $\left(\mathscr{f}_G \otimes_{i/p} \mathscr{g}_G\right)^{\mathrm{c}} = \mathscr{f}_G \otimes_{u/d} \mathscr{g}_G$.

Proposition 3.21. Let $\mathscr{f}_G$, $\mathscr{g}_G$, and $\hbar_G$ be three $\mathcal{SS}$s. If $\mathscr{f}_G \tilde{\subseteq} \mathscr{g}_G$, then $\mathscr{g}_G \otimes_{i/p} \hbar_G \tilde{\subseteq} \mathscr{f}_G \otimes_{i/p} \hbar_G$ and $\hbar_G \otimes_{i/p} \mathscr{f}_G \tilde{\subseteq} \hbar_G \otimes_{i/p} \mathscr{g}_G$.

PROOF. Let $\mathscr{f}_G$, $\mathscr{g}_G$, and $\hbar_G$ be three $\mathcal{SS}$s such that $\mathscr{f}_G \tilde{\subseteq} \mathscr{g}_G$. Then, for all $x \in G$, $\mathscr{f}_G(x) \subseteq \mathscr{g}_G(x)$, and hence, $\left(\mathscr{g}_G(x)\right)' \subseteq \left(\mathscr{f}_G(x)\right)'$. Then, for all $x \in G$,

$$\left(\mathscr{g}_G \otimes_{i/p} \hbar_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{g}_G{}^c(y) \cup \hbar_G(z) \right)$$
$$\subseteq \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \hbar_G(z) \right)$$
$$= \left(\mathscr{f}_G \otimes_{i/p} \hbar_G\right)(x)$$

implying that $\mathscr{g}_G \otimes_{i/p} \hbar_G \tilde{\subseteq} \mathscr{f}_G \otimes_{i/p} \hbar_G$. Similarly, for all $x \in G$,

$$\left(\hbar_G \otimes_{i/p} \mathscr{f}_G\right)(x) = \bigcap_{x=yz} \left(\hbar_G{}^c(y) \cup \mathscr{f}_G(z) \right)$$
$$\subseteq \bigcap_{x=yz} \left(\hbar_G{}^c(y) \cup \mathscr{g}_G(z) \right)$$
$$= \left(\hbar_G \otimes_{i/p} \mathscr{g}_G\right)(x)$$

implying that $\hbar_G \otimes_{i/p} \mathscr{f}_G \tilde{\subseteq} \hbar_G \otimes_{i/p} \mathscr{g}_G$.□

Proposition 3.22. Let $\mathscr{f}_G$, $\mathscr{g}_G$, σ_G, and $\mathscr{k}_G$ be four $\mathcal{SS}$s. If $\mathscr{k}_G \tilde{\subseteq} \sigma_G$, and $\mathscr{f}_G \tilde{\subseteq} \mathscr{g}_G$, then $\sigma_G \otimes_{i/p} \mathscr{f}_G \tilde{\subseteq} \mathscr{k}_G \otimes_{i/p} \mathscr{g}_G$ and $\mathscr{g}_G \otimes_{i/p} \mathscr{k}_G \tilde{\subseteq} \mathscr{f}_G \otimes_{i/p} \sigma_G$.

PROOF. Let $\mathscr{f}_G$, $\mathscr{g}_G$, σ_G, and $\mathscr{k}_G$ be four $\mathcal{SS}$s such that $\mathscr{k}_G \tilde{\subseteq} \sigma_G$, and $\mathscr{f}_G \tilde{\subseteq} \mathscr{g}_G$. Then, for all $x \in G$, $\mathscr{k}_G(x) \subseteq \sigma_G(x)$, $\mathscr{f}_G(x) \subseteq \mathscr{g}_G(x)$, and thus, $\sigma_G{}^c(x) \subseteq \mathscr{k}_G{}^c(x)$, $\mathscr{g}_G{}^c(x) \subseteq \mathscr{f}_G{}^c(x)$, for all $x \in G$. Then, for all $x \in G$,

$$\left(\sigma_G \otimes_{i/p} \mathscr{f}_G\right)(x) = \bigcap_{x=yz} \left(\sigma_G{}^c(y) \cup \mathscr{f}_G(z) \right)$$
$$\subseteq \bigcap_{x=yz} \left(\mathscr{k}_G{}^c(y) \cup \mathscr{g}_G(z) \right)$$
$$= \left(\mathscr{k}_G \otimes_{i/p} \mathscr{g}_G\right)(x)$$

implying that $\sigma_G \otimes_{i/p} \mathscr{f}_G \tilde{\subseteq} \mathscr{k}_G \otimes_{i/p} \mathscr{g}_G$. Similarly, for all $x \in G$,

$$\left(\mathscr{g}_G \otimes_{i/p} \mathscr{k}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{g}_G{}^c(y) \cup \mathscr{k}_G(z) \right)$$

$$\subseteq \bigcap_{x=yz}\left(f_G{}^c(y) \cup \sigma_G(z)\right)$$
$$= \left(f_G \otimes_{i/p} \sigma_G\right)(x)$$

implying that $g_G \otimes_{i/p} \hbar_G \,\tilde{\subseteq}\, f_G \otimes_{i/p} \sigma_G$. □

Proposition 3.23. The soft intersection-plus product distributes over the intersection operation of $\mathcal{SS}$s from the left side.

PROOF. Let f_G, g_G, and $\hbar_G$ be three $\mathcal{SS}$s. Then, for all $x \in G$,

$$\left(f_G \otimes_{i/p} (g_G \,\tilde{\cap}\, \hbar_G)\right)(x) = \bigcap_{x=yz}\left(f_G{}^c(y) \cup (g_G \,\tilde{\cap}\, \hbar_G)(z)\right)$$
$$= \bigcap_{x=yz}\left(f_G{}^c(y) \cup \left(g_G(z) \cap \hbar_G(z)\right)\right)$$
$$= \bigcap_{x=yz}\left(\left(f_G{}^c(y) \cup g_G(z)\right) \cap \left(f_G{}^c(y) \cup \hbar_G(z)\right)\right)$$
$$= \left[\bigcap_{x=yz}\left(f_G{}^c(y) \cup g_G(z)\right)\right] \cap \left[\bigcap_{x=yz}\left(f_G{}^c(y) \cup \hbar_G(z)\right)\right]$$
$$= \left(f_G \otimes_{i/p} g_G\right)(x) \cap \left(f_G \otimes_{i/p} \hbar_G\right)(x)$$

Thus, $f_G \otimes_{i/p} (g_G \,\tilde{\cap}\, \hbar_G) = \left(f_G \otimes_{i/p} g_G\right) \tilde{\cap} \left(f_G \otimes_{i/p} \hbar_G\right)$.□

Example 3.24. Consider the group G in Example 3.3. Let f_G, g_G, and $\hbar_G$ be three $\mathcal{SS}$s over $U = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\mathfrak{Q}, \{e, y\}), (\mathfrak{b}, \{yx\})\},\ g_G = \{(\mathfrak{Q}, \{x, y\}), (\mathfrak{b}, \{e, x\})\},\ \hbar_G = \{(\mathfrak{Q}, \{x\}), (\mathfrak{b}, \{y, yx\})\}$$

Since $f_G \otimes_{i/p} g_G = \{(\mathfrak{Q}, \{x, y\}), (\mathfrak{b}, \{e, x\})\}$ and $f_G \otimes_{i/p} \hbar_G = \{(\mathfrak{Q}, \{x, yx\}), (\mathfrak{b}, \{x, y\})\}$, then

$$\left(f_G \otimes_{i/p} g_G\right) \tilde{\cap} \left(f_G \otimes_{i/p} \hbar_G\right) = \{(\mathfrak{Q}, \{x\}), (\mathfrak{b}, \{x\})\}$$

Moreover, since $g_G \,\tilde{\cap}\, \hbar_G = \{(\mathfrak{Q}, \{x\}), (\mathfrak{b}, \emptyset)\}$

$$f_G \otimes_{i/p} (g_G \,\tilde{\cap}\, \hbar_G) = \{(\mathfrak{Q}, \{x\}), (\mathfrak{b}, \{x\})\}$$

Thus, $f_G \otimes_{i/p} (g_G \,\tilde{\cap}\, \hbar_G) = \left(f_G \otimes_{i/p} g_G\right) \tilde{\cap} \left(f_G \otimes_{i/p} \hbar_G\right)$.□

Proposition 3.25. The soft intersection-plus product does not distribute over the intersection operation of $\mathcal{SS}$s from the right side.

PROOF. Consider the group G in Example 3.3. Let f_G, g_G, and $\hbar_G$ be three $\mathcal{SS}$s over $U = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\mathfrak{Q}, \{e, y\}), (\mathfrak{b}, \{yx\})\},\ g_G = \{(\mathfrak{Q}, \{x, y\}), (\mathfrak{b}, \{e, x\})\},\ \hbar_G = \{(\mathfrak{Q}, \{x\}), (\mathfrak{b}, \{y, yx\})\}$$

Since $f_G \otimes_{i/p} \hbar_G = \{(\mathfrak{Q}, \{x, yx\}), (\mathfrak{b}, \{x, y\})\}$ and $g_G \otimes_{i/p} \hbar_G = \{(\mathfrak{Q}, \{yx\}), (\mathfrak{b}, \{y, yx\})\}$, then

$$\left(f_G \otimes_{i/p} \hbar_G\right) \tilde{\cap} \left(g_G \otimes_{i/p} \hbar_G\right) = \{(\mathfrak{Q}, \{yx\}), (\mathfrak{b}, \{y\})\}$$

Moreover, since $f_G \,\tilde{\cap}\, g_G = \{(\mathfrak{Q}, \{y\}), (\mathfrak{b}, \emptyset)\}$

$$(\mathscr{f}_G \tilde{\cap} \mathscr{g}_G)\otimes_{i/p}\hbar_G = \{(\mathfrak{Q}, \{e, x, yx\}), (\mathfrak{b}, U)\}$$

Thus, $(\mathscr{f}_G \tilde{\cap} \mathscr{g}_G)\otimes_{i/p}\hbar_G \neq (\mathscr{f}_G\otimes_{i/p}\hbar_G) \tilde{\cap} (\mathscr{g}_G\otimes_{i/p}\hbar_G)$.□

Remark 3.26. The soft intersection-plus product does not distribute over the intersection operation of $\mathcal{SS}$s from both sides.

4. Conclusion

This study begins with the formal introduction of a novel binary operation on soft sets, designated as the soft intersection–plus product, constructed over parameter domains endowed with an intrinsic group-theoretic structure. Grounded in this foundational formulation, we embark on a comprehensive algebraic investigation of the operation, with particular emphasis on its structural behavior across various hierarchies of soft subsethood and its alignment with generalized soft equality relations. The operation is further subjected to a rigorous comparative analysis with the previously established soft binary products, systematically embedded within the stratified lattice of soft subset classifications. This comparative framework yields sharpened theoretical insights into the relative expressive capacities and algebraic compatibilities of alternative soft operations. In parallel, we undertake a detailed structural analysis of the proposed product's interaction with both the null and absolute soft sets, as well as with existing binary soft operations defined over group-structured parameter spaces. These investigations further elucidate the operation's foundational role within the broader algebraic topology of soft systems. The algebraic treatment is conducted within a strictly axiomatic setting, adhering to core principles of abstract algebra wherein properties such as closure, associativity, commutativity, idempotency, distributivity over other soft set operations, and the presence or absence of identity, inverse, and absorbing elements serve as critical invariants in the classification of algebraic structures. The regularities and algebraic phenomena revealed by this analysis confirm the internal coherence and formal integrity of the soft intersection–plus product and underscore its potential to extend classical algebraic paradigms into the domain of soft set theory. In particular, the operation serves as a conceptual and structural cornerstone for the development of a generalized soft group theory, wherein soft sets defined over group-parameterized domains emulate the axiomatic signatures of classical group constructs through rigorously defined soft operations. Beyond its foundational contributions, the algebraic framework developed herein offers fertile ground for future research—both in the synthesis of new algebraic operations within soft environments and in the refinement of generalized soft equalities—thereby expanding the theoretical boundaries and practical applicability of soft set theory across algebraic modeling, computational abstraction, and uncertainty-oriented decision science.

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