A concrete stress-strain model for analysis of high strength reinforced concrete columns

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Abstract

A concrete stress-strain model for analysis of eccentrically loaded both short and slender high strength reinforced concrete columns is proposed in this paper. The mathematical model has been developed based on the test data to represent the complete nonlinear stress-strain relationship of high strength concrete ranging from 40 to 90 MPa. Several slender high strength concrete columns available in the literature have been analysed using both proposed nonlinear stress-strain model and the equivalent rectangular stress block model to confirm the reliability and validity of the proposed model. Good agreement has been achieved between the computed theoretical ultimate strength capacities and the test results of high strength concrete columns.

Key Words: Biaxial bending, High-strength concrete column, Stress-strain relationship, Ultimate strength.

Yüksek dayanımlı betonarme kolonların analizi için beton gerilme-birim deformasyon modeli

Özet

Sunulan çalışmada, eksantrik yüklemeye maruz yüksek dayanımlı kısa ve narin betonarme kolonların analizi için beton gerilme-birim deformasyon modeli önerilmektedir. Beton dayanımı 40 ile 90 MPa arasında değişen ve doğrusal olmayan beton gerilme-birim deformasyon modelini temsil eden matematiksel model deneysel verilere dayandırılarak geliştirilmiştir. Beton modelinin güvenilirliğini ve geçerliliğini doğrulamak için literatürde bulunan birçok deneysel kolon önerilen beton modeli ve dikdörtgen gerilme dağılım modeli kullanılarak analiz edilmiştir. Hesaplanan teorik yüksek dayanımlı kolon taşıma gücü değerlerinin test değerleri ile uyumlu olduğu elde edilmiştir.

Anahtar Kelimeler: İki eksenli eğilme, Yüksek dayanımlı betonarme kolon, Gerilme-şekil değiştirme ilişkisi, Taşıma gücü.

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1. Introduction

High-strength concrete offers performance and economy advantages using especially in high rise buildings and bridges. High strength concrete allows a reduction in column size and provides member stiffness and high modulus of elasticity when compared with normal strength concrete. On the other hand, high-strength concrete is very brittle material. Thus, it is very difficult to obtain complete stress-strain relationship under compression, particularly after the peak load. The mechanical behaviour of concrete plays very significant role on analysis and design of high strength concrete columns. Therefore, it is necessary to develop an analytical model to represent the complete stress-strain relationship of high strength concrete material.

It is widely believed that mechanical behaviour of concrete alters with concrete strength. Hence, the stress-strain relationship suggested for normal strength concrete cannot be applicable to high strength concrete. The ascending branch of the stress-strain curve for high strength concrete becomes more linear and steeper when compared with normal strength concrete. A number of studies have been carried out on the investigation of the behaviour of high strength concrete under compression. Indeed, most of the models developed for high strength concrete are modified version of the models suggested for normal strength concrete. Various analytical models have been developed on the basis of test data to describe the stress-strain behaviour of both normal and high-strength concrete by several researchers (Hognestad et al. [1], Kent and Park [2], Popovics [3], Sheikh and Uzumeri [4], Ahmad and Shah [5], Fafitis and Shah [6], Sheikh and Yeh [7], Hsu and Hsu [8], Saatcioglu and Razvi [9], Nagashima et al. [10], Muguruma et al. [11], Cusson and Paultre [12], Razvi and Saatcioglu [13], Binici [14]).

In practice, columns are commonly subjected to eccentric compression, particularly located at the corner of the buildings. It is important to understand the behaviour of these columns for rational analysis and design. Several experimental and theoretical studies carried out on eccentrically loaded high-strength concrete columns. Yong et al. [15] investigated the behaviour of confined high-strength concrete columns by means of experimental tests. Mander et al. [16] researched eccentrically loaded tests of reinforced concrete columns to observe the stress-strain behaviour of concrete columns. Cusson and Paultre [17] presented an experimental investigation of the behaviour of large scale high-strength concrete columns. Polat [18] reported an experimental and analytical investigation of high strength concrete columns including confinement effect. Razvi and Saatcioglu [19] investigated strength and deformability of high strength concrete columns. Hsu et al. [20] reported experimental and theoretical load-deflection behaviour of square section slender high strength concrete columns. Lloyd and Rangan [21,22] presented experimental and theoretical study on the behaviour of high strength concrete columns subjected to uniaxial bending. Ibrahim and MacGregor [23] reported test results of high strength concrete columns under axial loads with small eccentricities to study the flexural behaviour of rectangular and triangular compression zones of high strength concrete. Saatcioglu and Razvi [24] researched experimental behaviour of square high-strength concrete columns confined by rectilinear reinforcement. Chuang and Kong [25] presented a numerical method to analyse the failure load of slender high strength concrete columns under uniaxial bending and axial load. Rangan [26] conducted experimental investigation of high strength concrete columns under uniaxial and biaxial bending. Stewart and Attard [27] investigated the structural reliabilities and model accuracy of normal strength and high strength concrete short columns design by using the rectangular stress distribution. Lee and Son [28] studied structural behaviour and strength of eccentrically loaded high strength reinforced concrete tied columns. Sarker and Rangan [29] reported experimental and analytical investigation of high strength concrete columns subjected to equal and unequal load eccentricities. Diniz and Frangopol [30] presented an analysis of the reliability of slender high strength concrete columns including long term effects under eccentric loading conditions. Canbay et al. [31], Tokgoz [32], Dundar and Tokgoz [33] and Jin et al. [34] studied the experimental behaviour of high strength concrete tied columns under eccentric load.

The primary aim of this paper is to develop a mathematical stress-strain model based on the test data for high strength concrete. In the study, the proposed model has been used in the analysis of eccentrically loaded short and slender high strength concrete columns. Computed theoretical ultimate strength capacities of high strength concrete columns have been compared with the test results available in the literature and discussed in the paper.

2. Proposed stress-strain model for high strength concrete

In this study, a mathematical model has been developed to represent the complete stress-strain relationship of high strength concrete ranging from 40 to 90 MPa. The proposed stress-strain relationship consists of a nonlinear ascending branch up to peak stress and a linear descending branch beyond the peak. Empirical equations suggested in the model have been constituted based on the experimental verifications of high strength concrete.

Reported data indicated that the ascending branch of the stress-strain curve becomes steeper with increasing concrete strength. The shape of this portion alters almost a second order parabola for normal strength concrete, but a linear variation for high strength concrete (Ozbakkaloglu [35], ACI 441 [36]). From this point of view, the ascending branch of the proposed stress-strain relation was developed by introducing a factor (1+γ) to the Hognestad's [1] curve. The mathematical expression for the ascending branch of the proposed model is recommended below:

$$\sigma_c = f_c\left[\frac{2\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right]^{1+\gamma} \tag{1}$$

where σ_c and ε_c are concrete stress and concrete strain in general, respectively; f_c is the concrete compressive strength; ε_{co} is the concrete strain at peak stress and γ is the shape factor. Some experimental concrete stress-strain diagrams obtained in this study are illustrated in Fig. 1. The experimental concrete stress-strain relationships revealed that the concrete strain at peak stress (ε_{co}) alters with concrete strength as clearly shown in Fig. 1.

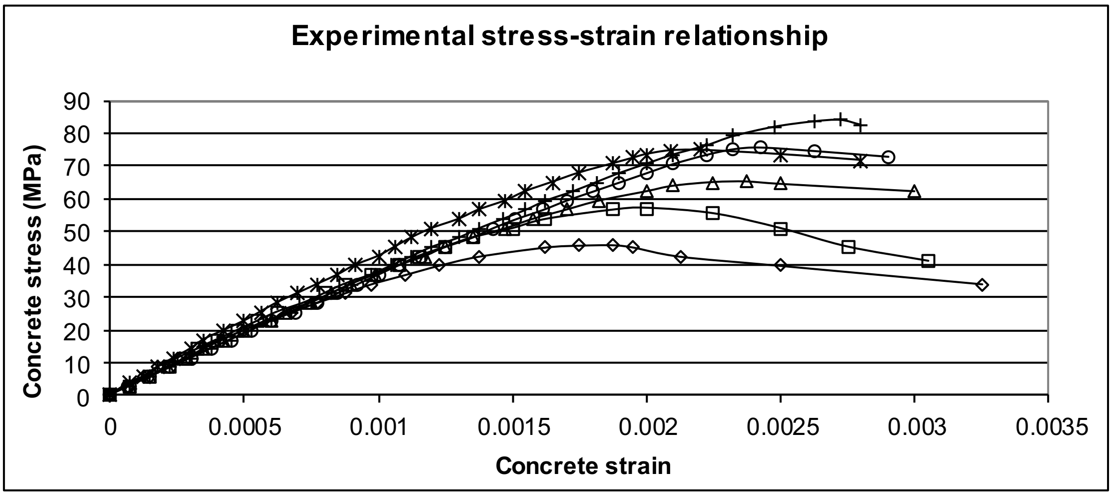


Fig. 1. Experimental stress-strain relationships of high strength concrete

Therefore, a regression analysis based on the experimental data obtained from axial tests of high strength concrete cylinder specimens was performed to establish a relation between the concrete strain ε_{co} and the concrete compressive strength (Fig. 2).

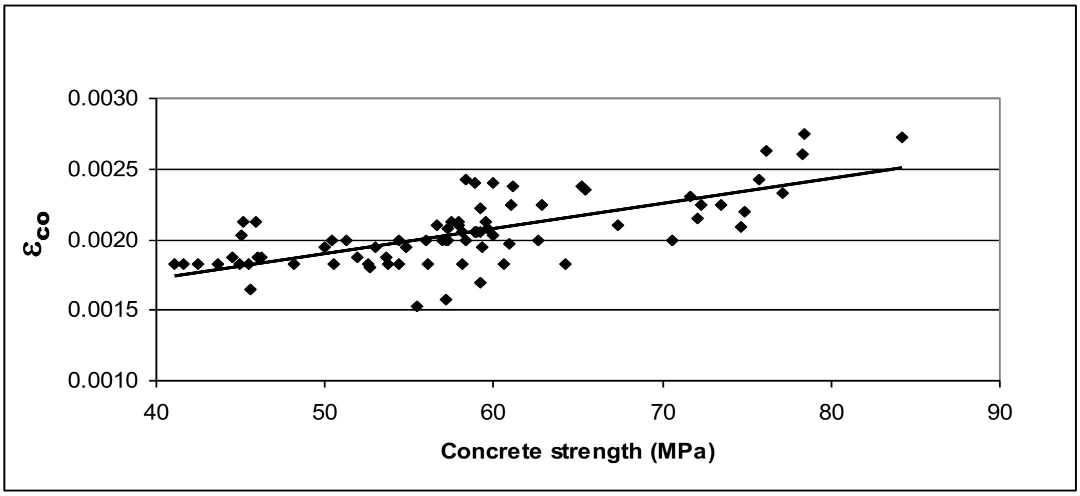


Fig. 2. Variation of strain at peak stress (ε_{co})

The parameter ε_{co} was derived from regression analysis of test data as a function of concrete compressive strength ranging from 40 to 90 MPa as follows:

$$\varepsilon_{co}=2\times10^{-5}\ f_c+0.001 \qquad (2)$$

The γ values were computed from the solution of Eq. (1) for each experimental concrete strain and corresponding stress data by using the experimental stress-strain curves. An exponential regression analysis was performed based on the computed data to establish an expression for the parameter γ (Fig. 3).

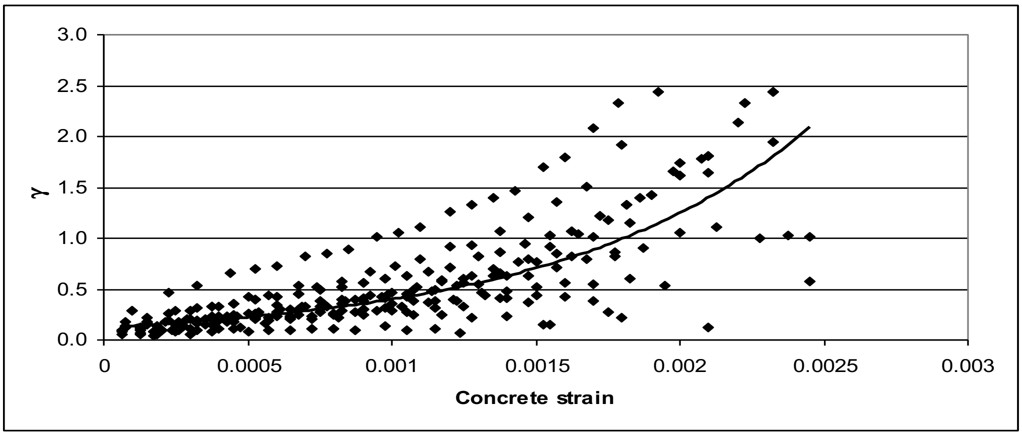
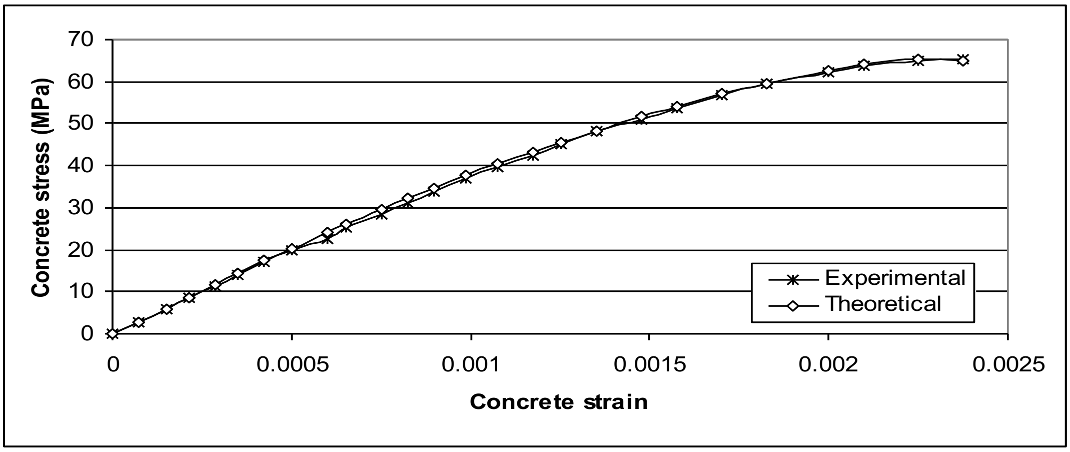


Fig. 3. Variation of parameter γ with concrete strain

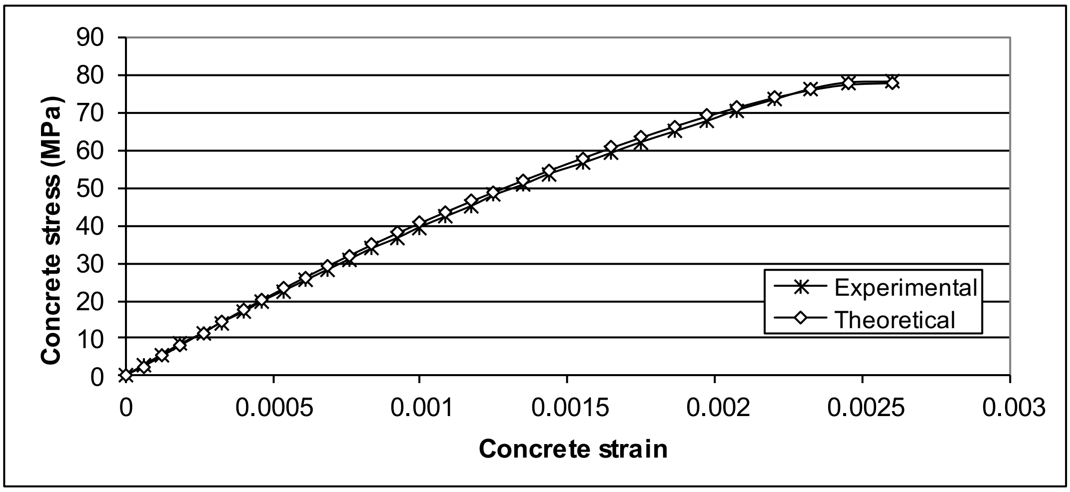
The following mathematical expression was obtained for the parameter γ as a function of concrete strain:

$$\gamma = 0.1278e^{1140.3(\varepsilon_c)} \qquad (3)$$

The typical theoretical ascending branch of the stress-strain curve was formed by using the proposed Eq. (1) for the concrete strengths 65 and 78 MPa separately and illustrated with experimental curves for comparison in Fig. 4(a,b).



(a)



(b)

Fig. 4(a,b). Typical ascending part of experimental and theoretical stress-strain relations

It is seen in the diagrams that good correlation has been observed between the experimental and the theoretical curves. Moreover, the proposed Eq. (1) was verified with various concrete strengths and the results were found to be in good agreement with the test results.

The descending portion of the stress-strain curve was adopted to be linear as similarly suggested by Hognestad [1]. The empirical equation to represent the descending part of the curve is given as follows:

$$\sigma_c = f_c\left[1 - 0.15\left(\frac{\varepsilon_c - \varepsilon_{co}}{\varepsilon_{cu} - \varepsilon_{co}}\right)\right] \tag{4}$$

where ε_{cu} is the ultimate concrete compressive fiber strain. This parameter plays significant role to predict the ultimate flexural strength of high strength concrete members. The experimental verifications indicated that ε_{cu} value changed with concrete strength. In the proposed model, ε_{cu} parameter was assumed as a function of concrete strength as follows ([13, 35]):

$$\varepsilon_{cu}=0.0036-(f_c-30)\times10^{-5}\ f_c \geq 0.0027 \tag{5}$$

where, f_c is in MPa.

The Eq. (5) is in accordance with the experimental values of ultimate concrete compressive fiber strain obtained from cylinder specimens. The proposed complete stress-strain relationship for high strength concrete is illustrated in Fig. 5.

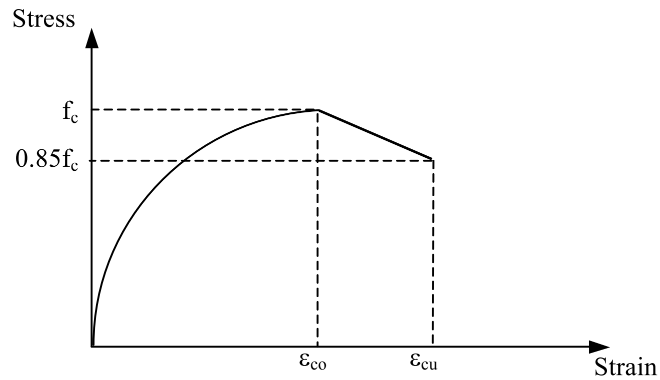


Fig. 5. Proposed stress-strain model

The proposed stress-strain model has been used for the analysis of eccentrically loaded high strength reinforced concrete columns.

3. Method of column analysis

The analysis method of high strength reinforced concrete columns has been reported previously by Tokgoz [32]. In this study, the brief description of the algorithm has been presented. The proposed method is based on the following assumptions.

3.1 Fundamental assumptions

(i) Plane sections remain plane before and after bending.
(ii) Both arbitrary monotonic stress distribution and equivalent rectangular stress distribution can be assumed for the concrete compression zone.
(iii) The reinforcing steel bars are assumed to be elastic-perfectly plastic.
(iv) There is perfect bond between steel and concrete.
(v) Creep and shrinkage effects and the tensile strength of concrete are neglected.
(vi) Shear deformation is ignored.

3.2. Formulation of the analysis method

A reinforced concrete column cross section subjected to compressive axial load N with the eccentricities e_x and e_y is shown in Fig. 6. The strain at any point in the cross section (x_i,y_i) can be defined according to Bernoulli's assumption as follows:

$$\varepsilon_i = \varepsilon_{cu}\left[\left(\frac{y_i}{c}+\frac{x_i}{a}\right)-1\right] \tag{6}$$

where a and c are the parameters that describe the neutral axis of the column (Fig. 6). In the analysis procedure, the stress resultants of the concrete material are computed at the centroid of each segment by using the Eq. (6) and assumed stress-strain model. Similarly, reinforcing steel stress resultants are obtained using its own stress-strain relation.

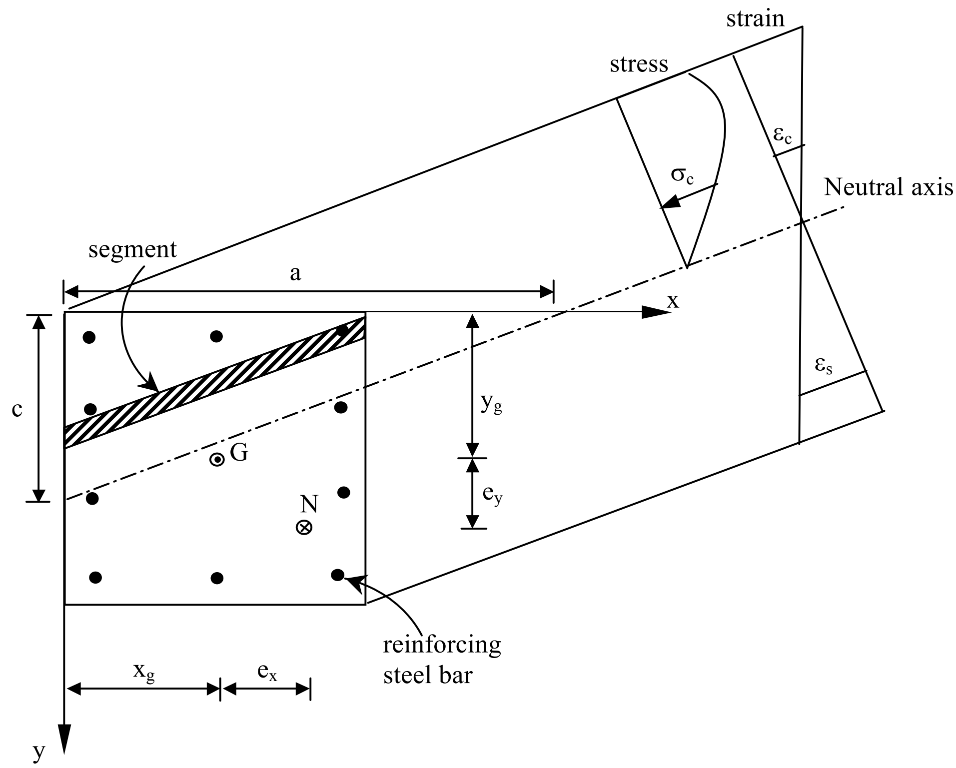


Fig. 6. A reinforced concrete column cross section

3.3. Equilibrium equations

The equilibrium equations in the cross section can be written as follows:

$$N = \sum A_c \sigma_c + \sum A_s \sigma_s \tag{7}$$

$$M_x = \sum A_c \sigma_c y_c + \sum A_s \sigma_s y_s \tag{8}$$

$$M_y = \sum A_c \sigma_c x_c + \sum A_s \sigma_s x_s \tag{9}$$

where, A_c and A_s represent the segmental area of concrete and reinforcing steel area, respectively; σ_c and σ_s indicate concrete stress and reinforcing steel stress in general, respectively; (x_c, y_c) and (x_s, y_s) denote the distances of the centre of concrete segment and reinforcing steel bar with respect to the centre of the cross section in *x-y* axis system, respectively.

More details for the analysis of both short and slender reinforced concrete columns can be obtained from the previous studies reported by Dundar et al. [37] and Tokgoz [32].

4. Computer analysis of high strength concrete columns

4.1 Analysis of high strength concrete columns reported by Lloyd and Rangan [22]

Lloyd and Rangan [22] tested pin ended 175×175 mm square and 300×100 mm rectangular section of slender high strength concrete columns under uniaxial bending and axial load. The column specimens were designed as 1680 mm long. The column specimens consisted of seven series and each series had three high strength concrete columns. The longitudinal reinforcements consisted of six or four deformed 12 mm in diameter bars had the yield strength of 430 MPa. The lateral reinforcements arranged by using 4 mm in diameter deformed bars at 60 mm spacing. The cross section details and reinforcement configuration of test columns are shown in Fig. 7.

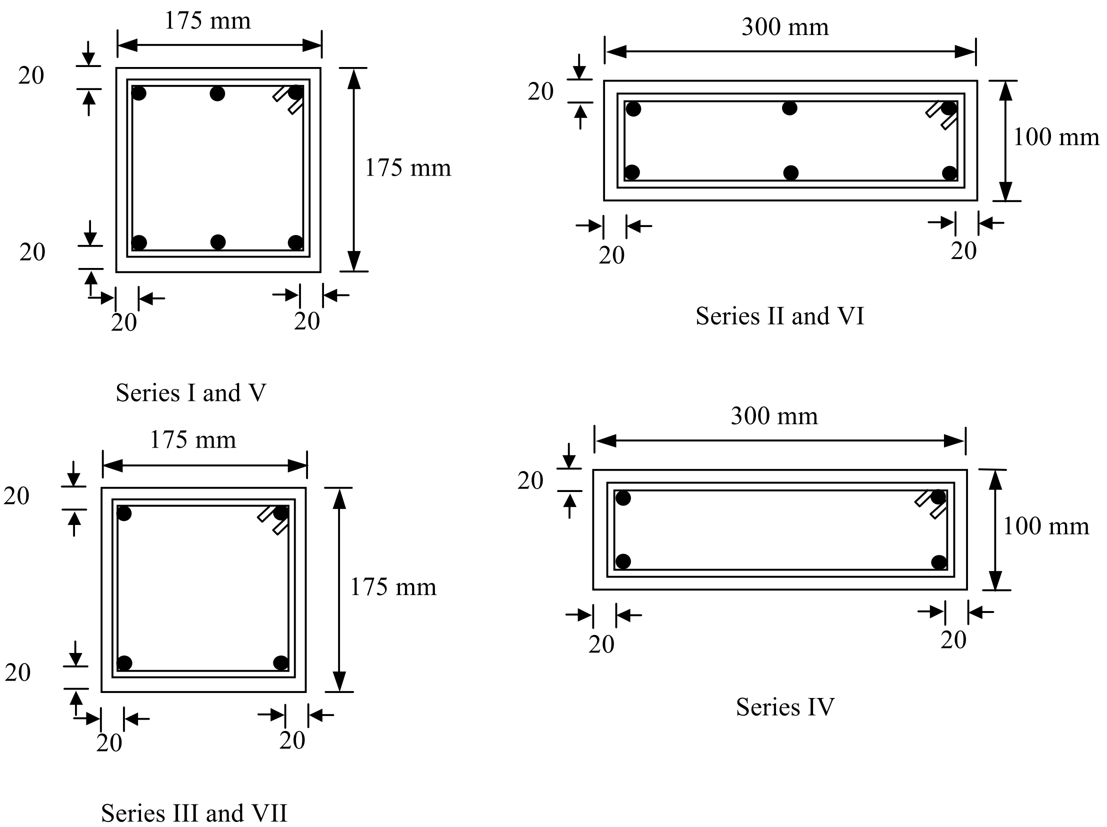


Fig. 7. Cross section details of high strength concrete columns

The specimen details of the columns are summarized in Table 1.

The column specimens were analysed with the computer program developed based on the suggested theoretical method using the proposed stress-strain relationship for the concrete compression zone. Specimen features, obtained theoretical strength results and comparative values of the theoretical load to test loads are given in Table 1.

Table 1. Ultimate strength results and comparative values of the columns

Column no.	Column properties		Ultimate loads			Ratio
	f_c (MPa)	e (mm)	N_{exp} (kN)	N_u (kN)	M_u (kN-cm)	N_u/N_{exp}
I A	58	15	1477	1329.29	3623.87	0.900
I B	58	50	830	790.11	5357.89	0.952
I C	58	65	661	651.82	5412.65	0.986
II A	58	10	1192	1130.12	2260.48	0.948
II B	58	30	437	452.98	2445.79	1.037
II C	58	40	343	345.22	2195.71	1.006
III A	58	15	1141	1271.38	3342.95	1.114
III B	58	50	724	715.44	4705.29	0.988
III C	58	65	512	558.39	4485.74	1.091
IV A	58	10	916	1066.96	2135.86	1.165
IV B	58	30	426	403.25	2056.01	0.947
IV C	58	40	262	297.01	1781.39	1.134
V A	92	15	1704	1807.45	5252.46	1.061
V B	92	50	1018	992.98	6793.14	0.975
V C	92	65	795	796.33	6591.14	1.002
VI A	92	10	1189	1141.12	3438.11	0.960
VI B	92	30	471	541.8	2924.19	1.150
VI C	92	40	422	443.36	2662.05	1.051
VII A	92	15	1745	1729.63	4901.45	0.991
VII B	92	50	908	922.72	6151.58	1.016
VII C	92	65	663	700.54	5626.95	1.057
Mean ratio						1.025

Table 1 shows that good degree of correlation was obtained between the predicted theoretical loads and experimental ultimate strength capacities of the slender high strength concrete columns reported by Lloyd and Rangan [22]. The flexural rigidity and ultimate compressive fiber strain parameters played significant role on the computation of the column flexural strength capacities. It is widely accepted that the ultimate concrete compressive fiber strain decreases with increasing concrete strength. Therefore, the ultimate concrete fiber strain proposed for normal strength concrete gives overestimate flexural strength results and thus, it cannot be applicable in the analysis of high strength concrete columns. The results indicate that the ultimate concrete compressive fiber strain assumed in this study (Eq. (5)) gives reasonable accuracy in predicting the ultimate strength capacity of the analysed high strength concrete columns.

4.2 Analysis of high strength concrete columns reported by Hsu et al. [20]

Hsu et al. [20] tested a total of nine square slender high strength reinforced concrete column specimens (L1–L9) under biaxial bending and axial load. The column specimens were 76.2×76.2 mm square and 1220 mm long. The cross section details and steel arrangements of the column specimens are shown in Fig. 8.

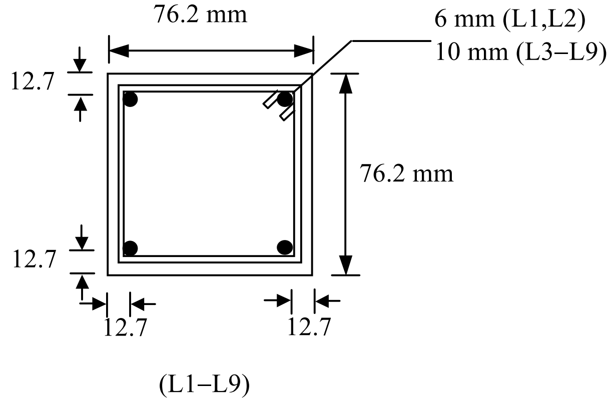


Fig. 8. Cross section details of column specimens L1–L9

The longitudinal reinforcements consisted of four 6 mm (L1 and L2) or 10 mm (L3–L9) deformed bars located at each corner of the section. The lateral reinforcements were arranged with 12 gauge steel wires (2.67 mm) had the yield strength of 456 MPa. The specimen properties of concrete compressive strength (f_c), yield strength of longitudinal reinforcement (f_y), tie spacing (s) and the eccentricities of applied axial load (e_x, e_y) are presented in Table 2.

Table 2. Details of column specimens L1–L9

Column no.	Column specimen properties				
	f_c (MPa)	f_y (MPa)	s (mm)	e_x (mm)	e_y (mm)
L1	76.95	434.4	76.2	23.47	9.73
L2	68.95	434.4	76.2	17.96	17.96
L3	73.91	531.3	76.2	17.96	17.96
L4	78.40	531.3	76.2	53.87	53.87
L5	71.02	531.3	76.2	43.99	25.4
L6	75.85	531.3	76.2	43.99	25.4
L7	76.05	531.3	76.2	65.99	38.1
L8	69.09	531.3	50.8	17.96	17.96
L9	71.71	420.6	50.8	53.87	53.87

The column specimens were solved by using the proposed stress-strain model for the concrete compression zone of the slender high strength concrete columns. In order to show the reliability and validity of the proposed nonlinear stress-strain model, the column specimens were also analysed using the rectangular stress block model (RSB) suggested by Ozbakkaloglu and Saatcioglu [35] for high strength concrete.

ACI 318-08 [38] defines the equivalent rectangular stress block, having $\alpha_1 f_c$ width and $\beta_1 c$ depth, for analysis and design of reinforced concrete members. The model parameters α_1 and β_1 were defined by Ozbakkaloglu and Saatcioglu [35] based on the test data for high strength concrete as follows:

$$\alpha_1=0.85-0.0014(f_c-30)\geq 0.72 \quad (10)$$

$$\beta_1=0.85-0.002(f_c-30)\geq 0.67 \quad (11)$$

In the analysis, the concrete stress which takes part in the equilibrium equations has been taken as ($\alpha_1 f_c$) to provide the rectangular stress block distribution. In addition, the neutral axis is changed by multiplying with β_1 parameter.

The obtained ultimate strength results based on the rectangular stress block (N_{RSB}) and proposed model (N_u) as well as the comparative values of the slender high strength concrete column specimens are presented in Table 3.

Table 3. Ultimate strength capacities and comparative results of the columns L1–L9

Column	Test results	Computer analysis results				Ratio
no.	N_{exp} (kN)	N_{RSB} (kN)	N_u (kN)	M_{ux} (kN-cm)	M_{uy} (kN-cm)	N_u/N_{exp}
L1	107.1	83.58	85.02	130.35	314.39	0.794
L2	100.1	79.57	80.99	230.27	230.27	0.809
L3	130.9	115.14	119.32	364.25	364.25	0.912
L4	41.9	50.47	50.02	351.08	351.08	1.194
L5	62.4	69.29	70.66	262.61	454.85	1.132
L6	68.7	70.66	71.95	265.54	459.94	1.047
L7	50.7	51.21	51.84	254.59	441.02	1.022
L8	125.9	114.76	117.75	359.25	359.25	0.935
L9	40.9	45.47	45.27	314.52	314.52	1.107
Mean ratio						0.995

The comparisons reveal a good agreement between the computed theoretical loads and experimental strength values. The results indicate that the use of the proposed stress-strain relationship gives reasonable accuracy in the analysis of high strength concrete columns. In addition, the model is approved well by the rectangular stress block model suggested by Ozbakkaloglu and Saatcioglu [35] (Table 3). The study indicate that the flexural rigidity has significantly effect on the prediction of slender high strength reinforced concrete columns. Table 3 shows that the theoretical ultimate strength capacities (N_{RSB} and N_u) compare well with the test results of slender high strength concrete columns reported by Hsu et al. [20].

5. **Conclusions**

In this study, a complete mathematical stress-strain model developed based on the test data is suggested for high strength concrete ranging from 40 to 90 MPa. The main principles used by Hognestad [1] were employed in developing the stress-strain relationship. An iterative numerical procedure considering the nonlinear behaviour of the materials is presented for analysis of reinforced concrete columns including slenderness effect. The proposed stress-strain model has been verified with test results of eccentrically loaded slender high strength reinforced concrete columns available in the literature.

It is concluded from the presented study that the use of the proposed stress-strain relationship for high strength concrete gives reasonable accuracy for the computation of high strength concrete columns. Furthermore, the model is confirmed by the equivalent rectangular stress block model. The ultimate concrete compressive fiber strain ε_{cu} diminishes with increasing concrete strength because of the brittle nature of high strength material. The parameter ε_{cu} determined for normal strength concrete gives overestimate results when used in the analysis high strength reinforced concrete columns. The ultimate concrete compressive fiber strain is expressed as a function of concrete strength. This parameter has been provided reasonable accuracy between the theoretical and tests results of column strength capacities.

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